



“ शिक्षा मानव को बन्धनों से मुक्त करती है और आज के युग में तो यह लोकतंत्र की भावना का आधार भी है। जन्म तथा अन्य कारणों से उत्पन्न जाति एवं वर्गगत विषमताओं को दूर करते हुए मनुष्य को इन सबसे ऊपर उठाती है।”

–इन्दिरा गांधी

VOLUME I

BLOCK 1: WAVES - AN INTRODUCTION

BLOCK 2: INTERFERENCE

“ Education is a liberating force, and in our age it is also a democratising force, cutting across the barriers of caste and class, smoothing out inequalities imposed by birth and other circumstances.”

– Indira Gandhi

Block

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INTERFERENCE

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Course Design Committee

Prof. A. K. Ghatak, *Retd.*
IIT Delhi, New Delhi

Prof. Suresh Garg, *Retd.*
School of Sciences
IGNOU, New Delhi
Vice Chancellor,
Usha Martin University

Prof. R.M. Mehra, *Retd.*
Dept. of Electronics,
South Campus,
University of Delhi, Delhi

Dr. Ashok Goyal, *Retd.*
Dept. of Physics, Hansraj College
University of Delhi, Delhi

Dr. Parthasarathy
Dept. of Physics,
Maharaja Agrasen College,
University of Delhi, Delhi

Prof. M.S. Nathawat
Former Director,
School of Sciences,
IGNOU, New Delhi

Prof. Vijayshri
School of Sciences
IGNOU, New Delhi

Prof. Sudip Ranjan Jha
School of Sciences
IGNOU, New Delhi

Prof. Shubha Gokhale
School of Sciences
IGNOU, New Delhi

Dr. Sanjay Gupta
School of Sciences
IGNOU, New Delhi

Dr. Subhalakshmi Lamba
School of Sciences
IGNOU, New Delhi

Block Preparation Team

Prof. Sanjay Gupta (Units 6-7)
School of Sciences
IGNOU, New Delhi

Prof. S.R. Jha (Unit 8)
School of Sciences
IGNOU, New Delhi

Units of this block are based on the Optics (PHE-09) course of the earlier B.Sc. programme of IGNOU.

Course Coordinators: Prof. S.R. Jha, Prof. Sanjay Gupta

Block Production Team

Mr. Rajiv Girdhar
Assistant Registrar
MPDD, IGNOU, New Delhi

Mr. Hemant Kumar Parida
Section Officer
MPDD, IGNOU, New Delhi

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BLOCK 2 : INTERFERENCE

In Block 1 of the course, you studied about waves and their characteristic features. You learnt the basics of wave motion by considering mechanical waves such as transverse waves on a string and sound waves. You also learnt how Maxwell's electromagnetic theory predicted that light is also a wave phenomenon. However, unlike waves on a string, light is an electromagnetic wave which comprises time-varying electric and magnetic fields. The wave nature of light provides us a basis for understanding many experimentally observed phenomena such as polarisation, interference and diffraction of light.

In this block, as the title suggests, you will study about interference. Interference of light refers to the phenomenon wherein two or more waves superpose and give rise to a fringe pattern comprising bright and dark fringes. The superposition of waves is the prerequisite for all interference effects. Another necessary requirement for observing interference effects is that the superposing light waves should be emanating from coherent sources. So, for observing interference fringes, light from a source is divided by some suitable apparatus into two coherent beams and then superposed; the intensity in the region of superposition vary from point to point between maxima, which exceed the sum of intensities of the superposing beams, and minima, which may be zero.

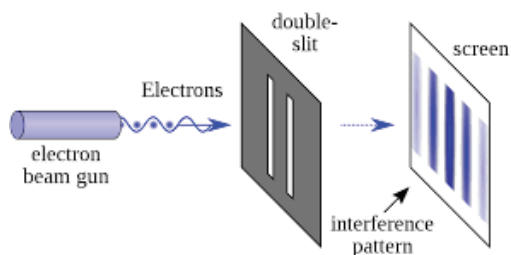
In general, there are two methods of obtaining coherent sources from a single beam of light. These methods provide a basis of classifying the arrangement/apparatus used for observing interference. In one method, a beam of light is divided by passage through apertures placed very close to each other side by side. The light coming out from these closely placed apertures are coherent and can produce interference pattern. This method is called division of wavefront. In another method, coherent sources are obtained from a single beam of light when it is reflected by one or more partially reflecting surfaces. This method is called division of amplitude.

In **Unit 6**, we have discussed in detail the phenomenon of interference produced due to coherent sources obtained by the method of division of wavefront. You will learn the Young's double slit experiment which is of historical significance because it provided a strong experimental evidence in support of the wave theory of light. You will also learn to derive expressions for the intensity of the bright and dark fringes, fringe width and displacement of fringes.

Unit 7 deals with the formation of interference fringe pattern due to coherent sources obtained by division of amplitude. You will also learn how reflection of light from thin films produce interference patterns. Such studies have many practical applications.

In the last unit (**Unit 8**) of the Block, you will learn about Michelson interferometer. In fact, the phenomenon of interference has given us a basis to design a variety of interferometers which are scientific instruments used for precise measurements. The discussion on Michelson interferometer is intended to familiarise you with the practical use of the interference of light.

We hope you enjoy studying this Block.



Set-up to analyse the interference pattern.

(Source: en.wikipedia.org)

INTERFERENCE BY DIVISION OF WAVEFRONT

Structure

- 6.1 Introduction
 - Expected Learning Outcomes
- 6.2 Review of Concepts of Simple Harmonic Motion
 - Relation between Phase Difference and Path Difference
 - Superposition of Two Waves of Same Frequency but having Constant Phase Difference
- 6.3 Division of Wavefront
 - Young's Double-Slit Experiment
 - White Light Fringes
 - Displacement of Fringes

- 6.4 Fresnel's Biprism
- 6.5 Lloyd's Mirror
- 6.6 Summary
- 6.7 Terminal Questions
- 6.8 Solutions and Answers

Appendix 6A: Determination of d between two Coherent Virtual Sources for Fresnel Biprism

STUDY GUIDE

In your, you have learnt basic concepts about wave nature of light. From Block 4 of the BPHCT-133 course entitled Electricity and Magnetism, you may recall that light is a transverse electromagnetic wave. You have also studied the principle of superposition of waves in Block 4 of BPHCT-131 (Mechanics) on oscillations. So, you know that superposition of sound waves helps us to understand the phenomenon of beats and stationary waves. However, superposition of light waves leads to interference by division of wavefront and division of amplitude. In this unit, you will learn about interference produced by division of wavefront. When light from two closely spaced holes/slits having constant phase difference superpose, we obtain beautiful interference fringes. You will learn to derive expressions for fringe width and intensity distribution of the fringe pattern. In these derivations, you will need a clear understanding of the basic concepts of trigonometry and calculus. We advise you to refresh your knowledge of +2 school mathematics. We have included many examples and interspersed SAQs in the text to help you grasp the concepts well. Although the answers to all these questions are given at the end of unit, we advise you to solve them yourself for better understanding.

"The science of today is the technology of tomorrow."

Edward Teller

6.1 INTRODUCTION

From +2 school physics classes, you may be familiar with some of the topics related to wave optics. In Block 1 of this course and Block 4 of the BPHCT-133 course entitled Electricity and Magnetism, you have learnt that light is a transverse electromagnetic wave. In this unit, you will learn that superposition of light waves produces interference.

The interference of light is as real an effect as interference of water or sound waves. You must have seen **bright colours of a thin film of oil spread out on water surface**. There are two reasons why the interference of light is not observed in daily life situations. One, light waves have very short wavelengths – the visible part of the spectrum extends from 400 nm (for violet light) to 700 nm (for red light). Two, every natural source of light emits light waves only as short trains of random pulses so that any interference that occurs is averaged out during the period of observation by the eye. For such reasons, we have to adopt special procedures to observe interference of light waves.

Like standing waves and beats, the interference is produced due to superposition of two or more individual waves having constant phase difference. Interference can be constructive or destructive, depending on the phase difference between interfering waves. In a spatial display, the interference is said to produce a pattern of fringes as in the double slit interference pattern. The same condition may lead to enhancement of one colour at the expense of the other, producing interference colours as in the case of oil slicks and soap film about which you will study in next unit.

In this unit, we consider the interference pattern produced by waves originating from two point sources. However, in case of light, we cannot observe interference between the waves from two independent sources (although the interference does take place). Therefore, we invariably derive the interfering waves from a single wave so that the constant phase difference is maintained between the interfering waves. This can be achieved in two ways: By allowing a beam of light to fall on two closely spaced holes, and the waves emanating from these holes interfere. This method is known as *division of wavefront* and will be discussed in detail in this unit. In the other method, known as *division of amplitude*, a beam is divided at two or more reflecting surfaces, and the reflected waves interfere. This is discussed in the next unit.

As the phenomenon of interference can be successfully explained by treating light as a wave motion, it is necessary to understand the fundamentals of wave motion. We understand that you have learnt the basic concepts associated with waves in your class +2 Physics as well as in the BPHCT-131 course entitled “Mechanics”.

It is important to mention here that study of wave motion is important because waves are present all around us and are responsible for our communication with the outside world—seeing in particular. We are able to enjoy life because light makes us to see. The red of the rising sun, blue of the sky and green of the emerald involve light. Experience shows that vision stimulates our thinking and enriches life. We become aware of the infiniteness of Mother Nature because we are able to see.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ explain the basic concepts of simple harmonic motion;
- ❖ describe the origin of the interference pattern produced by double slit;
- ❖ discuss the intensity distribution in interference pattern;
- ❖ express fringe-width in terms of wavelength of light;
- ❖ describe arrangements for producing interference by division of wavefront; and
- ❖ state the differences between Biprism and Lloyd's mirror fringes.

6.2 REVIEW OF CONCEPTS OF SIMPLE HARMONIC MOTION

You have already learnt about simple harmonic motion in the previous course BPHCT-131 entitled "Mechanics". You may recall that total energy (intensity) is proportional to square of amplitude, and the energy is proportional to square of frequency. If we denote the intensity associated with a light wave by I and the amplitude of wave by a , then $I \propto a^2$.

In Unit 1 of this course, you have learnt that a harmonic wave (Fig. 6.1) can be represented as

$$y(x, t) = a \sin \omega(t - x/v) \quad (6.1)$$

or $y(x, t) = a \sin(kx - \omega t) \quad (6.2)$

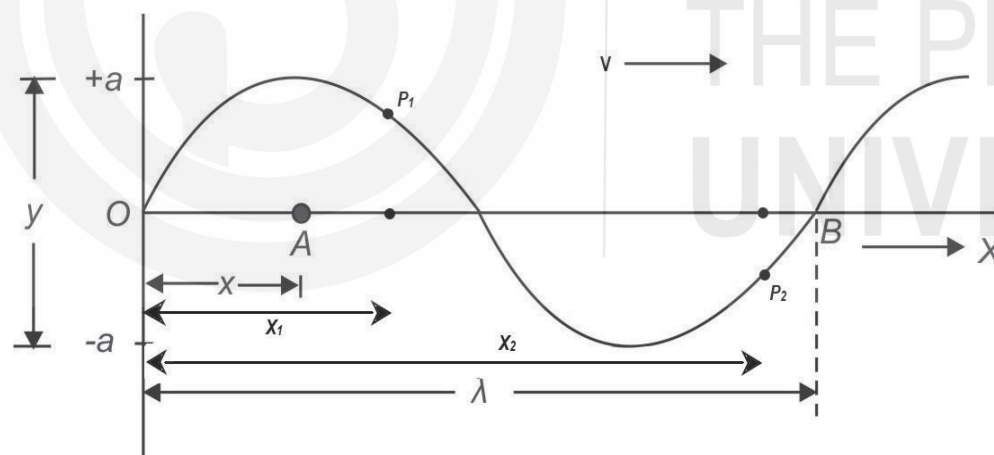


Fig. 6.1: A harmonic wave travelling in the positive x-direction.

Eq. (6.2) represents the displacement of a particle at a distance x from a fixed point at time t in a direction normal to the direction of propagation. This is, therefore, the equation of the wave. Such types of waves are called transverse waves. But you now know that light is a transverse wave, so we can say that Eq. (6.2) represents a light wave.

The expression
Phase difference = $\frac{2\pi}{\lambda} \times (\text{path difference})$
can be obtained in a less formal manner by remembering that a difference in phase of 2π corresponds to a path difference of one wavelength.

6.2.1 Relation between Phase Difference and Path Difference

To relate phase difference and path difference, we concentrate on two particles P_1 and P_2 on the wave described by Eq. (6.2) at distances x_1 and

x_2 from the origin, we can write the relation between phase difference and path difference as (read margin remark)

$$\text{Phase difference between } P_1 \text{ and } P_2 = \frac{2\pi}{\lambda}(x_2 - x_1)$$

or

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} \times (\text{path difference}) \quad (6.3)$$

$$\begin{aligned} y &= y_1 + y_2 \\ &= a_1 \sin \omega t + a_2 \sin(\omega t + \delta) \end{aligned}$$

where δ is a constant phase difference.

Using the trigonometry identity, we can write

$$\begin{aligned} &= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta \\ &= \sin \omega t (a_1 + a_2 \cos \delta) + \cos \omega t (a_2 \sin \delta) \dots (i) \end{aligned}$$

Let us write

$$a_1 + a_2 \cos \delta = A \cos \theta \dots (ii)$$

$$a_2 \sin \delta = A \sin \theta \dots (iii)$$

On substituting Eq. (ii) and (iii) in Eq. (i), we get,

$$y = \sin \omega t A \cos \theta + \cos \omega t A \sin \theta$$

or

$$y = y_1 + y_2 = A \sin(\omega t + \theta)$$

where A and θ are new constants. The constant A is given by

$$I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

This is the resultant intensity I , which is proportional to the square of the resultant amplitude A .

When two or more waves overlap in some region of space, interesting effects are observed. Let us consider a mechanical activity: Drop two stones simultaneously in a quiet pool. You will note that two sets of waves are created. What do you observe in the region where these waves cross? You will note that there are some places where the disturbance is almost zero, and there are places where the disturbance is greater than that due to either wave. These effects can be understood in terms of the principle of superposition, which states that when two or more waves can travel simultaneously, the resultant displacement of each particle of the medium is merely the vector sum of displacements due to each wave separately. You may like to know: Does superposition of waves influence the amplitude, frequency and such other characteristics of a wave? To discover answer to this question, let us consider superposition of two waves of the same frequency but having constant phase difference.

6.2.2 Superposition of Two Waves of Same Frequency but having Constant Phase Difference

Consider two sine waves of the same frequency but having constant phase difference, say δ moving in the same line. Therefore, for the displacements we can write

$$y_1 = a_1 \sin \omega t \quad (6.4a)$$

$$\text{and } y_2 = a_2 \sin(\omega t + \delta) \quad (6.4b)$$

where a_1 and a_2 are amplitudes of the motions, and ω is angular frequency of the two waves. By the principle of superposition, we can write the resultant displacement as (read margin remarks)

$$\begin{aligned} y &= y_1 + y_2 \\ y &= A \sin(\omega t + \theta) \end{aligned} \quad (6.5)$$

where A and θ are new constants. Note that the resultant displacement is simple harmonic and of amplitude A . The expression for resultant intensity I is obtained by noting that it is proportional to the square of the resultant amplitude. Hence

$$I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta \quad (6.6)$$

(Here we have chosen the constant of proportionality as unity for simplicity).

Note that the resultant intensity is not equal to the sum of the intensities due to separate waves i.e., $(a_1^2 + a_2^2)$. Since the intensity of a wave is proportional to square of amplitude, we can write $I_1 \propto a_1^2$ and $I_2 \propto a_2^2$, where we have

taken the proportionality constant as unity as before. In terms of intensities of individual waves, we can rewrite Eq. (6.6) as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta \quad (6.7)$$

Please go through Example 6.1 to check your understanding of concepts discussed so far.

EXAMPLE 6.1: INTENSITY OF TWO WAVES

Two coherent waves of same frequency and constant phase difference having intensities I and $4I$, respectively interfere. Calculate the resultant intensity when the phase difference between these two waves is $\pi/2$ and π ?

SOLUTION ■ From Eq. (6.7), we can write

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Given:

$I_1 = I$ and $I_2 = 4I$, so that expression for resultant intensity takes the form

$$\begin{aligned} I &= 5I + 2I\sqrt{4} \cos \delta \\ &= 5I + 4I \cos \delta \end{aligned}$$

Hence

$$I_{\pi/2} = 5I + 4I \cos 90^\circ = 5I$$

and

$$I_\pi = 5I + 4I \cos \pi = I$$

This result shows that when two waves interfere, the resultant intensity is seen to vary.

By again referring to Eq. (6.7), you will note that the intensity is maximum when phase difference is $\delta = 2n\pi$ (even multiple of π):

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

That is, the resultant intensity is greater than the sum of the separate intensities. For $I_1 = I_2$

$$I_{\max} = 4I_1$$

The intensity I is minimum when $\cos \delta = -1$, i.e., when δ is an odd multiple of π , i.e., $\delta = (2n + 1)\pi$. Eq. (6.7) shows that

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

The resultant intensity is thus less than the sum of two separate intensities. If $I_1 = I_2$, then $I_{\max} = 0$, which mean that there is no light. Such interference is known as destructive interference.

Before proceeding further, we would like you to answer a SAQ.

SAQ 1 - Intensity of two waves

Two waves of same frequency and constant phase difference have intensities in the ratio 81:1. They produce interference fringes. Deduce the ratio of the maximum to minimum intensity.

In general, for waves of same intensity and having a constant phase difference of δ , the resultant intensity is given by

$$\begin{aligned} I &= 2I_1 + 2I_1 \cos \delta \\ &= 4I_1 \cos^2 \frac{\delta}{2} \end{aligned} \quad (6.8)$$

Usually, when two light waves are made to interfere, we get alternate dark and bright bands of a regular or irregular shape. These are called interference fringes.

This result shows that when two waves of the same frequency travel in the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space. The non-uniform distribution of the light intensity due to the superposition of two waves is called **interference**. At some points the intensity is maximum and the interference at these points is called *constructive interference*. At points where the intensity is minimum, the interference is called *destructive interference*. To understand this further, answer SAQ 2.

SAQ 2 - Interference due to superposition of waves

Fig. 6.2 shows two waves emanating from two sources, A and B, arriving at point C and interferes. Which situation indicates constructive interference and which one corresponds to destructive interference? Justify your answer.

(Hint: (Eq. 6.3) will help you in answering this question.)

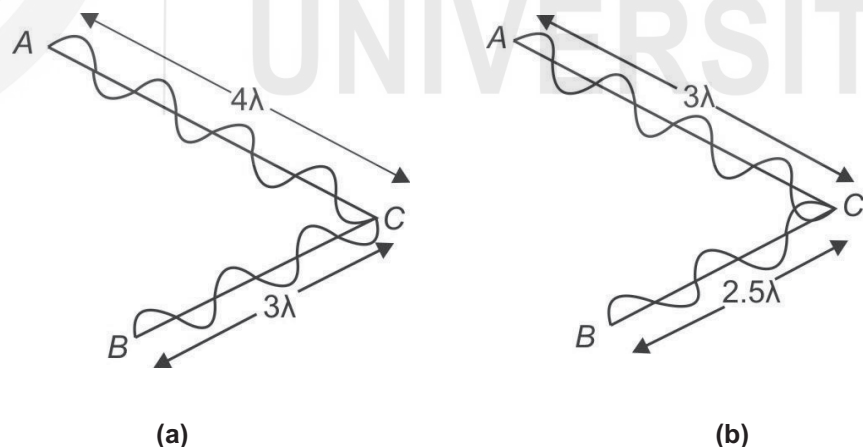


Fig. 6.2: Superposition of waves

What inference do you draw on solving SAQ 2. We note that for constructive interference

$$\text{path difference} = n\lambda, \quad \text{where } n = 0, 1, 2, 3, \dots, \quad (6.9)$$

and for destructive interference

$$\text{path difference} = n\frac{\lambda}{2}, \quad \text{where } n = 1, 3, 5, \dots \quad (6.10)$$

Let us sum up what you have learnt so far.

For production of stationary interference patterns, i.e. definite regions of constructive and destructive interference, the interfering waves must have (1) the same frequency, and (2) a constant phase difference (and they must be travelling in the same direction). When these conditions are satisfied, we say that the sources emitting waves are **coherent**.

Recap

Recall that mechanical sources of same frequency can be readily obtained using a vibrating tuning fork. However, the phase relationship between waves may vary with time. In the case of light, the waves are emitted by a very large number of independent atoms and each atom emits light for a very small time, of the order of a nano second. It means that such waves bear definite phase relationship with each other for less than one-billionth of a second implying that interference pattern will change every billionth of a second. However, due to limitations imposed by persistence of vision, we cannot take note of these changes and observe uniform intensity on the screen ; It means that light waves brought together from different light sources are coherent over very short periods of time and do not produce stationary interference pattern.

In the following section we will discuss the experiment of Young who observed interference phenomenon for the first time using division of wavefront.

6.3 DIVISION OF WAVEFRONT

6.3.1 Young's Double-slit Experiment

One of the earliest demonstrations of interference produced by division of wavefront was first done by Young in 1801, establishing the wave character of light. Young devised a simple experiment by dividing a single wavefront in two wavefronts, which behaved as if these resulted from two sources having a fixed phase relationship. Superposition of these waves led to a stationary interference pattern.

In the actual experiment, Young made sunlight to fall on a pinhole S_0 , punched in screen A , as shown in Fig. 6.3 The light coming from this pinhole was made to fall on another screen having two pinholes S_1 and S_2 , punched in the screen B , close to one another. These sources behaved as a pair of coherent sources and light reached there in the same phase. Do you know the reason? It is because the pair of sources in second screen was equidistant from the pinhole in the first screen. Superposition of waves from these pinholes during their propagation produced a symmetrical interference pattern on the observation screen C . It consisted of alternate dark and bright fringes. The interference produced by this method is said to have arisen by **division of wavefront**. Note that in this method, waves having constant phase difference are obtained from two closely spaced pinholes.

It may be mentioned here that this method is useful while working with the sufficiently small sources. In a physics laboratory, the pinholes are invariably substituted by narrow slits and light from a monochromatic source such as sodium is used.

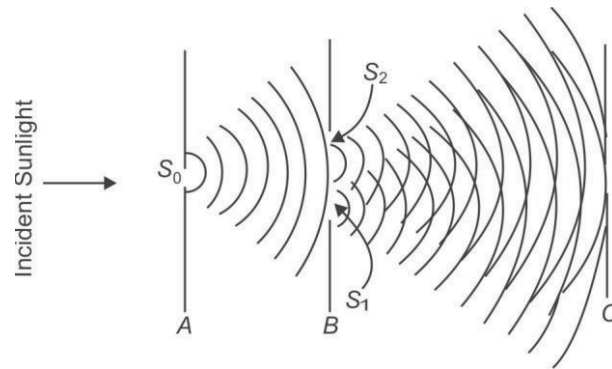


Fig. 6.3: Young's double slit experiment. The pinholes S_1 and S_2 act as coherent sources and an interference pattern is observed on the screen C .

