Block 3
Imperfect Competition-II

UNIT 5 OLIGOPOLY

Structure

5.0 OBJECTIVES

After reading and studying the unit, you should be able to:

- understand the nature of strategic interaction in an Oligopolistic market structure in terms of various models;
- find the equilibrium under various oligopoly models;
- compute Cournot equilibrium using the Residual demand curve and by way of Reaction curves;
- throw light on the possibility of Collusions and Cartels among the rivals in an Oligopoly;
- appreciate the Bertrand model of oligopoly as an alternative to the Cournot approach;
- find the Stackelberg equilibrium; and
- explain the Dominant firm model.

5.1 INTRODUCTION

We have so far considered three different market structures, viz. Perfect competition, Monopoly and Monopolistic competition. Perfect competition and monopoly are the two extreme forms of market structures. In a perfect competitive market, there is free entry and exit, many buyers and sellers, perfect information, etc. Such an idealised market does not exist in reality. Moreover, except for some natural monopolies, one hardly finds any example of a monopolistic market structure. A step closer to the reality is the monopolistic competition market structure which assumes no barriers to entry and non-price competition (in the form of product differentiation) between firms. But firms under monopolistic competition are naïve and therefore do not involve in strategic interactions. This makes the market structure under monopolistic competition rarely experienced in reality. The present unit is a move towards a relatively more realistic market structure. Most markets are better described as oligopolies. These are markets where there exist more than one market player, yet where each firm is large enough to enjoy some *monopoly power*. There are barriers to entry which result in a market with a small number of dominant players dependent strategically on each other's decisions about output or price. The prominent example of an oligopolistic market structure is that of soft drink suppliers, Pepsi and Coca-cola. For instance, if price of Pepsi is lowered to attract more sales, it will necessarily attract a reaction from Coca-cola suppliers whose customers will get lured away by the lower Pepsi price. Unless Coca-cola is willing to bear the revenue loss (which is unlikely to happen), it will find itself obligated to respond to Pepsi's price cut, which in turn would affect Pepsi sales. Considering rival's reaction thus becomes significantly important in such a market condition. How do firms behave in an oligopolistic market is what will be covered in this unit.

5.2 OLIGOPOLY

In oligopoly market structure a few firms account for most or all of the total production. Barriers to entry and exit result in prevalence of a small number of dominant players that remain strategically dependent on each other. By strategically dependence it is meant, a firm while taking its optimising decisions must consider the expected reaction of its rival. This is because, output and/or price decision by one impacts output sales and hence the revenue earnings of the other players in the market. Like in perfect competition firms in oligopoly cannot take market price as it comes, neither do they have the price-setting power like that with a monopolist who can set the price and worry about no prospective retaliation. In oligopoly, firms possess certain degree of price-setting power (or the monopoly power, i.e., the rate at which it can set the price above its marginal cost) depending on their market structure, as they have to worry about retaliation from competitors. Any action by a firm in oligopolistic market is followed by a counter reaction from other firms in that market. This compels each firm to consider its rivals' expected reactions as it decides about output and pricing. It follows that accurately portraying the interactions between firms across a wide range of possibilities is the critical task of any model of oligopoly.

There are two different ways in which firms might interact with each other in an oligopolistic market structure. They may cooperate or choose not to cooperate. If they collude, that is, choose to cooperate with each other so as to maximise their joint profits, they form a cartel. For example, the Organisation of Petroleum Exporting Countries (OPEC) is a cartel of oil producing countries that cooperate in how much oil to produce in an attempt to move the price of oil up or down in the market. On the other hand, if they behave non-cooperatively, acting in their own self-interest, they take into account the actions of other firms. Examples of a non-collusive oligopoly model include— the Cournot model, the Bertrand model, the Stackelberg model and the Dominant firm model. We will be covering these models in the present Unit.

5.2.1 Equilibrium in an Oligopolistic Market

A firm's equilibrium position refers to its profit optimising price and output decisions in different market situations. Under perfect competition and monopoly, outcomes are more or less certain with the former optimising at the point where market determined price equals the marginal cost of production; in the case of the latter, however, the firm possessing the market power optimises by setting marginal revenue equal to the marginal cost of production. However, under oligopoly no such certainty exists due to the presence of a small number of dominant players who control the major share of the market and who are strategically interdependent in terms of pricing and output decisions. There exist several ways in which individual oligopolists may respond to rivals' price and output decisions. Consequently, several different models of oligopoly, viz. the Cournot model, the Cartel, the Bertrand model, the Stackelberg model, the Dominant firm model, have been developed, underpinned by different analytical approaches and assumptions about the nature of oligopolistic market behaviour. The discussion of these models is provided in the subsequent sections. However before that, let us try to understand the nature of monopoly power of the oligopolistic firm. Let us assume there are only two symmetric firms in the market having the same constant marginal cost C (and zero fixed cost) and the firms are involved in quantity competition. The demand function faced by a firm $(i = 1, 2)$: $P = D_i(Q_1, Q_2)$, where P represents the market price and Q_1, Q_2 are the quantity produced by firm 1 and 2, respectively. Let the profit function of the firm i $(i = 1, 2)$ be:

$$\pi_i(Q_1, Q_2) = P \times Q_i - C(Q_i) = D_i(Q_1, Q_2)Q_i - C(Q_i)$$

Where $C(Q_i)$ is the cost function faced by firm i. First-order condition for profit maximisation for any of the firm i $(i = 1, 2)$ involves differentiating the profit function with respect to Q_i and put it equal to 0.

$$\Rightarrow \frac{\partial \pi_i\ (Q_1, Q_2)}{\partial Q_i} = 0 \Rightarrow \frac{\partial D_i(Q_1, Q_2)}{\partial Q}\frac{\partial Q}{\partial Q_i}Q_i + \ D_i(Q_1, Q_2) - C = 0$$

$$\Rightarrow \frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{\partial Q}{\partial Q_i}Q_i + P - C\ = 0$$

$$[\because\ P = D_i(Q_1, Q_2)]$$

Multiplying and dividing the first term on the LHS with Q×P, we get

$$\Rightarrow \left[\frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{Q}{P}\left(\frac{PQ_i}{Q}\right) + P - C\right] = 0$$

$$[\because \frac{\partial Q}{\partial Q_i} = 1\ ]$$

Taking first term of the LHS on the RHS and dividing both the sides by P

$$\Rightarrow \frac{P - C}{P} = \left(-\frac{\partial D_i(Q_1, Q_2)}{\partial Q}\frac{Q}{P}\right)\left(\frac{PQ_i}{Q}\right)\frac{1}{P}$$

$$\Rightarrow \frac{P-C}{P} = \ -\frac{1}{\varepsilon_d}s_i$$

⟹This is the equation for Lerner' s Index for Oligopoly.

Here, $s_i = \frac{PQ_i}{PQ}$ is the share of firm i in the total value of output; the reciprocal of market elasticity of demand is given by : $\frac{\partial D_i(Q_1,Q_2)}{\partial Q}\frac{Q}{P} = \frac{1}{\varepsilon_d}$. The term $\frac{1}{\varepsilon_d}s_i$ measures how far the i^{th} firm can raise the price above the marginal cost C. Recall we have derived a similar equation for Lerner's Index in the case of Monopoly (Unit 3) which is $\frac{P-C}{P} = -\frac{1}{\varepsilon_d}$. Given that s_i is the share of the i^{th} firm in the total value of market output

$\Rightarrow\ \ 0 \leq s_i \leq 1$. Thus the monopoly power (as measured by $\frac{P-C}{P}$) of an oligopolistic firm is less than that of the Monopoly.

5.3 THE COURNOT MODEL

A Cournot model is an Oligopoly model in which all the firms decide on their profit maximising output simultaneously with each firm assuming that its rivals will continue producing their current output levels. Given the rival's quantity, each firm attains equilibrium by producing the quantity where marginal revenue equals marginal cost, i.e. MR = MC. Introductory Microeconomics course (BECC 101) of Semester 1 introduced the Cournot model by considering two firms in the market. In such case both the firms decide simultaneously about their profit-maximising level of output. Each firm considers its rival's output as given while making its output decision. The relationship between a firm's profit-maximising output and the given output of its rival's is summarised by a reaction function. These functions are first obtained separately for each firm, and then solved simultaneously to obtain Nash equilibrium. The market price is then determined using the total output of both firms.

5.3.1 Equilibrium using Residual Demand Curve

A Residual demand curve for the first firm is that portion of the market demand curve that remains for this firm assuming that the second firm supplies a fixed amount of quantity Q_2 in the market. From the Fig. 5.1, residual demand curve facing the first firm is ascertained by shifting the vertical axis to the right by the amount of output assumed to be sold by the second firm (i.e. Q_2). Let the market demand curve (DD' in Fig. 5.1) be given by $P = A - BQ$, where $Q = Q_1 + Q_2$ with Q_1 and Q_2 be the respective quantities supplied by firm 1 and 2, P be the market price, and A and B any constants. Then residual demand curve faced by the first firm will be given by

$$P = A - BQ$$

$$P = A - B(Q_1 + Q_2)$$

$$P = [A - BQ_2] - BQ_1$$

where $[A - BQ_2]$ represent the price intercept of the residual demand curve facing the first firm. The MR curve will be given by first finding the total revenue (TR) function of the first firm, which equals quantity × price, that is,

$$TR_1: P \times Q_1 = [AQ_1 - BQ_2\, Q_1] - BQ_1^2$$

and then taking the first derivative of this function with respect to Q_1 to get MR_1

$$MR_1 = A - BQ_2 - 2BQ_1 \qquad (1)$$

In the Fig. 5.1, MR_1 originates from the point of intersection of the vertical line $Q_1 = 0$ with the market demand curve, $P = A - BQ$. At point A, MR_1 equals 0 which should be the case as at that point firm 1 produces no output. Now, on assuming marginal cost to be 0, we get the profit maximising quantity of firm 1 at point E where $MR_1 = MC$. Using (1) and that MC = 0, in equilibrium, we get

$$A - BQ_2 - 2BQ_1 = 0$$

Now, we solve for profit maximising level of Q_1 (let it be denoted by Q_1^*) from the above equation, which we get as a function of given quantity of Q_2 (also known as the reaction curve for firm 1):

$$Q_1^* = \frac{A}{2B} - \frac{1}{2}Q_2$$

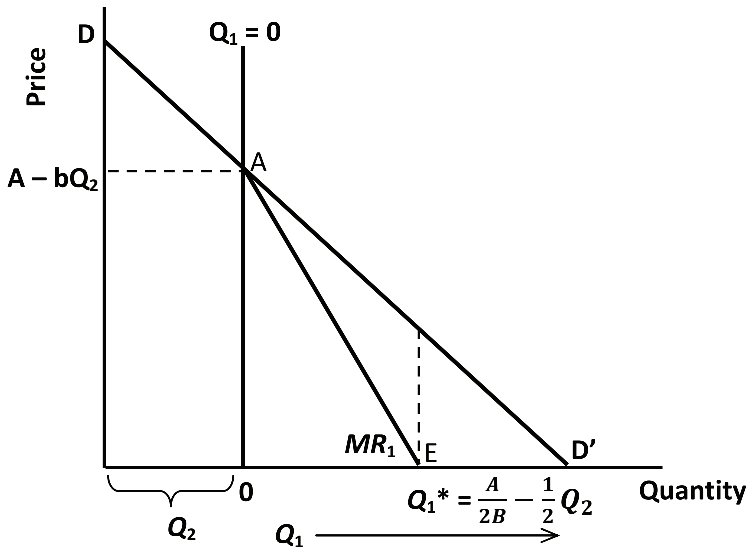

Fig. 5.1 : Equilibrium under Cournot Model using Residual Demand Curves

Similarly, profit maximising level of Q_2 can be obtained (assuming first firm supplies a fixed amount of quantity Q_1 in the market) as a function of Q_1. Both the functions can then be solved to get the optimising quantities supplied by each firm in the market.

5.3.2 Equilibrium using Reaction Curves

Reaction curves show relationship between an Oligopolist's profit maximising output and the amount it thinks its rivals will produce. If there are only two firms in the model, then firm 1^{st} profit maximising output will depend upon what it thinks firm 2^{nd} will produce, and this relationship will derive firm 1's reaction curve. Similarly firm 2^{nd} reaction curve shows its output as a function of how much it thinks firm 1^{st} will produce. The equation of the reaction curve is given by the profit maximising condition, MR = MC. In the above case, the reaction curve equation for firm 1, considering MC = 0, will be given by,

$$R_1(Q_2): MR_1 = MC \Rightarrow Q_1 = \frac{A}{2B} - \frac{1}{2}Q_2 \qquad (2)$$

where, R_1 (Q_2) represents the reaction function of firm 1 which gives the optimal amount of output supplied by firm 1 as a function of given quantity supplied by firm 2 (Q_2). Similarly, the reaction curve equation for firm 2 will be given by,

$$R_2(Q_1): MR_2 = MC \Rightarrow Q_2 = \frac{A}{2B} - \frac{1}{2}Q_1 \qquad (3)$$

To note here is that if both demand and cost functions are linear, reaction function will be linear as well. In Fig. 5.2, we plot these reaction functions by marking output supplied by firm 1 and firm 2 on the vertical and the horizontal axis, respectively. Reaction curve of firm 1, given by Equation (2) has the vertical intercept given by $Q_1 = \frac{A}{2B}$ when $Q_2 = 0$, and horizontal intercept given by $Q_2 = \frac{A}{B}$ when $Q_1 = 0$. Similarly, we plot reaction curve of firm 2.

