

Block

4**NUCLEAR PHYSICS**

UNIT 13**Radioactivity**

7**UNIT 14****The Atomic Nucleus**

37**UNIT 15****Applied Nuclear Science**

57

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BLOCK 4 : NUCLEAR PHYSICS

In Block 3, you have learnt how to solve Schrodinger equation for simple systems such as a free particle in a box, step potential, potential barrier and potential well. In the last block of this course, you will study nuclear physics. In this block, we intend to provide you with a bird's eye view of the physics of the atomic nucleus.

The march towards our understanding the physics of nucleus can be said to have begun about a hundred years ago, the chance discovery of radioactivity by Becquerel and subsequent work by Curies helped to identify natural sources of radiation. This discovery led to the fundamental developments in nuclear physics. Radioactivity leads to spontaneous transformation of a nucleus with the emission of alpha, beta or gamma particles. In Unit 13, entitled *Radioactivity* you will learn about nuclear stability, radioactivity and laws of radioactive decay. Successive radioactive disintegration and the condition of radioactive equilibrium are also discussed.

Alpha particles from radioactive nuclei were used by Rutherford to 'see' the atomic structure. These classic scattering experiments of alpha particles by atoms led to the discovery of the nuclear model of atom. This opened the flood gates of investigations into the atomic nucleus. The composition and structure of the nucleus, its properties and the forces of interaction between its constituents and nuclear density are discussed in Unit 14 entitled *The Atomic Nucleus*. We also discussed binding energy of nuclei.

Science and its applications are bound together as the fruit to the tree which bears it. This statement of Louis Pasteur beautifully captures the spirit in which we teach physics in our courses. Thus, having discussed the fundamentals of nuclear physics in the first two units of this block, in Unit 15 entitled *Applied Nuclear Science*, we focus our attention on the applied aspects of nuclear physics. We have discussed how nuclear energy is produced in fission and fusion in a nuclear reactor.

One final word about how to study the block. At the risk of repeating ourselves, we would like to remind you that while studying the units you should answer the SAQs and Terminal Questions on your own. Do not give into the temptation of looking up the answers before attempting the questions.

We hope that you will enjoy this journey into the world of atomic nucleus as much as we have.

We wish you good luck!





Nuclear waste contains radioactive material and requires safe disposal.

Source: <https://pixabay.com/photos/nuclear-waste-radioactive-trash-1471361/>

UNIT 13

RADIOACTIVITY |

Structure

- | | |
|--|--|
| <p>13.1 Introduction
Expected Learning Outcomes</p> <p>13.2 Nuclear Stability</p> <p>13.3 Law of Radioactive Decay
Preliminaries of Radioactivity
Radioactive Series
Half-Life, Mean-Life and Activity</p> | <p>13.4 Radioactive Equilibria
Growth and Decay of Radioactivity
Successive Radioactive Transformations
Radioactive Equilibrium</p> <p>13.5 Alpha Decay, Beta Decay and Gamma Decay</p> <p>13.6 Summary</p> <p>13.7 Terminal Questions</p> <p>13.8 Solutions and Answers</p> |
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STUDY GUIDE

In this unit, you will learn about nuclear stability and radioactivity. You will learn how to calculate half-life, mean-life and activity of a radioactive element. Radioactivity leads to spontaneous transformation of a nucleus with the emission of a α , β or γ -particle. In this unit, you will learn the laws of radioactive decay as well as α -decay, β -decay and γ -decay. You should be familiar with some of these concepts from your school +2 physics classes. We expect that, your prior knowledge will help you understand various concepts with greater clarity. This unit involves simple calculus and solution of first order ODEs and it will be advisable to refresh your knowledge of the subject from school mathematics course and Unit 3 of BPHCT-131. This unit has many sections and we advise you to master each topic one at a time. The solved examples and SAQs will help you fix your ideas and check your progress. Moreover, if you answer Terminal Questions and solve numerical problems given in the unit texts yourself, you will enjoy the subject.

“Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less.”