Fig. 6.4 shows the sections of the wavefronts on the plane containing S_0 , S_1 and S_2 . Since the waves emanating from S_1 and S_2 are coherent, we will see alternate bright and dark fringes, called interference fringes.

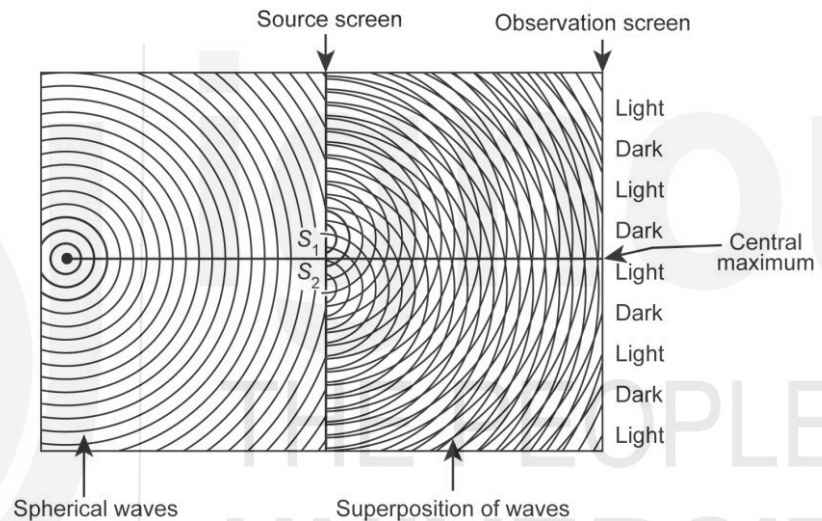


Fig. 6.4: Sections of the spherical wavefronts coming out of S_0 , S_1 and S_2 .

To analyse the interference pattern and investigate the spacing of the interference fringes, let us consider the geometry shown in Fig. 6.5. S is a narrow slit illuminated by monochromatic light, and S_1 and S_2 two parallel narrow slits very close to each other and equidistance from S . The spacing between these slits should be about 0.2 mm. (You can perform this experiment by using photographic plates also.) The light waves from S arrive at S_1 and S_2 in the same phase.

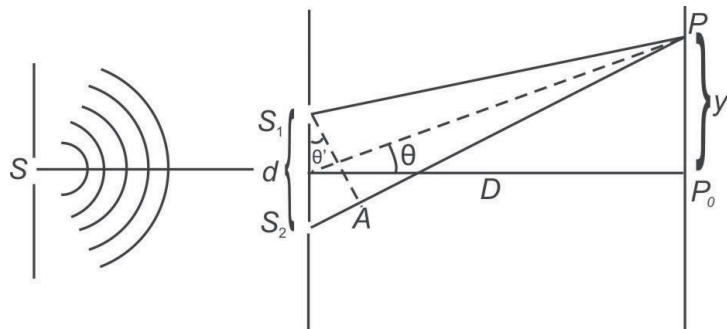


Fig. 6.5: The geometry of Young's experiment: The path difference of the light from the slits arriving at P on the screen is $d \sin \theta$.

It is assumed that the amplitudes of the emanating waves are the same, because S_1 and S_2 are the same size slits and very close to each other.

Hence these waves produce an interference pattern on a screen placed parallel to S_1 and S_2 .

To obtain an expression for the intensity at a point P on the screen, we join S_1P and S_2P . The waves arriving at P from S_1 and S_2 traverse different paths S_1P and S_2P . The path difference is $S_2P - S_1P$. Suppose

y = distance of P from P_0 , the central point on the screen

d = separation of two slits S_1 and S_2 .

D = distance of slits from the screen.

The path difference = S_2A , as shown in Fig. 6.5, since S_1A is normal drawn from S_1 . It helps to make S_1 and A equidistance from P . Since in Young's experiment $D \gg d$ or y , the angles θ and θ' are small and nearly the same. Since triangle S_1AS_2 is a right-angled triangle and

$S_2A = d \sin \theta' = d \sin \theta = d \tan \theta$, for small θ , $\sin \theta = \tan \theta$. From Fig. 6.5 we can write, $\tan \theta = y / D$.

$$\therefore S_2P - S_1P = S_2A = d \frac{y}{D} \quad (6.11)$$

To obtain the location of fringes from the central spot and fringe width, we recall that the intensity at the point P will be maximum or minimum depending on whether the path difference $S_2P - S_1P$ is an integral multiple of wavelength or an odd multiple of half wavelength (See Eq. (6.9) and Eq. (6.10)). Hence, for bright fringes (maxima), we can write

$$S_2P - S_1P = \frac{yd}{D} = 0, \lambda, 2\lambda, 3\lambda, \dots = n\lambda$$

where $n = 0, 1, 2, \dots$. That is, we will obtain constructive interference which will be seen in the form of bright fringes on the observation screen:

$$y = nD \lambda / d \quad (\text{bright fringes}) \quad (6.12)$$

The number n is called the order of the fringe. Thus the fringes with $n = 0, 1, 2, \dots$ etc. are called zeroth order, first order, second order, ... etc. The zeroth order fringe corresponds to the central maximum while the first order fringe ($n = 1$) corresponds to the first bright fringe on either side of the central maximum, and so on.

For dark fringes (minima),

$$S_2P - S_1P = \frac{yd}{D} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \dots = \left(n + \frac{1}{2}\right)\lambda$$

where $n = 0, 1, 2, \dots$. That is, we will obtain destructive interference, which manifests in the form of dark fringes:

$$y = \left(n + \frac{1}{2}\right) \frac{\lambda D}{d} \quad (\text{dark fringes}) \quad (6.13)$$

We can use either Eq. (6.12) or Eq. (6.13) to obtain an expression for the distance y_n of the n th order bright (or dark) fringe as well as fringe width. But before that we would like you to solve the following SAQ.

SAQ 3 - Young's experiment

Monochromatic light passes through two narrow slits 0.40 mm apart. The third-order bright fringe of the interference pattern, observed on a screen 1.0 m from the slits, is 3.6 mm from the centre of the central maximum. Calculate the wavelength of light.

Fringe width

If y_n and y_{n+1} denote the distance of n th and $(n + 1)$ th bright fringes, then from Eq. (6.12), we can write

$$y_n = \frac{D}{d} n\lambda$$

and
$$y_{n+1} = \frac{D}{d} (n + 1)\lambda$$

The spacing between the n th and $(n + 1)$ th bright fringes is given by

$$y_{n+1} - y_n = \frac{D}{d} (n + 1)\lambda - \frac{D}{d} n\lambda = D\lambda / d$$

Note that the distance between the n th and $(n + 1)$ th bright fringes is independent of n . It means that the spacing between any two consecutive bright fringes is the same. Similarly, by starting from Eq. (6.13), you can convince yourself that the spacing between two dark fringes is also equal to $\frac{D}{d}\lambda$. The distance between any two consecutive bright or dark fringes is known as the **fringe-width** and is usually denoted by β . Thus, we can write

$$\beta = \frac{D}{d}\lambda \quad (6.14)$$

This result shows that fringe-width varies

- i) directly with D and the wavelength of the light used, and
- ii) inversely with the distance d between the slits.

These results are confirmed by experiments.

It may be mention here that the fringes are so thin that to see them, we usually use a magnifier.

To make certain that you have understood the concept of fringe width, answer the following SAQs.

SAQ 4 - Fringe width

In a two-slit interference pattern performed with light of wavelength $\lambda = 6000 \text{ \AA}$, the zero order and tenth order maxima fall at 12.34 mm and 14.73 mm respectively from the central fringe. Calculate the fringe width.

SAQ 5 - Interference fringes

If in the SAQ 4, λ is changed to $5000 \text{ \AA}$, deduce the positions of the zero order and twentieth order fringes, other arrangements remaining the same.

So far we have discussed how interference pattern arises when two waves of same frequency travelling in the same direction are made to superimpose. We also obtained expression for fringe width. Now you may ask: What is the shape of fringes in an interference pattern? We seek answer to this question in the following paragraphs.

Shape of the Interference Fringes

Refer to Fig. 6.6 where S_1 and S_2 are two coherent sources. We can now say that at the point P , there will be maximum or minimum intensity according as

$$S_2P - S_1P = n\lambda$$

or
$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$$

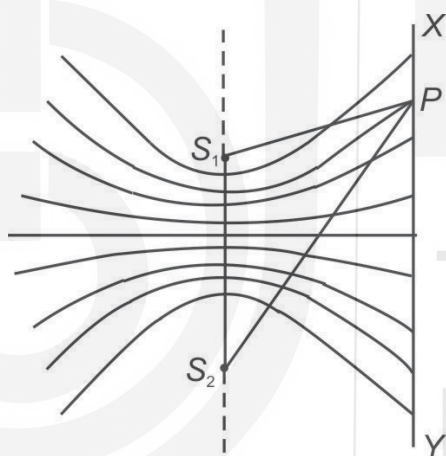


Fig. 6.6: Shape of the fringes.

From this we can say that for a given value of n , the locus of points of maximum or minimum intensity is given by

$$S_2P - S_1P = \text{constant},$$

Without going into details we just quote the result. The interference pattern will be a hyperboloid of revolution, obtained by revolving the hyperbola about the line S_1S_2 . In practice, fringes are observed on a screen in a plane normal to the plane of the figure and parallel to the line joining S_1S_2 . Hence the fringes that are observed are simply the sections of the hyperboloids by this plane, i.e. they are hyperbolae. Since the wavelength of light is extremely small (of the order of 10^{-5} cm), the value of $(S_2P - S_1P)$ is also of that order. Hence these hyperbolae fringes appear, more or less, as straight lines.

Intensity Distribution in the Fringe System

To know the intensity distribution in the fringe-system, we take $a_1 = a_2$, and then Eq. (6.6) takes the form

$$I = A^2 = 2a^2(1 + \cos \delta)$$

$$= 4a^2 \cos^2 \frac{\delta}{2}$$

From this result, we note that if the phase difference between the waves is such that $\delta = 0, 2\pi, 4\pi, \dots$, then we obtain $I = 4a^2$. That is, the intensity is 4 times the intensity of either beam. This corresponds to constructive interference. However, when $\delta = \pi, 3\pi, 5\pi, \dots$, the intensity is zero and we say that interference is destructive. That is, when the two light waves arrive at a point on the screen completely out of phase, they interfere destructively and the resultant intensity is zero.

In between these extremes, the intensity varies as $\cos^2 \delta/2$. This is shown in Fig. 6.7 where we have plotted intensity versus the phase difference.

You may now ask: What happens to the energy of two waves. The law of conservation of energy tells us that energy cannot be destroyed; it can only redistribute; energy seems to disappear at the minima but reappear at maxima. There is no violation of the law of conservation of energy in the interference phenomenon.

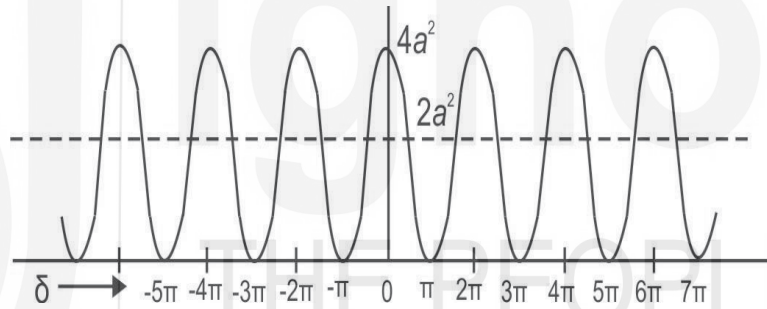


Fig. 6.7: Intensity distribution of interference fringes when two waves of the same frequency and amplitude superpose.

The average intensity on the screen is exactly what would exist in the absence of interference. Thus, as shown in Fig. 6.7, the intensity of the interference pattern varies between $4a^2$ and zero. Acting separately, each beam would contribute a^2 to intensity when there is no interference and we would have a uniform intensity of $2a^2$, as indicated by the broken line. The average intensity on the screen when phase difference between the waves is π is given by

$$I_{\text{average}} = \frac{\int_0^\pi I d\delta}{\int_0^\pi d\delta}$$

To evaluate the integral in the numerator, we substitute for I and get

$$I_{\text{average}} = \frac{\int_0^\pi \left(4a^2 \cos^2 \frac{\delta}{2} \right) d\delta}{\int_0^\pi d\delta}$$

$$\begin{aligned}
 &= \frac{\int_0^\pi (2a^2 + 2a^2 \cos \delta) d\delta}{\int_0^\pi d\delta} \quad \left(\because 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right) \\
 &= \frac{[2a^2 \delta + 2a^2 \sin \delta]_0^\pi}{[\delta]_0^\pi} \\
 &= \frac{2a^2 \pi}{\pi} \\
 &= 2a^2.
 \end{aligned}$$

This result shows that the average intensity is equal to the sum of the separate intensities.

In our discussion so far, we have considered interference pattern produced when a monochromatic light from a narrow slit falls on two parallel slits. You may now ask a logical question: What happens if white light is used to illuminate the slits? You will get answer to this question in the following sub-section.

6.3.2 White Light Fringes

If white light is used to illuminate the slits we obtain an interference pattern consisting of a central 'white' fringe, having on both sides a few coloured fringes and then a general illumination.

A pair of white light coherent sources is equivalent to a number of pairs of monochromatic sources. Each monochromatic pair produces its own system of fringes with a different fringe-width β , since β depends on λ $\left(\beta = \frac{D\lambda}{d} \right)$.

At the centre of the pattern, the path difference between the interfering waves is zero. This is true for all wavelengths of the white light. These combine to produce a white fringe at the centre. This is said to be the 'zero order fringe'.

As we move on either side of the centre, the path difference gradually increases from zero. At a point where it becomes equal to half the wavelength of the wave having the smallest wavelength, i.e., violet, we obtain the first dark fringe of violet. Beyond this, we obtain the first minimum of blue, green, yellow and of red in that order. It may be mentioned here that the inner edge of the first dark fringe, which is the first minimum for violet, receives sufficient intensity due to red, hence it is reddish. The outer edge of the first dark fringe, which is minimum for red, receives sufficient intensity due to violet, and is therefore, violet. The same applies to every other dark fringe. Hence, we obtain a few coloured fringes on both sides of the central fringe.

As we move further away from the centre, the path difference becomes quite large. Then, from the range 7000 – 4000 Å, a large number of wavelengths (colours) will produce maximum intensity at a given point, and an equally large number will produce minimum intensity at that point. For example, at any point P , we may have

For maxima,
path difference = $n\lambda$,
where $n = 0, 1, 2, \dots$

For minima,
path difference

$$= \left(n + \frac{1}{2} \right) \lambda,$$

where $n = 0, 1, 2, \dots$

$$\text{path difference} \begin{cases} = 11\lambda_1 = 12\lambda_2 = 13\lambda_3 = \dots \text{ etc. (maxima)} \\ = \left(11 + \frac{1}{2}\right)\lambda'_1 = \left(12 + \frac{1}{2}\right)\lambda'_2 = \left(13 + \frac{1}{2}\right)\lambda'_3 = \dots \text{ etc. (minima)} \end{cases}$$

Thus, at P , we shall have 11^{th} , 12^{th} , 13^{th} , etc., bright fringes of $\lambda_1, \lambda_2, \lambda_3, \dots$ etc., and 11^{th} , 12^{th} , 13^{th} , ... etc. dark fringes of $\lambda'_1, \lambda'_2, \lambda'_3, \dots$ etc.

Hence, the resultant colour at P is very nearly white. This happens at all points, for which the path difference is large. Hence, in the region of large path difference, uniform white illumination is obtained.

To ensure that you have gained grasp over the concepts discussed in this section, we would advise you to solve the following SAQ.

SAQ 6 - White light fringes

Suppose the path difference $S_1P - S_2P = 30 \times 10^{-5} \text{ cm}$. For which values of wavelength the point P will be a maximum?

6.3.3 Displacement of Fringes

We will now discuss the change in the interference pattern produced when a thin transparent plate, say of glass or mica, is introduced in the path of one of the two interfering waves, as shown in Fig. 6.8. It is observed that the entire fringe-pattern is displaced towards the path of the wave in which the plate is introduced.

Suppose that a thin transparent plate of thickness t and refractive index μ is introduced in the path of one of the constituent interfering waves (say in the path of S_1P , shown in Fig. 6.8). It means that before reaching the observation screen, light from S_1 travels partly in air and partly in the plate. It means that for the light travelling along S_1P , the distance travelled in air is $(S_1P - t)$, and that in the plate is t . If c and v be the velocities of light in the air and in the

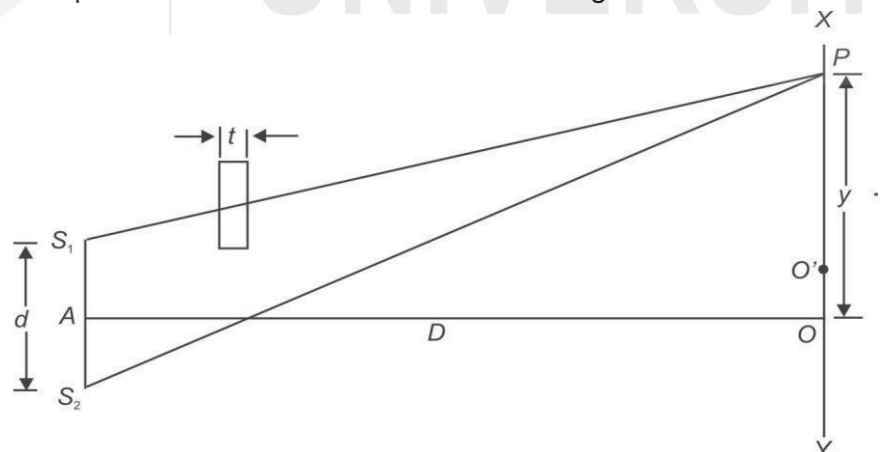


Fig. 6.8: Shift in the fringe pattern when a thin transparent sheet (of thickness t) is introduced in the path of one of the waves.

glass plate, respectively the time taken by light to reach P from S_1 is T , then we can write

$$T = \frac{S_1P - t}{c} + \frac{t}{v}$$

By expressing v in terms of c , we can express T in terms of the ratio of path travelled in air and speed of light in air:

$$T = \frac{S_1P - t}{c} + \frac{\mu t}{c} \quad \left(\because v = \frac{c}{\mu} \right)$$

$$= \frac{S_1P + (\mu - 1)t}{c}$$

That is, the effective path in air from S_1 to P is $[S_1P + (\mu - 1)t]$, i.e., the air path S_1P increases by $(\mu - 1)t$, due to the introduction of the plate of glass of refractive index, μ .

Suppose O is the position of the central bright fringe in the absence of the plate. Then the optical paths S_1O and S_2O will be equal. On introducing the plate, the two optical paths become unequal. Therefore, the central fringe is shifted towards O' , such that at O' the two optical paths become equal. A similar argument applies to all the fringes. Now, at any point P , the effective path difference between the waves reaching the point of observation is given by $S_2P - [S_1P + (\mu - 1)t] = (S_2P - S_1P) - (\mu - 1)t$.

From Eq. (6.11) we recall that $S_2P - S_1P = \frac{d}{D} y$

$\therefore$ Effective path difference between the waves reaching the point of observation at $P = \frac{d}{D} y - (\mu - 1)t$

If the point P is to be the centre of the n th bright fringe, the effective path difference should be equal to $n\lambda$ i.e.,

$$\frac{d}{D} y_n - (\mu - 1)t = n\lambda$$

or

$$y_n = \frac{D}{d} [n\lambda + (\mu - 1)t] \quad (6.15)$$

From this result, we note that in the absence of the plate ($t = 0$), the distance of the n th bright fringe from O is given by $\frac{D}{d} n\lambda$.

$\therefore$ Displacement y_0 of the n th bright fringe is given by

$$y_0 = \frac{D}{d} [n\lambda + (\mu - 1)t] - \frac{D}{d} n\lambda$$

$$y_0 = \frac{D}{d} (\mu - 1)t \quad (6.16)$$

Note that the shift is independent of the order of the fringe. It means that the fringe shift is the same for all the bright fringes. Similarly, it can be shown that the displacement of any dark fringe is also given by Eq. (6.16). It means that the entire fringe-system is displaced through a distance $\frac{D}{d} (\mu - 1)t$ towards the side in which the plate is placed. However, the fringe width is not affected with introduction of the plate. To show this we recall that fringe width is given by:

$$\beta = y_{n+1} - y_0$$

$$= \frac{D}{d} [(n+1)\lambda + (\mu-1)t] - \frac{D}{d} [n\lambda + (\mu-1)t] \quad (\text{see Eq. (6.15)})$$

$$= \frac{D\lambda}{d}.$$

Eq. (6.16) shows that we can determine the thickness of extremely thin transparent sheets (like that of mica) by measuring the shift of the fringe system.

Now, we would like you to apply this yourself to answer SAQ 7.

SAQ 7 - Displacement of fringes

In a double slit interference arrangement one of the slits is covered by a thin mica sheet whose refractive index is 1.58. The distances S_1S_2 and AO (see Fig. 6.8) are 0.1 cm and 50 cm, respectively. Due to the introduction of the mica sheet, the central fringe gets shifted by 0.2 cm. Determine the thickness of the mica sheet.

6.4 FRESNEL'S BIPRISM

In the initial years of Young's double slit experiment, an objection was raised about the authenticity of the interference pattern. It was argued that the bright fringes observed by Young could be due to some complicated modification of the light by the edges of the slits rather than due to interference. However, soon after, Fresnel devised a series of arrangements to produce the interference of light which was free from this criticism. The schematic of one of the experimental arrangements, known as Fresnel's Biprism arrangement, is shown in Fig. 6.9. This work put wave theory on very sound footing.

S is a narrow vertical slit illuminated by monochromatic light. The light from S is allowed to fall symmetrically on the Biprism P , placed at a small distance from S and having its refracting edges parallel to the slit. The light emerging from the upper and lower halves of the prism appears to start from two virtual sources, S_1 and S_2 , which act as coherent sources. The cones of light bS_1e and aS_2c , diverging from S_1 and S_2 , when superposed form interference fringes.

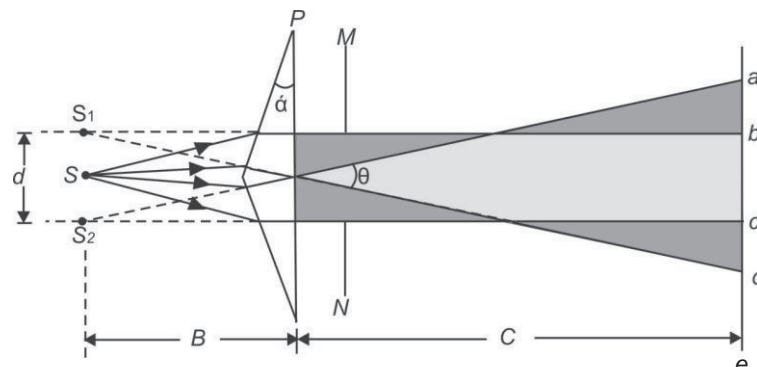


Fig. 6.9: Interference produced by Fresnel's Biprism.