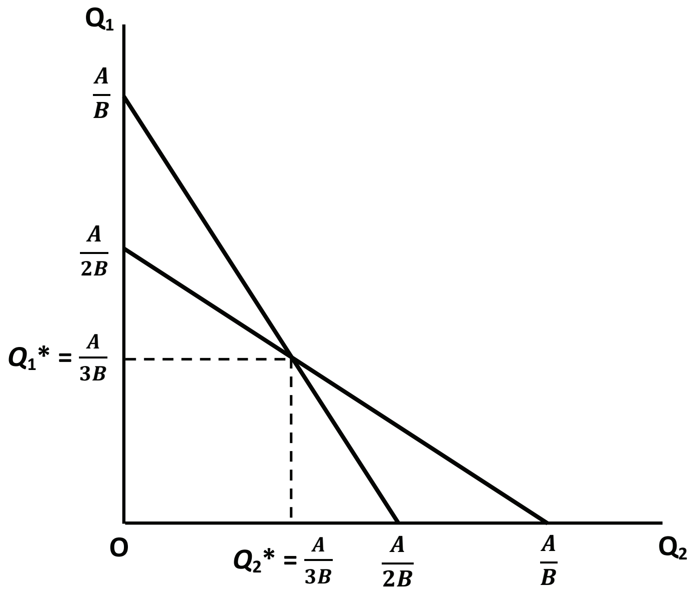

Fig. 5.2: Equilibrium under Cournot Model using Reaction Curves

Now, equilibrium output for firm 1 and firm 2 will be given by the intersection point of the two reaction curves. This is also called "Cournot-Nash Equilibrium", as each firm is doing the best it can given the behaviour of rival firms. This can be ascertained by substituting reaction curve for Firm 2 into the reaction curve for firm 1:

$$Q_1 = \frac{A}{2B} - \frac{1}{2}\left[\frac{A}{2B} - \frac{1}{2}Q_1\right]$$

$$Q_1 = \frac{A}{2B} - \frac{A}{4B} + \frac{1}{4}Q_1 \Rightarrow \; Q_1^* = \frac{A}{3B}$$

Inserting the equilibrium value of Q_1^* in equation (3), we get

$$Q_2 = \frac{A}{2B} - \frac{1}{2}\left(\frac{A}{3B}\right) \Rightarrow Q_2^* = \frac{A}{3B}$$

Total equilibrium quantity, $Q^* = Q_1^* + Q_2^* \Rightarrow Q^* = \frac{2A}{3B}$. Insert this in market demand curve to get Cournot price: $P = A - BQ$

$$P = A - B\left(\frac{2A}{3B}\right) \Rightarrow P = \frac{A}{3}$$

Cournot equilibrium profit of the firm 1 and 2 :

$$\pi_1(Q_1, Q_2) = PQ_1 = \pi_2(Q_1, Q_2) = PQ_2 = \left(\frac{A}{3} \times \frac{A}{3B}\right) = \frac{A^2}{9B}$$

Cournot equilibrium profit of the industry: $\pi_1(Q_1, Q_2) + \pi_2(Q_1, Q_2) = \frac{2A^2}{9B}$

If firm act as a monopoly, then the optimising output will be given by setting MR = MC:

Total revenue (TR): $P \times Q = (A - BQ) \times Q$;
Marginal Revenue (MR): $\frac{dTR}{dQ} = A - 2BQ$

For monopoly equilibrium, we set MR = MC $\Rightarrow A - 2BQ = 0 \Rightarrow Q_M = \frac{A}{2B}$, where Q_M gives the profit maximising monopoly output. Inserting this in the demand curve equation, we get monopoly price $P_M = \frac{A}{2}$. So the Monopoly profit : $P_M \times Q_M = \frac{A}{2B} \times \frac{A}{2} = \frac{A^2}{4B}$

Hence, we find that Cournot duopoly model has a higher total quantity and a lower price as compared to Monopoly quantity and price, respectively. Moreover firms under the Cournot competition earns less profit (even at the industry level) as compared to the monopoly $\Longrightarrow \frac{A^2}{4B} > \frac{A^2}{9B}$ and $\frac{A^2}{4B} > \frac{2A^2}{9B}$.

Example

Consider a duopoly of firm 1 and 2 producing a homogenous product, the demand of which is described by the following demand function:

$$Q = \frac{1}{2}(100 - P)$$

Where Q is total production of both firms (*i.e.*, $Q = Q_1 + Q_2$). Also let the marginal cost of production faced by both firms be Rs. 40, *i.e.*, $MC_1 = MC_2 = 40$. Calculate the residual demand function for both the firms. Using them ascertain their reaction curves and the Cournot-Nash equilibrium quantity produced by each firm?

Solution

Residual demand function of a firm is ascertained by fixing the quantity produced by the other firm.

We will first be finding the inverse demand function from the given demand function. Given, $Q = \frac{1}{2}(100 - P)$, the inverse demand function would be

$$P = 100 - 2Q \Rightarrow P = 100 - 2(Q_1 + Q_2)$$

The residual demand function faced by firm 1 will be given by assuming quantity Q_2 produced by firm 2 as fixed:

$$P = [100 - 2Q_2] - 2Q_1$$

where $[100-2Q_2]$ represent the price intercept. Now, MR_1 will be given by $\frac{\partial TR_1}{\partial Q_1}$

TR_1: $P \times Q_1 = 100Q_1 - 2Q_2Q_1 - 2Q_1^2$; MR_1: $\frac{\partial TR_1}{\partial Q_1} = 100 - 2Q_2 - 4Q_1$

For reaction function, we make use of the condition, $MR_1 = MC$

$$100 - 2Q_2 - 4Q_1 = 40 \quad \Rightarrow \quad Q_1 = 15 - \frac{1}{2}Q_2 \text{ [Reaction curve for firm 1]}$$

Similarly, residual demand function faced by firm 2 will be given by:

$$P = [100 - 2Q_1] - 2Q_2$$

And the corresponding reaction curve for firm 2 will be: $Q_2 = 15 - \frac{1}{2}Q_1$

We solve both the reaction curves to get the Cournot-Nash equilibrium,

$Q_1^* = Q_2^* = 10$, and considering the demand function, we can derive the equilibrium price, P* = 60

Profit by firm 1 = Profit by firm 2 = Total Revenue – Total Cost

$$= (60 \times 10) - (40 \times 10) = 200$$

Hence total industry profit = 400

Let us compare this result with the monopoly outcome. For the monopolist, the aggregate inverse demand curve will be $P = 100 - 2Q$.

Total revenue (TR): $P \times Q = 100Q - 2Q^2$;

Marginal Revenue (MR): $\frac{\partial TR}{\partial Q} = 100 - 4Q$

For monopoly equilibrium, we set MR = MC $\Rightarrow 100 - 4Q = 40 \Rightarrow Q_M = 15$, where Q_M gives the profit maximising monopoly output. Inserting this in the demand curve equation, we get monopoly price $P_M = 70$.

Monopoly profit = total revenue – total cost

$$= (15 \times 70) - (15 \times 40) = 450$$

Hence, we find that Cournot duopoly model has a higher total quantity (= 20) and a lower price (= 60) as compared to Monopoly quantity (= 15) and price (= 70), respectively. Also the total industry profit in case of Cournot model (= 400) is less than that of a monopolistic industry profit (= 450).

5.3.3 Cournot Equilibrium with Different Costs

In the above example we assumed that the two firms are symmetric i.e., having the same marginal cost of production as 40. However, cost conditions may vary from firm to firm. That is, a firm may be more cost efficient as compared to the other firm.

Considering the demand function $P = 50 - 2Q$. Let the marginal cost faced by firm 1 be Rs. 2, and that faced by firm 2 be Rs. 8. Find the Cournot-Nash equilibrium in such a case.

We make use of the optimising condition to find the reaction curves for each of the firms.

TR_1: $P \times Q_1 = 50Q_1 - 2Q_2Q_1 - 2Q_1^2$; MR_1: $\frac{\partial TR_1}{\partial Q_1} = 50 - 2Q_2 - 4Q_1$

For reaction function, we make use of the condition, MR_1 = MC

$$50 - 2Q_2 - 4Q_1 = 2 \quad \Rightarrow \quad Q_1 = 12 - \frac{1}{2} Q_2 \text{ [Reaction curve for Firm 1]}$$

Similarly, reaction function for firm 2 is given by: $Q_2 = \frac{21 - Q_1}{2}$

On solving the two reaction functions, we get the Cournot-Nash solution as $Q_1 = 9$, $Q_2 = 6$, Q (total quantity) = 15, P (Cournot price) = 20, with profit of Firm 1 = 152, profit of Firm 2= 60.

Hence, when firms have different costs, they choose different output levels, with the firm having low-cost (here firm 1) enjoying higher share of the market and making higher profits than the firm incurring high-cost (here firm 2).

Check Your Progress 1

1) Consider a Cournot duopoly of firm 1 and 2 producing output Q_1 and Q_2, respectively. Let market demand curve be $P = 60 - Q$, where $Q = Q_1 + Q_2$. Assume marginal cost of production is null for both the firms. Calculate Cournot-Nash equilibrium output of both the firms.

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2) Let market demand curve be $P = a - bQ$, where $Q = Q_1 + Q_2$. Let there be two firms 1 and 2 producing output Q_1 and Q_2, respectively at a constant marginal cost of 'c'. Calculate output of both the firms under the Cournot model.

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5.4 COLLUSION AND CARTELS

Collusion is said to take place when rival firms enter into an agreement in terms of price, market share, etc. in an attempt to realise higher profits than what they would have realised individually in an independent scenario. Firms may explicitly enter into an agreement resulting in an explicit collusion, or the collusion may take place without a formal agreement, which then is referred to as a Tacit collusion. Cartel results when few firms formally agree upon a certain level of output or a certain price of the goods in order to maximise joint profits of the industry, which they can share among themselves on a mutually binding agreement. Let us consider a case of two firms under Cournot market structure as well as under collusion. Coming back to the earlier example where demand curve is as follows:

$$P = A - BQ, \text{ with } Q = Q_1 + Q_2$$

along with the assumption that MC = 0.

If both the firms produce individually under Cournot market structure then (we know from our earlier section) the equilibrium outputs of each firm: $Q_1 = Q_2 = \frac{A}{3B}$

If the two firms collude and form a cartel, then total Revenue of the cartel will be given by:

TR = $(A - BQ)\, Q = AQ - BQ^2$; Marginal Revenue (MR): $\frac{\partial TR}{\partial Q} = A - 2BQ$

For cartel equilibrium, the two firms jointly behave like a monopolist and jointly control the entire market share. So from the equilibrium condition:

MR = MC $\Rightarrow A - 2BQ = 0 \;\Rightarrow Q^* = \frac{A}{2B}$ (same as the monopoly solution). Therefore total equilibrium quantity is Q = $\frac{A}{2B}$. Using the demand function, we get the equilibrium price as $P = \frac{A}{2}$.

The goal of the cartel is to set the industry output at a level that maximises industry profits. A rule governing the cartel behaviour specifies how the industry output and profits must be shared among the cartel members. In our example, we assume the two firms are producing homogeneous goods and are sharing similar cost structure which enabled the firms to equally share the industry output and thus profits, which means $Q_1 = Q_2 = \frac{A}{4B}$. This is less than the Cournot equilibrium output of $\frac{A}{3B}$ units produced by each firm. By independently maximising their own profits, firms produce more total output than they would if they collusively maximised industry profits. This pursuit of self interest does not typically maximise the well being of the industry as well as their individual profits. In terms of our numerical example, we saw that the total industry output of the duopoly was higher than that of the monopolistic industry, but total industry profit of monopolistic industry (Cartel) was higher as compared to the duopoly.

Conditions in oligopolistic industries tend to promote collusion since the number of firms is small and firms recognise their interdependence. The advantages can be more profits and decreased uncertainty. However it is hard to retain the collusion and firms tend to cheat each other and thus collusive arrangement often breaks down. Consequently, as long as a cartel is not maintained by legal provisions, there is a constant threat to its existence.

5.5 THE BERTRAND MODEL

In the Cournot model, each firm takes quantity as a strategic variable, and the resulting total output determines the market price. However there exists an alternative model, the Bertrand model where firms take price as the strategic variable, with each firm selecting a price and standing ready to meet all the demand it faces given the prices chosen by all the other firms. However unlike Cournot competition, each firm faces separate demand function based on the price charged by it and its rivals. Let a homogeneous good is produced by '*n*' firms in the industry competing on price, with each firm producing Q_i units of a good at a cost of C_i (Q_i), where i = 1,2...*n*. The model further assumes that if firms set different prices, all demand shifts to the firm charging the lowest price, which in turn produces enough output to meet this demand. Further price rationing rule states that with more than one firm charging the lowest price, output demand is shared between those firms equally. Any firm charging above the lowest price charged in the market, receives no market share.

Let us assume a Bertrand model of duopoly where there are two firms, 1 and 2, producing homogenous product at a constant marginal cost "C". The firms choose prices P_1 and P_2, simultaneously. All sales go to the firm with the lowest price. Sales are split equally if $P_1 = P_2$. In contrast to the Cournot or Stackelberg models, the only Nash Equilibrium is the perfectly competitive outcome i.e., $P_1 = P_2 = C$. This results from the fact that each firm in the Bertrand model has an incentive to undercut price as long as production remains profitable. If one firm cut its price than the rival's price, then even for a slightest undercutting, it can appropriate the entire market share, thereby, inducing the rival out of the market. Similarly, the same firm may face zero market share if its rival outcompetes it by a slightest undercutting of their price. Given that each firm has an incentive to undercut its price in order to grab the market share, the firms would engage in reciprocal price undercutting until the price gets pushed down to the level of marginal cost "C". At this price, economic profits would be zero. Hence firms would have no incentive to undercut its price further. This would be Nash equilibrium as there would be no incentive for either firm to change its price once $\mathrm{P}_1 = \mathrm{P}_2 = \mathrm{C}$. If either firm lowers their price below marginal cost, they would incur losses. If either raised their price, then it would be no better off, because it would lose the entire market share to the rival. Thus, quantity sold by each firm depends on both prices. If $P_1 < P_2$, then firm 1 serves the entire market demand $D(P_1)$; the opposite happens, that is, firm 2 serving the entire market demand $D(P_2)$ when $P_1 > P_2$; and *when* $P_1 = P_2$, each firm supplies half of the total market demand, that is, $\frac{D(P)}{2}$. If $P_1 = P_2 > C$, even though the firms share the market equally, it is not an equilibrium strategy. In this case either of the firm has an incentive to undercut the price and grab the entire market and still enjoy positive profit.