Marie Curie

13.1 INTRODUCTION

Towards the end of the nineteenth century, physicists thought that the era of exciting discoveries in physics was over. However, the chance discovery of radioactivity by Becquerel in 1896 overturned this view, and acted as precursor of fundamental developments in nuclear physics in the early part of the twentieth century. This was made possible as alpha, beta and gamma rays – spontaneously emitted radiations in radioactive decay – could be used as convenient tools to probe matter. For instance, alpha-particles from radioactive nuclei were used by Rutherford to discover the nuclear model of atom. It further led to discoveries of artificial transmutation and production of transuranic elements as well as radioisotopes. These find wide use in medical diagnosis and therapy, research, agriculture, carbon dating of archaeological specimen, etc. Studies of beta decay also led to the discovery of neutrino.

In Sec.13.2, we discuss nuclear stability and in Sec. 13.3 we explain the theory of radioactive decay in detail. Here you will learn the preliminaries of radioactivity as well as radioactive series. This will include detailed discussion of half-life, law of radioactive decay and mean/average life. The theory of radioactive decay can be used to explain the growth and decay of radioactivity in a given radioactive sample. This is the subject of Sec. 13.4. Successive radioactive disintegrations and the condition of radioactive equilibrium among different members of a radioactive series are also discussed in Sec. 13.4. In Sec. 13.5, you will learn about alpha decay, beta decay and gamma decay.

In the next unit, you will learn the physics of atomic nucleus.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ explain nuclear stability;
- ❖ identify types of radioactive radiations;
- ❖ state the laws of radioactive transformation;
- ❖ calculate half-life, mean life and disintegration constant of a radioactive substance;
- ❖ explain the growth and decay of radioactivity in a given sample;
- ❖ explain radioactive equilibrium among different members of a radioactive series;
- ❖ list different elements belonging to naturally occurring radioactive series; and
- ❖ explain alpha decay, beta decay and gamma decay.

13.2 NUCLEAR STABILITY

From your senior school physics, you may recall that in Rutherford's model of atom, the central core, called nucleus, consists of protons (Z) and neutrons (N). The only exception is the hydrogen atom which has only one proton in the nucleus. Since protons are positively charged, there would exist a repulsive coulomb force between them, which will try to push them apart. So,

how does the nucleus does not decay and remain stable? The stability of the nucleus was explained by proposing the existence of *strong nuclear forces*. It was observed though that some nuclei transform into other nuclei by emitting nuclear radiations spontaneously and attain stability. It means that not all combinations of neutrons and protons lead to stable nuclei.

Now refer to Fig.13.1, where we have plotted the number of neutrons (on y-axis) against the number of protons (on x-axis) for naturally occurring nuclei. The unstable (radioactive) nuclei are shown by blank circles.