If screens M and N are placed, as shown in Fig. 6.9, interference fringes are observed only in the region bc . When the screen ae is replaced by a photographic plate, equally spaced interference fringes are obtained as shown in the Fig. (6.10).



Fig. 6.10: Interference fringes produced in the Fresnel biprism experiment.

With such an experiment, Fresnel was able to show the interference effect without the diffracted beams through the two slits. Just like Young's double slit experiment, this arrangement could also be used to determine the wavelength of monochromatic light by viewing interference fringes through the eye-piece. The fringe-width β can be determined by means of micrometer attached to the eye piece. If D is the distance between source and screen, and d the distance between the virtual images S_1 and S_2 , the wavelength is given by

$$\lambda = \frac{\beta d}{D} \quad (6.17)$$

The distances d and D can easily be determined by placing a convex lens between the Biprism and the eyepiece. For a fixed position of the eyepiece, there will be two positions of the lens, shown as L_1 and L_2 in Fig. 6.11 where the images of S_1 and S_2 can be seen at the eyepiece. Let d_1 be the distance

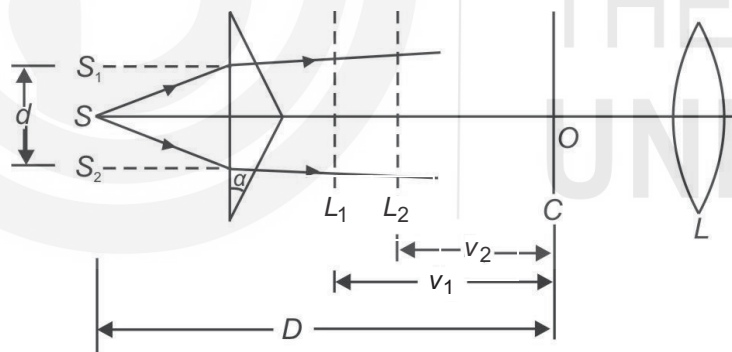


Fig. 6.11: Fresnel's biprism arrangement. C and L represents the position of cross wires and the eyepiece respectively. In order to determine d a lens is introduced between the biprism and cross wires. L_1 and L_2 represent the two positions of the lens where the slits are clearly seen.

between the two images, when the lens is at the position L_1 (at a distance v_1 from the eyepiece). Let d_2 and v_2 be the corresponding distances, when the lens is at L_2 . Then it can easily be shown that

$$d = \sqrt{d_1 d_2} \quad (6.18a)$$

$$\text{and } D = v_1 + v_2 \quad (6.18b)$$

For derivation of Eq. (6.18a), you may refer Appendix 6A.

Before proceeding further, you should solve the following SAQ using Eqs. (6.17) and (6.18).

SAQ 8 - Fresnel's biprism

In a Fresnel's Biprism experiment, the eyepiece is at a distance of 100 cm from the slit. A convex lens inserted between the Biprism and the eyepiece gives two images of the slit in two positions. In one case, the two images of the slit are 4.05 mm apart, and in the other case 2.10 mm apart. If sodium light of wavelength $5893 \text{ \AA}$ is used, calculate the thickness of the fringes.

In the preceding section, we have discussed how Fresnel's biprism can be used to study interference phenomenon. Two other devices-- Fresnel's two mirror arrangement and Lloyd's single mirror—can also be used to observe interference fringes. You will learn about Lloyd's mirror in the following section.

6.5 LLOYD'S MIRROR

Two interfering beams can be brought together in several other ways to produce interference.

Lloyd's mirror arrangement helps us to obtain two coherent sources of light to produce a stationary interference pattern. It consists of a plane mirror MN (Fig. 6.12) polished on the front surface and blackened at the back (to avoid multiple reflections). S_1 is a narrow slit, illuminated by monochromatic light, and placed with its length parallel to the surface of the mirror. Light from S_1 falls on the mirror at nearly grazing incidence, and the reflected beam appears to diverge from S_2 , which is virtual image of S_1 . Thus S_1 and S_2 act as coherent sources. The direct cone of light AS_1E and the reflected cone of light BS_2C when superposed lead to formation of interference fringes in the overlapping region BC on the screen.

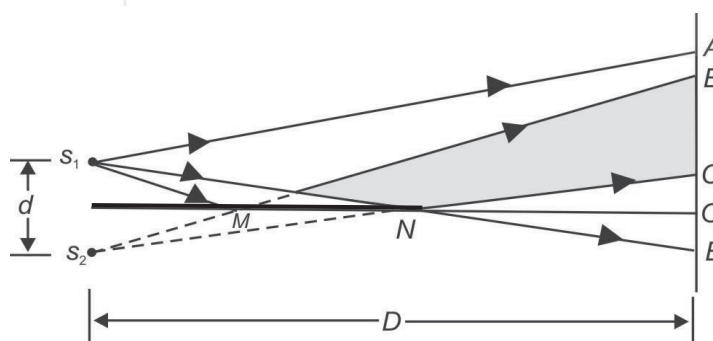


Fig. 6.12: Production of interference fringes in Lloyd's mirror arrangement.

Zero-order Fringe

The central zero-order fringe, which is expected to lie at O (the perpendicular bisector of S_1S_2) is not usually seen since only the direct light, and not the reflected light, reaches O . (It can be seen by introducing a thin sheet of mica in the path of light from S_1 , when the entire fringe system is displaced in the upward direction.)

It may be mentioned here that expressions for fringe width etc. are same in this case as for Young's double slit experiment.

SAQ 9 - Lloyd's mirror

Interference bands are obtained with a Lloyd's mirror with light of wavelength $5.45 \times 10^{-5} \text{ cm}$. A thin plate of glass of refractive index 1.5 is then placed normally in the path of one of the interfering beams. The central dark band is found to move into the position previously occupied by the third dark band from the centre. Calculate the thickness of the glass plate.

With white light the central fringe is expected to be white, but actually it is found to be 'dark'. This is because the light suffers a phase change of π or a path-difference of $\frac{\lambda}{2}$ on reflection by a plane mirror. Therefore, the path difference between the interfering waves at the position of zero-order fringe becomes $\frac{\lambda}{2}$ (instead of zero), which is a condition for a minimum. that is why the central fringe is dark.

Difference between Biprism and Lloyd's Mirror Fringes

The main differences between the biprism and Lloyd's mirror fringes are as given below:

1. In biprism, the complete pattern of fringes is obtained. In Lloyd's mirror, ordinarily, only a few fringes on one side of the central fringe are visible.
2. In biprism the central fringe is bright, while in Lloyd's mirror it is dark.
3. The central fringe in biprism is less sharp than that in Lloyd's mirror.

6.6 SUMMARY

Concept	Description
Simple harmonic motion	■ The relation between phase difference and path difference is: $\text{phase difference} = \frac{2\pi}{\lambda} (\text{path difference})$
Principle of superposition	■ If two waves of same frequency and of amplitudes a_1 and a_2 and phase difference δ are superposed then, according to principle of superposition, the amplitude A of the resultant wave is given by $A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$
Coherent sources	■ Two sources are said to be coherent if they emit light waves with no or constant phase difference.

Interference

- When two waves of the same frequency travel in approximately the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space. This non-uniform distribution of the light intensity is due to the phenomenon of interference.

- For constructive interference

$$\text{path difference} = n\lambda, \text{ where } n = 0, 1, 2, \dots$$

and for destructive interference

$$\text{path difference} = n\frac{\lambda}{2}, \text{ where } n = 1, 3, 5, 7$$

Fringe width in interference

- In an interference pattern, the distance between any two consecutive maxima or minima is given by

$$\beta = \frac{D\lambda}{d}$$

where β is called the fringe-width, λ is the wavelength of light used, d is the distance between the two coherent sources, and D is the distance between the sources and the screen.

Displacement of fringes

- When a thin transparent plate of thickness t and refractive index μ is introduced in the path of one of the constituent interfering beams of light, the entire fringe system is displaced through a distance $\frac{D}{d}(\mu - 1)t$.

Fresnel biprism

- Just as in Young's double slit experiment, the wavelength of light can be determined from measurement of fringe-width produced by the biprism by the following relation:

$$\lambda = \frac{\beta d}{D}$$

where $d = \sqrt{d_1 d_2}$ and $D = v_1 + v_2$

d_1 is the distance between the two images, when the lens is at the position L_1 at a distance v_1 from the eyepiece. d_2 and v_2 are the corresponding distances when the lens is at L_2 .

Lloyd's mirror

- The other device used for producing coherent sources is a Lloyd's mirror.
- A system of white and dark fringes, without any colours, obtained by white light are known as achromatic fringes.
- Lloyd's mirror produces achromatic fringes.

6.7 TERMINAL QUESTIONS

1. Young's experiment is performed with light of the green mercury line. If the fringes are measured with a micrometer eye-piece 80 cm behind the double slit, it is found that 20 of them occupy a distance of 10.92 mm. Find the distance between two slits. Given that the wavelength of green mercury line is $5460 \text{ \AA}$.

2. In a certain Young's experiment, the slits are 0.2 mm apart. An interference pattern is observed on a screen 0.5 m away. The wavelength of light is $5000 \text{ \AA}$. Calculate the distance between the central maxima and the third minima on the screen.
3. A Lloyd's mirror of length 5 cm, is illuminated with monochromatic light ($\lambda = 5460 \text{ \AA}$) from a narrow slit 0.1 cm from its plane, and 5 cm, measured in that plane, from its near edge. Find the separation of the fringes at a distance of 120 cm from the slit, and the total width of the pattern observed.
4. In a Young's double-slit experiment, a monochromatic light of wavelength 600 nm is used. One of the slits is covered by a transparent sheet of thickness $1.7 \times 10^{-5} \text{ m}$ of refractive index 1.6. Determine the number of fringes that will shift due to introduction of the sheet.
5. A thin glass sheet of refractive index (μ) 1.5 is introduced normally in the path of one of the two interfering waves. The central bright fringe is observed to shift to the position originally occupied by the eight bright fringes. The source of light is monochromatic and emits light of wavelength (λ) 5890 Å. Determine the thickness of the glass sheet.

6.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. We have

$$\frac{I_{\max}}{I_{\min}} = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}}{I_1 + I_2 - 2\sqrt{I_1 I_2}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

It is given that,

$$\frac{I_1}{I_2} = \frac{81}{1} \quad \text{or} \quad \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{9}{1}$$

$$\text{or} \quad \sqrt{I_1} = 9\sqrt{I_2}$$

Hence

$$\begin{aligned} \frac{I_{\max}}{I_{\min}} &= \frac{(9\sqrt{I_2} + \sqrt{I_2})^2}{(9\sqrt{I_2} - \sqrt{I_2})^2} \\ &= \frac{(10)^2 I_2}{(8)^2 I_2} = \frac{100}{64} = \frac{25}{16} \end{aligned}$$

2. The phase difference is related to the path difference by Eq. (6.3) as follows:

$$\begin{aligned} \text{phase difference} &= \frac{2\pi}{\lambda}(AC - BC) \\ &= \frac{2\pi}{\lambda}(4 - 3)\lambda = 2\pi \end{aligned}$$

This is the condition of maximum intensity. So, the waves interfere constructively in Fig. 6.2(a).

In case of Fig. 6.2(b)

$$\begin{aligned}\text{phase difference} &= \frac{2\pi}{\lambda}(AC - BC) \\ &= \frac{2\pi}{\lambda}(3 - 2.5)\lambda = \pi\end{aligned}$$

This is the condition of minimum intensity. Here the waves are completely out of phase and destructive interference occurs.

3. Given : $d = 0.40 \text{ mm}$, $D = 10^3 \text{ mm}$, $y = 3.6 \text{ mm}$, and $m = 3$. Using Eq. (6.12), we get

$$\lambda = \frac{yd}{nD} = \frac{(3.6)(0.40)}{3 \times 10^3} = 4.8 \times 10^{-4} \text{ mm} = 4.8 \times 10^{-5} \text{ cm}$$

Hence, the light is in the blue-green region of the visible spectrum.

4. With $\lambda = 6000 \text{ \AA}$, the distance between zero-order and tenth order fringe is $14.73 \text{ mm} - 12.34 \text{ mm} = 2.39 \text{ mm}$, so that the fringe width is $2.39 \text{ mm} / 10 = 0.239 \text{ mm}$.

5. $\beta = \frac{D\lambda}{d}$. Therefore

$$\frac{(\beta)_{6000}}{(\beta)_{5000}} = \frac{6000 \text{ \AA}}{5000 \text{ \AA}} = \frac{6}{5}$$

$$\therefore (\beta)_{5000} = \frac{5}{6} \times (\beta)_{6000} = \frac{5}{6} \times 0.239 = 0.199 \text{ mm}$$

Thus, with $\lambda = 5000 \text{ \AA}$, the zero-order fringe will still be at 12.34 mm , while the twentieth order fringe will be at

$$12.34 \text{ mm} + (0.199 \text{ mm} \times 20) = 16.32 \text{ mm}$$

6. For maxima, the path difference $= n\lambda$

$$\text{or } 30 \times 10^{-5} \text{ cm} = n\lambda$$

$$\therefore \lambda = \frac{30 \times 10^{-5}}{n} \text{ cm}$$

where $n = 1, 2, 3, 4, \dots$

7. $y_0 = 0.2 \text{ cm}$, $d = 0.1 \text{ cm}$, $D = 50 \text{ cm}$

Hence

$$t = \frac{dy_0}{D(\mu - 1)} = \frac{0.1 \times 0.2}{50 \times 0.58} = 6.7 \times 10^{-4} \text{ cm}$$

8. The fringe-width is given by

$$\beta = \frac{D\lambda}{d}, \text{ where } d = \sqrt{d_1 \times d_2}$$

Here $d_1 = 4.05 \text{ mm} = 0.405 \text{ cm}$ and $d_2 = 2.10 \text{ mm} = 0.210 \text{ cm}$.

$$\therefore d = \sqrt{0.405 \times 0.210} = 0.292 \text{ cm}$$

Also $D = 100 \text{ cm}$ and $\lambda = 5893 \text{ \AA} = 5893 \times 10^{-8} \text{ cm}$.

$$\therefore \beta = \frac{100 \times 5893 \times 10^{-8}}{0.292} = 0.0202 \text{ cm}$$

9. By introducing a glass plate of thickness t in one of the interfering beams, $t \text{ cm}$ of air ($\mu = 1$) are replaced by $t \text{ cm}$ of glass ($\mu = 1.5$). $t \text{ cm}$ of glass are optically equivalent to μt or $1.5 t \text{ cm}$ of air. The increase in the length of the path $= \mu t - t = 0.5 t$. This produces a shift of 2 in the interference bands

$$\therefore 0.5 t = 2\lambda = 2 \times 5.45 \times 10^{-5}$$

$$\text{and } t = \frac{2 \times 5.45 \times 10^{-5}}{0.5} = 21.8 \times 10^{-5} \text{ cm.}$$

Terminal Questions

1. The fringe width β in Young's experiment is $\beta = \lambda D/d$.

Since 20 fringes occupy a distance of 10.92 mm, the fringe width β is

$$\beta = (10.92/20) \text{ mm} = (10.92 \times 10^{-3}/20) \text{ m}$$

Also $D = 80 \text{ cm} = 0.8 \text{ m}$, and $\lambda = 5.460 \times 10^{-7} \text{ m}$

$$\begin{aligned} \therefore d &= \frac{5.460 \times 10^{-7} \times 0.8 \times 20}{10.92 \times 10^{-3}} \text{ m} \\ &= 8 \times 10^{-4} \text{ m} \end{aligned}$$

2. See Fig. 6.5. Suppose the required distance on the screen is y .

Here $d = 2 \times 10^{-4} \text{ m}$ (slit separation)

$\lambda = 5 \times 10^{-7} \text{ m}$ (wavelength)

$D = 5 \times 10^{-1} \text{ m}$ (distance between slit to screen)

The minima is observed when the phase difference between the two waves is an odd multiple of π , i.e., when

$$\delta = \pi, 3\pi, 5\pi, 7\pi, \dots$$

At the third minimum, $\delta = 5\pi$.

From Eq. (6.3), phase difference $= \frac{2\pi}{\lambda}$ (path difference)

But from Fig. 6.5, the path difference between the waves arriving at P is $d \sin \theta$.

Hence

$$5\pi = \frac{2\pi}{\lambda}(d \sin \theta)$$

$$\begin{aligned} \text{or } \sin \theta &= \frac{\lambda}{2\pi d} (5\pi) = \frac{5\pi \times 5 \times 10^{-7}}{2\pi \times 2 \times 10^{-4}} \\ &= 6.25 \times 10^{-3} \end{aligned}$$

From Fig. 6.5, the required distance on the screen $y = D \tan \theta$.

$$\begin{aligned} &= D \sin \theta = 5 \times 10^{-1} \times 6.25 \times 10^{-3} \quad (\because \tan \theta = \sin \theta) \\ &= 3.1 \times 10^{-3} \text{ m.} \end{aligned}$$

3. Let MM' , (Fig. 6.12) the Lloyd's mirror be 5 cm long. The source S_1 is as shown in the figure. The interference pattern is observed in the region AB .

The fringe width β is given by $\beta = \lambda D/d$

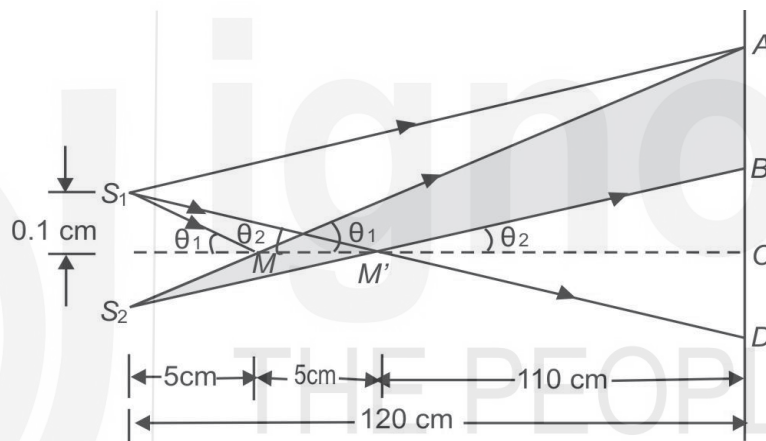


Fig. 6.13

Given $\lambda = 5460 \text{ \AA} = 5.460 \times 10^{-7} \text{ m}$; $D = 120 \text{ cm} = 1.20 \text{ m}$ and

$$d = 0.2 \text{ cm} = 2 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \therefore \beta &= \frac{5.460 \times 10^{-7} \times 1.20}{2 \times 10^{-3}} \text{ m} = 3.276 \times 10^{-4} \text{ m} \\ &= 0.3276 \text{ mm} \end{aligned}$$

The total width of interference pattern is obviously AB . From Fig. (6.13),

$$\tan \theta_1 = 0.1/5, \text{ and } \tan \theta_2 = 0.1/10$$

Also from right angled $\triangle AMC$

$$\begin{aligned} AC/MC &= \tan \theta_1 \text{ or } AC = 115 \times \tan \theta_1 \\ &= 115 \times 0.1/5 = 2.3 \text{ cm} \end{aligned}$$

From right angled $\triangle BM'C$

$$BC/M'C = \tan \theta_2 = \tan \theta_2 \text{ or } BC = 110 \times (0.1/10) = 1 \text{ cm}$$

$$\therefore AB = AC - BC = 2.3 - 1.1 = 1.2 \text{ cm} = 1.2 \times 10^{-3} \text{ m}$$

4. Using Eq. (6.16), we can write the expression for

$$\text{the total fringe shift} = \frac{\beta}{\lambda} (\mu - 1) t$$

The expression for fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

$\therefore$ Hence, the number of fringes that will shift = $\frac{\text{Total fringe shift}}{\text{fringe width}}$

$$= \frac{\frac{\beta}{\lambda} (\mu - 1) t}{\beta} = \frac{(\mu - 1) t}{\lambda}$$

Substituting the values, we get

$$= \frac{(1.6 - 1) \times 1.7 \times 10^{-5} \text{m}}{600 \times 10^{-9} \text{m}} = 17$$

5. Let y_0 represents displacement of the images, therefore,

$$y_0 = 8\beta$$

where β is the fringe width.

$$\therefore \frac{D}{d} (\mu - 1) t = 8 \times \frac{\lambda D}{d}$$

Substituting the values, we get

$$t = \frac{8\lambda}{\mu - 1} = \frac{8 \times 5890 \times 10^{-8} \text{cm}}{1.5 - 1}$$

$$t = 9.42 \times 10^{-4} \text{cm}$$

APPENDIX 6A: DETERMINATION OF d BETWEEN TWO COHERENT VIRTUAL SOURCES FOR FRESNEL BIPRISM

A biprism is made up of two identical prisms. When a light from a source S falls symmetrically on the biprism, as a result two virtual images (S_1 and S_2) arise on either side of S . A convex lens of short focal length ($f \sim 15$ cm to 20 cm) is introduced between the biprism and the eye-piece (Fig. 6A.1). The eye-piece is kept at a large distance from the slit ($5f > D > 4f$). This condition on D minimises the error in the measurement of d . The convex lens converges the two refracted beams. We can adjust its position to obtain clear well-defined images in the plane of the cross wires in the eye-piece. In fact, you can observe that once positions of slit, biprism and the eye-piece are fixed, there are two positions of the lens, shown as L_1 and L_2 in Fig. 6A.1, for which clear images of S_1 and S_2 are obtained in the eye-piece. When the lens is at one of these positions (say at L_1), we obtain magnified images I_1 and I_2 while in the other position (say L_2), we obtain diminished images I'_1 and I'_2 of the sources. Now, you may like to know how d (the distance between the two virtual images S_1 and S_2) is connected to the separation between the images d_1 and d_2 of the two virtual sources when the lens L_1 and L_2 are kept at different positions between the biprism and the eye-piece as shown in Fig. 6A.1.

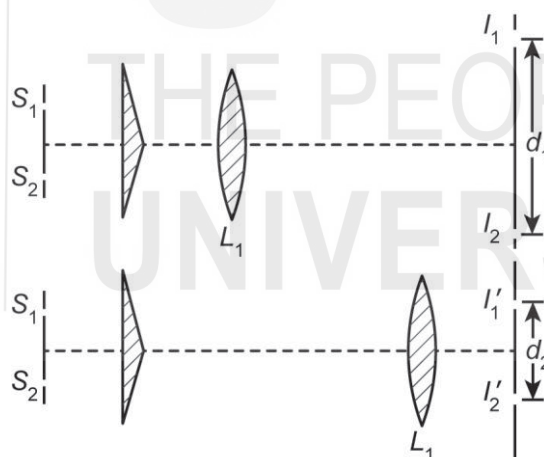


Fig. 6A.1: The positions of the lens between the biprism and the eye-piece correspond to enlarged images I_1, I_2 and diminished images I'_1, I'_2 of S_1 and S_2 .