Assuming Π_i denote the profit earned by firm *i,* where *i* = 1, 2. Then, Nash equilibrium will be given by the pair of price (P_1^*, P_2^*) such that,

$$\left.\begin{aligned} \Pi_1 (P_1^*, P_2^*) &\geq \Pi_1 (P_1, P_2^*)\ ;\ \forall\ P_1 \\ \Pi_2 (P_1^*, P_2^*) &\geq \Pi_2 (P_1^*, P_2)\ ;\ \forall\ P_2 \end{aligned}\right\} \qquad 5.1$$

where symbol ∀ stands for "*for all*".

5.5.1 Bertrand Paradox

The unique pair of prices satisfying condition 5.1 is given *by* $P_1 = P_2 = C$ (the given constant marginal cost) at which $\Pi_1 = \Pi_2 = 0$. This situation is referred to as the condition of *Bertrand Paradox*. The paradox results from the fact that just two firms are sufficient to dissipate the market power and yield an outcome similar to the perfectly competitive market (which usually assumes many sellers). In other words, with the number of firms rising from one to two, the price decreases from the monopoly price to the competitive price and stays at the same level even when the number of firms increases further. This is contrary to our perception, where we think that markets with a small number of firms possessing market power typically charge a price above the marginal cost.

5.5.1.1 Condition to be Satisfied for the Bertrand Paradox

For the Bertrand model to generate the Bertrand paradox, i.e., a situation when a perfect competitive outcome results with just two firms (engaged in reciprocal price undercutting above their marginal cost) are: 1. Firms involved in price competition must possess unlimited capacities. If initially the price condition is given by, $P_1 = P_2 = P$ (say) $> C$, with each firm sharing market equally, then either firm would be tempted to undercut its price slightly (say by $\varepsilon > 0$) and grab the entire market. In order to throw the rival out of the market and solely cater to the market demand, either firm needs a drastic expansion of capacity. The firm can satisfy this increased demand only when it faces no capacity constraints. If the firm faces a capacity constraint, price undercutting would not be profitable and it would not be able to drive its rival out of the market. Rather, that would leave some residual demand for the higher-priced rival firm and would decrease the incentive to undercut. Moreover Bertrand paradox prevails when firms are assumed to be in a homogeneous product industry i.e., the products of each firms are close substitutes, so that consumer cannot distinguish one firm's product from the other firms. If the firms are in the differentiated product industry (where consumer can exercise a brand preference as the products are similar but not exactly close substitutes) each firm enjoys some monopoly power and hence $P_1 = P_2 = C$ is not the Nash equilibrium. Rather the Nash equilibrium may be $P_1 \lesseqgtr P_2 > C$, depending upon their brand values.

5.5.2 Bertrand Equilibrium using Reaction Curves

In case of a Bertrand model, reaction function of a firm will be in price terms. In other words, a reaction function will give the optimum price at which firm chooses to supply its output, given the price of its rival's. Firm 1's reaction curve equation will be given by $R_1(P_2)$, giving the optimal price charged by firm 1 given that the price set by firm 2 is P_2. Similar description goes for the reaction curve equation of firm 2, that is, $R_2(P_1)$. Refer Fig. 5.3, where we represent Bertrand equilibrium using reaction curves assuming same marginal cost C is faced by both the firms.

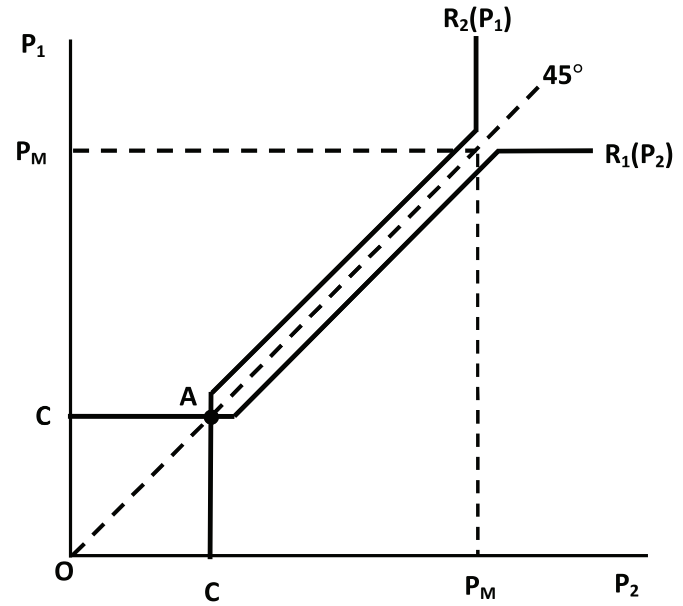

Fig. 5.3: Bertrand Equilibrium using Reaction Curves

In the above figure, firm 1's reaction function $R_1(P_2)$ shows when firm 2's price is below the marginal cost, i.e., when $P_2 < C$, it is optimal for firm 1 to set its price at the marginal cost, i.e., $P_1 = C$ as shown by the horizontal segment CA of the firm 1's reaction curve $R_1(P_2)$. The 45° line represents prices where $P_1 = P_2$. P_M represent the monopoly prices charged for the good when both the firms collude and act as a monopolist. Now, again look at the reaction curve of firm 1. When firm 2's price is above the marginal cost (C) but below the monopoly price (P_M), then it is optimal for firm 1 to set price just below firm 2, i.e. $P_1 < P_2$. Whereas, when firm 2's price is above the monopoly price, it is optimal for firm 1 to charge the monopoly price, i.e. $P_1 = P_M$. With both facing the same marginal cost (C), reaction function of firm 2, $R_2(P_1)$ will be symmetrical with respect to the 45° line. Symbolically, we can represent a reaction function as follows:

$$R_1(P_2) = \begin{cases} P_M \quad \text{if} \quad P_2 > P_M \\ P_2 - \varepsilon \ \text{if} \ C < P_2 \leq P_M \\ C \ \text{if} \ P_2 \leq C \end{cases} \quad \text{and}$$

$$R_2(P_1) = \begin{cases} P_M \quad \text{if} \quad P_1 > P_M \\ P_1 - \varepsilon \ \text{if} \ C < P_1 \leq P_M \\ C \ \text{if} \ P_1 \leq C \end{cases}$$

where, ε represents a small positive number. Bertrand equilibrium is given by the intersection of both the reaction curves at Point A, where $P_1 = P_2 = C$, which is mutually best response for both the firms. At any other price above the marginal cost, either firm would always find it is in their best interest to undercut its rival's price a little (say by $\varepsilon > 0$) and serve the entire market itself. It is only at the price equals to the marginal cost that firms have no incentives to deviate from the equilibrium prices.

5.5.3 Comparison between the Cournot and the Bertrand Model

In the Cournot model, firms engage in quantity competition, so quantity is the strategic variable of the firm whereas in the Bertrand model firms engage in price competition, so price is the strategic variable. In the Cournot model the equilibrium price is generally above the marginal cost and the quantity approaches the perfectly competitive market situation only when the number of firms becomes large. In contrast to this, in the Bertrand model even with two firms, price competition dissipates the monopoly power and ensures competitive price which is equal to the marginal cost. Another difference is that in the Cournot model, the firms take their rival's output as given and cannot "steal" any consumers away from their rivals by lowering their prices. Whereas, a Bertrand rival believes that it can lure customers from its rivals by small cuts in price.

5.6 THE STACKELBERG MODEL

Similar to firms in Cournot model, in the Stackelberg model of oligopoly, firms produce homogeneous product and engage in quantity competition. The principal difference between the two models is that instead of simultaneous output or quantity choice of the rival firms (as in the Cournot model) the firms in Stackelberg model is based on sequential output or quantity choice. So instead of a static-move game like that of Cournot model, the firms in the Stackelberg model are engaged in the dynamic-move game. In case of a Stackelberg model of duopoly, we assume one of the firm moves first, followed by it the other firm. Thus, the model basically becomes a model of two periods, where first mover (say firm 1) decides about its quantity choice in period 1, then the second mover (say firm 2) after observing the firm 1's move, decides about its optimal quantity choice in period 2, and there the game ends with the appropriation of profits by both the firms. How does this affect the equilibrium of this game? We analyse the result using the example considered above in case of the Cournot model of duopoly. Now, let the firm 1, also referred to as the Stackelberg leader, moves first to produce quantity Q_1. Firm 2, the Stackelberg follower, observes firm 1's quantity choice Q_1, and then chooses to produce quantity Q_2. The quantity chosen by the follower must therefore be along its reaction function. Since, firm 1 knows that firm 2 will take firm 1's output as given and optimally decides its output along its reaction function, firm 1 (the Stackelberg leader) being the first mover enjoys a strategic advantage over firm 2 (the Stackelberg follower) by knowing the reaction function of the firm 2. Firm 1 can influence the behaviour of firm 2 by altering its own output and it takes into account the effects of its own output on firm 2's behaviour.

In sequential games, the optimal solution is obtained by the backward induction techniques, where, we first solve the firm 2's optimal quantity choice problem in the second period and then proceed backward to solve the firm 1's optimal quantity choice problem in the first period. Such approach is called backward induction as it is a process of reasoning backwards in time. In period 2, firm 2 chooses Q_2 given firm 1 has chosen Q_1 in period 1. This gives us the reaction curve of firm 2 as a function of Q_1, $R_2(Q_1)$. This reaction function of firm 2 is then considered by firm 1 in period 1 as given to decide its own profit maximising quantity Q_1. We illustrate this below:

Consider the same demand function that we considered in the earlier example of Cournot model. Here we assume firm 1 to be the leader and firm 2, to be the follower. The solution is worked out as follows:

$$P = A - BQ \qquad \text{[where, } Q = Q_1 + Q_2\text{]}$$

Given marginal cost is $MC_1 = MC_2 = 0$.

Backward Induction: Consider period 2 first. Here, firm 2 will choose its optimal output according to its own Reaction function i.e.

$$R_2(Q_1)\text{: } Q_2 = \frac{A}{2B} - \frac{1}{2}Q_1 \qquad (4)$$

(as derived in the earlier example)

Now consider period 1: Firm 1 will choose its output by maximising its own profit function and considering the reaction function of firm 2, $R_2(Q_1)$. The profit function of firm 1 (Π_1) is :

$$\Pi_1(Q_1, Q_2) = TR_1 - TC_1$$

where TR_1 is total revenue of firm 1 and TC_1 is total cost of firm 1

$$\Pi_1(Q_1, Q_2) = TR_1 - TC_1 = [A - B(Q_1 + Q_2)]\, Q_1 - 0 \times Q_1$$

$$\Rightarrow \Pi_1(Q_1, Q_2) = AQ_1 - BQ_1^2 - BQ_2 Q_1$$

$$= AQ_1 - BQ_1^2 - BQ_1 \times R_2(Q_1)$$

Firm 1 being the first mover knows the reaction function of firm 2 (as shown in Eq. 4) and therefore incorporate it in its own profit function in order to solve for its own optimal output choice:

$$\Pi_1(Q_1, Q_2) = AQ_1 - BQ_1^2 - BQ_1\left(\frac{A}{2B} - \frac{1}{2}Q_1\right)$$

$$= AQ_1 - BQ_1^2 - \frac{AQ_1}{2} + \frac{BQ_1^2}{2}$$

So the first order condition for the optimisation of firm 1's profit is obtained by setting the first - order partial derivative with respect to Q_1 equal to zero.

$$\frac{\partial \Pi_1(Q_1, Q_2)}{\partial Q_1} = 0 \;\Rightarrow A - 2BQ_1 - \frac{A}{2} + \frac{2BQ_1}{2} = 0$$

Solving for optimal Q_1, gives $Q_1 = \frac{A}{2B}$.

Now, substituting Q_1 back in Eq. (4), we get the optimal output of firm 2: $Q_2 = \frac{A}{4B}$. So the equilibrium output choice of firm 1 and 2 are:

$[Q_1 = \frac{A}{2B}, Q_2 = \frac{A}{4B}]$.

Equilibrium price under Stackelberg model : P = $[A - B(Q_1 + Q_2)]$= $[A - B\frac{3A}{4B}] = \frac{A}{4}$

Now profit of the leader (firm 1) : $\Pi_1(Q_1, Q_2) = [A - B(Q_1 + Q_2)]\, Q_1 =$
$[A - B\frac{3A}{4B}]\ \frac{A}{2B} \Rightarrow\ \Pi_1(Q_1, Q_2\ ) = \frac{A}{4}\frac{A}{2B} = \frac{A^2}{8B}$

Profit of the follower (firm 2): $\Pi_2(Q_1, Q_2) = [A - B(Q_1 + Q_2)]Q_2 = [A - B\frac{3A}{4B}]\ \frac{A}{4B} \Rightarrow \Pi_2(Q_1, Q_2) = \frac{A}{4}\frac{A}{4B} = \frac{A^2}{16B} \Rightarrow\ \Pi_1(Q_1, Q_2) > \Pi_2(Q_1, Q_2) \Rightarrow$ Thus the leader enjoys higher market share and higher profit than the follower in equilibrium.

Unlike the Cournot outcome which was symmetric, that is both the firms produced the same level of output, in the Stackelberg model, leader firm i.e., firm 1 enjoys greater market share (produces more output) as compared to the follower i.e., $Q_1 > Q_2$ in the equilibrium. One may compare the Stackelberg equilibrium outcome with that of the Cournot outcome and may check that the leader (firm 1) produces more than produced by a firm in Cournot equilibrium whereas the follower produces less than Cournot equilibrium quantity. Thus, there is an advantage of being the first mover.