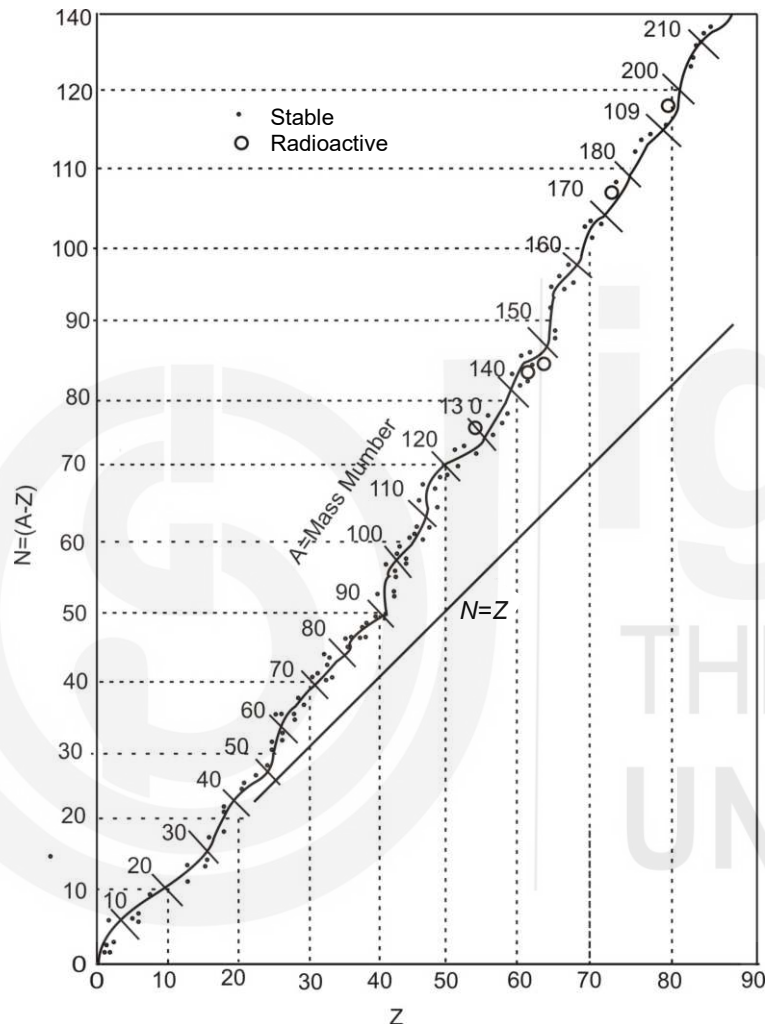


Fig. 13.1: Plot of the number of neutrons versus the number of protons for naturally occurring nuclei.

In Fig. 13.1, we note that

- the number of neutrons is nearly equal to the number of protons (Z) in the stable nuclei,
- light nuclei ($A < 20$) contain approximately equal number of neutrons and protons;
- for heavier nuclei (higher A), the neutron number is more than the proton number and this excess gradually increases with increasing A . It means that only certain combinations of protons and neutrons form stable nuclei;
- the stability curve is a straight line with $Z = N$ for nuclei with $Z \leq 20$. For $Z > 20$, the stability curve bends towards $N > Z$ and departs more and more

from the $N = Z$ line as Z increases. Such nuclei are unstable but tend to become stable by emitting alpha, beta or gamma radiations spontaneously. These processes are accompanied by changes in the values of Z and/or N . This process is known as *radioactivity* and its preliminary discussion forms the subject matter of the following section.

13.3 LAW OF RADIOACTIVE DECAY

As mentioned in Sec. 13.2, nuclei with $Z > 20$ tend to be unstable and spontaneously emit nuclear radiations in the form of alpha, beta or gamma rays in order to become stable. This was a chance discovery by Becquerel. We now recount the story of discovery of radioactivity and explain the preliminary concepts associated with it.

13.3.1 Preliminaries of Radioactivity

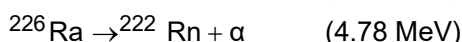
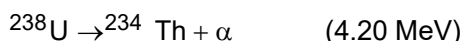
The story of discovery of radioactivity is very interesting. In 1896, Henri Becquerel was working on the phenomenon of fluorescence in which certain substances emitted visible light when they were exposed to ultra-violet radiations, say from the sun. In one of the drawers of his desk, Becquerel had kept a collection of various minerals, including some salts of uranium. He had also kept several cardboard boxes of photographic plates wrapped with thick black paper. A few days later, he took out one of the boxes of photographic plates for use. When he developed the plate, he was amazed to observe that it was heavily fogged. He tried other plates and found them also to be exposed. This puzzled him because all boxes were unused. Can you guess what had affected these photographic plates?

Becquerel conjectured very rightly that uranium salts, placed near *the* photographic plates must have emitted some new kind of radiation(s), which fogged the photographic plates. He then carried out extensive experiments with uranium salts and proved conclusively that such radiations, if passed through some *gas*, made it conduct electricity by virtue of their ionising power. The emission of ionising and penetrating radiation(s) from uranium was named **radioactivity**.

Madame Marie Curie extensively worked with elements like thorium, polonium and radium and discovered evidence of radioactivity in these elements. She also obtained convincing evidence that radioactivity was a nuclear property of an element and remained unaffected by its physical or chemical state.