Suppose that the separation between the two magnified images as seen in the eye-piece is d_1 . If the actual distance between the virtual sources S_1 and S_2 is d , the expression for magnification by the lens is given by

$$m_1 = \frac{d_1}{d} \quad (6A.1a)$$

And, if d_2 is the distance between diminished images of S_1 and S_2 as seen in the eye-piece, the magnification is given by

$$m_2 = \frac{d_2}{d} \quad (6A.1b)$$

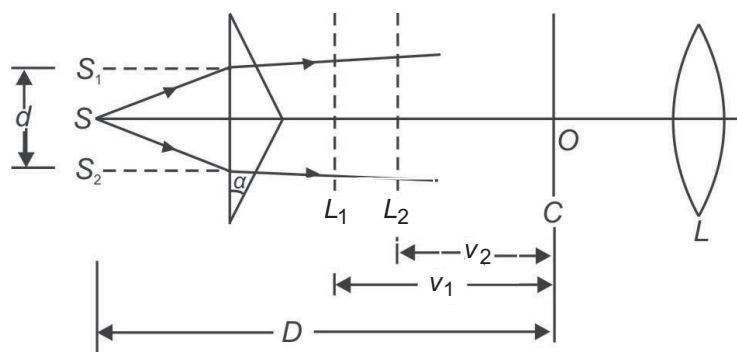


Fig. 6A.2: Displacement method to determine the distance d between the coherent virtual sources in Fresnel biprism experiment. Two positions of the lens between the biprism and the eye-piece are shown.

Now, if u and v are distances of the object and the image, respectively, we can write that

$$u + v = D$$

or, $u = D - v$

From the lens formula, we know that

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

On substituting for u , we can write

$$\frac{1}{v} + \frac{1}{D-v} = \frac{1}{f}$$

or $\frac{D-v+v}{v(D-v)} = \frac{1}{f} \Rightarrow \frac{D}{v(D-v)} = \frac{1}{f}$

This can be rewritten as

$$v^2 - Dv + fD = 0$$

For real roots of this quadratic equation, we must have

$$D^2 - 4fD > 0$$

or $D > 4f$ (6A.2)

Further, if the real roots are v_1 and v_2 , then the sum of the roots is

$$v_1 + v_2 = D$$
 (6A.3)

But, from Fig. 6A.2, we can write:

$$u_1 + v_1 = u_2 + v_2 = D$$
 (6A.4)

where u_1, v_1 are the object and image distances when lens is in position L_1 and u_2, v_2 are object and image distances when the lens is in the position L_2 .

On substituting for $v_1 = D - v_2$ from Eq. (6A.3) in Eq. (6A.4), we can write

$$u_1 + D - v_2 = D \Rightarrow u_1 = v_2$$

On eliminating v_2 by combining Eqs. (6A.3) and (6A.4), we can show that $u_2 = v_1$. Since

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The sum of the roots of a quadratic equation

$$ax^2 + bx + c = 0$$

is equal to $-b/a$. Here $b = D$ and $a = 1$. Therefore, sum of roots

$$v_1 + v_2 = D$$

$$m_1 = \frac{v_1}{u_1} \Rightarrow m_1 = \frac{v_1}{v_2}$$

Similarly, you can show that

$$m_2 = \frac{v_2}{u_2} = \frac{v_2}{v_1}$$

Hence, $m_1 \times m_2 = 1$

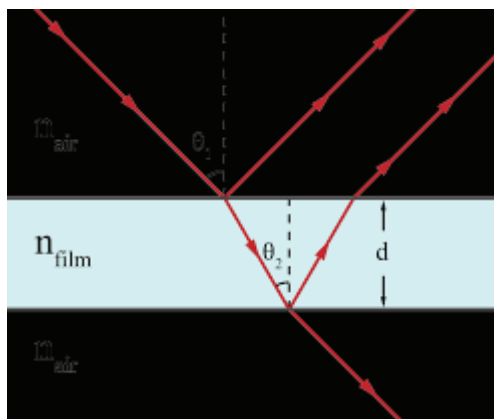
or $m_1 = \frac{1}{m_2}$ (6A.5)

On combining Eqs. (6A.1a), (6A.1b) and (6A.5), we can write

$$d = \sqrt{d_1 d_2} \quad (6A.6)$$

That is d is geometric mean of d_1 and d_2 .





Light incident on a thin film gives rise to a beautiful interference pattern by division of amplitude. The details are discussed in this Unit.

(Source: commons.wikimedia.org)

INTERFERENCE BY DIVISION OF AMPLITUDE

Structure

- | | | | |
|-----|--|-----|-------------------------------------|
| 7.1 | Introduction | 7.4 | Interference by a Wedge-shaped Film |
| | Expected Learning Outcomes | 7.5 | Newton's Rings |
| 7.2 | Stokes' Analysis of Phase Change on Reflection | 7.6 | Summary |
| | | 7.7 | Terminal Questions |
| 7.3 | Interference in Thin Films | 7.8 | Solutions and Answers |

STUDY GUIDE

In the preceding unit, you studied the interference produced by division of wavefront. You learnt to obtain expressions for fringe width and intensity distribution in the fringe-system. You also studied how Fresnel Biprism and Lloyd's mirror produce interference by division of wavefront. We begin this unit by discussing how interference takes place by division of amplitude of light wave. Then we shall describe the formation, shape and location of interference fringes obtained from a thin wedge-shaped film. The SAQs, TQs, and examples should help you to fix your ideas, grasp of the concepts and self-monitor your progress. The mathematics used in derivations is quite simple and we have solved the difficult mathematical steps for you to seamlessly progress through this unit. We hope you will enjoy this unit.

"Science knows no country, because knowledge belongs to humanity, and is the torch which illuminates the world."

Louis Pasteur

7.1 INTRODUCTION

In the preceding unit, we discussed interference of light by division of wavefront wherein different portions of the wavefront propagate in different directions and then made to recombine. You may recall that in Young's double slit experiment, light coming out of a pin hole was allowed to fall on two holes, and the light waves emanating from these two holes interfered to produce the interference pattern. But interference of light responsible for the brilliant colour of thin oil films on water involves two light beams derived from a single incident beam by division of amplitude. When light is incident on a thin transparent plane-parallel film, the waves are reflected partially from the upper surface interfere with the waves partially reflected from the lower surface giving rise to beautiful colours. However, in this process, the amplitudes are reduced. This phenomenon is also responsible for colours in soap bubbles and cracks in glass.

You may recall from the previous unit that in Lloyd's mirror, the interference takes place due to superposition of waves coming directly from the source and those reflected from an optically denser medium. In this arrangement, the interference pattern had a dark central fringe. This observation was explained by arguing that a phase change of π took place when light waves were reflected at the surface of an optically "denser" medium. We begin this unit by giving proof of this statement in Sec. 7.2 based on the principle of reversibility of light. In Sec. 7.3, we have discussed interference in thin transparent films with particular reference to fringes of equal inclination (Haidinger fringes) and fringes of equal thickness (Fizeau fringes). Interference by a wedge-shaped film is discussed in Sec. 7.4. In Sec. 7.5, we have discussed Newton's rings and how these rings are used to determine the wavelength of light.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ prove that when a light wave is reflected at the surface of an optically denser medium, it undergoes a phase change of π ;
- ❖ explain the origin of interference pattern produced by a parallel thin transparent film;
- ❖ describe the formation, shape and location of interference fringes obtained from a thin wedge-shaped film;
- ❖ discuss how Newton's rings are produced and used to determine wavelength of light;
- ❖ explain why a thin coating of a suitable substance minimizes the reflection of light from a glass surface; and
- ❖ distinguish between fringes of equal inclination and fringes of equal thickness.

7.2 STOKES' ANALYSIS OF PHASE CHANGE ON REFLECTION

From the preceding unit you may recall that In Lloyd's mirror arrangement, the interference pattern had a dark central fringe. This observation was explained

by arguing that a phase change of π took place when light waves were reflected at the surface of an optically “denser” medium. Stokes used the principle of optical reversibility to verify this observation. The principle of optical reversibility states that a reflected or refracted light (ray) retraces its original path, if its direction is reversed, provided there is no absorption of light.

Refer to Fig. 7.1(a), which shows the surface MN separating two optically different media (air and glass); glass being denser. We denote air as medium 1 and glass as medium 2.

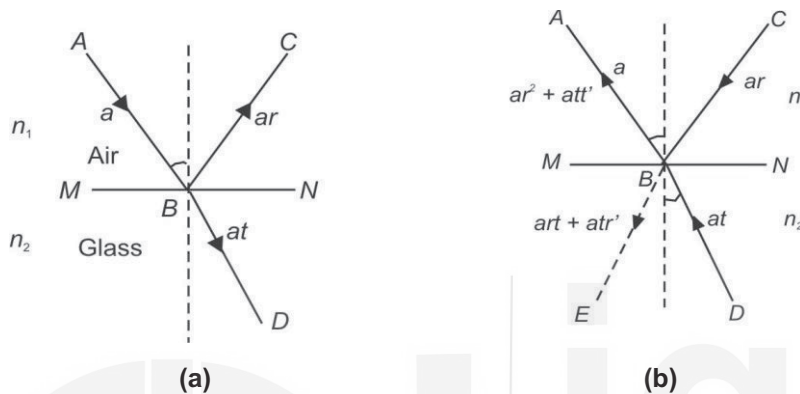


Fig. 7.1: a) Partial reflection and refraction of light at an air-glass interface; b) the optically reversed situation; the two rays in the lower left must cancel. In both cases, $n_2 > n_1$ (n_1 and n_2 are the refractive indices of the media).

An incident light wave, denoted by ray AB , is partly reflected along BC and partly transmitted (refracted) along BD . Suppose the amplitude of the incident wave AB is a , the fraction of the amplitude reflected is r , and the fraction transmitted when the wave is travelling from medium 1 to 2 is t . Then the amplitudes of the wave travelling along BC and BD will be ar and at , respectively.

Now, suppose that the directions of the reflected and transmitted (refracted) waves are reversed. As shown in Fig. 7.1(b), the wave BC , on reversal, gives a reflected wave along BA , and a transmitted wave along BE . You may now like to know the amplitudes of waves reflected along BA and transmitted wave along BE . You will agree that the amplitude of reflected wave along BA will be $ar \cdot r = ar^2$ and the amplitude of transmitted wave along BE will be art . Further, the wave BD , on reversal, gives a transmitted wave along BA and a reflected beam along BE . Let us denote the fractions of amplitude reflected and transmitted when the wave is travelling from a denser medium (2) to a rarer medium (1) be r' and t' . This will contribute att' to the amplitude of the transmitted wave along BA and atr' to the amplitude of reflected wave along BE . According to the principle of reversibility of light, the reflected and transmitted waves BC and BD , when reversed, should give rise to the original wave of amplitude a along BA . It means that there should be no wave along BE , i. e. component along BE should be zero. Similarly, the amplitude of the wave along BA should be equal to a . Mathematically, we write

$$art + atr' = 0 \quad (7.1)$$

$$\text{and} \quad ar^2 + att' = a \quad (7.2)$$

By rearranging terms in Eqs. (7.1) and (7.2), we get

$$r' = -r \quad (7.3)$$

$$\text{and} \quad tt' = 1 - r^2 \quad (7.4)$$

These equations are known as **Stokes' relations**.

Eq. (7.3) shows that the fraction of amplitude reflected when incident wave is travelling from a rarer to denser medium (r) is numerically equal to the fraction of amplitude r' when incident wave is travelling from a denser to a rarer medium but have opposite sign. It means that these components are exactly out of phase with one another, i.e., their phase difference is ' π '. Now two situations arise: No phase change occurs when a light wave is reflected by a denser medium but a phase change of π occurs when a light wave is reflected by a rarer medium – and conversely, no phase change occurs when a light wave is reflected by a rarer medium then there must be a phase change of π when a light wave is reflected by a denser medium. Now, out of these two alternatives, the second one is correct because it has been experimentally observed (see Sec 6.5 on Lloyd's mirror) that the phase change of π occurs when light reaches the boundary from the rarer medium. Hence, light reflected by a material of higher refractive index than the medium in which the waves are travelling undergoes a phase change of 180° (or π).

Reflection by a material of refractive index lower than the medium in which the rays are travelling causes no phase change.

The following SAQ will provide a useful check of your understanding of this section.

SAQ 1 - Principle of reversibility of light

In Fig. 7.2, we have illustrated the situations when the refractive index between the two surfaces is lower than that outside. Determine whether or not there will be a phase change. In Fig. 7.2, we have indicated the points by an arrow where phase change is taking place. Redraw Fig. 7.2, indicating the phase change taking place at the points marked by an arrow.

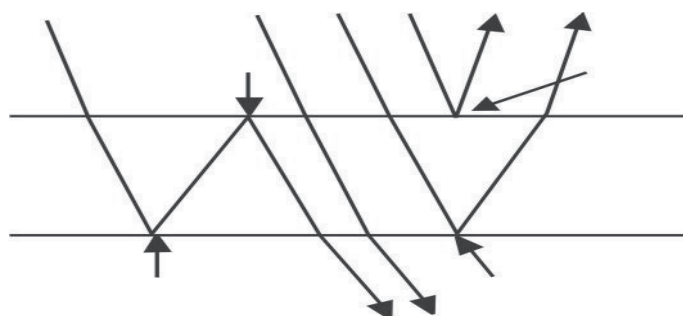


Fig. 7.2

7.3 INTERFERENCE IN THIN FILMS

Refer to Fig. 7.3(a). Light from a source S is incident on a thin film of soapy water, at A . A part of this will be reflected as ray (1) and part refracted along the direction AB . On arrival at B , a part of the latter will be reflected to C , and a part refracted along BT_1 . At C , the wave will again get partly reflected along

You must be aware that a transverse wave on a string undergoes a 180° phase change when reflected from a rigid support. A similar phase change occurs for reflection of a light wave from the boundary of a medium, having a greater index of refraction. The optically denser medium corresponds to a rigid support. A light wave reflected from the boundary of a medium whose index of refraction is greater than that of the medium in which the incident wave travels undergoes a 180° phase change.

CD and refracted as ray (2) along CR_2 . Note that continuation of this process yields two sets of parallel rays, one on each side of the film. In each of these sets, of course, the amplitude of wave decreases rapidly in each step. Considering only the first two reflected rays (1) and (2) we observe that these two rays are in a position to interfere. This is because, if we assume S to be a monochromatic point source, the film serves as an amplitude-splitting device. Ray (1) and (2) may be considered as arising from the point source S due to reflection at the top and bottom surfaces of the thin film. But these two rays may respectively be considered as arising from two coherent virtual sources S' and S'' which are obtained by producing the two rays behind the film as shown in Fig. 7.3 (b).

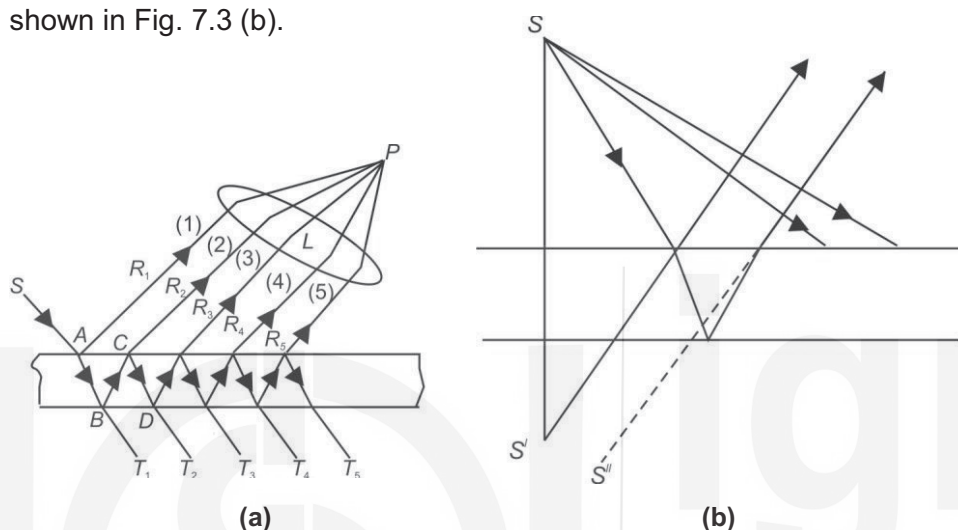


Fig. 7.3: a) Multiple reflections in a soap film; b) The interference pattern produced due to rays (1) and (2) is approximately the same as would have been produced by two coherent point sources S' and S'' .

If the set of waves (shown as parallel reflected rays) is now collected by a lens and focussed at P , each wave would have travelled a different distance. So the phase relationship between them may be such as to produce destructive or constructive interference at P giving rise to colours when seen by unaided eyes.

Let us now obtain an expression for the path difference between reflected waves.

Path difference in reflected light

Refer to Fig. 7.4. It shows that light is incident on the thin film of soapy water at A at an angle i . Suppose the thickness of the film is t and refractive index of the material of the film μ is greater than one. At A , light is partly reflected along AR_1 giving the ray (1) and partly refracted along AB at an angle r . At B it is again partly reflected along BC and partly refracted along BT_1 . Similarly, reflections and refractions occur at C . Since, the rays (1) and (2) have been derived from the same incident ray, they constitute a pair of coherent lights and interfere. Let CN and BM be perpendicular to AR_1 and AC . Since the paths of the rays AR_1 and CR_2 beyond CN are equal, the path difference between ray (1) and (2) is given by path ABC in film – path AN in air. We can therefore write

$$\text{path difference} = \mu (AB + BC) - AN \quad (7.5)$$

Note that

$$AB = BC = \frac{BM}{\cos r} = \frac{t}{\cos r}$$

and $AN = AC \sin i$

Since $AC = AM + MC$, we can write

$$\begin{aligned} AC &= BM \tan r + BM \tan r \\ &= 2t \tan r \end{aligned}$$

Using this result in the relation for AN , we get

$$\begin{aligned} \therefore AN &= 2t \tan r \sin i \\ &= 2t \frac{\sin r}{\cos r} (\sin i) \\ &= 2t \frac{\sin r}{\cos r} (\mu \sin r) \quad \left[\because \frac{\sin i}{\sin r} = \mu \right] \end{aligned}$$

$$\text{or } AN = 2\mu t \frac{\sin^2 r}{\cos r}$$

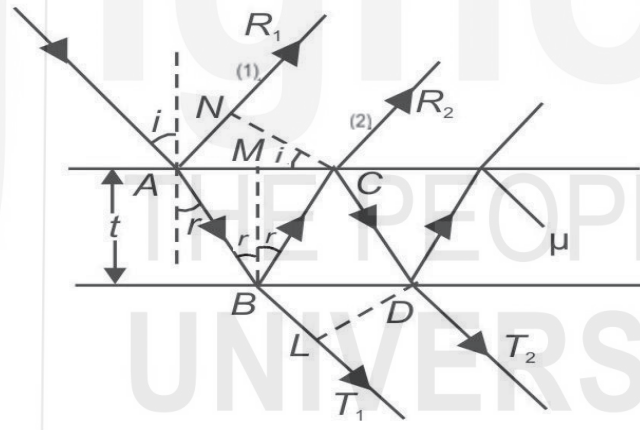


Fig. 7.4: Optical path difference between two consecutive rays arising in multiple reflections.

On substituting the values of AB, BC and AN in Eq. (7.5), we get

$$\begin{aligned} \text{path difference} &= \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2\mu t \frac{\sin^2 r}{\cos r} \\ &= \frac{2\mu t}{\cos r} (1 - \sin^2 r) \\ &= 2\mu t \cos r \end{aligned} \quad (7.6)$$

Recall that ray (1) undergoes a phase change of π on reflection, while ray (2) does not, since it is internally reflected (See SAQ 1). So we must include this fact in the expression for path difference obtained in Eq. (7.6). Since phase change of π is equivalent to a path difference of $\frac{\lambda}{2}$, the effective path difference between rays (1) and (2) is given by

$$\text{path difference} = 2\mu t \cos r + \frac{\lambda}{2} \quad (7.7)$$

Note that the sign of phase change is immaterial.

From Unit 6 of this course, you may recall that if this path difference is an odd multiple of $\frac{\lambda}{2}$, we might expect rays (1) and (2) to be out of phase, and

produce destructive interference. Thus, the condition for **destructive interference** for light reflected by a thin film can be expressed as

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \dots$$

$$\text{or } 2\mu t \cos r = n\lambda \quad (7.8)$$

Next, let us examine the phases of the rays, (3), (4), (5), ... Since the geometry is the same, the path difference between rays (3) and (2) will also be given by Eq. (7.6). Since only internal reflections are involved in this case, the effective path difference will still be given by Eq. (7.6). Hence, if the condition given by Eq. (7.8) is fulfilled, ray (3) will be in the same phase as ray (2). The same holds true for all succeeding pairs, and so we conclude that, Eq. (7.8) expresses the condition that rays (1) and (2) will be out of phase, but rays (2), (3), (4), ..., will be in phase with each other. Further, since ray (1) has considerably greater amplitude than ray (2), it is only logical to think that these will not annul each other completely. That is, the condition given by Eq. (7.8) may not produce complete darkness. But it is not so because addition of rays (3), (4), (5), ..., which are all in phase with ray (2), gives a net amplitude which is just sufficient to make up for the difference and to produce complete darkness.

To prove that light reflected by a thin film can lead to complete darkness, refer to Fig. 7.5, which shows the amplitudes of successive rays when light undergoes multiple reflections.

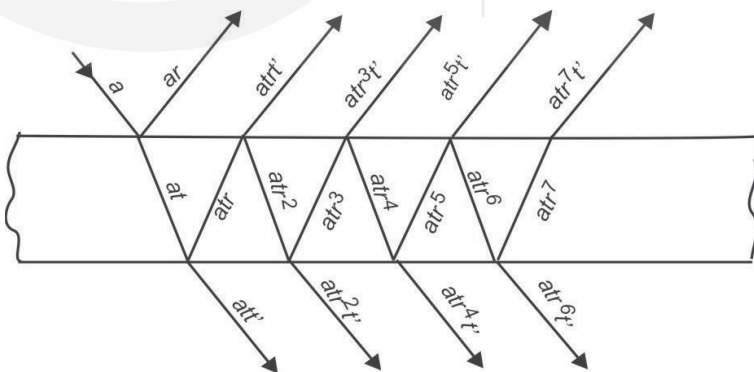


Fig. 7.5: Amplitude of successive rays when light undergoes multiple reflections.

On adding the amplitudes of all but the first reflected rays, on the upper side of the film, we obtain the resultant amplitude:

$$\begin{aligned} A &= atrt' + atr^3t' + atr^5t' + atr^7t' + \dots \\ &= atrt'(1 + r^2 + r^4 + r^6 + \dots) \end{aligned}$$

Do you recognise the series within the parentheses in this expression? For r less than one, it represents a geometrical series whose sum is finite, equal to $1/(1-r^2)$, giving

$$A = atrt' \frac{1}{(1-r^2)}$$

But from Eq. (7.4) we recall that $tt' = 1-r^2$. Using this result in the above relation, we get a compact relation for resultant amplitude of 2,3,4.... waves:

$$A = ar \quad (7.9)$$

That is, the resultant amplitudes of all, but the first, reflected wave, on the upper side of the film, is just equal to the amplitude of the first reflected wave. Hence, we can conclude that when Eq. (7.8) is satisfied, there will be complete destructive interference. On the other hand, if the path difference given by Eq. (7.7) is an integral multiple of λ , waves denoted by rays (1) and (2) will be in phase and lead to **constructive interference**, i.e., when

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda, \text{ where } n = 0, 1, 2, \dots \text{ etc.}$$

$$\text{or} \quad 2\mu t \cos r = (2n+1) \frac{\lambda}{2} \quad (7.10)$$

But rays (3), (5), (7), ..., will be out of phase with rays (2), (4), (6), ... Since (2) is more intense than (3), (4) is more intense than (5), etc., these pairs cannot cancel each other. As the stronger series combines with ray (1), the strongest of all, there will be maximum of intensity.