Now, the total industry output will be $Q_1 + Q_2 = \frac{3A}{4B}$, which is more than the total industry output under Cournot equilibrium of $\frac{2A}{3B}$ for a given value of A and B. Profit of the Stackelberg leader is higher than the firm under Cournot competition $\Longrightarrow\ \Pi_1(Q_1, Q_2)^S > \Pi_1(Q_1, Q_2)^C \Longrightarrow\ \frac{A^2}{8B} > \frac{A^2}{9B}$. Moreover profit of the Stackelberg follower is lower than the firm under Cournot competition $\Longrightarrow\ \Pi_2(Q_1, Q_2)^S < \Pi_2(Q_1, Q_2)^C \Longrightarrow\ \frac{A^2}{16B} < \frac{A^2}{9B}$. Industry profit under Stackelberg model is greater than that of the Cournot competition

$$\Longrightarrow \Pi_1(Q_1, Q_2)^S + \Pi_2(Q_1, Q_2)^S > \Pi_1(Q_1, Q_2)^C + \Pi_2(Q_1, Q_2)^C$$

$$\Longrightarrow \frac{3A^2}{16B}\left(= \frac{A^2}{8B} + \frac{A^2}{16B}\right) > \frac{A^2}{9B}$$

5.7 THE DOMINANT FIRM MODEL

In some oligopolistic models, one large firm dominates the market share and many small fringe firms are the followers catering the residual demand and acting competitively. A Dominant firm (also known as the leader), typically having a larger share in market, behaves as a price-setter that faces smaller price-taking firms (also known as Fringe firms) each having a very small share in the market. The leader or the dominant firm sets the price for the commodity that maximises its own profits and assumes that its rivals will behave as competitive firms or in other words as price-takers that will take the price set by the dominant firms as given in determining their output in that price. Let us explain how this market structure operates with the help of a diagram below.

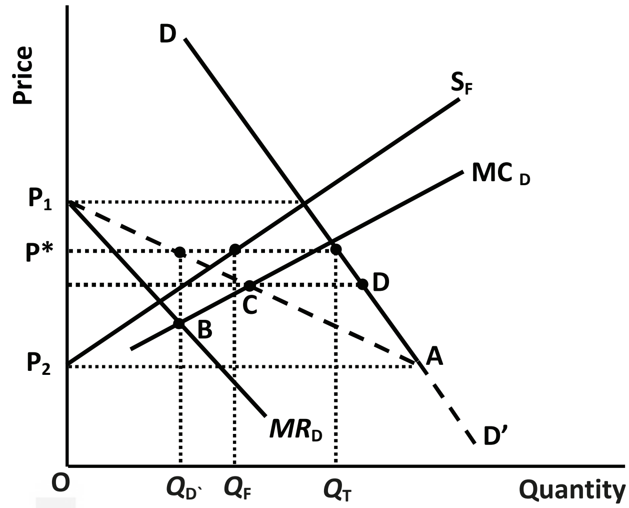

Fig. 5.4: Dominant Firm Model

In Fig. 5.4 DD' curve represents the market demand which is served by the dominant firm and the fringe firms collectively. Fringe firms are assumed to behave as competitive firms supplying where their marginal cost equals the price set by the dominant firm. This way fringe firms collectively behave as a competitive industry whose output can be determined by the supply curve S_F. The dominant firm's problem is to find a price which maximises its own profits. To solve this problem we need demand curve of the dominant firm. The extent of demand that the dominant firm gets to serve is equal to the total quantity demanded at that price (given by DD') minus the quantity the fringe firms supply at that price (given by S_F).This is represented by the residual demand curve P_1AD'. In other words, P_1AD' represents the horizontal difference between the fringe firm's supply curve S_F and the market demand curve DD'. At price P_1, dominant firm produces nothing with fringe firms serving the entire market demand at that price. On the other hand, at a price as low as P_2 dominant firm gets to serve the entire market demand with supply by fringe firms falling to zero. The dominant firm's residual demand curve between these two extreme prices is given by P_1A. Corresponding to the residual demand curve of the dominant firm, we have a marginal revenue curve represented by MR_D. MC_D likewise represents the marginal cost curve of the dominant firm.

The dominant firm optimise at point B where marginal revenue equals marginal cost resulting in profit maximising output Q_D sold at the market price P*. At this price fringe firms will supply Q_F amount of output as will be given by their supply curve. Total output at P* equals Q_T which is the sum of Q_D and Q_F. Total output generated in this model will be less than competitive output. This is clear by the figure above. Dominant firm optimise at a point where its price is above the marginal cost it faces. An efficient optimum in this case will be given by the point C where MC_D intersects the residual demand curve resulting in higher total output as is given by the point D. In this case, not only fringe firms but also the dominant firm is producing where MC equals price.

Check Your Progress 2

1) Explain the difference between the Bertrand model and the Stackelberg model of Oligopoly.

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2) For the following demand curve P = a – 2b and constant marginal cost curve ‘c’, find equilibrium price, quantity and profit according to:

   i) Stackelberg Model

   ii) Bertrand Model

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5.8 LET US SUM UP

The present unit is an attempt to move relatively closer to the real market conditions. The extreme market structures like perfect competition and monopoly have already been taken into account in order to discuss a relatively real market situation of that of an oligopolistic market. Oligopoly is a form of market structure where only a few firms account for most or all of production. The market also assumes barriers to entry which allows a few firms to act as dominant players. These few firms are characterised by strategic interdependence on output and pricing decisions. The unit discussed several oligopolistic models with varying assumptions, decision parameters and consequently the equilibrium outcome. In oligopolistic market, the concept of Nash equilibrium is more appropriate as it gives due consideration to the strategic interdependence among the firms in their decision-making. A Nash-equilibrium is a situation in which each firm adopts the best response strategy, given the strategy of its rival firm. In the Cournot model, each firm set its profit maximising quantity assuming its rival’s output as given. On the other hand, in Bertrand model, each firm set the profit maximising price assuming its rival’s price as given. Then after these two simultaneous-move models, we discussed a sequential-move model, i.e. the Stackelberg models, where we have a leader and a follower, with leader deciding first about the optimum output giving due consideration to the reaction function of the follower. Subsequently, the Dominant firm model is described, where a Dominant firm (the leader), behaves as a price-setter, with the smaller price-taking firms (Fringe firms) taking the price set by the dominant firms as given in determining their output in that price. Each model has its own significance with respect to the assumptions and the respective equilibrium outcome.

5.9 SOME USEFUL REFERENCES

- Hal R. Varian, *Intermediate Microeconomics,* a Modern Approach, W.W. Norton and company/Affiliated East- West Press (India), 8th Edition, 2010.
- Pindyck R. S and Rubinfeld D. L, *Microeconomics,* Pearson India, 2009
- C. Snyder and W. Nicholson, *Fundamentals of Microeconomics,* Cengage Learning (India), 2010.

5.10 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) Firm 1

$TR_1 = P\,Q_1 = (60 - Q)\,Q_1$

$= 60\,Q_1 - (Q_1 + Q_2)\,Q_1$

$= 60\,Q_1 - Q_1^2 - Q_2\,Q_1$

$\therefore MR_1 = 60 - 2Q_1 - Q_2$

Setting $MR_1 = MC_1$

Firm 1's Reaction curve is: $Q_1 = 30 - \frac{1}{2}Q_2$

Similarly we can get Firm 2's Reaction curve as $Q_2 = 30 - \frac{1}{2}\,Q_1$

The equilibrium levels of Q_1 and Q_2 are given by solving the two reaction curves.

$\therefore$ Cournot-Nash equilibrium $Q_1 = Q_2 = 20$, and Cournot price, $P = 20$

2) Solve the question like the above question.

$Q_1 = Q_2 = \frac{a-c}{3b}$, total industry output, $Q = \frac{2(a-c)}{3b}$, Cournot price, $P = \frac{a+2c}{3}$

Check Your Progress 2

1) Bertrand model is based on price competition, where the only Nash equilibrium is when $P_1 = P_2 = C$. In the Stackelberg model, the first mover, firm 1 has an advantage, it has a higher market share, hence higher profits than firm 2, price is also above marginal cost.

2)

Market form	Firms output	Price
Stackelberg	$Q_1 = \frac{a-c}{2b}$ $Q_2 = \frac{a-c}{4b}$	$P = \frac{a+3c}{4}$
Bertrand	$Q_1 = Q_2 = \frac{a-c}{2b}$	$P = C$

UNIT 6 GAME THEORY AND ITS APPLICATIONS

Structure

6.0 OBJECTIVES

After reading the unit, you should be able to:

- define the concept of a Game in the Economic theory;
- describe basic elements of a Game, viz. (1) players, (2) strategies and (3) payoffs;
- discuss different types of Games;
- appreciate two different forms of representing a Game;

- appreciate various basic concepts required to solve a Game theoretic problem;
- elucidate different equilibrium concepts related to the Game theory;
- explain the notion of a sequential game; and
- discuss the application of the Game theory.

6.1 INTRODUCTION

In Unit 5 we discussed the market structure of oligopoly where firms' decision problem exhibit strategic interdependence. In such a market, output or price decision by one firm impacts output (sales) or price decision and hence the profit of the other firms in the market. As a result, optimisation by firms in this market involves taking care of their strategic interdependence. Game theory is nothing but the study of interactive decision-making. A group of people (or players, teams, firms, countries) are in a game if their decision-making problem is interdependent. Thus in a multi-agent framework the behaviour of individual agents are contingent to the fact that action of one agent affects the payoff of the other agent(s). In the previous unit we discussed game theory in the context of firm competition. One can come across many real life situations involving strategic interdependence where more than one agent is involved in decision-making (like football, soccer, baseball are games). Economists apply various general tools to arrive at the solution of the problems involving strategic situations. In fact game theory has been used in many fields in recent decades (apart from Economics) like Political Science, Sociology, Computer Science, Biology. The present unit will be elaborating upon that.

6.2 THE GAME THEORY

The early beginnings of Game Theory can be traced to analysis of imperfectly competitive market by Augustine Cournot (1838). The first systematic attempt is Neumann's and Morgenstern's *Theory of games and Economic Behaviour* published in 1944. The next great advance is by John Nash who introduced the concept of 'Nash Equilibrium'. Game theory was invented as an attempt to find a theoretical solution to the problems posed by uncertainty in games of chance where rational players take decision in an interdependent set up. Basically it comprises a formal methodology and a set of techniques to study the interaction of rational agents in strategic settings. By 'rational agent' it is meant an individual who is assumed to take into account all the available information, probabilities of events, and potential costs and benefits to perform the action with the optimal expected outcome for itself from among all feasible actions. A 'strategic scenario' is defined as the one where actions of one individual affects the payoff (or reward) or utility of other individuals. Game theory can be used to model a wide variety of human behaviour in economic, political, and social settings. Game theory models seek to portray complex strategic situations in a simplified setting. Like in case of Perfect competition, Monopoly and Imperfect competition, game theory involves generalising the methods used earlier and with the help of some specific language arrive at a mathematical representation of the strategic situation. Let us now discuss some preliminaries related to the topic.

6.3 A GAME

A 'Game' is an abstract of a strategic situation involving interdependence. A simplest form of game is defined by: players, actions or strategies and payoffs. In game theory, players are the agents who are involved in the decision-making. Each player has a number of strategies or action to choose from. The strategies chosen by each player determine the outcome of the game, with each possible outcome resulting in a payoff to each player.

i) **Players:** Players are the agents playing the game. They may be firms, individuals, countries, or just about anything else that is capable of executing a strategy. In the duopoly game, for instance, the players are the two firms. Generally, in an *n*-player game, players are numbered from 1 to *n,* with any arbitrary player to be called player *i*.

ii) **Strategies:** Strategies are the actions or the set of actions available to the players. For instance, in case of Cournot game, each firm's strategy is to choose its quantity, taking as given the quantity of its rivals. In case of an *n*-player game we assume any player *i* has a strategy set S_i consisting of different strategies, with s_i referring to any arbitrary element of the set S_i, i.e. $s_i \in S_i$.

iii) **Payoffs:** Payoffs are the returns to the players at the conclusion of the game. For instance, payoffs are the profits in case of profit maximising firms. In an *n*-player game, payoff function for any arbitrary player *i* will be given by U_i $(s_1,s_2, s_3....s_n)$, where $(s_1, s_2, s_3....s_n)$ represents the combination of strategies (one for each player) chosen by the n-players. Here U_i is the payoff to player *i* as a function of $(s_1, s_2, s_3....s_n)$.

Besides the above set of three elements, a game is modelled by specifying 'information' that each player has when it decides its course of action. It is usually assumed that there is common knowledge, that is, each player knows not only about the ''rules of the game'' called common knowledge of the game, but also what the other player knows, and so forth called the common knowledge of rationality. Other aspects of information vary from game to game, depending on timing of moves and other issues.

A Game can be static (called simultaneous-move game) or dynamic (called sequential-move game) depending upon the information base of the players or the time of move of the players. In simultaneous-move games, neither player knows the other's action when she(he) decides her own move. In sequential-move games, the players move in sequence and the first-mover does not know the second's action but the second-mover knows what the first did and the payoffs are decided at the end.

6.4 TYPES OF GAMES

6.4.1 Non-cooperative versus Cooperative Games

There are two branches of the game theory, viz. cooperative and non-cooperative game theory. Under the cooperative game theory, groups or sub-sets of the players make a binding agreement to reach an outcome that is best for the group as a whole and is shared equally among the members. In contrast to this, under non-cooperative game theory, players cannot write binding contract. Players are guided by self-interest, each player acts as an individual who is normally assumed to maximise his own utility without caring about the effects of his choice on other players in the game. The outcome of the game, however, is jointly determined by the strategies chosen by all players in the game. As a result, each player's welfare depends, in part, on the decisions of other players in the game. An example of cooperative game is two firms negotiating a joint investment to develop a new technology. An example of non-cooperative game is two competing firms taking into account each other's behaviour when setting their prices independently.

Self-interested behaviour does not always lead to an outcome that is best for the players as a group. This we will come across when we discuss different illustrations of the games. Non-cooperative game theory is more widely used by economist; nevertheless, cooperative game theory has been used to model bargaining games and political processes.