By studying the ionising and penetrating powers of these radiations, Rutherford established the existence of two distinct entities: **Alpha rays** (α -rays) and **Beta rays** (β -rays). He also proved that α -rays were more easily absorbed in matter compared to β -rays but had comparatively greater ionising power. However, the penetrating power of β -rays was about 100 times more than that of α -rays. The existence of the third entity, **gamma rays**, which were appreciably more penetrating than *the* alpha and beta rays, was established by P. Villars. By subjecting these radiations to a magnetic field, it was also established that the gamma rays were electrically neutral, whereas the alpha-rays were positively charged and the beta-rays were negatively charged particles. We now know that α -rays can be identified with

helium nuclei and β -rays with electrons. Moreover, α -particles affect photographic films and excite fluorescence in many substances. Also, they produce phosphorescence and their speed is of the order of 1/100th of the speed of light. On the other hand, the speed of β -particles is about 1/10th of the speed of light. Gamma rays (γ) are very short wavelength electromagnetic radiations emitted from nuclei of radioactive substances and travel with the speed of light. Further studies revealed that radioactive emanations had energies in the MeV range. We give below some examples of radioactive decays which involve emission of α and β -rays:

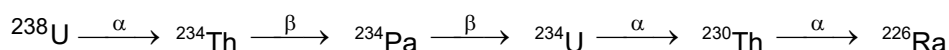


Note that β -decay is accompanied by the emission of neutrinos. But these are very difficult to detect. You will agree that in all these reactions, one element changes into another: uranium decays to thorium, radium to ruthenium, cobalt to nickel and strontium to yttrium. This process is known as **transmutation**. Do you know that in many cases of α and β decays, the daughter nuclei is in an excited state and subsequently undergoes a transition to a lower energy state by emitting a γ -ray? Note that a nucleus in an excited state can also make a transition to a lower state by transferring its excitation energy to an electron. Such energy transfer is called **internal conversion**. The electron gaining this energy may be ejected from the atom as a β -ray.

13.3.2 Radioactive Series

You now know that a radioactive element disintegrates spontaneously and its nucleus emits either an α - or a β -particle. As mentioned earlier, the nucleus of the new element so formed could also be unstable. The first decay is then succeeded by subsequent decays leading to a sequential series. Such a decay process continues till a stable isotope is produced.

We illustrate this with an example. Let us consider decay of ^{238}U .

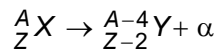


Note that ^{238}U decays through a succession of α and β decays to Radium (^{226}Ra) with a half-life of 4.5×10^9 yr, which, in turn, decays by the emission of α -particle to Radon with a half-life of 1620 yr. The decays continue until a stable isotope is produced. In the series shown above, the stable decay product is lead. ^{238}U is called the **parent nucleus** while ^{226}Ra is **daughter nucleus**.

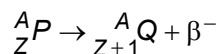
You may now like to know if other such series also exist? The answer to this question is, yes, they do. We have **four** such series in all: *thorium series*, *neptunium series*, *uranium series* and *actinium series* (see Table 13.1). You should discover the mass numbers of the members of these series.

Note that in a radioactive series, the parent radioactive element is of large atomic number and very long half-life. It successively emits α - or β -particle

giving rise to daughter nuclei of radioactive elements in the series. We can express such transformations by an expression of the form



The emission of alpha particle from the nucleus X of mass number A and charge number Z results in an element Y with mass number reduced by 4 units and the charge number reduced by 2 units. On the other hand, emission of a beta particle increases the charge by 1 unit without changing the mass number of the daughter nuclei as shown below:



Refer to Table 13.1, which shows four radioactive series with their characteristics. Note that the initial amount of the parent nuclei decreases gradually with time.

Table 13.1: Some Radioactive Series with their Characteristics

Radioactive species	Thorium	Neptunium	Uranium	Actinium
Parent element	${}^{232}_{90}\text{Th}$	${}^{241}_{94}\text{Pu}$	${}^{238}_{92}\text{U}$	${}^{235}_{92}\text{U}$
Half life (Yr)	1.39×10^{10}	2.25×10^6	4.47×10^9	7.07×10^8
Number of nucleons in the parent	$4n$	$4n+1$	$4n+2$	$4n+3$
Important intermediate daughter element	${}^{228}_{90}\text{Th}$	${}^{237}_{93}\text{Np}$	${}^{226}_{88}\text{Ra}$	${}^{234}_{90}\text{Pa}$
End product	${}^{208}_{82}\text{Pb}$	${}^{209}_{83}\text{Bi}$	${}^{206}_{82}\text{Pb}$	${}^{207}_{82}\text{Pb}$
Number of α and β emissions	$\alpha = 6;$ $\beta = 4$	$\alpha = 8;$ $\beta = 5$	$\alpha = 8;$ $\beta = 6$	$\alpha = 7;$ $\beta = 4$

To test your grasp of various concepts discussed in this section, we would like you to now answer a SAQ.