Thus, when a thin film is illuminated by monochromatic light, and seen in reflected light, it appears bright or dark according as $2\mu t \cos r$ is an odd multiple of $\frac{\lambda}{2}$ or an integral multiple of λ , respectively.

$$2\mu t \cos r = (2n+1) \frac{\lambda}{2} \text{ (condition of maxima)} \quad (7.11a)$$

$$2\mu t \cos r = n\lambda \text{ (condition of minima)} \quad (7.11b)$$

Before proceeding further, answer the following SAQ.

SAQ 2 - Thin films

Based on Eq. (7.7), state whether the following statement is true or false. Justify your answer.

“An **excessively thin** film seen in reflected light appears perfectly black”.

Now, we can discuss the reason for production of colours in thin films.

Colours in thin films

When light is incident on a thin film, we receive rays of light reflected at its top and at the bottom surface. These rays interfere and depending on the path difference between the interfering rays, as given by Eq. (7.7), we note that it depends on t , the thickness of the film and r , the fraction of amplitude reflected r , and, hence, upon inclination of the incident rays. Note that

inclination is determined by the position of the eye relative to the region of the film, which is being looked at. We know that sunlight consists of a continuous range of wavelengths (colours). At a particular point of the film, and for a particular position of the eye (i.e., for a particular t and a particular r), the rays of only certain wavelengths will have path difference, which satisfies the condition of maxima. Hence, only these wavelengths (colours) will be present with the maximum intensity. While some others, which satisfy the condition of the minima will be missing. Hence, the point of the film being viewed will appear coloured.

We now work out an example to make sure that you grasp the physics of the phenomenon of production of colours in thin films.

EXAMPLE 7.1: COLOURS IN THIN FILMS

A thin film of thickness 4×10^{-5} cm is illuminated by white light normal to its surface ($r = 0^\circ$). Its refractive index is 1.5. Of what colour will the thin film appear in reflected light?

SOLUTION ■ The condition for constructive interference of light reflected from a film is

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \dots$$

Hence $\mu = 1.5$; $t = 4 \times 10^{-5}$ cm and $r = 0^\circ$ (since light falls normally) so that $\cos r = 1$. On substituting the given values, we get

$$\begin{aligned} 2 \times 1.5 \times 4 \times 10^{-5} &= (2n + 1) \frac{\lambda}{2} \\ \text{or } \lambda &= \frac{2 \times 2 \times 1.5 \times 4 \times 10^{-5}}{2n + 1} \\ \lambda &= \frac{24 \times 10^{-5} \text{ cm}}{2n + 1} = \frac{24,000 \text{ \AA}}{2n + 1} \end{aligned}$$

Taking $n = 0, 1, 2, 3, \dots$ we get

$$\lambda = 24000 \text{ \AA}, 8000 \text{ \AA}, 4800 \text{ \AA}, 3431 \text{ \AA}, \dots$$

These wavelengths are reflected most strongly. Of these, the wavelength lying in the visible region is $4800 \text{ \AA}$ which corresponds to blue colour.

So far, we have considered viewing of thin film in reflected light. Suppose the eye is now situated on the lower side of the film, shown in Fig. 7.3 and Fig. 7.5. The light emerging from the lower side of the film can also be made to interfere. You may now like to Know: what colours will arise?

Let us now discuss, as to which colours will arise when the film is viewed in refracted (transmitted) light. As before, we first calculate the path difference between the transmitted waves, shown as rays in Fig. 7.3.

The path difference between the transmitted rays BT_1 and DT_2 is given by Eq. (7.6), i.e.,

$$\mu(BC + CD) - BL = 2\mu t \cos r$$

Note that in this case, there is no phase change due to reflections at B and C because in both cases light is reflected at the denser-rarer medium interface from the denser side (see SAQ 1). Hence, the effective path difference between BT_1 and DT_2 is also $2\mu t \cos r$.

The two rays BT_1 and DT_2 will reinforce each other, if

$$2\mu t \cos r = n\lambda \quad (\text{condition of maxima}) \quad (7.12a)$$

where $n = 1, 2, 3, \dots$

In this case, the film will appear brighter in the transmitted light.

The two rays will annihilate the effect mutually if

$$2\mu t \cos r = (2n + 1) \frac{\lambda}{2} \quad (\text{condition of minima}) \quad (7.12b)$$

where $n = 0, 1, 2, \dots$ and the film will appear dark in transmitted light.

Let us now reflect on the results that we have obtained in this section. Eqs. (7.11a), (7.11b), (7.12a) and (7.12b) show that the conditions for the maxima and minima for the reflected light are just reverse of those in transmitted light. It means that only those colours will be visible in transmitted light which are missed in reflected light. In other words, the appearances of colours in the two cases are complimentary. Hence, if the film appears bright in reflected light, it will appear dark in transmitted light and vice versa.

Interference fringes produced by thin films are classified into two types: Fringes of equal inclination and fringes of equal thickness.

Fringes of Equal Inclination (Haidinger fringes)

If a small aperture lens is used to focus light reflected by a thin film and incident from a point source, the interference fringes will be visible only from a small portion of the film. This is because light from only a small part of the film will reach the lens and be seen (see Fig. 7.6a). However, if an extended source is used, light will reach the lens from various directions, and the fringe pattern will spread out over a larger area of the film, as shown in Fig. 7.6b.

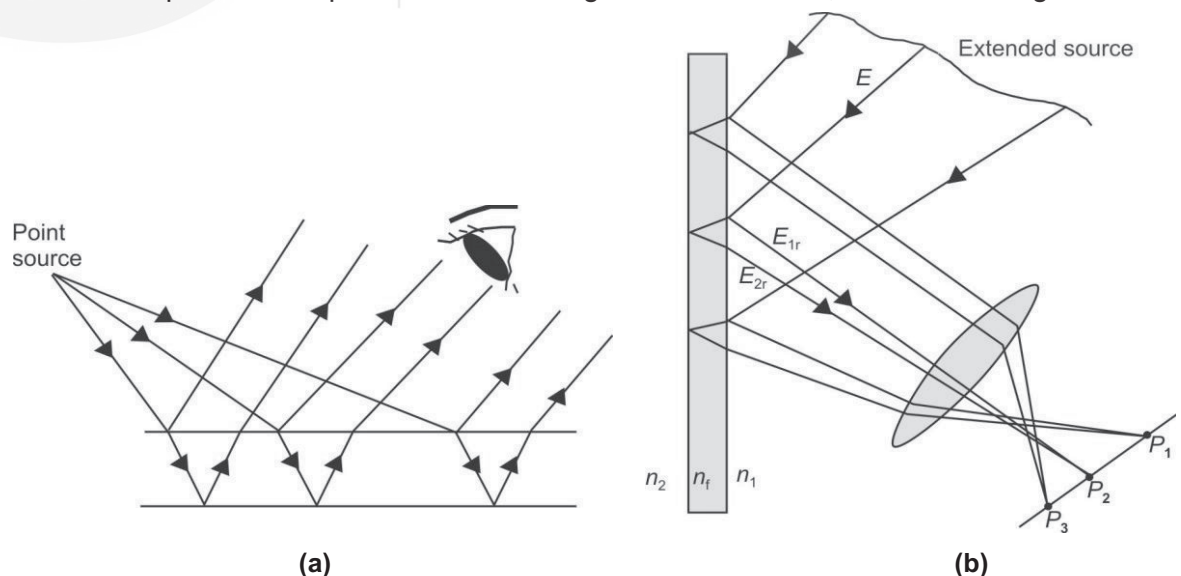


Fig. 7.6: Fringes are respectively seen from a small portion of the film and a larger region of the film if the source is a) a point source, and b) an extended source.

Note that the angle of incidence i or equivalently angle of refraction r , controls the path difference. The fringes appearing at point P_1 and P_2 in Fig. 7.7 are, accordingly, known as **fringes of equal inclination**.

It may be noted that as the film becomes thicker, the separation AC in Fig. 7.4 between rays (1) and (2) also increases, since $AC = 2t \tan r$. When only one of the two rays enters the eye, the interference pattern disappears. However, the larger lens of a telescope should then be used to gather both rays so that the pattern becomes visible. The separation between rays (1) and (2) can also be reduced by reducing i , i.e., by viewing the film at nearly normal incidence.

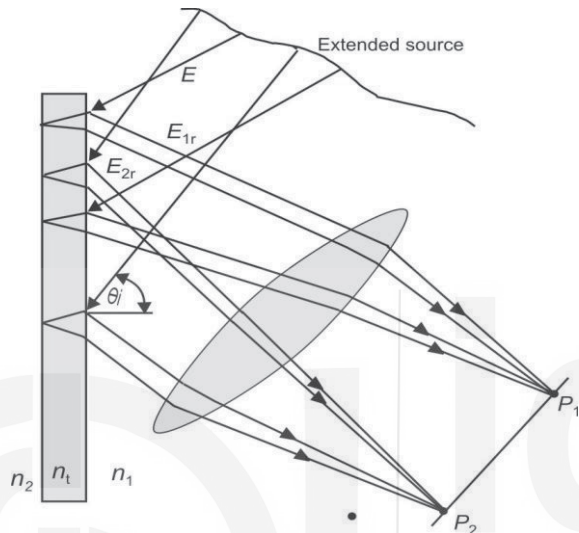


Fig. 7.7: Haidinger fringes: All rays inclined at the same angle arrive at the same point.

The equal inclination fringes obtained in thick plates are known as **Haidinger fringes**. With an extended source, the symmetry of the set up requires that the interference pattern consists of a series of concentric circular bands centred on the perpendicular drawn from the eye to the film, as shown in Fig. 7.8

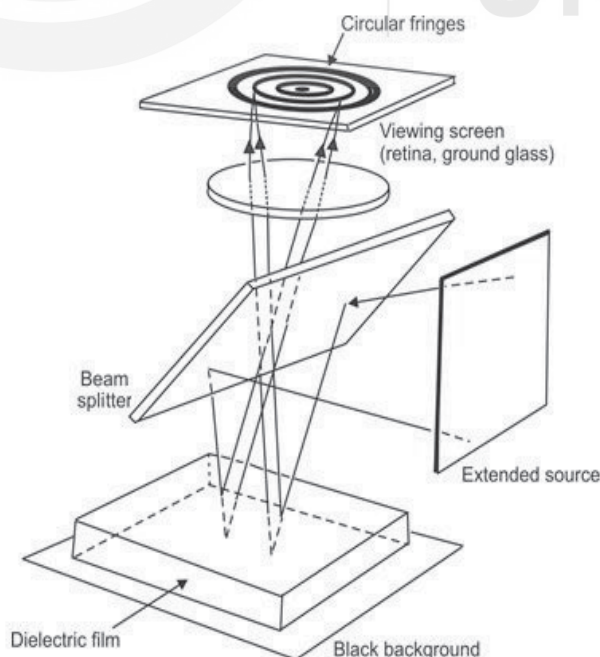


Fig. 7.8: Circular Haidinger fringes centred on the axis of the lens.

Haidinger fringes are formed at infinity and are observed using a telescope focussed at infinity. Such fringes are observed in Michelson interferometer, about which you will learn in the next unit.

Fringes of Equal Thickness (Fizeau fringes)

Interference fringes, for which thickness of the film rather than the angle of refraction is the dominating parameter, are referred to as **fringes of equal thickness**. The film is not plane-parallel i.e. the film has continuously varying thickness. When such a film is illuminated by a monochromatic light source than equidistant straight line interference fringes parallel to the thin edge of the wedge are formed. Each such fringe is the locus of all points in the film for which thickness is a constant. Such fringes are localised on the film itself and are observed by a microscope focussed on the film. Fringes due to the wedge-shaped film belong to this class. You will learn about these in the next section.

Fringes of equal thickness can be distinguished from Haidinger fringes by the manner in which the diameters of the rings vary with order n . The central region in the Haidinger pattern corresponds to the maximum value of n , whereas just the opposite applies to fringes of equal inclination.

7.4 INTERFERENCE BY A WEDGE-SHAPED FILM

So Far, we have assumed the film to be of uniform thickness. However, many a time, the fringes are produced by films of varying thickness, i.e., the film is not plane-parallel. Such films find practical application for testing optical surfaces for flatness.

We now discuss the interference pattern produced by a wedge-shaped film, whose plane surfaces are not parallel (Fig. 7.9a and 7.9b). Note that the interfering rays do not enter the eye parallel to each other but seem to diverge from a point near the film.

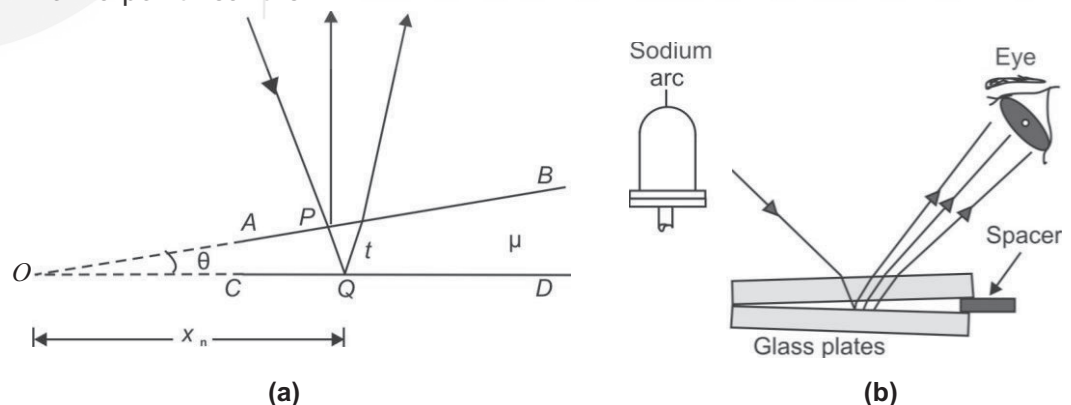


Fig. 7.9: Fringes of equal thickness: a) method of visual observations; b) a parallel beam of light incident on a wedge-shaped film.

To obtain conditions for maxima and minima of the interference pattern produce by a thin wedge-shaped film of refractive index μ , let us consider a film bounded by two plane surfaces AB and CD , inclined at an angle θ as shown in Fig. 7.9a. When produced backwards, the lines corresponding to these surfaces meet at point O . The thickness of the film increases from A to

B. Suppose the film is illuminated by a monochromatic source of light taken in the form of a slit held parallel to the edge of the wedge and light is incident perpendicularly on the surface AB. (The edge is the line passing through O and normal to the plane of the paper.) Interference occurs between the rays emerging from the upper and lower surfaces of the film. In this case, the path difference for a given pair of rays is essentially the same as that given by Eq. (7.6). But, if it is assumed that wedge angle is small, and since the light is assumed to be incident almost normally at a point P on the film, the factor $\cos r$ may be taken to be equal to unity. Then, the path difference between the rays reflected at the upper and the lower surfaces will be same, equal to $2\mu t$, where t is the thickness of the film at P .

Recall that an additional path difference of $\frac{\lambda}{2}$ is introduced for the light reflected from the upper surface. So, the effective path difference between the two reflected rays will be

$$2\mu t - \frac{\lambda}{2} \quad (7.13)$$

and the condition for bright fringes can be expressed as

$$2\mu t - \frac{\lambda}{2} = n\lambda$$

or $2\mu t = (2n + 1)\frac{\lambda}{2} \quad (7.14)$

Similarly, the condition for dark fringe is given by

$$2\mu t = n\lambda \quad (7.15)$$

From these results, it is clear that to observe a bright or dark fringe of a particular order for a wedge shaped film with small wedge angle and normal incidence of light, t must be constant. Note that in a wedge-shaped film, t remains constant only along lines parallel to the thin edge of the wedge. Therefore, the bright and dark fringes are obtained as straight lines parallel to the thin edge of the wedge. At the thin edge, $t = 0$ and therefore path difference = $\frac{\lambda}{2}$, which corresponds to the condition of minimum intensity. It

means that the edge of the wedge-shaped film will be dark. The resulting fringes resemble the localized fringes in the Michelson interferometer (this you will study in next unit) and appear to be formed in the film itself.

Fringe width

You now know that a wedge shaped film with small wedge angle gives rise to an interference pattern comprising dark and bright straight line fringes when light is incident almost normally. You may now ask: Is the width of these fringes the same? To discover answer to this question, we note that for the n th dark fringe, we have

$$2\mu t = n\lambda$$

Suppose this fringe is obtained at a distance x_n from the thin edge. Then, $t = x_n \tan \theta = x_n \theta$ (when θ is small). Hence, for the n th dark fringe, we can write

$$2\mu x_n \theta = n\lambda \quad (7.16)$$

Similarly, if the $(n + 1)$ th dark fringe is obtained at a distance x_{n+1} from the thin edge, then we can write

$$2\mu x_{n+1} \theta = (n + 1)\lambda \quad (7.17)$$

On subtracting Eq. (7.16) from Eq. (7.17), we get

$$2\mu \theta (x_{n+1} - x_n) = \lambda$$

Hence, we obtain the following expression for fringe width (β) :

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\mu\theta} \quad (7.18)$$

Note that θ is measured in radians. Such fringes are referred to as **fringes of equal thickness**. This result shows that fringe width depends on the wavelength of incident light, the refractive index of the film and the angle of wedge. It means that if we use mercury lamp, instead of a sodium lamp, to observe interference pattern, it will have coloured fringes.

You can convince yourself that the spacing between two consecutive bright fringes (fringe width) is equal to $\frac{\lambda}{2\mu\theta}$.

To check your progress, we would like you to solve a SAQ.

SAQ 3 - Interference by thin wedge-shaped film

Using sodium light ($\lambda = 5893 \text{ \AA}$), interference fringes are formed by reflection from a thin air wedge. When viewed perpendicularly, 10 fringes are observed in a distance of 1 cm. Calculate the angle of the wedge.

You now know that when light is incident almost perpendicularly on a plane surface of a wedge shaped film with small wedge angle, we obtain an interference pattern comprising dark and bright straight line fringes. You may now like to know: What would be the nature of fringes when the upper surface of the wedge shaped air film is curved, say as an air film enclosed between a convex surface of a lens and a plane glass surface. You will learn that the fringes will be circular in shape because the thickness of air film remains constant on the circumference of a circle whose centre is at the point of contact. The ring-shaped fringes, thus produced, were first studied in detail by Newton. In the next section, you will learn about Newton's rings.

7.5 NEWTON'S RINGS

Refer to Fig. 7.10, which shows a plano-convex lens of large radius of curvature placed with its convex surface in contact with a plane glass plate. Due to curved surface of the lens, an air-film is formed between the lower surface of the lens (LOL') and the upper surface of the plate (POQ). The thickness of the air film is zero at the point of contact O , and it increases as we move away from the point of contact. When monochromatic light falls normally on this film, reflection takes place at both the top as well as the bottom of the film. As a result of superposition of light waves reflected from the

upper and lower surfaces of the air film, an interference pattern is formed which comprises concentric dark and bright fringes. Since thickness of the air film increases with distance from the point of contact O , therefore, the fringe pattern consists of concentric dark and bright circular rings.

For rings observed in reflected light, the effective path difference between the interfering rays 1 and 2 is given by Eq. (7.13).

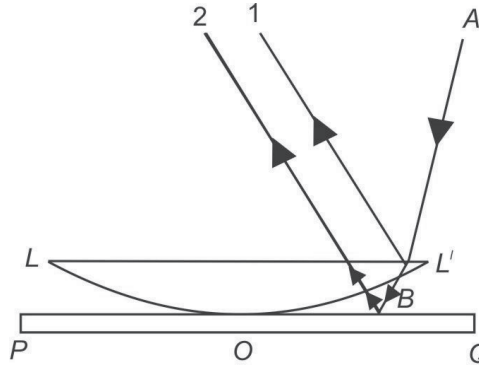


Fig. 7.10: A wedge shaped air film and formation of Newton's rings.

For air-film, $\mu = 1$ and the condition for the bright rings can be written as [Eq. (7.14)]:

$$2t = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2, 3, \dots \quad (7.19)$$

and the condition for the dark ring can be written as (Eq. (7.15)) :

$$2t = n\lambda \quad n = 1, 2, 3, \dots \quad (7.20)$$

Let us now obtain the relation between the radii of the rings and the wavelength of the light used. Refer to Fig. 7.11, where LOL' denotes the lens placed on the glass plate AOB . Suppose the radius of curvature of the curved surface of the lens is R and radius of the n th Newton's ring is r_n . Suppose that thickness of air film where n th fringe is formed is t and corresponds to point P . From the property of a circle, we can say that angle OPE is 90 degree. Draw perpendicular PN on the hypotenuse of right angled Δ . Then, from similarity of $\Delta s EPN$ and CPO , we can write

$$PN^2 = ON \times NE$$

or
$$r_n^2 = t(2R - t)$$

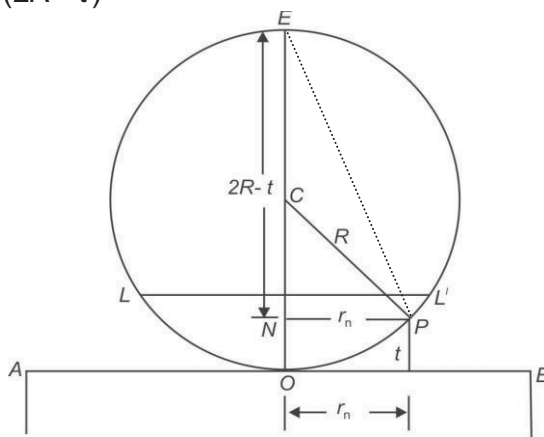


Fig. 7.11: Cross-sectional view of a Newton's ring. r_n denotes the radius of the n th dark ring and thickness of air film (where the n th dark ring is formed) is t .

Since t is small compared to R , in the above relation we can ignore t^2 .

Hence, we can write

$$r_n^2 = 2Rt$$

$$\text{or} \quad 2t = \frac{r_n^2}{R} \quad (7.21)$$

On using this result in Eq. (7.19), we obtain the condition for a bright ring as

$$2t = (2n+1) \frac{\lambda}{2}$$

Since $2t = \frac{r_n^2}{R}$ (Eq. (7.21)), we can write the relation between the radius of bright ring obtained in Newton's rings arrangement and radius of curvature of the plano-convex lens as

$$\therefore \frac{r_n^2}{R} = (2n+1) \frac{\lambda}{2}$$

$$\text{or} \quad r_n^2 = (2n+1) \frac{\lambda R}{2} \quad n = 0, 1, 2, 3, \dots$$

This shows that the radii of the rings vary as the square-root of odd natural numbers. It implies that the rings will get closer and closer as their order increases. Further, since radii of Newton's rings depend on the wavelength of light used, the monochromatic light will produce an extensive fringe system such as that shown in Fig. 7.12.