6.4.2 Games of Complete and Incomplete Information

In the games of complete information, the payoffs, strategies and types of players are common knowledge. Complete information is the concept that each player in the game is aware of the sequence, strategies, and payoffs throughout the game. Given this information, the players have the ability to plan accordingly based on the information to maximise their own rewards or payoff at the end of the game. The equilibrium solution concepts are Nash equilibrium or Sub-game perfect Nash equilibrium depending upon whether the game is simultaneous-move (static) or sequential-move (dynamic). We will introduce these concepts in the subsequent sections.

Inversely, in a game with incomplete information, players do not possess full information about their opponents. Some players possess private information, a fact that the others should take into account when forming expectations about how those players will behave. A typical example is an auction: each player knows his own utility function (valuation for the item), but does not know the utility function of the other players. The equilibrium solution concepts are Bayesian Nash equilibrium or Perfect Bayesian equilibrium depending upon whether the game is simultaneous-move (static) or sequential-move (dynamic).

6.4.3 Zero-sum versus Non-Zero Sum Games

A zero-sum game is the one in which the gain of one player comes at the expense of the other player and is exactly equal to the loss of the other player. In other words, the sum of the payoffs of the two players always adds to zero. An economic application can be the transaction between a buyer and a seller at the cost price. A non-zero sum game is when gain or loss does not come at the expense of the other player. An example of this might arise if increased advertisement leads to higher profits for both the firms.

6.4.4 Simultaneous-move versus Sequential-move Games

The order of moves is significant in the game theory. Players in a game may move simultaneously or sequentially which in turn results in different outcomes of the game. A simultaneous-move game is a game in which neither player knows the other's action when moving, that is, players take their action simultaneously without knowing the action that have been chosen by the other player(s). For instance, in Cournot model of oligopoly, each firm decides its profit maximising levels of output simultaneously. In contrast, in sequential-move games, the order of moves comes into picture. In this case, one player moves first which is then observed by his opponent. The player(s) who moves afterwards gets to observe and learn information about the course of the game up to that point, including what actions other players have chosen. These observations can then be used by that player to decide his (her) own optimal strategies than simply choosing an action. This way, strategies of the players depend on what the other player(s) before have done already.

6.5 ALTERNATIVE FORMS OF REPRESENTING A GAME

There are two principal representations of the rules of a game, i) the Normal or Strategic form, and ii) the Extensive form. The normal form of representation of a game is by using a payoff matrix in the form of rows and columns. By convention, rows correspond to represent player 1 and columns correspond to player 2, whenever we have a two player game and an entry in a cell shows the payoff of two players for that specific combination of strategies chosen. The extensive form is the pictorial representation of the rules. The main pictorial form is called the game tree. The game starts from player1 from a node and the choices available are represented by branches emanating from that node. At the end of a branch, player 2 makes his choice and the branch will split into further branches. The game specifies the order in which the players make choices, how many times each gets to choose and the eventual payoffs to each player for any sequence of choices. Normal form is usually used for simultaneous move games, whereas, an extensive form through a game tree may be used to represent either a simultaneous-move or a sequential-move game. Let us illustrate both of these presentation forms using the very famous example of Prisoner's Dilemma game.

Prisoner's Dilemma

A well known example of a non-cooperative and a game of complete information is the Prisoners' Dilemma game. Consider the following set-up of the game: A crime is committed for which there is no eye witness. Suspects 1 and 2 are caught and imprisoned in two separate cells. Thus each prisoner is in a solitary confinement with no means of communicating with each other. The magistrate speaks to each prisoner separately, and asks them to act as an informer. If one of them confesses the crime, he will be freed but the other one will spend 4 years in prison. If both confess, each will spend 3 years in prison. If both stay quiet and do not confess, the crime cannot be probed, so they will get nominal punishment by spending only one year in prison. Thus each player then has two possible strategies: Not confess (N) or Confess (C) and they decide simultaneously.

6.5.1 Normal-form Representation of a Game

The situation may be modelled as a strategic (normal) form game with the following elements:

Players: Two suspects, prisoner 1 and 2.

Strategy: each player's strategy set is {Not confess (N), Confess (C)}

Payoff: Number of years of prison sentence

		Player 2	
		Not Confess	Confess
Player 1	Not Confess	(−1, −1)	(−4, 0)
	Confess	(0, −4)	(−3, −3)

Fig. 6.1: Payoff Matrix

Entry in each cell of the above figure represents (Player 1's, Player 2's) payoff in terms of number of years in jail (a negative payoff with a negative sign) from each of the two strategies. In other words, the first numerical figure in each cell of the matrix corresponds to payoff of player 1 and the second figure corresponds to payoff of player 2. For example, if we look at the action [Not Confess, Not confess] the payoffs associated are (−1, −1). This means when both the players simultaneously decide not to confess, both will be spending one year in jail. The next entry in first row, second column is (Not confess, Confess), this means player 1 will get 4 years of imprisonment and player 2, freedom.

6.5.2 Extensive-form Representation of a Game

A simultaneous game can also be represented by the extensive form using a game tree. In Fig. 6.2 each dark dot is called the decision node for the player indicated there. The first move belongs to Player 1, who can choose to confess or not confess. The next move belongs to Player 2, who can also choose among the two options (confess or not confess). Payoffs are decided at the end of the tree towards the extreme right. To reflect the fact that the Prisoners' Dilemma is a simultaneous move game, a dotted oval is drawn around Player 2's decision nodes to reflect the fact that Player 2 does not know which of the two decision nodes he is at since he does not observe which action Player 1 has chosen, that is, he does not know whether the first decision by Player 1 was to confess or not confess. This dotted oval around the two nodes of Player 2 indicates his lack of specific information. An information set of a Player is a collection of nodes such that the same player (here Player 2) moves at each of these nodes; and the same moves (here Not Confess, Confess) are available at each of these nodes.

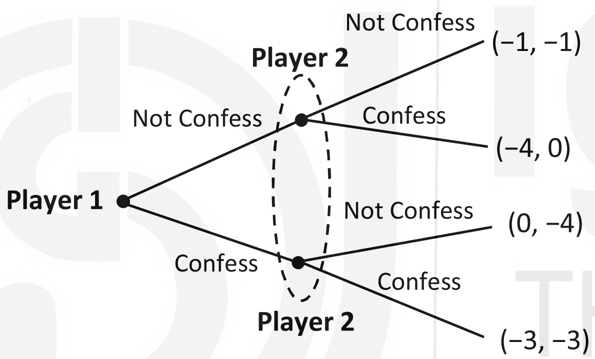

Fig. 6.2

Fig. 6.2 is consistent with the game being a simultaneous game. If we were to use the game tree to illustrate the above game as a sequential one in which Player 1 moves first which then is observed and reacted upon by Player 2, then the game would be more correctly drawn without the ellipse as:

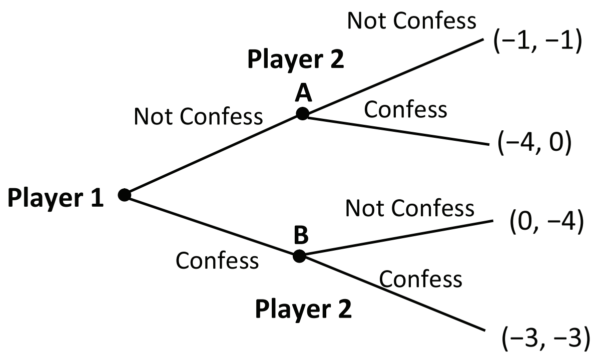

Fig. 6.3

The two nodes (say A and B) which signifies the move of Player 2 represent the information set of Player 2. Clearly the information base of Player 2 at A and at B are not same in this case of sequential-move game. The Prisoner's Dilemma is widely studied throughout the Social Sciences. It is a compelling scenario because the tensions it portrays between an individual player's self-interest and the group's self-interest shows up in many different ways around us. Understanding this shows us the reality of counter-productive outcomes in a non-cooperative situation.

6.6 SOLVING A GAME THEORY PROBLEM

Having covered different representation forms of a game theory problem in the previous section, let us now discuss how to find solution of game-theoretic problem. We start with discussing some basic concepts regarding different kinds of strategies.

6.6.1 Dominated Strategies

We consider the same illustration of the Prisoners' Dilemma. When we consider the payoffs associated with different decision options, we realise, if one player chooses to confess, then the other player would prefer to confess and be in the prison for three years, rather than opting to not confess and suffer imprisonment for four years. In the same lines, when one chooses to not confess, the other would prefer to confess and so get released immediately rather than opting to not confess and suffer imprisonment for a year. In both the situations and for both the players we observe that option "not confess" is dominated by the option "confess". That is, for each strategy that Player 1 (or Player 2) opts, payoff to Player 2 (or Player 1) from not confessing would be less than payoff to him from confessing. Hence, 'Not Confess' strategy is dominated by the 'Confess' strategy. Rational players in an attempt to maximise their positive payoffs will not opt for a strictly dominated strategy. In Prisoners' dilemma game, rational players will choose to confess, and hence the equilibrium outcome of the game will be (Confess, Confess) resulting in the payoff of (−3, −3). Such a result in not optimal when compared with the payoff (−1, −1) associated with the strategy profile (Not Confess, Not Confess). In other words, both players will suffer only one year of imprisonment when they opt for the strategy 'not confess', but they end up opting for the strategy 'confess' and suffer imprisonment of three years each. We will discuss this issue later in some more detail when we will come across the concept of nash equilibrium.

Generally, in an n-player game, among strategies s'_i and s''_i each belonging to strategy set S_i of an arbitrary player i, strategy s'_i is dominated by strategy s''_i if player i's payoff from playing s'_i is lower than the payoff he attains from playing s''_i for each feasible combination of other players' strategies (i.e., whatever be the strategy played by the other players).

6.6.2 Dominant Strategies

A dominant strategy for a player is the one that yields best payoff for that player no matter what the other player does. That is, a dominant strategy is a player's best response to all the feasible strategies of the other player. Generally, strategy s_i* will be referred to as the dominant strategy for player i if he is strictly better off playing s_i* rather than any other strategy regardless of what his opponent plays.

Please note: If a player has a dominant strategy then for this player all other feasible strategies become dominated by this dominant strategy. But in many situations, a player may not have a dominant strategy and yet may have dominated strategies.

6.6.3 Dominant Strategy Equilibrium

A rational player is expected to play his dominant strategy, and thus, if all players have a dominant strategy, then it is rational for them to choose the dominant strategies and this way we reach at the *dominant strategy equilibrium*. In terms of the Prisoners' Dilemma, both players have 'confess' as a dominant strategy, that is, in that case, each player's best response would be to play confess regardless of what the other player might opt for. Thus, we have (Confess, Confess) resulting in a payoff of (−3, −3) as the *dominant strategy equilibrium* in case of Prisoners' Dilemma.

6.6.4 Iterated Elimination of Strictly Dominated Strategies

Now, suppose players (one or both) have no dominant strategy. In that case, we come to a method of reaching an equilibrium position of a game-theoretic problem by way of elimination of strictly dominated strategies. We discussed what is meant by dominated strategies in Sub-section 6.6.1. Further this dominance could be strict or weak. Among strategies s'_i and s''_i each belonging to strategy set S_i of an arbitrary player i, strategy s'_i is strictly dominated by strategy s''_i if for every strategy choice of the opponent, player i's payoff from choosing s''_i is strictly greater than player i's payoff from choosing s'i. On the other hand, strategy s'_i will be said to be weakly dominated by strategy s''_i if, i's payoff from choosing s''_i is at least as good as i's payoff from choosing s'i. Here, s'i is called dominated strategy while s''_i is referred to as dominant strategy.

Iterated elimination of strictly dominated strategies involves elimination of strategies which are dominated relative to opponents' strategies which have not yet been eliminated until there are no strictly dominated strategies left. The principle behind this method is that rational players of the game never play a dominated strategy since they can get higher payoffs by playing another strategy, no matter what the other players are playing. Hence, a dominated strategy can be safely discarded to play a reduced form of game with a smaller number of strategies. Consider the following payoff matrix where we have two players 1 and 2 with their respective strategies P, Q (for player 1) and A, B, C (for player 2) and associated payoffs.

		Player 2		
		A	B	C
Player 1	P	(6,6)	(1, 11)	(4, 5)
	Q	(4,1)	(3,3)	(5, 6)

We can begin eliminating the dominated strategy of either of the player. For instance, for Player 2, there are three strategies A, B and C. Irrespective of Player 1's strategy (either P or Q), strategy A for Player 2 is strictly dominated by strategy B. This is because if Player 1 plays P, Player 2 gets 6 from playing A whereas 11 from playing B (6 < 11). Similarly if Player 1 plays Q, Player 2 gets 1 from playing A whereas 3 from playing B (1 < 3). Hence, strategy A can be eliminated for Player 2 so that the game reduces to:

		Player 2	
		B	C
Player 1	P	(1, 11)	(4, 5)
	Q	(3, 3)	(5, 6)

Now irrespective of Player 2's strategy (either B or C), strategy P for Player 1 is strictly dominated by Q. If Player 2 plays B, Player 1 gets 1 from playing P and 3 from playing Q (1 < 3). Similarly, if Player 2 plays C, Player 1 gets 4 from playing P and 5 from playing Q (4 < 5). So player 1 being rational won't play the strictly dominated strategy P, so P can be eliminated so that the game to reduce to:

		Player 2	
		B	C
Player 1	Q	(3, 3)	(5, 6)

Now Player 1 is left with one strategy Q. Player 2 will choose to play strategy C for a payoff of 6 which is higher than the payoff of 3 from strategy B. Thus, we arrive at the equilibrium solution strategy profile of (Q, C) yielding a payoff of (5, 6) through iterated elimination of dominated strategy.

6.6.5 Nash Equilibrium

Presence of dominated strategies allows iteratively eliminating such strategies from the game and successively reaching the equilibrium solution. In many games, however, there are no dominated strategies or no dominant-strategy, and hence ruling out any outcome is not an option. Hence deriving the equilibrium is not possible. A more general approach of deriving the equilibrium solution is a Nash Equilibrium. Nash equilibrium refers to a set of mutually best response strategies, where each player chooses his best (optimal) strategy given the strategy of the other players. Thus at Nash equilibrium no player has incentive to deviate from the chosen strategy, given the Nash equilibrium strategy of its rivals. In other words, no player can obtain a higher payoff by switching to a different strategy given that all the other players stick to their Nash equilibrium strategy. For instance, if two firms are selling a particular commodity, Nash equilibrium will be an array of production levels, one for each firm, such that neither firm can raise its profits by unilaterally deviating and making a different choice, in other words, no firm will be able to gain by changing its production level, given the strategy choice of the other player.