SAQ 1 – Radioactive decay

An element characterised with $A=232$ and $Z=90$ loses 6α and 4β particles in a 10 stage decay process. What will be the final stable isotope?

13.3.3 Half-life, Mean-Life and Activity

We now know that a given amount of radioisotope gradually decreases with time. Experiments show that the law describing the decay process is very simple. To understand this, refer to Fig. 13.2. It shows a plot of decay of radioactive strontium (on the logarithmic scale) as a function of time. You will Note that

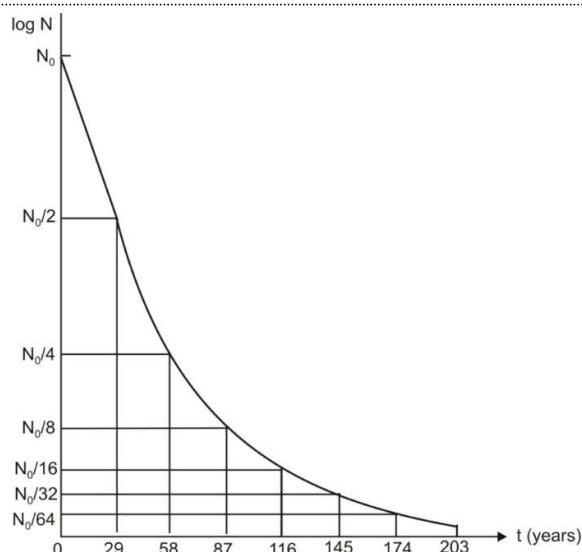


Fig.13.2: Decay of radioactive strontium as a function of time.

- it takes 29 years for one-half of the initial amount of strontium to decay.
- In the next 29 years, one-half of the remainder of strontium will decay, i.e. we will be left with $\frac{1}{2} \left(\frac{N_0}{2} \right) = \frac{N_0}{4}$ nuclei.

It means that the fraction of parent radioactive substance (left) after 29, 58, 87 years will be $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ of the initial amount. Do you recognize this series? It

forms a geometric progression with $r = \frac{1}{2}$. So, if we denote the number of strontium nuclei surviving decay at time t as $N(t)$ and if N_0 is the number at $t = 0$, then we can write

$$N(t) = N_0 \left(\frac{1}{2} \right)^{t/(29\text{yr})} \quad (13.1)$$

The time required for one-half of the parent nuclei (material) to decay is called **half-life**. We denote it by the symbol $T_{1/2}$. The half-lives of some important radioisotopes are given in Table 13.2. Note that the values of $T_{1/2}$ show a very wide range: from 4.5×10^9 yr for ^{238}U to 3×10^{-9} s for ^{212}Po .

Table 13.2: Half-lives of a few typical radioisotopes

Radioisotope	$T_{1/2}$
^{238}U	4.5×10^9 yr
^{14}C	5730 yr
^{40}K	1.3×10^9 yr
^{60}Co	5.24 yr
^{90}Sr	28.8 yr
^{131}I	8.05 d
^{212}Po	3×10^{-9} s

In terms of the half-life, you can rewrite Eq. (13.1) as

$$N(t) = N_0 \left(\frac{1}{2} \right)^{t/T_{1/2}} \quad (13.2)$$

Let us pause for some time and seek the physical implication of this result. This relation tells us how the given quantity of a radioactive sample shall survive disintegrations as time passes. You may also like to know: Does this relation hold only for the times $t = 0, T_{1/2}, \dots$, etc.? The answer to this question is: Eq. (13.2) holds for all times. Further, the general appearance of the decay curve shown in Fig.13.2 is essentially the same for all radioactive elements. However, each element takes its own characteristic time to decay.