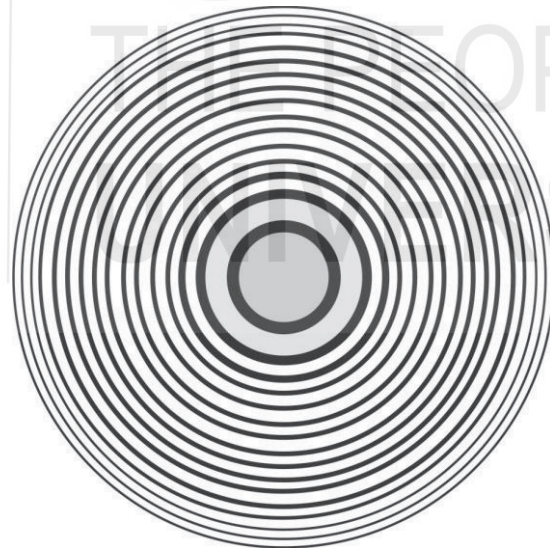


Fig. 7.12: Newton's rings observed in reflected light.

Usually, it is more convenient to measure the diameter of a ring than its radius. We therefore, rewrite the condition for bright ring in terms of its diameter. If we denote the diameter of the n th bright ring as D_n , then

$$D_n = 2r_n \text{ or } r_n = \frac{D_n}{2}. \text{ On substituting this in the expression for the condition}$$

obtained for a bright ring, we get

$$D_n^2 = 2(2n+1)\lambda R \quad (7.22)$$

$$\text{or} \quad D_n = \sqrt{2\lambda R} \sqrt{2n+1}$$

For constant values of λ and R , we get

$$D_n \propto \sqrt{2n+1} \quad (7.23)$$

Before proceeding further, answer the following SAQ.

SAQ 4 - Newton's rings

Using Eqs. (7.20) and (7.21), prove that the radii of dark rings are proportional to the square-root of the natural numbers.

Having answered SAQ4, you now know that the radii of dark rings are proportional to the square-root of the natural numbers. It means that when the contact between lens and glass is perfect, the central spot will be black. Physically, this is direct evidence of the relative phase change of π between air-to-glass and glass-to-air, reflections. If there were no such phase change, the rays reflected from the two surfaces in contact would have been in the same phase, and we should have obtained a bright spot at the centre. However, the central spot was made bright by a slight modification. In an interesting modification of Newton's experimental arrangement, Thomas Young, introduced the lower plate of higher index of refraction than that of the lens, and the film in-between was filled with an oil of intermediate index. As a result both reflections occurred at "rare-to-dense" surfaces. (In this situation, no relative phase change occurs, and the central fringe of the reflected system is bright.)

From Eq. (7.21) we recall that expression for diameter of the n th bright ring is given by

$$D_n^2 = 2(2n+1)\lambda R$$

If D_{n+p} denotes the diameter of the $(n+p)$ th bright ring, then we can write

$$D_{n+p}^2 = 2[2(n+p)+1]\lambda R \quad (7.24)$$

On subtracting Eq. (7.22) from Eq. (7.24), we get

$$\begin{aligned} D_{n+p}^2 - D_n^2 &= 2[2(n+p)+1]\lambda R - 2(2n+1)\lambda R \\ &= 4p\lambda R \end{aligned}$$

$$\text{or } \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \quad (7.25)$$

From this result, we note that on measuring the diameters of the rings and the radius of curvature R of the lens, we can determine the wavelength λ of light used. In physics laboratory, you can measure the radius of curvature fairly accurately with the help of a spherometer.

If a liquid of refractive index μ is introduced between the lens and the glass plate, then the expression for path difference between two interfering rays will also include μ . Then the radii of the dark rings will be given by

$$r_n = \left(\frac{n\lambda R}{\mu} \right)^{1/2} \quad (7.26)$$

Thus, when a little water is introduced between the lens and the plate, the rings contract according to the relation

$\frac{\text{diameter of a ring in water – film}}{\text{diameter of the same ring in air – film}} = \frac{1}{\sqrt{\mu}}$	(7.27)
---	--------

where μ is the refractive index of water.

A ring system is also observed in the light transmitted by Newton's ring plates. However, there are two differences between rings obtained in the reflected and transmitted lights:

- i) The rings observed in transmitted light are exactly complementary to those seen in the reflected light, so that the central spot is now bright.
- ii) The rings in transmitted light are much poorer in contrast to those in reflected light.

Now solve the following SAQ to check your understanding.

SAQ 5 - Diameter of Newton's rings

If in a Newton's ring experiment, the air in the inter space is replaced by a liquid of refractive index 1.33. In what proportion would the diameters of the rings change?

Let us sum up what you have learnt in this unit.

7.6 SUMMARY

Concept	Description
Stokes' analysis	- When the light wave is reflected from a boundary, there is an abrupt change of phase. When the light ray is reflected while going from a rarer to a denser medium, it suffers a phase change of π. But there is no phase change when the light ray is reflected while going from a denser to a rarer medium. - Length ℓ in a medium of refractive index μ is optically equivalent to length $\mu\ell$ in a vacuum. $\mu\ell$ is called the optical path length of distance ℓ in the medium.
Interference in thin films	- For a thin film in reflected light, the conditions for constructive and destructive interference are: $2\mu t \cos r = (2n+1) \frac{\lambda}{2} \text{ (maxima)}$ $2\mu t \cos r = n\lambda \text{ (minima)}$ where μ is the refractive index of the film, t is its thickness and r is the angle of refraction in the film. - For a thin film in transmitted light, the conditions for constructive and destructive interference are: $2\mu t \cos r = n\lambda \text{ (maxima)}$ $2\mu t \cos r = (2n+1) \frac{\lambda}{2} \text{ (minima)}$ - The basic formula for the path difference between the interfering rays, obtained due to division of amplitude by a film of thickness t and refractive index μ, is $2\mu t \cos r$, where r is the inclination of ray inside the film. If the thickness of the film is uniform, the path difference $2\mu t \cos r$ varies only with inclination r, and gives rise to the "fringes of equal inclination". On the other hand, if the thickness of the film is rapidly varying, the path difference $2\mu t \cos r$ changes mainly due to changes in μ. This gives rise to the "fringes of equal thickness".

**Interference by
wedge-shaped film**

- The spacing β between two consecutive bright (or dark) fringes produced by wedge-shaped film is given by

$$\beta = \frac{\lambda}{2\mu\theta}$$

where λ is the wavelength of light being used for illuminating the film, μ the refractive index of the film, and θ (measured in radians) the angle between the two plane surfaces, which form the wedge-shaped film.

- The diameters of the bright rings are proportional to the square-roots of the odd natural numbers, whereas the diameters of dark rings are proportional to the square-roots of natural numbers, provided the contact is perfect.

Newton's ring

- On measuring the diameters of Newton's rings and the radius of curvature R , the wavelength can be calculated with the help of the following relation:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

7.7 TERMINAL QUESTIONS

1. White light is reflected normally from a uniform oil film ($\mu = 1.33$). An interference maximum for $6000 \text{ \AA}$ and a minimum for $4500 \text{ \AA}$, with no minimum in between, are observed. Calculate the thickness of the film.
2. Light of wavelength (λ) $6300 \text{ \AA}$ falls normally on a thin wedge-shaped film ($\mu = 1.5$). There are ten bright and nine dark fringes over the length of the film. By how much does the film thickness change over this length?
3. Two glass plates 12 cm long touch at one end, and are separated by a wire 0.048 mm in diameter at the other. How many bright fringes will be observed over the 12 cm distance in the light ($\lambda = 6800 \text{ \AA}$) reflected normally from the plates? (Take $\mu = 1$)
4. Newton's rings are formed in reflected light of wavelength $5895 \times 10^{-8} \text{ cm}$ with a liquid between the plane and curved surfaces. The diameter of the fifth ring is 0.3 cm and the radius of curvature of the curved surface is 100 cm. Calculate the refractive index of the liquid, when the ring is
i) bright, ii) dark.
5. A Newton's rings arrangement is used with a source emitting two wavelengths

$$\lambda_1 = 6.0 \times 10^{-5} \text{ cm} \quad \text{and} \quad \lambda_2 = 4.5 \times 10^{-5} \text{ cm}$$

and it is found that the n th dark ring due to λ_1 coincides with the $(n+1)$ th dark ring due to λ_2 . If the radius of curvature of the curved surface is 90 cm, find the diameter of the n th dark ring for λ_1 .

7.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. See Fig. 7.13.

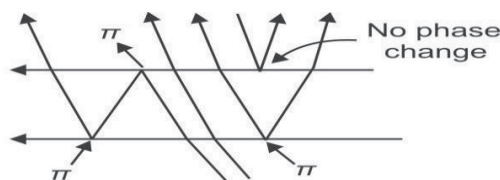


Fig. 7.13

2. According to Eq. (7.7) the path difference between the interfering rays in reflected light is $2\mu t \cos r + \frac{\lambda}{2}$. When the film is excessively thin, t is very small, and $2\mu t \cos r$ is almost zero. Hence the path difference, in such a case becomes $\frac{\lambda}{2}$. This is a condition of minimum intensity. Hence, the film will appear black in the reflected light.
3. Let θ radian be the angle of the air-wedge. For normal incidence, the fringe-width is given by

$$\beta = \frac{\lambda}{2\theta} \quad (\because \mu = 1 \text{ for air})$$

Here $\lambda = 5893 \times 10^{-8} \text{ cm}$ and $\beta = 1/10 \text{ cm}$.

$$\therefore \theta = \frac{\lambda}{2\beta} = \frac{5893 \times 10^{-8}}{2 \times 1/10} = 2.95 \times 10^{-4} \text{ radian.}$$

4. According to Eq. (7.20), the condition for the dark ring is

$$2t = n\lambda$$

But from Eq. (7.21), $2t = \frac{r_n^2}{R}$

$$\therefore \frac{r_n^2}{R} = n\lambda$$

If D_n be the diameter of the n th dark ring, $r_n = \frac{D_n}{2}$.

$$\therefore \frac{D_n^2}{4R} = n\lambda$$

$$\text{or } D_n = \sqrt{4nR\lambda}$$

$$\text{or } D_n = \sqrt{4R\lambda} \sqrt{n}$$

$$\text{or } D_n \propto \sqrt{n}$$

Thus, the diameters of the dark rings are proportional to the square root of the natural number.

$$5. \quad \frac{(D_n)_{\text{air}}^2}{(D_n)_{\text{liquid}}^2} = \mu$$

$$\text{or } \frac{D_{\text{liquid}}}{D_{\text{air}}} = \frac{1}{\sqrt{\mu}} = \frac{1}{\sqrt{1.33}} = 0.867$$

The rings are contracted to 0.867 their previous diameters.

Terminal Questions

1. The condition for an interference maximum in the light reflected normally from an oil film of thickness t is

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda; \text{ where } n = 0, 1, 2, \dots$$

and that for an interference minimum is

$$2\mu t = n\lambda; \text{ where } n = 1, 2, 3, \dots$$

Here $\mu = 1.33$. Now there is a maximum for $\lambda = 6000 \text{ \AA}$.

We can write

$$2 \times 1.33 \times t = \left(n + \frac{1}{2}\right) 6000 \text{ \AA} \quad (\text{i})$$

$$2 \times 1.33 \times t = (n + 1) 4500 \text{ \AA} \quad (\text{ii})$$

In view of Eq. (i) we have taken the integer $(n + 1)$ rather than n in Eq. (ii).

Comparing Eq. (i) and (ii), we get

$$\left(n + \frac{1}{2}\right) 6000 = (n + 1) 4500$$

$$\therefore n = 1.$$

Substituting $n = 1$ in Eq. (i), we get

$$2 \times 1.33 \times t = \frac{3}{2} \times 6000 \text{ \AA}$$

$$\therefore t = \frac{3 \times 6000}{2 \times 2 \times 1.33} = 3383 \text{ \AA}$$

2. The condition of destructive interference in light reflected from a film is

$$2\mu t \cos r = n\lambda$$

Suppose the film thickness changes over this length by Δt . Let n be the order of the dark fringe appearing at one end of the film. The order of the dark fringe at the other end will be $(n + 9)$. We, therefore, have

$$2\mu t \cos r = n\lambda,$$

$$\text{and } 2\mu(t + \Delta t) \cos r = (n + 9)\lambda$$

Subtracting, we get

$$2\mu(\Delta t) \cos r = 9\lambda$$

$$\therefore t = \frac{9\lambda}{2\mu \cos r}$$

If the fringes are seen normally, then $\cos r = 1$.

$$\begin{aligned} \therefore t &= \frac{9\lambda}{2\mu} = \frac{9 \times 6300}{2 \times 1.5} = 18900 \text{ \AA} \\ &= 1.89 \times 10^{-4} \text{ cm} \end{aligned}$$

3. Let t be the thickness of the wire and ℓ the length of the wedge, as shown in Fig. 7.14. The wedge angle is

$$\theta = \frac{t}{\ell} \text{ radian.}$$

$$\text{Now, fringe-width } \beta = \frac{\lambda}{2\theta}.$$

Putting value of θ from above we get

$$\beta = \frac{\ell\lambda}{2t}.$$

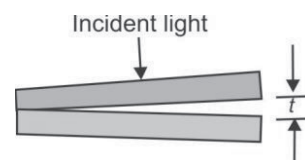


Fig. 7.14

Since N fringes are seen, $\ell = N\beta$. Thus

$$\beta = \frac{N\beta\lambda}{2t}$$

$$\therefore N = \frac{2t}{\lambda}$$

But $\lambda = 6800 \text{ \AA} = 6800 \times 10^{-8} \text{ cm}$ and $t = 0.048 \text{ mm} = 0.0048 \text{ cm}$

$$\therefore N = \frac{2 \times 0.0048}{6800 \times 10^{-8}} = 141.$$

4. i) The diameter D_n of the n th bright ring is given by

$$D_n^2 = \frac{2(2n+1)\lambda R}{\mu}$$

$$\therefore \mu = \frac{2(2n+1)\lambda R}{D_n^2}$$

Here $n = 4$, $\lambda = 5895 \times 10^{-8} \text{ cm}$, $R = 100 \text{ cm}$ and $D_n = 0.3 \text{ cm}$

$$\therefore \mu = \frac{2(8+1) \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.18$$

ii) The diameter of the n th dark ring is given by

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$\therefore \mu = \frac{4n\lambda R}{D_n^2} = \frac{4 \times 5 \times 5895 \times 10^{-8} \times 100}{(0.3)^2} = 1.31.$$

5. $D_n^2 = 4nR\lambda$

where D_n = diameter of n th ring, R = the radius of curved surface and λ = the wavelength of light.

If D_n and D_{n+1} be two diameters,

$$D_n^2 = 4nR\lambda_1 \quad (i)$$

$$D_{n+1}^2 = 4(n+1)R\lambda_2$$

But $D_n = D_{n+1}$

$$\therefore 4nR\lambda_1 = 4(n+1)R\lambda_2$$

or $4nR(\lambda_1 - \lambda_2) = 4R\lambda_2$

or $n = \frac{4R\lambda_2}{4R(\lambda_1 - \lambda_2)}$

$$= \frac{\lambda_2}{(\lambda_1 - \lambda_2)}$$

$$= \frac{4.5 \times 10^{-5}}{(6 - 4.5) \times 10^{-5}} = 3$$

Putting $n = 3$ in (i)

$$D_3 = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$= 648 \times 10^{-4}$$

$$= 25.45 \times 10^{-2} \text{ cm}$$



This is the photo of LIGO (Laser Interferometer Gravitational-Wave Observatory) India being set up in India. It is a large-scale physics experiment and observatory to detect cosmic gravitational waves. The working principle of interferometer is at the core of the design and functioning of LIGO. (Source: wikimedia.org)

MICHELSON INTERFEROMETER

Structure

- | | | | |
|-----|--|-----|--|
| 8.1 | Introduction | 8.5 | Applications of Michelson Interferometer |
| | Expected Learning Outcomes | | Wavelength of Light |
| 8.2 | Working of Michelson Interferometer | | Difference in Wavelengths |
| 8.3 | Forms of Fringes | | Refractive Index of a Thin Plate |
| | Circular Fringes | 8.6 | Summary |
| | Localised Fringes (Straight Fringes) | 8.7 | Terminal Questions |
| | White Light Fringes | 8.8 | Solutions and Answers |
| 8.4 | Adjustment of Michelson Interferometer | | |

STUDY GUIDE

In this unit, you will study the construction, working principle and applications of Michelson interferometer. An interferometer is a scientific instrument designed to carry out measurements with extraordinary precision. The Michelson interferometer is the first interferometer designed by Albert A. Michelson in the year 1880 which demonstrated that the principle of interference of light can be employed as a highly precise measurement tool. The basic principle behind the working of interferometer is the interference of light waves. Thus, while discussing Michelson interferometer, we will refer to the concepts and results of Unit 6 and 7 of this course. You should, therefore, revise those units before studying this one. Further, using an interferometer, one can carry out precise determination of the characteristics parameters of the interfering waves themselves and also of the materials with which light waves interact in the process of producing interference fringe patterns. All these measurements are based on the careful study and measurement of the fringe patterns. So, while studying this unit, you should focus of the nature of fringes obtained under different adjustments of the interferometer.

You are advised to solve SAQs and TQs of your own to check your understanding of the concepts discussed in the unit.

“The nature of light is a subject of no material importance to the concerns of life or to the practice of the arts, but it is in many other respects extremely interesting.”

**Thomas
Young**

8.1 INTRODUCTION

In the previous two units, you have studied about interference of light. You know that when two coherent beams of light superpose, they give rise to an interference pattern comprising bright and dark fringes. You also know that coherent beams of light can be obtained by either of the two methods: division of wavefront and division of amplitude. In the famous Young's double slit experiment, the two coherent sources of light were obtained by division of wavefront.

Young's double slit experiment was important experimental evidence in favour of the wave theory of light. In addition, the findings of this experiment generated a considerable activity in the scientific community to explore the possibility of using the interference phenomenon for some practical applications. This led to the development of a scientific instrument called interferometer which is a powerful tool for precise measurements. The measurements which can be done by an interferometer may pertain to the characteristics such as wavelength of the interfering waves themselves or to the characteristics such as refractive index of the medium with which the interfering light waves interact in the process of producing the interference fringe pattern. **The interferometer designed to exploit the interference of light and the fringe patterns that result from optical path differences is called an optical interferometer.** In this unit, we will discuss the working and applications of Michelson interferometer which is an optical interferometer.

Michelson interferometer was developed by Albert A. Michelson in 1880. This was the first interferometer and it demonstrated that the interference of light can be employed as a highly precise measurement tool. Apart from being the first interferometer, Michelson interferometer was the main scientific instrument in the famous Michelson-Morley experiment. You may be aware that this experiment disproved the existence of the all pervading medium called "the ether" and helped Einstein in developing the theory of relativity.

In order to achieve interference between two coherent beams of light, an interferometer divides an initial beam into two or more parts that travel diverse optical paths and then superpose to produce an interference pattern. One criterion for broadly classifying interferometers distinguishes the manner in which the initial beam is split into two coherent beams. Wavefront division interferometer divide an incident wavefront of a coherent beam of light, as in the case of Young's double slit, Lloyd's mirror or Fresnel's biprism arrangement discussed in Unit 6. Amplitude-division interferometers, instead, use some type of beam-splitter that divides the initial beam into two parts. The Michelson interferometer is of this type. Usually, the beam splitting is managed by a semi-reflecting metallic film. In this interferometer, the two interfering beams are widely separated, and the path difference between them can be varied at will by moving the mirror or by introducing a refracting material in one of the beams. Corresponding to these two ways of changing

the optical path, there are two important applications of this interferometer, which you will study in this unit.

We begin this unit with a description of the Michelson Interferometer in Sec. 8.2. You will learn about various components of the instrument and their functions in obtain coherent beams of light and the observable interference pattern. You will also find that, depending upon the position and alignment of the two mirrors and the source of light, fringes of different forms are obtained. These forms of fringes have also been discussed in this section. Further, different components of the interferometer needs to be finely adjusted to obtain desired type of fringe. You will learn about these adjustments in Sec. 8.3. In Sec. 8.4, you will learn how Michelson interferometer is used to determine the wavelength of light, difference in wavelengths and the refractive index of a transparent material.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ describe the functions of various components of Michelson interferometer;
- ❖ understand how interference fringe patterns are formed in a Michelson interferometer;
- ❖ explain how Michelson interferometer produces different types of fringes, viz., circular, localised (or straight) and white light fringes;
- ❖ describe how Michelson interferometer is used for the determination of the wavelength and wavelength difference of light waves;
- ❖ explain the use of Michelson interferometer for determining the refractive index of a material; and
- ❖ solve numerical problems based on the concepts discussed in this unit.

8.2 WORKING OF MICHELSON INTERFEROMETER

Michelson interferometer is a precision instrument to obtain interference fringes of various shapes which have a number of applications in optics. The interference fringes are obtained in this interferometer by splitting a beam of light into two parts and then recombine them after they have travelled different optical paths. The method of division of amplitude is used in this interferometer for obtaining two coherent beams of light with the help of an arrangement of mirrors and a beam splitter.

The instrument comprises a collection of glass plates and mirrors in a particular configuration to obtain the desired fringe pattern. Let us now learn about its components and their configuration.

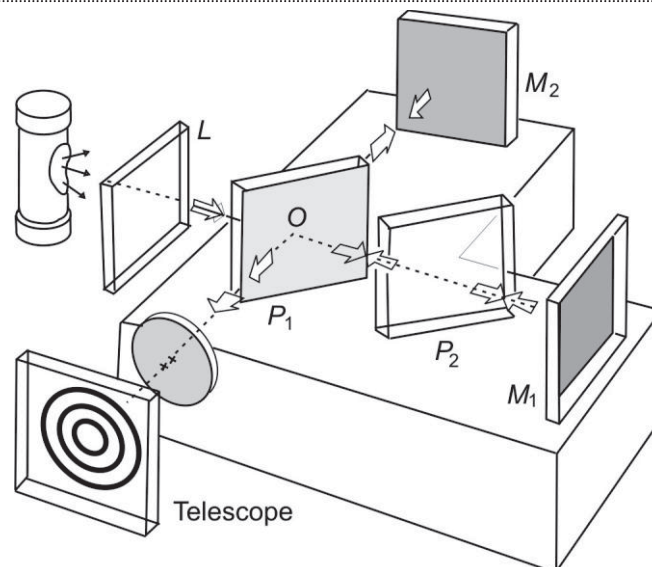


Fig. 8.1: A schematic diagram of Michelson interferometer.

Refer to Fig. 8.1 which depicts a schematic diagram of Michelson interferometer. Its main optical components are two plane mirrors M_1 and M_2 and two similar optically-plane parallel glass plates P_1 and P_2 . The plane mirrors M_1 and M_2 are silvered on their front surfaces and are mounted vertically on two arms of the interferometer at right angles to each other. To obtain fringes, the mirrors M_1 and M_2 are made exactly perpendicular to each other by means of screws. The mirror M_2 is mounted on a carriage which can be moved towards or away from the glass plate P_1 . The plates P_1 and P_2 are mounted exactly parallel to each other, and inclined at 45° to M_1 and M_2 . The surface of P_1 towards P_2 is **partially silvered**. The plate P_1 is called **beam splitter** and plate P_2 is called **compensator plate**.