Generally, in an n-player game, strategy profile (s_1*, s_2*, s_3*...., s_n*) is said to be a Nash equilibrium if for each player i, where s_i* is player i's best response to the given strategies of all the other n − 1 players. Hence, $(s_i{}^*, s_{-i}{}^*)$ is the Nash equilibrium strategy if: $U_i\ (s_i{}^*, s_{-i}{}^*) \geq U_i(s_i, s_{-i}{}^*)$; for all $s_i \in S_i$, where $(s_{-i}{}^*)$ is the Nash equilibrium strategy of all the (n – 1) players and U_i is the payoff function. This shows that if player 1 deviates from the Nash equilibrium strategy given the remaining (n – 1) players' strategy, she (he) won't gain. Same applied for other (n – 1) players. So it is a mutually best response strategy.

Let us attempt to find the Nash equilibrium in case of the Prisoners' Dilemma game. Consider the normal form of the game as given below:

		Player 2	
		Not Confess	Confess
Player 1	Not Confess	(−1, −1)	(−4, 0)
	Confess	(0, −4)	(−3, −3)

Now we proceed by underlining the best response of each player to a given strategy of the other player. For instance, if Player 2 believes that Player 1 will choose *not confess*, then Player 2's best response will be to play *confess* because with the strategy (Not confess, Not confess) the payoff of (−1, −1) implies Player 2 will suffer 1 years of imprisonment, while with the strategy (Not confess, confess) payoff of (–4,0) implies Player 2 will be set free for telling truth. We mark this best response of Player 2 to the strategy (not confess) of Player 1 by underlining 0. This way we underline best responses of each player to the given strategy of the other player as shown below:

		Player 2	
		Not Confess	Confess
Player 1	Not Confess	(−1, −1)	(−4, **0**)
	Confess	(**0**, −4)	**(−3, −3)**

From the above payoff matrix, mutually best response strategy of both the players is the Nash equilibrium in pure strategy i.e., each move involved is the best response to the other moves. In other words, a cell in the normal form is a Nash equilibrium in pure strategy if each entry is marked (underlined) as being the best response to the other moves. This way, (Confess, Confess) yielding a payoff of (−3,−3) will be the Nash equilibrium in the above game.

But one may also note that both the players would have been better off if both played 'not confess'. When both play 'not confess' the resulting payoff is one year of imprisonment. This is certainly better than the three year of imprisonment that they suffer when both choose to play 'confess'. Thus, (*Not Confess, Not Confess*) is the Pareto optimal outcome of this game. However, both players playing *'not confess'* is not an equilibrium, because given (say) Player 1 plays *Not Confess,* Player 2 always has an incentive to deviate towards the strategy '*confess'* and vice versa in the absence of binding agreement. If binding agreement is possible, then both would agree on the (*Not confess, Not confess)* combination, reaching a higher payoff. Thus, rules of the games that do not allow binding agreements may induce the players towards the strategy that results in lower payoffs for both of them. Game-theoretic conditions like Prisoners' Dilemma arise in many real world settings. For instance, in a cartel agreement among suppliers of steel to restrict output would lead to higher prices and profits if it could be sustained, but such an agreement may be unstable because it may be too tempting for an individual steel supplier to sell more output at the high price.

Remember:

1) In game theory, every dominant strategy equilibrium is a Nash Equilibrium. However, a Nash Equilibrium may or may not be a dominant strategy equilibrium.

2) With just two strategies for each player if one strategy is dominant, the other must be dominated. However with more than two strategies available, a player might have dominated strategies but no dominant strategy. If neither player has a dominant strategy, we can deduce equilibrium by iteratively eliminating the dominated strategies and successively moving towards the reduced form of game.

3) Nash equilibrium is widely used as an equilibrium definition because it exists for all games. For games that at first appear not to have a Nash equilibrium in pure strategy will end up having one in mixed strategies (we will discuss this in the subsequent section).

6.6.6 Multiple Equilibria

Nash equilibrium is useful for it being stable and existing for all games. However, an unpleasant situation arises when a game has multiple Nash equilibria. The problem arises due to the fact that a unique outcome cannot be predicted in such a case. To illustrate the possibility of multiple equilibria we consider yet another classic game, the *Battle of the Sexes.*

Battle of the Sexes

The game involves two players, a husband and a wife, planning an evening out with both preferring to go together rather than going alone. The wife wants to listen to an Opera performance, while the husband wants to watch a Boxing match. The normal form for the game is given below:

		Husband: Opera	Husband: Boxing
Wife	Opera	(3, 1)	(0, 0)
	Boxing	(0,0)	(1, 3)

When both wife and husband end up at Opera, woman receives a payoff of 3 while her husband 1, on the other hand, when they end up attending a boxing match, woman receives a payoff of 1 while her husband receives a payoff of 3. When they both go to different locations, their payoff reduces to 0. There is no dominant (or dominated) strategy for any of the player in this game. A player would rather go for Opera if the other player chooses to go for Opera, while the former would go for a boxing match if the later chooses to go for a boxing match. It illustrates the fact that every game cannot be solved using the iterated elimination of dominated strategies. For Nash equilibrium we mark the best responses of the husband and the wife.

		Husband: Opera	**Husband**: Boxing
Wife	Opera	**(<u>3</u>, <u>1</u>)**	(0, 0)
	Boxing	(0,0)	**(<u>1</u>, <u>3</u>)**

For both of them, best response is to play the same action as the other. This results in multiple pure strategy Nash equilibria, (Opera, Opera) and (Boxing, Boxing) yielding payoffs of (3, 1) and (1, 3), respectively. It is not possible to say which is pareto superior as they are symmetric.

6.7 MIXED STRATEGIES

The game of Prisoner's Dilemma we considered so far involved playing a single strategy with certainty or with a probability of 1, known as a pure strategy. Some games involve playing more complicated strategies in the form of mixed strategies than simply choosing a single strategy with certainty. In mixed strategies, players' choice of strategies follows a probability distribution from among several possible strategies. Consider a classic game known as the Matching Pennies to understand the concept of mixed strategies.

Matching Pennies

This is a game where two players 1 and 2, each possessing a coin (penny) decides to simultaneously display their coins with either heads or tails facing up. If the face of the coins matches with either both head or both tail faced up, Player 2 gives his penny to Player 1; if face of the coins do not matches, Player 1 gives his penny to Player 2. The normal form of the game is as follows:

Player 2

		Head	Tail
Player 1	Head	(1, −1)	(−1,1)
	Tail	(−1,1)	(1, −1)

Such a game is called a zero-sum game due to the fact that sum of the payoffs of the players in each box add to zero. We will employ the same method of underlining the best responses we applied in case of Prisoner's Dilemma to solve for the Nash equilibrium. We get the following result:

Player 2

		Head	Tail
Player 1	Head	($\underline{1}$, −1)	(−1,$\underline{1}$)
	Tail	(−1,$\underline{1}$)	($\underline{1}$, −1)

There is no Nash equilibrium in pure strategies in this case as whatever Player 2 opts for, Player 1 would want to choose the same option, but then Player 2 would want to deviate to the other option, and this process of moving to a different option would be endless. Thus, as long as one player knows where the other player is, the latter gets a bad payoff. Therefore, each would want to be unpredictable with their choices. Such a strategy is referred to as a mixed strategy, and the resulting strategy profile with associated probability for actions is called mixed strategy Nash equilibrium.

In a mixed strategy equilibrium, each of the player equate their expected payoff from each of the strategy (Head and Tail) in equilibrium. In other words, the players must be indifferent between the actions which they choose to play. If a player was not indifferent between the available actions, this would imply that one particular action yields a higher payoff than the others, and the player would play that action with probability 1 rather than mixing strategies with certain probability distribution.

To solve the Mixed Strategy Nash equilibrium, suppose that Player 2 opts for Head with probability p, and chooses Tails with probability 1−p. Similarly suppose Player 1 plays the game with a probability distribution q and (1−q) for Head and Tail. Now, the expected payoff of Player 1 for playing a pure strategy of Head when Player 2 plays a mixed strategy with a probability distribution p and (1−p) is:

$$E_1(H) = p(1) + (1-p)(-1) = 2p - 1$$

Similarly, the expected payoff of Player 1 for playing a pure strategy of Tail when Player 2 plays a mixed strategy with a probability distribution p and (1−p) is:

$$E_1(T) = p(-1) + (1-p)(1) = -2p + 1$$

If the Player 1 plays a mixed strategy, in the equilibrium, he should be indifferent between the two expected payoffs from Head and Tail. Thus:

$$E_1(H) = E_1(T) \Longrightarrow\ 2p - 1 = -2p + 1 \Longrightarrow p = \frac{1}{2} \Longrightarrow (1-p) = \frac{1}{2}$$

Thus, Player 2 will play the game with a mixed strategy for Head and Tail with a probability distribution $p = \frac{1}{2}$ and $(1-p) = \frac{1}{2}$, i.e., to keep Player 1 guessing, Player 2 must opt for Head with a probability of 1/2 and Tail with the probability of 1/2. Similarly, we can find Player 1's equilibrium mixed strategy. Now, the expected payoff of Player 2 for playing a pure strategy of Head when Player 1 plays a mixed strategy with a probability distribution q and (1 – q) is:

$$E_2(H) = q(-1) + (1-q)(1) = -2q + 1$$

Similarly, the expected payoff of Player 2 for playing a pure strategy of Tail when Player 1 plays a mixed strategy with a probability distribution q and (1−q) is:

$$E_2(T) = q(1) + (1-q)(-1) = 2q - 1$$

If Player 1 plays a mixed strategy, in the equilibrium, he should be indifferent between the two expected payoffs from Head and Tail. Thus:

$$E_2(H) = E_2(T) \Longrightarrow\ -2q + 1 = 2q - 1 \Longrightarrow q = \frac{1}{2} \Longrightarrow (1-q) = \frac{1}{2}$$

Thus, Player 1 will play the game with a mixed strategy for Head and Tail with a probability distribution $q = \frac{1}{2}$ and $(1-q) = \frac{1}{2}$, i.e., to keep Player 1 guessing, Player 2 must opt for Head with a probability of ½ and Tail with the probability of 1/2.

Thus, in mixed strategy Nash equilibrium, Player 1 would opt for Heads with probability 1/2, and Tails with probability 1/2 and Player 2 would also play Heads and Tails with probabilities 1/2 and 1/2 respectively. Such an equilibrium can be represented as p*=1/2 and q*=1/2.

Note that, suppose $E_1(H) \gtrless E_1(T)$, that is, expected payoff of the Player 1 from Head is greater (lesser) than Tail. Obviously in that case a rational player will not play mixed strategy but will play Head (Tail) with certainty.

Thus if the player has to play Mixed strategy, in equilibrium the expected payoffs from the different strategies should be equal.

Check Your Progress 1

1) Following is the payoff matrix in strategic form in which two players have two strategies each.

		Player 2	
		Left	Right
Player 1	Top	(7,3)	(5,3)
	Bottom	(7,0)	(3, −1)

A) Write the game in extensive form.

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B) Find Nash equilibrium of the game.

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C) Find dominant strategies of both players.

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2) Following is a game in which the players have three strategies each.

		Player 2		
		Left	Centre	Right
Player 1	Top	(4,5)	(1,6)	(5, 6)
	Middle	(3,5)	(2,5)	(5, 4)
	Bottom	(2, 5)	(2, 0)	(7, 0)

a) Write the game in extensive form.

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b) Find Nash equilibrium in pure strategy of the game.

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3) Are dominant strategy always Nash equilibrium? Is the converse true?

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4) From the following payoff matrix where the payoffs are the profits or losses of the two firms, determine

		Firm B	
		Low Price	High Price
Firm A	Low Price	(2,2)	(4,−1)
	High Price	(−1,4)	(6,3)

a) Whether firm A has a dominant strategy?

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b) Whether firm B has a dominant strategy?

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c) Find the Nash equilibrium in pure strategy, if there is one.

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5) From the following payoff matrix, where the payoffs refer to the profits firms earn by cheating and not cheating in a cartel

		Firm B	
		Cheat	Don't Cheat
Firm A	Cheat	(3,2)	(9,1)
	Don't Cheat	(2,6)	(7,4)

a) Determine the Nash equilibrium in pure strategy.

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b) What if we change the payoff of the bottom left cell to (4, 4)?

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7) In the Battle of Sexes game (given in the Section 6.6), two pure strategy Nash equilibria are derived. Derive the mixed strategy Nash equilibrium of the game (if there is any).

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8) If any of the player in a game is having a dominant strategy, do you think that the player will ever play mixed strategy ? Why ?and Why not?

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6.8 SEQUENTIAL GAMES

So far, we have been considering games in which players make decisions simultaneously, that is, where all the players (possessing no prior information about the actions of their opponents) act at the same time. In such games, time plays no role. So those games are often called Static games. In many games, however, one player can move before the other players. Such games are referred to as sequential-move games where players take action at well-defined turns (over time), and have perfect knowledge about what the other player(s) did at previous turns. In other words, in a two-player sequential-move game, one player (the first mover) takes an action before another player (the second mover). The second mover observes the action taken by the first mover before it decides what action it should take. Now consider the same game of the Battle of the Sexes with the same strategies and payoffs, but with sequential moves. We assume, instead of them both moving simultaneously, the wife is the first to move and makes a choice between Opera or Boxing, on observing which the husband decides next between the two options. Illustration of the game is presented in the following extensive form:

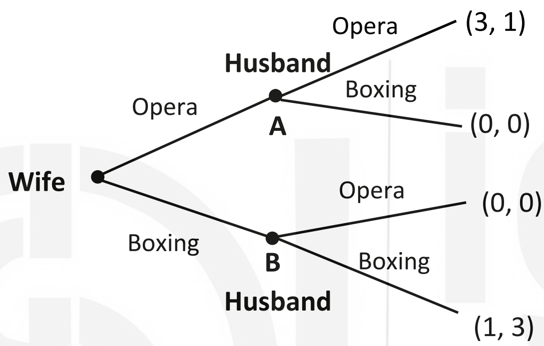

Fig. 6.4

As one may notice, the oval around the decision nodes of Player 2 (that was there in case of simultaneous-move game) has been removed. This is because with sequential moves, husband (second mover) can observe his wife's (first mover) action and hence is aware of the decision node he is on before he takes his action. The wife's possible strategies remain unchanged at Opera or Boxing. But the husband's set of possible strategies has expanded to four strategies; as for each of the wife's two actions, he can choose one of two actions. So the husband has the option to move either from node A or node B, that is, he is not in the same information base. Note that to define the husband's strategies completely we must define an action at each information set that he makes a choice at, even if that information is not actually reached in the game. Thus all strategies in his strategy profile, include an action for both the top and bottom information sets. To solve for the Nash equilibria, we will consider the normal form of the game and use the method of underlining payoffs for best responses:

		Husband			
		Always Opera (Opera|Opera) (Opera|Boxing)	Follows Wife (Opera|Opera) (Boxing|Boxing)	Opposite to Wife (Boxing|Opera) (Opera|Boxing)	Always Boxing (Boxing|Opera) (Boxing|Boxing)
Wife	Opera	(3, 1)	(3, 1)	(0, 0)	(0, 0)
	Boxing	(0,0)	(1, 3)	(0, 0)	(1, 3)

Now we have a 2 × 4 matrix, where wife has a strategy profile S_W = {Opera, Boxing}, while husband has the strategy profile S_H= {(Opera|Opera) (Opera|Boxing), (Opera|Opera) (Boxing|Boxing), (Boxing|Opera) (Opera|Boxing), (Boxing|Opera) (Boxing|Boxing)}. Here, (Opera|Opera) (Opers|Boxing) for instance means that husband opts for Opera given that wife has chosen Opera while husband opts for Opera after wife has chosen to go for a Boxing match. Now the underlining of the best responses ensuring we underline the payoffs for all the strategies that tie for the best response as well is shown below:

		Husband			
		Always Opera (Opera|Opera) (Opera|Boxing)	Follows Wife (Opera|Opera) (Boxing|Boxing)	Opposite to Wife (Boxing|Opera) (Opera|Boxing)	Always Boxing (Boxing|Opera) (Boxing|Boxing)
Wife	Opera	($\underline{3}$, $\underline{1}$)	($\underline{3}$, $\underline{1}$)	($\underline{0}$, 0)	(0, 0)
	Boxing	(0,0)	(1, $\underline{3}$)	($\underline{0}$, 0)	($\underline{1}$, $\underline{3}$)

This way, we get multiple Nash equilibria:

i) Wife opts Opera, husband opts (Opera|Opera) (Opera|Boxing)

ii) Wife opts Opera, husband opts (Opera|Opera) (Boxing|Boxing)

iii) Wife opts Boxing, husband opts (Boxing|Opera) (Boxing|Boxing)

Among the three equilibria let us analyse the plausibility of each. We start with the first Nash equilibrium. Husband's strategy (Opera|Opera) (Opera|Boxing) involves a threat to wife that he will choose Opera when his wife will go for a Boxing match. However, this is an empty threat. This results from the fact that if the wife really were to choose a boxing match first, then husband would be giving up a payoff of 3 by choosing Opera rather than Boxing, which is certainly hard to believe. Thus, husband's threat to his wife to opt for Opera if the wife goes for a boxing match is *non-credible*. On the similar lines, the husband's strategy in the third Nash equilibrium (Boxing|Opera) (Boxing|Boxing) involves an empty threat of him choosing to go for a boxing match when his wife opts for Opera in the first move. As for this he will be giving up a payoff of 1 by choosing Boxing rather than Opera.

6.8.1 Sub-game Perfect Nash Equilibrium (SPNE)

Nash equilibrium in sequential games may result in strategy profiles which are not very plausible. A formal procedure for selecting a reasonable Nash equilibrium strategy profile (in a way that rules out empty threats) is done using the concept of Sub-game Perfect Nash equilibrium (SPNE) which requires that for equilibrium to be rational it should be an equilibrium not just for the game as a whole, but also for each subgame of the game. A subgame is a part of the extensive form beginning from any node of the game and including everything that branches out below it. A decision node initiates a subgame if neither it nor any of its successors are in an information set that contains nodes that are not successors to it. Such a subgame is termed a proper subgame. This way, the sequential Battle of the Sexes game has three subgames, viz. the game itself, and the two lower subgames beginning from the nodes where the husband gets to decide. We mark these subgames with a dashed rectangle as shown in the figure:

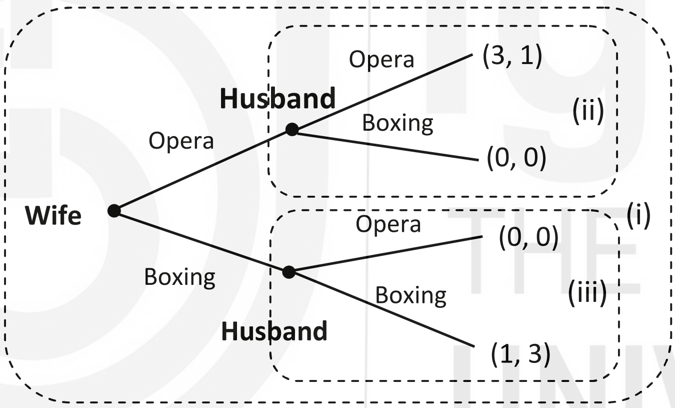

Fig. 6.5

A sub-game Perfect Nash Equilibrium is a set of strategies, one for each player, that induce a Nash equilibrium in every proper subgame. As far as the sequential Battle of the Sexes game is concerned, for a strategy profile to be termed as a SPNE, in addition to it being a Nash equilibrium on the whole game, it must be the Nash equilibrium of the two other proper subgames. In the above set-up, given that wife chooses Opera, in the subgame (ii) the husband has to decide between Opera (resulting in payoff of 1) and Boxing (yielding a payoff of 0). His best response in this subgame will be to go for Opera. Similarly, given that wife chooses to go for a Boxing match, husband in the subgame (iii) is to decide between Opera (resulting in payoff of 0) and Boxing (yielding payoff of 3). His best response would be to go for a Boxing match. Thus, (Opera|Opera) (Boxing|Boxing) is the only strategy profile of the husband that forms a part of the SPNE. The other two strategy profiles like (Opera|Opera) (Opera|Boxing) and (Boxing|Opera) (Boxing|Boxing) results in him playing something that is not a Nash equilibrium on some proper subgame. Thus, among the three Nash equilibria we came across for the game in Section 6.8, only the second one is subgame perfect Nash equilibrium while the first and the third are not.

6.8.2 Backward Induction

In the previous sub-sections, we solved for the equilibrium in the sequential Battle of the Sexes game by finding the Nash equilibria using the normal form and then look for a subgame perfect Nash equilibrium among them. Another method providing a somewhat direct way of solving for the subgame perfect Nash equilibrium in such a setting is the method of Backward Induction. In this method we start with the subgames at the bottom of the extensive form, and determine the Nash equilibrium of these subgames. These subgames are then replaced by their respective Nash equilibrium. This process of replacing a subgame with the associated Nash equilibrium is then continued up to the next level of subgames till we reach a subgame perfect Nash equilibrium. The process is illustrated below:

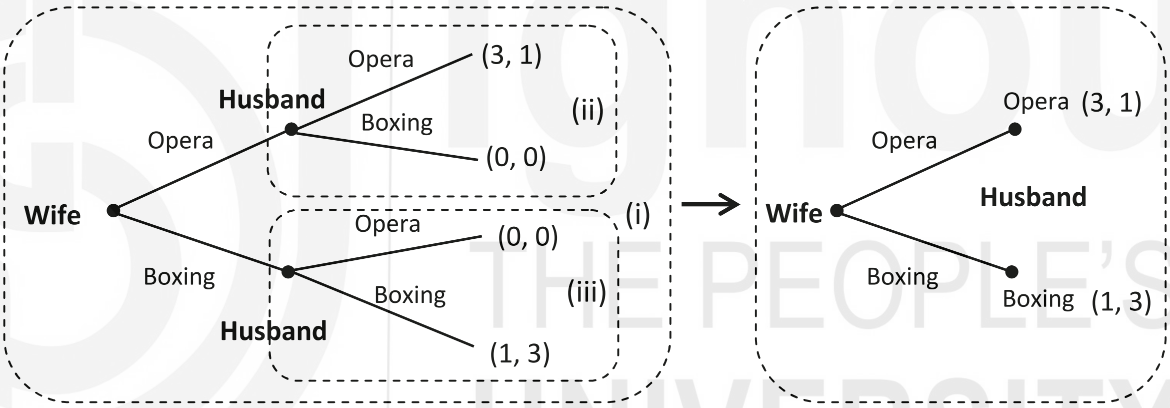

Fig. 6.6

In Fig. 6.6 the method of Backward Induction involves first solving the two subgames (ii and iii) for Nash equilibrium. In subgame (ii), given that wife opts for Opera, husband's best response would be to choose Opera for a payoff of 1 rather than going for Boxing match resulting in payoff of 0. Similarly, in subgame (iii), given that wife opts for Boxing, husband's best response will be to go for a Boxing match. Now, we replace the two subgames with their respective Nash equilibrium strategies to get a simple game where wife is to decide. Wife gets a payoff of 3 if she goes for Opera, while Boxing results in a payoff of 1. Nash equilibrium strategy yielding the higher payoff will be for her to go for Opera. So the wife's best response is to go for Opera. Thus, we get the subgame perfect Nash equilibrium outcome as (Opera|Opera).

6.9 APPLICATIONS OF THE GAME THEORY

We also came across important equilibrium concepts and a few examples of games in the previous sections. The purpose of this section is to give an overview of its application in order to capture models of conflict and cooperation in the field of Economics. Also this theory is widely applied in the field of, Biology, Sociology, Political Sciences, etc. to predict important trends. For instance, firms operating in an Oligopolistic market structure face nothing but a game theoretic situation where firms are interdependent on each other. The concept of Nash equilibrium which we came across in the present unit, can help us to define the Cournot equilibrium, described in Unit 6. The Cournot equilibrium has the property that each firm is choosing its profit-maximising optimal output, given the choice of the other firm. Similarly, we have the Bertrand equilibrium, as the Nash equilibrium in pricing strategies, which is the mutually best response price strategy. Each firm chooses the optimal price that maximises its profit, given the price that it thinks the other firm will set. We consider a few other examples below:

Example 1: Consider a game that discusses Prisoner's Dilemma in the context of firms attempting to make decisions about how they should operate without knowing about their competitor's actions. The simplest oligopoly model is by Cournot (1838), known as Cournot duopoly model, where there are two competing firms, say i=1,2 producing q_i amounts of output each. There is a single homogeneous good being produced by the two firms, with demand function p(Q), where p stands for price which is a function of total quantity $Q = q_1 + q_2$, produced by the firms. According to Cournot, each firm simultaneously takes output decision, without actually knowing the output choice of rival firm. We seek the non-cooperative equilibrium of the game, where firms cannot write binding contract and uses the output as their strategy variable.

Example 2: Consider two symmetric firms, A and B, having same constant average cost of Rs. 2 per unit. The firms face a total market demand of 100 and 180 units corresponding to two different prices i.e., either a high price of Rs. 10 or at a low price of Rs. 5 respectively. In both these cases, the market demand is splitted between the two firms. If one firm sets high price and other low price, the low priced firm sells 150 units and high priced firm sells only 20 units of the commodity. Now we calculate the profits of these firms and derive the equilibrium set of strategies using Prisoner's Dilemma game.

We have the following possible strategy profiles for {Firm A, Firm B} and their associated payoffs as (Firm A, Firm B):

a) {**High price, High price**}**:** In this case, total demand = 100, so each firm will sell 50. Total revenue of an individual firm = 50 × 10= 500 and Total cost = 50 × 2= 100. Thereby Profit= 500 − 100 = 400 for each firm.

b) {**low price, low price**}**:** Now, total demand = 180, so each firm will sell 90. Total revenue of an individual firm = 90 × 5 = 450 and Total cost = 90 × 2 = 180. Thereby Profit = 450 − 180 = 270 for each firm.

c) {**High price, low price**}**:** Suppose Firm A sells at high price, its total demand is 20 units, so Revenue of Firm A = 20 × 10 = 200 and Total cost = 20 × 2 = 40. Thereby Profit = 200 − 40 = 160 for firm A. Firm B sell at low price, its demand is 150, so Revenue of Firm B = 150 × 5 = 750 and Total cost = 150 × 2 = 300. Thereby Profit = 750 – 300 = 450 for firm B.

d) {**low price, high price**}**:** Contrary to above point, assume firm A sells at low price and B sells at high price. Then with similar calculation as above, Firm A profit is Rs. 450 and firm B profit is Rs. 160.

The payoff (profit) matrix therefore is

		Firm B's Profit	
		High Price	Low Price
Firm A's Profit	High Price	(400,400)	(160,450)
	Low Price	(450,160)	**(270, 270)**

Each player has a dominant strategy of charging **low price**. The equilibrium is therefore {low price, low price} yielding payoff (270, 270). It is a scenario similar to Prisoner's Dilemma game, where the equilibrium outcome is the one which gives lowest joint-payoffs (= 270 + 270 = 540). The strategy *{Low Price, Low Price}* is the Nash equilibrium as well as Dominant strategy Equilibrium. However the outcome *{high price, high price}* with a highest joint-payoffs (= 400 + 400 = 800) is a Pareto improvement over the Nash equilibrium outcome.