Working: An extended source (e.g., a diffusing ground glass plate L illuminated by a discharge lamp) emits light waves in different directions, part of which travels to the right and falls on beam splitter P_1 . The light wave incident on P_1 is partly reflected and partly transmitted. Thus, the incident wave gets divided into two waves, viz., the transmitted wave 1 and the reflected wave 2. The waves 1 and 2 travel to M_1 and M_2 , respectively. After reflection at M_1 and M_2 , the two waves return to P_1 . Part of the wave coming from M_2 passes through P_1 and travels towards the telescope, and part of the wave coming from M_1 gets reflected by P_1 toward the telescope. Since the waves entering the telescope are derived from the same incident wave, they are coherent and hence, in a position to produce interference fringes. The interference fringes can be seen in the telescope.

In contrast to the Young double slit experiment, which uses light from two very narrow sources, Michelson interferometer uses light from an extended (e.g. broad spread out) source.

You must be eager to know the purpose of the compensator plate P_2 , because till now we have not mentioned anything about it.

Function of the compensator plate P_2 : Note that the reflection at P_1 occurs at its rear surface at point O , as shown in Fig. 8.1. Thus, the light reflected at M_2 will pass through P_1 three times while the light reflected at M_1 will pass through it only once. **Thus, the paths of waves 1 and 2 in glass (plate P_1) are not equal if P_2 is not used.** However, each wave will pass through the same thickness of glass if a compensator plate P_2 , of the same thickness and

inclination as P_1 , is inserted in the path of wave 1. **The compensator plate P_2 is an exact duplicate of P_1 with the exception that it is not partially silvered.** With the compensator plate in place, any optical path difference which arises between the two waves 1 and 2 when they travel towards the telescope is due to the actual path difference.

The superposition of light waves reflected by mirrors M_1 and M_2 gives rise to interference fringe pattern in Michelson interferometer. The nature and form of fringes depend on various factors. Let us now learn about it.

8.3 FORMS OF FRINGES

In Michelson interferometer, we can obtain circular as well as localised (straight) fringes depending upon the mutual alignment of mirrors M_1 and M_2 . To understand how these fringes are formed, refer to the Fig. 8.2, where these components of the interferometer are represented somewhat differently. An observer at the position of the telescope will, simultaneously, see both mirrors M_1 and M_2 along with the source L , formed by reflection in the partially silvered surface of the glass plate P_1 . **Accordingly, we can redraw the interferometer as if all the elements were in a straight line as shown in Fig. 8.2.** Note that M'_1 corresponds to the image of mirror M_1 formed by reflection at the silvered surface of the glass plate P_1 so that $OM_1 = OM'_1$. Depending on the positions of the mirrors, image M'_1 may be in front of, behind or exactly coincident with mirror M_2 . The surfaces L_1 and L_2 are images of the source L in mirrors M_1 and M_2 , respectively.

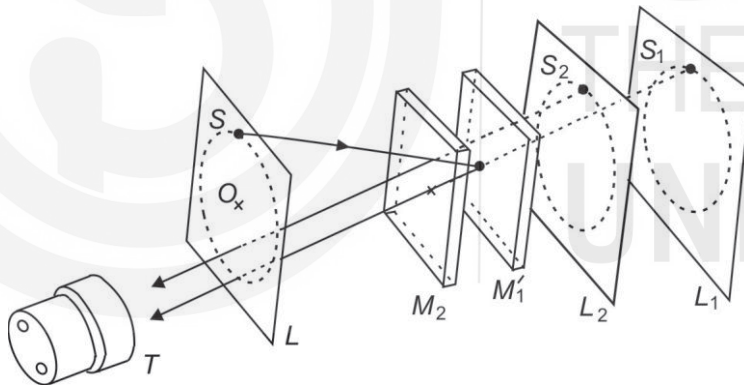


Fig. 8.2: The components of Michelson interferometer shown along a straight line.

In Fig. 8.1, if we consider a single point S on the extended source L , emitting light in all directions, then on reaching O , it gets split, and thereafter its segments get reflected by M_1 and M_2 . In Fig. 8.2, we represent this by reflecting the ray off both M'_1 and M_2 . Thus, the interference fringes may be regarded to be formed by light reflected from the surface of M'_1 and M_2 . Note that S_1 and S_2 on L_1 and L_2 , respectively, act as coherent point sources, because to an observer at telescope T , the two reflected rays will appear to have come from the image points S_1 and S_2 . **The mirror M_2 and the virtual image M'_1 of M_1 play the same role as the two surfaces of a thin film.**

Thus, we should expect same sort of interference fringes as we obtain due to light reflected by thin films as discussed in Unit 7.

Now, let us discuss, qualitatively, the conditions under which various types of fringes, viz., circular fringes, localised fringes and white light fringes are formed in a Michelson interferometer.

8.3.1 Circular Fringes

These fringes are observed when the mirror M_1 is exactly perpendicular to the mirror M_2 . Keeping the two mirrors perpendicular to each other, the distance of the mirrors M_1 and M_2 from the plate P_1 can be varied. Let us now consider the various possible positions of the mirrors M_1 and M_2 , and, eventually, see how it gives rise to circular fringes.

- i) Note that, if the two mirrors have the same axial distance from the rear face of P_1 , and if they are perpendicular to each other, the image M'_1 of mirror M_1 will be coincident with M_2 in Fig. 8.2. At the coincidence position, the two paths are of equal length. Thus, we expect the light waves to reinforce each other (that is, give rise to constructive interference) and to form a maximum. But this is not so. Can you guess why? You know that when light is reflected from a denser medium, there is a phase change of π . In the present case, light coming from mirror M_1 and getting reflected by P_1 suffers such a phase change (Fig. 8.1). However, the light coming from M_2 does not suffer any such phase change because no phase change occurs on transmission or refraction. Look again at Fig. 8.1 and note that it is the light that comes from M_1 and goes to the observer that is reflected, air-to-glass, at O , and undergoes the phase change of π . This means that at the coincidence position, the two light waves will reach the telescope out of phase and there will be a minimum: **the centre of the field of view in the telescope will be dark.**

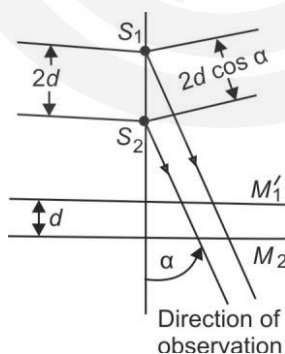


Fig. 8.3: Looking off-axis into the Michelson interferometer.

- ii) Now, let one of the mirrors be moved. If the mirror is moved through a distance of quarter of wavelength, $d = \lambda/4$, the path difference between the two waves changes by $\lambda/2$. It is so because if d is separation between M'_1 and M_2 , then $2d$ is the separation between S_1 and S_2 . Thus, the two waves getting out of phase by 180° , the phase change compensates, and we have a maximum. So, the dark field of view in the telescope turns bright. As the mirror is moved an additional distance of $\lambda/4$, the bright field of view again turns dark. Thus, we have

$$2d = n\lambda, \quad \text{where, } n = 0, 1, 2, \dots \quad (8.1)$$

Eq. (8.1) is called **Michelson's interferometer equation**. Note that this equation holds for the situation when light waves are reflected normal to the mirror.

- iii) Next, let us consider the superposition of light waves reflected by the mirrors at some arbitrary angle. Let us assume that we look obliquely into the interferometer and that our line of sight makes an angle α with the axis (Fig. 8.3). Ordinarily, when the two planes M'_1 and M_2 are at a distance d apart, the two virtual images, S_1 and S_2 are separated by $2d$. But, for oblique incidence, as we see from Fig. 8.3, the path difference between the light coming from S_1 and S_2 along the lines of sight becomes less and instead of Eq. (8.1), we get

$$2d \cos \alpha = n\lambda; \quad \text{where, } n = 0, 1, 2, \dots \quad (8.2)$$

If the mirror separation is d , wavelength λ and angle α is constant for a given order n of fringe and the contour of such points in the plane of observation forms a ring. Therefore, the maxima or minima will lie in the form of circles about the foot of the perpendicular from the eye to the mirrors. These circular fringes will look like the ones shown in Fig. 8.4. Fringes of this kind, where parallel beams are brought to interfere with a phase difference determined by the angle of inclination α , are referred to as **fringes of equal inclination**. These fringes are also known as **Haidinger fringes**. Note that these fringes differ from the fringes of equal inclination discussed in Unit 7 because here there are no multiple reflections.



Fig. 8.4: Circular fringes observed in a Michelson interferometer.

Now, let us see how does the separation d between the mirrors of the interferometer impact the nature of the circular fringes. When the separation between M_2 and M'_1 is a few centimetres, the fringe system will have the general appearance as shown in Fig. 8.5a with the rings very closely spaced. If M_2 is now moved slowly toward M'_1 , so that d is decreased, Eq. (8.2) shows that a given ring, characterized by a given value of the order n , must decrease its radius, because the product $2d \cos \alpha$ must remain constant. The rings, therefore, shrink and vanish at the centre. Thus, if the decrease in the value of d is such that $2d$ decreases by λ , (or d by $\lambda/2$), a ring will disappear. This follows from the fact that at the centre, $\cos \alpha = 1$, so that Eq. (8.2) becomes

$$2d = n\lambda$$

which is Eq.(8.1).

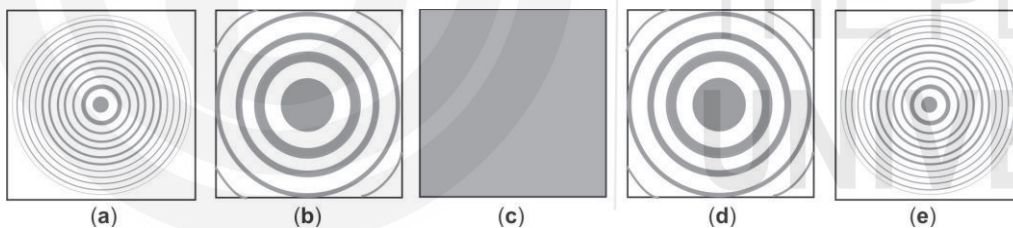


Fig. 8.5: Appearance of the circular fringes observed in the Michelson interferometer for different values of separation between the mirrors; note that the path difference increases outward, in both directions, from the centre.

Above equation indicates that to change n by unity, d must change by $\lambda/2$. Now as M_2 approaches M'_1 , d decreases further and the angle α must decrease more rapidly to keep n , the order of the fringe fixed. As a result, the fringes (or the rings) become more widely spaced as indicated in Fig. 8.5b. Finally, when d decreases zero, that is, when M_2 and M'_1 are exactly coincident, the central fringe spreads out to cover the whole field of view, as shown in Fig. 8.5c. This happens because under this condition, the path difference is zero for all angles of incidence. If the mirror M_2 is moved still farther, it effectively passes through M'_1 , and new widely spaced fringes appear, growing out from the centre. These will gradually become more closely spaced, when the path difference increases, as indicated in Fig. 8.5d and 8.5e.

So, you have learnt that we observe circular fringes in Michelson interferometer if the two mirrors are perpendicular to each other. The nature of these circular fringes is determined by the separation d between the mirrors.

Another situation of interest is: What will be the nature of fringes if the two mirrors are not exactly perpendicular to each other? In such situations, we obtained what is called localised fringes. Let us now learn about it.

8.3.2 Localised Fringes (Straight Fringes)

If the mirrors M_1 and M_2 are not exactly perpendicular to each other, then mirror M_2 and the image M'_1 of mirror M_1 are not exactly parallel. As a result, the air film between M'_1 and M_2 is wedge-shaped, as shown in Fig. 8.6.

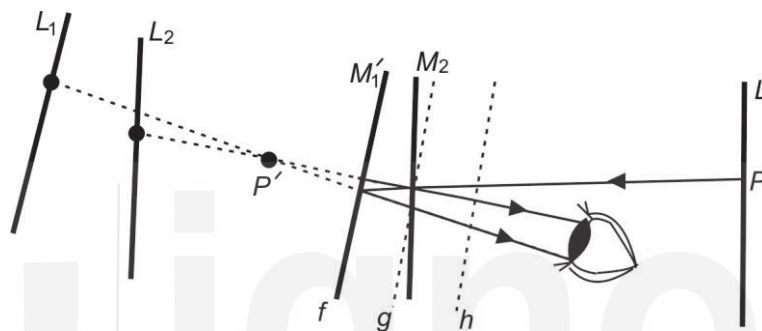


Fig. 8.6: Schematic diagram for observing fringes with inclined mirrors in the Michelson interferometer.

Note from Fig. 8.6 that when one of the mirrors is tilted, the ray of light from point P on the source is reflected by the mirrors M'_1 and M_2 in such a manner that they appear to diverge from point P' near the mirrors. That is, the reflected rays are no longer parallel. Also note that for various positions of P on the extended source, the path difference between the two reflected rays remains constant, but the distance of P' from mirrors changes. In this situation, we obtain localised fringes which are, practically, straight line fringes. The eye needs to be focussed at point P' to observe the localised fringes. The localised fringes are practically straight because the variation of the path difference between the light rays is now due primarily to the variation of the thickness of the “air film” between the mirrors. And, you know that in a wedge-shaped film, the locus of the points of equal thickness is a straight line, parallel to the edge of the wedge. (Refer to Unit 7 where we have discussed the nature of fringes produced by wedge-shaped films in detail.) If the distance d between mirrors has an appreciable value, the fringes are not exactly straight. It is so because there is also some variation of the path difference with angle. They are, in general, curved and are always convex toward the thin edge of the wedge.

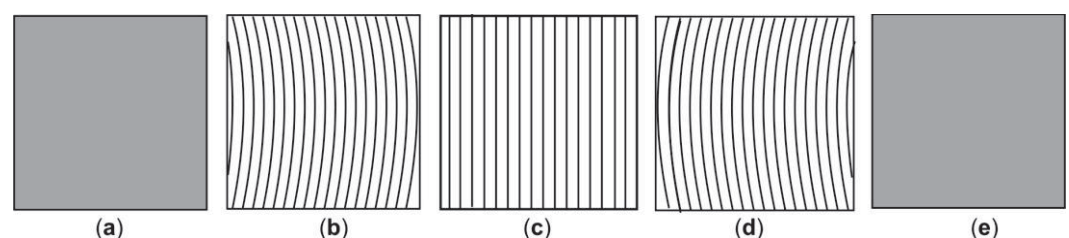


Fig. 8.7: Localised (straight) fringes in Michelson interferometer.

Interference fringes obtained in Michelson interferometer are classified into two broad categories namely non-localised and localised fringes. Non-localised fringes, such as circular fringes in Michelson interferometer, are real and exist everywhere within an extended region of space; that is, the fringe pattern is not restricted to some small region of space. On the other hand, localised fringes can be seen clearly only over a particular surface. The surface on which localised fringes can be observed may be near the thin air film formed by mirrors or at infinity.

Thus, with a certain value of d , we might observe fringes shaped like those shown in Fig. 8.7b. M'_1 could then be in position such as f of Fig. 8.6. If the separation of the mirrors is decreased, the fringes will move to the left across the field, such that a new fringe crossing the centre each time d changes by $\lambda/2$. As we approach the zero path difference, the fringes become straighter until the point is reached where M_2 actually intersects M'_1 (position of M'_1 shown by dotted line g in Fig. 8.6), when they are perfectly straight, as shown in Fig. 8.7c. Beyond this point, as the separation between the mirrors again begin to increase in opposite direction, the fringes begin to curve in the opposite direction, as shown in Fig. 8.7d. The blank fields shown in Figs. 8.7a and 8.7e indicate that this type of fringe cannot be observed for large path differences. As the principle variation of path difference results from a change of the thickness d , these fringes are called **fringes of equal thickness**.

Note that we have considered a monochromatic light source while discussing formation of circular and localised fringes. You would like to know: Can we observe fringes if a white light source are used in Michelson interferometer? Let us now find answer of this question.

8.3.3 White Light Fringes

If a source of white light is used in Michelson interferometer, no fringes will be seen at all except for a path difference so small that it does not exceed a few wavelengths. In observing these fringes, the mirrors are tilted slightly as for localized fringes, and the position of M_2 is found where it intersects M'_1 . Under this configuration of mirrors, a fringe pattern is observed having a central dark fringe, bordered on either side by 8 or 10 coloured fringes. This position is normally difficult to find, using white light source. Thus, to locate this position for the white light, we first locate this position approximately for monochromatic light. Then, replacing the monochromatic source with white light and slowly moving M_1 through this region, these fringes are brought into the field of view.

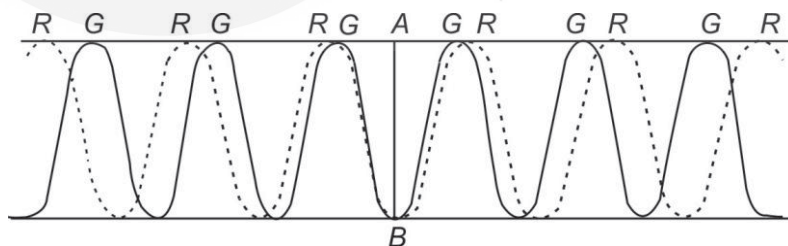


Fig. 8.8: The intensity distribution in fringes produced by white light with a dark fringe AB at the centre.

You may ask: **Why only a few fringes are observed with white light?** You know that white light contains all wavelengths between 400 nm and 750 nm. Each constituent wavelength of white light will produce its own fringe pattern. However, for the higher wavelengths, the fringes are more widely spaced. The fringes produced by different colours will only coincide for $d = 0$, as indicated by line AB in Fig. 8.8. The solid curve represents the intensity distribution in the fringes for the green light (G), and the broken curve for the red light (R). Clearly, only the central fringe will be uncoloured, and the fringes of different

colours will begin to separate at once on either side. After 8 or 10 fringes, so many colours are present at a given point that the resultant colour is essentially white. White light fringes are important in the Michelson interferometer because they are used for locating the position of zero path difference.

Visibility of Fringes

When the bright and dark fringes in an interference pattern have good contrast, we say that the visibility of fringes is high. Mathematically, the visibility of fringes is defined as

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where, $I_{\max}$ and $I_{\min}$ are the intensities of the bright and dark fringes, respectively. You may recall from Unit 6 that the intensity of the bright or dark fringe depends on the amplitudes of the interfering light waves. If the amplitudes are same, the visibility of the fringe pattern will be unity (maximum) because, in that case, $I_{\min} = 0$.

In Michelson interferometer with monochromatic source, the circular fringes have good visibility. The visibility of fringes gives us very useful information when the source of light is not monochromatic. To understand this, let us assume that the source of light used in Michelson interferometer emits light of two different wavelengths (such as a sodium vapour lamp). Since the condition for bright and dark fringes depend on the path difference between the two light waves being integral (or half integral) multiple of the wavelength of the light, both wavelength emitted by the source will form their own fringe patterns. It is found that, for certain values of separation d between the two mirrors of the interferometer, the path difference $2d \cos \alpha$ is an integral multiple of the wavelength for both the wavelengths emitted by the source. Therefore, the two fringe patterns coincide with each other for these values of d over an appreciable range of angles α . Thus, the result of this coincidence of fringe patterns is fringes of good visibility.

On the other hand, for certain intermediate values of d , the fringe pattern due to one wavelength will be exactly complementary to the fringe pattern due to the other wavelength. That is, the bright fringes due to one wavelength coincide with the dark fringes due to the other. As a result, for such values of d , fringe visibility becomes zero. So, by gradually increasing d from zero and noting down the values of d corresponding to the maximum and minimum visibility of the fringes, one can estimate the values of the two wavelengths emitted by the source.

Based on the analysis of the change in the visibility of fringes, Michelson interferometer is used as a tool for determining the wavelength(s) emitted by a non-monochromatic source. That is, this instrument can be used for spectral analysis. However, we shall not go into the details of this application of Michelson interferometer as it is beyond the scope of this course.

So far, you have learnt about the conditions under which different forms of fringes are produced in Michelson interferometer. Before proceeding further, let us recapitulate what you have learnt in this section.

Recap

- In Michelson interferometer, interference fringes are produced due to superposition of light waves emitted from two coherent sources obtained by division of amplitude. A light wave is split into two coherent light waves and when they superpose after reflection from mirrors, interference fringes are produced.
- When the two mirrors of the interferometer are exactly perpendicular to each other, circular fringes are formed. For oblique incidence, Michelson interferometer equation is

$$2d \cos \alpha = n\lambda; \quad n = 0, 1, 2, \dots$$

- When the two mirrors of the interferometer are not exactly perpendicular, a wedge-shaped air film is formed and, under this condition, we obtain localised fringes. For small separation d between the mirrors, the localised fringes are straight line fringes.
- When white light source is used in Michelson interferometer, we obtain a fringe pattern having central dark band and few coloured fringes on both sides.

Michelson interferometer is a very sensitive instrument because the fringe patterns observed in it are dependent upon the alignment of the two mirrors and the separation between them. You have learnt that the form of the fringes changes if there is slight change in the alignment of mirrors or when the separation d is changed by a distance of the order of the wavelength of light used. So, the mirrors are to be moved by distances of the order of nano-metre, the wavelength of the light typically used as source. Therefore, for adjusting different optical components of Michelson interferometer, one needs to be very attentive about certain important points. Let us now briefly discuss some of them.

8.4 ADJUSTMENT OF MICHELSON INTERFEROMETER

For carrying out measurements using Michelson interferometer, adjustment of its mirrors M_1 and M_2 (refer Fig. 8.1) is most crucial. These mirrors are provided with levelling screws to adjust their alignments. In addition, one of the mirrors, say, M_2 is also provided with arrangement of screws which is used to move it along a straight line towards or away from the beam splitter, P_1 . As you have learnt in the previous section, we obtain different types of fringes depending upon mutual alignment of the two mirrors and the path difference between the superposing light waves reflected by the mirrors.

Let us now learn how these adjustments are done in Michelson interferometer to achieve desired outcome.

- For Localising Fringes:** The distance of the mirrors M_1 and M_2 from the silvered surface of P_1 are first made as nearly equal as possible by moving the movable mirror M_2 . Now, a pin-hole is placed between the

lens and the plate P_1 (Fig. 8.9). If M_1 is not perpendicular to M_2 , four images of the pin-hole are obtained, two by reflection at the semi-silvered surface of P_1 and then by mirrors M_1 and M_2 and the other two by reflection at the other (un-silvered) surface of P_1 .

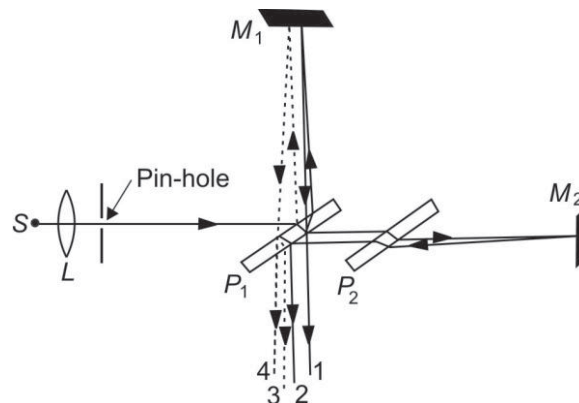


Fig. 8.9: Adjustment of Michelson interferometer.