Why this paradoxical situation arises in equilibrium? From the Dominant strategy equilibrium perspective obviously none of the firm will choose the dominated strategy *{High Price}* over the dominant strategy. Hence the Dominant strategy equilibrium is *{Low Price, Low Price}.* Let us now analyse it from the perspective of Nash equilibrium. From the payoff matrix, it is evident that even though each of the firm has an incentive to choose High Price strategy because the strategy *{high price, high price}* offers highest joint profit, but each firm knows that if it chooses *(High Price),* the rival firm has the incentive to deviate and its best response is to choose *(Low Price)* and appropriate higher profit of Rs. 450 (450 > 400). Thus there is a mutual threat for deviation for *Low Price* against the choice of *High Price.* This credible threat actually compels the firms to choose the mutually best response strategy, even though it is a sub optimal outcome *{Low Price, Low Price}.*

Example 3: Now consider a non-price competition where two firms are deciding whether to spend on an expensive advertising campaign. If none of the firm advertise they will earn a normal profit of Rs. 50 million. If one firm advertise, it will earn more profit say Rs. 75 million due to comparative advantage, whereas the other firm which did not advertise will have to face loss and will earn only Rs. 25 million. If one firm advertises and the other does not, the firm that advertised will gain profit despite advertising cost, as advertising will help this firm's to capture larger market size through product popularity, over the other firm. If both firms advertise, they will be in same situation in the market but have to bear extra loss of advertisement cost and hence their profit will be Rs. 20 million each. So payoff matrix is given by:

		Firm B's strategy	
		Advertise	Do not Advertise
Firm A's strategy	Advertise	(20,20)	**(75,25)**
	Do not Advertise	**(25,75)**	(50,50)

In this game, neither of the two firms will have a dominant strategy, hence no dominant strategy equilibrium. But the game will be having two Nash equilibria, viz. {Do not advertise, Advertise} and {Advertise, Do not advertise}.

Example 4: Product Choice Problem

Following is a product choice problem faced by two firms, who have to decide whether to produce a salty or a sweet snack. If both produce the same variant, the market will have excess supply and both will end up making losses. The payoff matrix is as follows:

		Firm 2	
		Salty	Sweet
Firm 1	Salty	(−3,−3)	(10, 15)
	Sweet	(15, 10)	(−3,−3)

Clearly in a static (or simultaneous move) game there are two Nash equilibria in pure strategies (Sweet, Salty) and (Salty, Sweet). Let us consider the sequential move (extensive form) of the game (see Fig. 6.7 and 6.8)

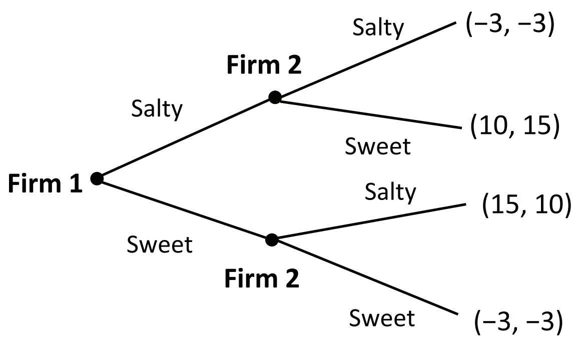

Fig. 6.7

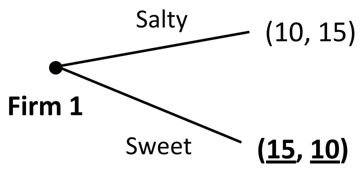

Fig. 6.8: (Reduced form of the Game)

Suppose firm 1 gets to move first, followed by firm 2. If it chooses the strategy ‘sweet’, firm 2 after observing firm 1’s choice, will choose ‘salty’. If firm 1 chooses the strategy ‘Salty’, firm 2 after observing firm 1’s choice will choose ‘sweet’. Notice, firm 2’s choices actually arise from firm 1’s choices, which the payoff matrix does not reveal. As it hides the fact that firm 2 gets to know choice of firm 1. Through the backward induction the Subgame perfect Nash equilibrium outcome of the game is (Sweet, Salty) and the Subgame perfect Nash equilibrium strategy is (Sweet, Salty) , (Salty Sweet), as shown in the reduced form of the game (Fig. 6.8)

Example 4: Business Game

Let there be a firm A which is trying to enter a business, either on a small scale or large scale. Firm B now has to decide whether to accommodate or start a price war. So the game essentially involves the two strategies of whether a firm enters business on a large scale or, small scale. Refer the following payoff (in millions of rupees) matrix for the game:

		Firm B	
		Accommodate	Price War
Firm A	Small	(2,10)	(0,8)
	Large	(4,5)	(1, 12)

As per the above payoff matrix, for firm A ‘large’ is the dominant strategy, given firm A choose this, firm B will respond by launching a ‘price war’. At this the Nash Equilibrium will be (Large, Price War) where firm A will have a payoff of 1 and firm B a payoff of 12. But firm A can do better if you turn this into a sequential-move game. Here firm A can commit to a scale of operation in advance. If firm A decide ‘large’, then firm B’s best response is to fight a ‘price war’, resulting in firm A’s payoff to be 1. But if firm A choose ‘small’ then firm B’s best response will be to accommodate yielding a payoff of 2 to firm A. Hence Sub game Perfect Nash Equilibrium strategy in this sequential move game is (Small, Accommodate, Large, Price War), where firm A enters ‘small’ and firm B ‘accommodates’ in the top information set, and engages in a price war in the bottom information set. Hence, equilibrium payoffs is (2,10).The Sub-game Perfect Nash Equilibrium outcome is (Small, Accommodate). This shows moving first can have strategic value.

The extensive form can be depicted as follows:

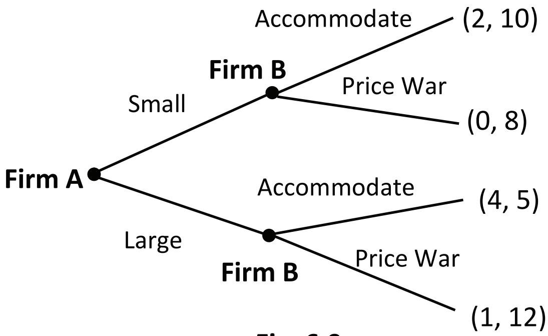

Fig. 6.9

Threats, Commitment and Credibility

Firms in oligopoly often adopt strategies to gain a competitive advantage over their rivals even if it means constraining their own behaviour. An Oligopolist must have a commitment, so that the threat it makes is credible. This can be explained by an example, let firm A produce cars and firm B produce car seats, where production decisions depend heavily on firm A.

		Firm A	
		Small Cars	Big Cars
Firm B	Small Car Seats	(2,6)	(2,0)
	Big Car seats	(1,1)	(10,3)

We have a sequential game, in which firm A is the first mover (leader), it can do best by making small cars and so firm B should make small car seats.

Can firm B threaten firm A to produce big car seats ? No, because if it makes big car seats and firm A makes small cars, firm B would earn a payoff of '1' and not '2'. Therefore this threat won't work out and it is called an empty threat. In other words, this threat is not a credible threat. However it can make its threat credible by shutting down small car seats factory. The new payoff matrix is now:

		Firm A	
		Small Cars	Big Cars
Firm B	Small Car Seats	(0,6)	(0,0)
	Big Car seats	(1,1)	(10,3)

Now firm A would be forced to produce big cars and firm B would produce big car seats and earn a payoff of '10'. But here one may question, what if firm A finds another producer to produce small car seats? Here, the role of reputation becomes very important.

Check Your Progress 2

1) Two firms are planning to enter into a market. Firm 1 is contemplating its capacity strategy, which can be "aggressive" or "accommodating". Firm 2 also has similar options. The payoff matrix is as follows:

		Firm 2	
		Aggressive	Accommodating
Firm 1	Aggressive	(7,2)	(10,3)
	Accommodating	(9,5)	(11,4)

a) If both firms decide their strategies simultaneously, what is the Nash Equilibrium in pure strategy ?

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b) Write the game in Extensive form and find the optimal strategy of firm 1, if it could move first. What would firm 2 do ?

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2) Two cereal manufacturers firm A and firm B are contemplating manufacturing a cereal made from either wheat or rice.

The payoff matrix is as follows:

		Firm 2	
		Wheat	Rice
Firm 1	Wheat	(10,12)	(6,8)
	Rice	(1,6)	(8,12)

a) If two firms choose simultaneously what is the Nash Equilibrium?

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b) If firm 1 chooses first, what is the optimal strategy of firm 2 ? Does firm 1 have any advantage in moving first ?

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6.10 LET US SUM UP

Game theory is a set of tools that economists use to analyse conflict and cooperation between firms or any other rational players or decision-making agents. Each firm adopts a strategy or plan of action to compete with other firms. The unit discusses various elements employed in representing a game. Every game theory model includes players, strategies and payoffs. The players are the decision-makers and the strategies are the potential choices. The payoff is the reward or profit or the outcome of each combination of strategies. The unit also discussed the two forms of representing a game, viz. the normal and the extensive form. It continued in explaining various basic concepts needed to solve a game theoretic problem. The dominant strategy is the optimal choice for a player, no matter what the strategy of the other player is. When both the players are having a dominant strategy, we reach a dominant strategy equilibrium. Nash Equilibrium occurs when player choose mutually best response strategy, given the strategy chosen by the other player. Examples of various games including, the Prisoners' Dilemma, the Battle of the Sexes, Matching Pennies, etc. were discussed in order to illustrate the concepts of equilibrium. The concept of the Mixed strategy Nash equilibrium arises when there is no Nash equilibrium in Pure strategies. From the simultaneous-move games, we moved next to a Sequential-move games where the time pattern of choices is important. The two additional concepts of that of a Sub-game Perfect Nash equilibrium and the method to reach such equilibrium known as the Backward induction method, were also covered with illustration. The unit concluded with a discussion on some application of the Game theory.

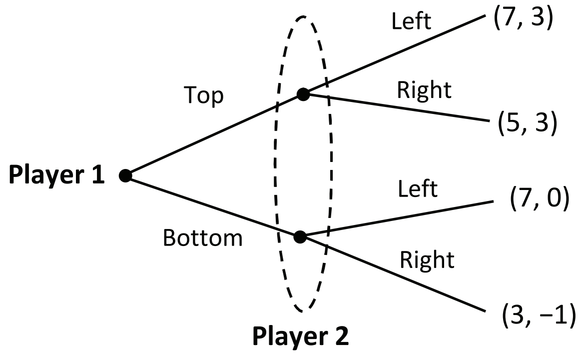
6.11 SOME USEFUL REFERENCES

- Varian, H. R. (2010). *Intermediate Microeconomics: A modern approach* (8th ed.). New York: W.W. Norton & Co.
- Gibbons, R. (1992). *Game Theory for Applied Economists.* Princeton, New Jersey: Princeton University Press.
- Nicholson & Snyder. *Intermediate Microeconomics and its Application* (11th ed).South-Western Cengage Learning.
- "Notes 6: Game theory". retrieved from http://www2.econ.iastate.edu/classes/econ335/lewin/games.pdf

6.12 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) a)



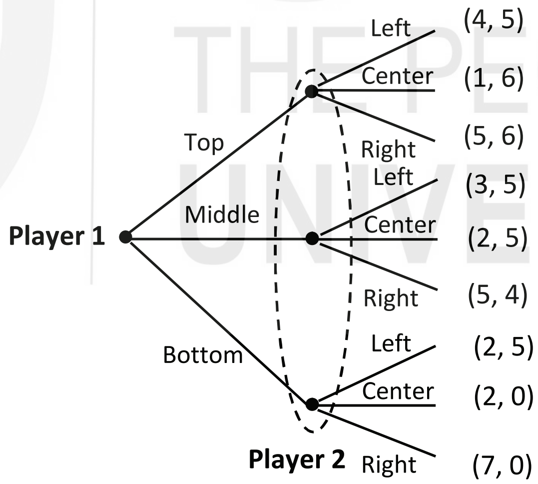
b) There are 3 Nash Equilibria

{Top, Left}, {Top, Right} and {Bottom, Left}

c) Dominant Strategy of player 1 is Top

Dominant Strategy of player 2 is Left

2) a)



b) Nash Equilibrium of the game is (Middle center)

3) a) Yes, if all players are using their dominant strategies then it would be the case that it is optimal given what the other players are doing and therefore a Nash Equilibrium. No, the converse is not true.

4) a) Firm A does not have a dominant strategy.

b) Firm B has a dominant strategy of low price.

c) Nash Equilibrium is {Low price, Low price}

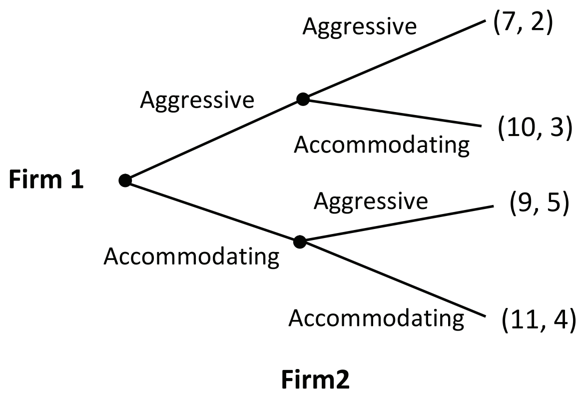
5) a) Nash Equilibrium is {Cheat, Cheat}

b) Nash Equilibrium would be {Don't Cheat, Cheat}

Check Your Progress 2

1) a) Nash Equilibrium would be {Accommodating, Aggressive}

   b) If firm 1 could move first it would opt to be 'Aggressive'followed by firm 2 to be choosing 'Accommodating'.



2) a) The two Nash Equilibrium are {wheat, wheat} and {Rice, Rice}.

   b) Firm 1 should choose wheat and firm 2 would also choose wheat, giving firm 1 a payoff of 10. Yes firm 1 has an advantage of moving first. If firm 2 were to move first, it would choose Rice,then firm 1 would also choose Rice giving firm 1 a payoff of only 8.

NOTES