The former pair is, naturally, brighter than the latter. The small screws at the back of the mirror, M_1 , are then adjusted until the two bright images appear to coincide. The pin-hole is now removed. The air-film between M_2 and M_1 would be wedge-shaped, and the localised fringes would appear.

- b) **For White Light Localised Fringes:** First, the localised fringes with monochromatic light are obtained. The mirror M_2 is then moved until the fringes become straight. Now, monochromatic light source is replaced by white light source. M_2 is further moved in the same direction until the central achromatic fringe is obtained in the field of view.
- c) **For Circular Fringes:** After localised fringes are obtained with a wedge shaped air film between mirrors (Fig. 8.6), we now need to adjust mirrors M_1' and M_2 exactly parallel to each other for obtaining circular fringes. To do that the screws of M_1 are adjusted so that the spacing between the localised fringes increases. This happens when the angle of the wedge decreases. If this adjustment be continued, at one stage, the angle of the wedge will become zero, and the air film will be of constant thickness. At this stage, circular fringes will appear.

So far, you have learnt about the construction and working of Michelson interferometer, the various types of fringes formed under different conditions and how to make adjustment of its different components to observe these fringes. We hope that you now have a fairly good idea about this interferometer and its working. Now, you are in a position to study some of the applications of this interferometer. Let us discuss how Michelson interferometer is used for precise determination of some physical quantities.

8.5 APPLICATIONS OF MICHELSON INTERFEROMETER

Michelson interferometer is a scientific instrument which enables us to carry out very precise measurements for the determination of the values of physical quantities. The first important use of this interferometer was done in the

famous Michelson-Morley experiment in 1887 which disproved the then widely held belief that there exists an all pervading medium called 'aether'. In the contemporary times, a large configuration of optical components similar to Michelson interferometer is being used in Laser Interferometer Gravitation-wave Observatory (LIGO). At this observatory, the interferometer enabled scientists to detect gravitation waves for the first time in the year 2015. The experimental detection of gravitational wave is a strong evidence in support of the general theory of relativity.

Apart from being a part of the above pioneering experiments, Michelson interferometer is also used for precise determination of the values of physical quantities such as wavelength of light and refractive index of a material. In the following, we discuss how Michelson interferometer is used for determination of the wavelength of light, difference in wavelength of the non-monochromatic light and refractive index of transparent materials.

8.5.1 Wavelength of Light

For determining the wavelength of the monochromatic light used as the source in Michelson interferometer, we first adjust the interferometer for circular fringes as described in Sec. 8.4. You know that circular fringes are obtained when mirrors M_1 and M_2 are perpendicular to each other. Now, we adjust the position of mirror M_2 to obtain a bright spot at the centre of the field of view. If d be the thickness of the air film between M'_1 and M_2 (see Fig. 8.2) and n , the order of the spot obtained, we have from Eq. (8.2):

$$2d \cos \alpha = n\lambda \quad (8.3)$$

But, at the centre, $\alpha = 0$; so that $\cos \alpha = 1$. Therefore, we get

$$2d = n\lambda \quad (8.4)$$

If M_2 be moved away now from M'_1 by a distance $\lambda/2$, the path difference $2d$ increases by λ . Therefore, $(n+1)^{\text{th}}$ bright spot replaces the n^{th} bright spot at the centre (see Sec. 8.3.1). Each time M_2 moves through a distance equal to $\lambda/2$, next bright spot appears at the centre.

Suppose, during the movement of M_2 through a distance x , N new fringes appear at the centre of the field of view. Then, we can write

$$x = N \frac{\lambda}{2} \Rightarrow \lambda = \frac{2x}{N} \quad (8.5)$$

Thus, from Eq. (8.5), it is evident that by measuring (with the micrometer) the distance x through which M_2 moved and counting the number N of fringes that appear at the centre, the value of wavelength, λ can be determined.

The determination of λ by this method is very accurate, because x can be measured to an accuracy of 10^{-4}mm , and the value of N can be sufficiently increased, because the circular fringes can be obtained up to large values of path difference.

Now, you should go through an example to concretise what you have learnt in this section.

EXAMPLE 8.1: DETERMINATION OF WAVELENGTH OF LIGHT

In a Michelson interferometer, circular fringes with a central dark disk surrounded by bright and dark rings are observed. If the distance of one mirror from the beam splitter is 2 cm larger than the other mirror, calculate the order of the central disk. Take the wavelength of the light used as 500 nm.

SOLUTION ■ We know that Michelson interferometer equation is

$$2d = mn\lambda$$

where, n is the order of the central disk. As per the problem, $d = 2 \text{ cm} = 0.02 \text{ m}$ and $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$. So, we can write

$$n = \frac{2d}{\lambda} = \frac{2 \times (0.02 \text{ m})}{500 \times 10^{-9} \text{ m}} = 80,000$$

SAQ 1 - Determining the wavelength of light

When the movable mirror of Michelson interferometer is moved through a distance 0.0589 mm, a shift of 200 fringes is observed. What is the wavelength of light used?

8.5.2 Difference in Wavelengths

Suppose that we have a non-monochromatic source of light which emit two close wavelengths λ_1 and λ_2 . An example of such a source is sodium vapour lamp which emits two closely spaced wavelengths called sodium D-lines. If we wish to determine the difference in the value of these two wavelengths, we can do that using a Michelson interferometer. We discussed the nature of fringes and their evolution in the previous section in the context of the visibility of fringes. When the source of light emits two wavelengths, say λ_1 and λ_2 whose values are very close to each other (such as sodium D_1 and D_2 lines), each wavelength produces its own system of circular fringes in the form of bright and dark rings. Let us assume that $\lambda_1 > \lambda_2$. When the thickness of the film between M_2 and M'_1 (refer to Fig. 8.2) is small, the bright and dark rings due to λ_1 and λ_2 almost coincide, since λ_1 and λ_2 are nearly equal.

Now, the mirror M_2 is moved away in such a manner that the value of d increases. Then, due to different spacing between the rings of λ_1 and λ_2 , the rings of λ_1 are gradually separated from those of λ_2 . When the thickness of the air-film between the mirrors (due to further increase in d) becomes such that dark rings of λ_1 coincides with bright rings of λ_2 , the rings will have maximum indistinctness. Due to closeness of λ_1 and λ_2 , the dark rings due to λ_1 will practically coincide with bright rings due to λ_2 in the entire field of view.

The mirror M_2 is moved further away through a distance, say, x until the rings, after becoming most distinct, once again become most indistinct. Clearly, during this movement of mirror M_2 between two successive maximum indistinctness, n fringes of λ_1 and $(n + 1)$ fringes of λ_2 have appeared at the centre (because, then the dark rings of λ_1 will again coincide with the bright rings of λ_2).

Since the movement of the mirror M_2 by a distance equal to one-half wavelength results in the appearance of one new fringe at the centre, we have

$$x = n \frac{\lambda_1}{2} = (n + 1) \frac{\lambda_2}{2}$$

or
$$n = \frac{2x}{\lambda_1} \text{ and } (n + 1) = \frac{2x}{\lambda_2}$$

Thus, we can write

$$\frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = 1 \Rightarrow \frac{2x(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2} = 1$$

or
$$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x}$$

Since λ_1 and λ_2 are close together, we can replace $\lambda_1 \lambda_2$ by λ^2 where, λ is the mean of λ_1 and λ_2 . Thus, we can write the above expression as

$$\lambda_1 - \lambda_2 = \Delta\lambda = \frac{\lambda^2}{2x} \quad (8.6)$$

Thus if we measure the distance x moved by M_2 between two consecutive positions of disappearance (or maximum indistinctness) of the fringe pattern and the mean wavelength is known, we can determine the difference $(\lambda_1 - \lambda_2)$ between the wavelengths.

Before proceeding further, you should solve an SAQ.

SAQ 2 - Determining difference in wavelength

In a Michelson interferometer experiment with a non-monochromatic source, the readings of the movable mirror for a pair of consecutive maximum indistinctness were found to be 0.6939 mm and 0.9884 mm. If the mean wavelength of the two component wavelengths emitted by the source of light be 5893 Å, calculate the difference between the component wavelengths.

8.5.3 Refractive Index of a Thin Plate

To determine the refractive index of a (transparent) material, we place a thin plate of thickness t of the material into the path of one of the interfering beams in the interferometer. As a result, the optical path for this beam is increased. Can you guess why does it happen? It is because of the fact that light travels more slowly in a material medium compared to air.

If the refractive index of the material of the thin plate is μ , then the optical path through the plate is μt , whereas it was practically t through the corresponding thickness of air ($\mu = 1$). Thus, the increase in the optical path due to insertion of the thin plate is $(\mu - 1)t$. The change in the optical path of one of the beams causes a change in the path difference between the two interfering beams of the interferometer and the fringe pattern shifts.

In practice, the insertion of a plate of glass in one of the beams of the interferometer produces a discontinuous shift of the fringes so that the number of fringes cannot be counted. With monochromatic fringes, it is impossible to tell which fringe in the displaced set corresponds to one in the original set. And, with white light, the displacement in the fringes of different colours is very different. Thus, it is necessary to adjust the interferometer to produce straight white light fringes. After having adjusted so, the cross-wire of the telescope is set on the achromatic fringe, which is perfectly straight. The given plate is now inserted in the path of one of the interfering beams. This increases the optical path of the beam by $(\mu - 1)t$. Since the beam traverses the plate twice, an extra path difference of $2(\mu - 1)t$ is introduced between the two interfering beams. The fringes get shifted. The movable mirror M_2 is moved till the fringes are brought back to their initial positions so that the achromatic fringe again coincides with the cross wire. If the displacement of M_2 is x , then we can write

$$2x = 2(\mu - 1)t$$

$$\text{or} \quad x = (\mu - 1)t \quad (8.7)$$

Alternatively, if N be the number of fringes shifted then

$$2(\mu - 1)t = N\lambda \quad (8.8)$$

Thus, measuring the displacement x of the movable mirror, we can determine the thickness t , of the thin plate if refractive index μ of the material of the plate known, or we can determine μ if t is known.

The method describe above for thin transparent plate can also be used to determine the refractive index of a gas. The gas is taken in an evacuated tube which is placed in the path of one of the interfering beams of the interferometer, and the experiment is carried out as described above.

***SAQ 3* - Determining the thickness of a thin plate**

A Michelson interferometer is illuminated with light of wavelength 4800 Å. A transparent thin plate of glass of refractive index 1.50 is introduced normally in the path of one of the interfering beams of the interferometer. This causes 500 dark fringes to sweep across the field of view. Determine the thickness of the film.

We now summarise what you have learnt in this unit.

8.6 SUMMARY

Concept	Description
Fringes in Michelson Interferometer	■ Interference fringes are formed in Michelson interferometer due to division of amplitude. A beam of light from an extended monochromatic source is split into two beams by a beam splitter. The two beams, after getting reflected by mirrors superpose and form interference fringe pattern.
Circular fringes	■ When the mirrors M_1 and M_2 of the interferometer are perpendicular to each other, i.e., when M_1' (the image of M_1) and M_2 are parallel, the fringes due to a monochromatic source are circular. For displacement d of the movable mirror, the path difference between the superposing beams is $2d$. So, for d equal to $\lambda/4$ the spot at the centre of the field of view changes from dark to bright or from bright to dark. Michelson Interferometer equation for normal incidence is $2d = n\lambda$.
Localised fringes	■ When the mirrors of the interferometer are inclined with respect to each other, i.e., when M_1 and M_2 are not perpendicular to each other, a pattern of localised (straight) parallel fringes are obtained. For white light source, the central fringe is achromatic with coloured fringes on both sides.
Fringe shift	■ When M_1 and M_2 are parallel or inclined, any fringe shift seen in Michelson interferometer may be due to either a change in thickness of the air film or a change in refractive index.
Wavelength and fringe shift	■ As the movable mirror is displaced by $(\lambda/2)$, each fringe will move to the position previously occupied by an adjacent fringe. If N is the number of fringes that have moved past a reference point, when the mirror is moved a distance x, then

$$x = N \frac{\lambda}{2}$$

Michelson interferometer can be used for the determination of the wavelength difference of two closely spaced wavelengths and the refractive index of a transparent material.

8.7 TERMINAL QUESTIONS

1. When one mirror of a Michelson interferometer is moved by a distance of 0.0112 cm, 500 fringes are observed to cross the field of view. Calculate the wavelength of light.
2. A thin plate of a transparent material having refractive index 1.434 is inserted in the path of one beam of a Michelson interferometer. For light of wavelength 580 nm, it is observed that the fringe pattern shifts by 30 fringes. Determine the thickness of the plate.
3. When one arm of a Michelson interferometer is lengthened slightly, 150 dark fringes sweep through the field of view. If the light used has $\lambda = 480$ nm, how far was the mirror in that arm moved?

4. Circular fringes are observed in a Michelson interferometer illuminated with light of wavelength $5896 \text{ \AA}$. When the distance between the mirrors M_1 and M_2 is 0.3 cm , the central fringe is bright. Calculate the angular diameter of the 7th bright fringe.
5. A Michelson interferometer is illuminated with monochromatic light. When one of its mirrors is moved $2.2 \times 10^{-5} \text{ m}$, 90 fringes cross the field of view. Determine the wavelength of incident light.

8.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. The distance, x , moved by the mirror when N fringes cross the field of view is given by

$$x = N \frac{\lambda}{2} \Rightarrow \lambda = \frac{2x}{N}$$

Here, $x = 0.00589 \text{ cm}$ and $N = 200$.

$$\therefore \lambda = \frac{2 \times 0.00589 \text{ cm}}{200} = 0.0000589 \text{ cm} = 5890 \text{ \AA}$$

2. If x be the distance moved by the movable mirror between two consecutive positions of maximum indistinctness (or distinctness), we have difference in wavelengths from Eq. (8.6):

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

Where, λ is the mean value of λ_1 and λ_2 . As per the problem,

$$\lambda = 5893 \text{ \AA} = 5893 \times 10^{-8} \text{ cm}$$

$$\text{and } x = 0.9884 - 0.6939 = 0.2945 \text{ mm} = 0.02945 \text{ cm}$$

$$\therefore \Delta\lambda = \frac{(5893 \times 10^{-8})^2}{2 \times 0.02945} = 5.896 \times 10^{-8} \text{ cm} = 5896 \text{ \AA}$$

3. Let t be the thickness of the film. When it is put in the path of one of the interfering beams of a Michelson interferometer, an additional path difference of $2(\mu - 1)t$ is introduced. If N be the number of fringes which shift across the field of view due to the introduction of the plate, we have

$$2(\mu - 1)t = N\lambda \Rightarrow t = \frac{N\lambda}{2(\mu - 1)}$$

As per the problem, $N = 500$; $\lambda = 4800 \times 10^{-8} \text{ cm}$ and $\mu = 1.50$. So, we have

$$\therefore t = \frac{500 \times 4800 \times 10^{-8} \text{ cm}}{2(1.50 - 1)}$$

$$= 0.024 \text{ cm}$$

Terminal Questions

1. If N fringes shift through the field of view due to displacement of movable mirror by x , we have

$$x = \frac{N\lambda}{2} \Rightarrow \lambda = \frac{2x}{N}$$

As per the problem, $N = 500$; $x = 0.0112$ cm. So,

$$\lambda = \frac{2 \times (0.0112 \text{ cm})}{500} = 4480 \text{ \AA}$$

2. Let t be the thickness of the thin plate. The additional path difference introduced due to this plate is $2(\mu - 1)t$. If due to this additional path difference, N fringes crosses the field of view, we have

$$2(\mu - 1)t = N\lambda \Rightarrow t = \frac{N\lambda}{2(\mu - 1)}$$

So,

$$t = \frac{30 \times 580 \times 10^{-9} \text{ m}}{2(1.434 - 1)} = 0.02 \text{ mm}$$

3. Let N fringes cross the field of view when the mirror be moved by a distance x . Then, we can write

$$x = \frac{N\lambda}{2}$$

As per the problem, $N=150$, $\lambda=480 \times 10^{-9}$ m. Thus, we have

$$x = (150) \left(\frac{\lambda}{2} \right) = (150)(240 \text{ nm}) = 36,000 \text{ nm} = 0.036 \text{ mm}$$

4. The Michelson interferometer equation for oblique incidence is

$$2d \cos \alpha = n\lambda \quad (i)$$

At the centre, $\alpha = 0$, so that

$$2d = n\lambda \quad (ii)$$

where n now stands for the order of the central bright fringe. The order of fringes decreases as we move outwards from the centre. Thus, the second bright fringe is of $(n - 1)^{\text{th}}$ order and the seventh bright fringe is of $(n - 6)^{\text{th}}$ order. Hence, if α be the angular radius of the 7th bright fringe, we can write from Eq. (i)

$$2d \cos \alpha = (n - 6) \lambda \quad (iii)$$

So, from Eqs. (ii) and (iii) we can write,

$$2d (1 - \cos \alpha) = 6\lambda$$

$$\text{or} \quad \cos \alpha = 1 - \frac{6\lambda}{2d}$$

Substituting the given values of λ and d , we get

$$\cos \alpha = 1 - \frac{6 \times (5896 \times 10^{-8} \text{ cm})}{2 \times 0.3 \text{ cm}}$$

$$= 1 - 0.0005896 = 0.9994$$

$$\therefore \alpha = \cos^{-1}(0.9994) = 2^\circ.$$

So, the angular diameter of the 7th bright ring is 4° .

5. When the movable mirror of Michelson interferometer is moved by a distance $(\lambda/2)$, one fringe (that is, a pair of fringe comprising one bright and one dark fringe) cross the field of view. So, the distance x by which the movable mirror needs to be moved so that N fringes crosses the field of view is

$$x = N \frac{\lambda}{2} \Rightarrow \lambda = \frac{2x}{N} = \frac{2 \times (2.2 \times 10^{-5} \text{ m})}{90} = 4888 \text{ \AA}$$



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FURTHER READINGS

1. **Optics**, Eugene Hecht, Fourth Edition, Pearson Education, Indian Reprint (2004).
2. **Optics**, Ajoy Ghatak, Seventh Edition (2021), McGraw Hill Education (India) Pvt. Ltd.



TABLE OF PHYSICAL CONSTANTS

Symbol	Quantity	Value
c	Speed of light in vacuum	$3.00 \times 10^8 \text{ ms}^{-1}$
μ_0	Permeability of free space	$1.26 \times 10^{-6} \text{ NA}^{-2}$
ϵ_0	Permittivity of free space	$8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
$1/4\pi\epsilon_0$		$8.99 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$
e	Charge of the proton	$1.60 \times 10^{-19} \text{ C}$
$-e$	Charge of the electron	$-1.60 \times 10^{-19} \text{ C}$
h	Planck's constant	$6.63 \times 10^{-34} \text{ Js}$
$\hbar$	$h / 2\pi$	$1.05 \times 10^{-34} \text{ Js}$
m_e	Electron rest mass	$9.11 \times 10^{-31} \text{ kg}$
$-e / m_e$	Electron charge to mass ratio	$-1.76 \times 10^{11} \text{ Ckg}^{-1}$
m_p	Proton rest mass	$1.67 \times 10^{-27} \text{ kg (1 amu)}$
m_n	Neutron rest mass	$1.68 \times 10^{-27} \text{ kg}$
a_0	Bohr radius	$5.29 \times 10^{-11} \text{ m}$
N_A	Avogadro constant	$6.02 \times 10^{23} \text{ mol}^{-1}$
R	Universal gas constant	$8.31 \text{ Jmol}^{-1} \text{ K}^{-1}$
k_B	Boltzmann constant	$1.38 \times 10^{-23} \text{ J K}^{-1}$
G	Universal gravitational constant	$6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

LIST OF BLOCKS AND UNITS: BPHCT-137

BLOCK 1: WAVES – AN INTRODUCTION

- Unit 1 Wave Motion
- Unit 2 Superposition of Waves
- Unit 3 Acoustic Wave
- Unit 4 Electromagnetic Wave
- Unit 5 Polarisation of Light

BLOCK 2: INTERFERENCE

- Unit 6 Interference by Division of Wavefront
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BLOCK 4: LASERS AND OPTICAL FIBRE

- Unit 12 Physics of Lasers
- Unit 13 Properties and Applications of Lasers
- Unit 14 Fibre Optics

Waves: Transverse waves on a string, wave formation and propagation, relation between wavelength and frequency, wave velocity, mathematical description of wave motion; types of waves – plane waves and spherical waves (*qualitative description only*), energy carried by waves, intensity of waves. Superposition principle for waves, travelling and standing waves on a string, normal modes of a string, group velocity, phase velocity. Acoustic waves, production of sound waves – forced vibrations and resonance, intensity and loudness of sound, decibels, intensity levels, musical notes, musical scale, acoustics of buildings. Electromagnetic waves, wave equation for electromagnetic waves, electromagnetic nature of light, definition and properties of wave front, Huygens principle. Polarisation, transverse nature of light waves, production and analysis plane polarised light, Malus' law, Brewster's law, double refraction, Nicol prism, wave plates, circular and elliptical polarisation.

Interference: Division of amplitude and division of wave front, interference by division of wave front, Young's double slit experiment, white light fringes, displacement of fringes, Lloyd's mirror and Fresnel's biprism. Interference by division of amplitude, phase change on reflection – Stokes' treatment, interference in thin films, parallel films, qualitative description of Fringes of equal inclination (Haidinger fringes) and fringes of equal thickness (Fizeau fringes), interference in wedge-shaped films, Newton's rings – measurement of wavelength and refractive index. Michelson's interferometer, qualitative idea of the form of fringes, determination of wavelength, wavelength difference, refractive index and visibility of fringes.

Diffraction: Fresnel diffraction, Fresnel construction, half-period element, the zone plate, diffraction pattern of a straight edge, a slit and a wire using half-period zone analysis. Fraunhofer diffraction, evolution from Fresnel to Fraunhofer diffraction, diffraction from a single slit - observed pattern, intensity distribution; diffraction from a double slit – observed pattern, intensity distribution, position of minima and maxima, missing orders. Diffraction grating, Fraunhofer diffraction from multiple slits, intensity distribution, principal maxima, minima and secondary maxima, angular half-width of principal maxima.

Lasers and Optical Fibre: Basic principles of lasers, emission and absorption of light, spontaneous and stimulated emission, Einstein's relations (general idea only), population inversion, optical pumping, pumping mechanisms, three and four level pumping schemes, feedback mechanism, optical resonant cavity. Types of lasers and their applications, solid state, liquid and gas lasers, Helium-Neon laser, applications of lasers in communication, medicine, industry and photography. Optical fibre, core and cladding materials and their refractive indices, propagation of light through optical fibre - total internal reflection, types of fibres – step index and gradient index fibres, refractive index profiles, optical communication through fibres, qualitative idea of pulse dispersion and its reduction, material dispersion and power loss.

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