Mean Life and Law of Radioactive Decay

Using the identity: $2 = e^{\ln 2} \cong e^{0.693}$, we can rewrite Eq. (13.2) in a more convenient form as

$$N(t) = N_0 \exp(-\ln 2 \ t / T_{1/2})$$

$$\text{or} \quad N(t) = N_0 \exp(-t / \tau) \quad (13.3)$$

where $\tau = \frac{T_{1/2}}{\ln 2}$ is called the **characteristic time constant**.

Eq. (13.3) expresses mathematically the **law of radioactive decay**. It shows that the number of atoms in a radioactive sample decreases exponentially with time as determined by the characteristic time constant τ .

Let us now obtain an expression for the number of nuclei decaying in time interval dt . Differentiating Eq. (13.3) with respect to t , we get:

$$dN = -\frac{N_0}{\tau} e^{-t/\tau} dt$$

The negative sign implies that the number of nuclei decreases with time due to their continuous disintegration.

Average/Mean Life Time

By definition, the average life time, $\bar{t}$, is defined as

$$\begin{aligned} \bar{t} &= \frac{\int_0^\infty t |dN|}{\int_0^\infty |dN|} = \frac{1}{N_0} \int_0^\infty t |dN| = \frac{1}{N_0} \int_0^\infty \frac{t}{\tau} N_0 e^{-t/\tau} dt \\ &= \frac{1}{\tau} \int_0^\infty t e^{-t/\tau} dt \end{aligned}$$

Using the method of integration by parts, we obtain

$$\bar{t} = \frac{1}{\tau} \left[-t\tau e^{-t/\tau} \Big|_0^\infty + \tau \int_0^\infty e^{-t/\tau} dt \right]$$

You can see that the first term on the RHS of the above equation vanishes at both the limits. So, the expression for mean life simplifies to:

$$\bar{t} = \int_0^\infty e^{-t/\tau} dt = \left[-\tau e^{-t/\tau} \right]_0^\infty = \tau \quad (13.4)$$

That is, **mean life** of radioactive nuclei is equal to τ . Note that like $T_{1/2}$, τ will also vary over a very wide range as these are directly related. Also note that

the measurement techniques over such a wide spread of time are bound to differ vastly. But the law of radioactive decay is common to all radioactive decay processes. The average/mean life time is the reciprocal of its decay probability per unit time.

$$\bar{t} = \tau = \frac{1}{\lambda}$$

We will now illustrate these concepts through a solved example.

EXAMPLE 13.1: HALF-LIFE

The half-life of radon is 3.8 days. After how many days will only 5% of radon be left over?

SOLUTION ■ We know that $T_{1/2} = 3.8$ days. Therefore

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} = \frac{3.8 \text{ days}}{0.693} = 5.48 \text{ days}$$

We are required to calculate the number of days in which only 5% radon is left. Therefore, we can write

$$\frac{N}{N_0} = 0.05.$$

Substituting $\tau = 5.48$ days and $\frac{N}{N_0} = 0.05$, we get

$$\frac{N}{N_0} = 0.05 = \exp\left(-\frac{t}{5.48 \text{ days}}\right) = \frac{1}{20}$$

We can write it as

$$\exp\left(\frac{t}{5.48 \text{ days}}\right) = 20$$

Taking natural logarithm of both sides, we find that

$$\frac{t}{5.48 \text{ days}} = \ln 20 = 2.303 \log_{10} 20 = 2.303 \times 1.3010 = 2.996$$

$$\therefore t = 2.996 \times 5.48 \text{ days} = 16.42 \text{ days}.$$

You may now like to solve an SAQ

SAQ 2 – Mean life

The mean life of a radioactive element is 14.43 months. Calculate the time required for 75% of the element to decay.

In practice, we are more interested in the *decay rate* of a material rather than its amount, since it determines the rate of emission of α , β or γ -rays.

Moreover, this information can also be used to estimate the age of any specimen. To this end, we differentiate Eq. (13.3) with respect to time and get

$$\frac{dN}{dt} = -\frac{N_0}{\tau} \exp(-t/\tau) = -\frac{N}{\tau} \quad (13.5)$$

That is, the decay rate at any time is proportional to the amount of radioactive material present at that instant.

Alternatively, we can write Eq. (13.5) as

$$\frac{dN}{dt} = -\lambda N \quad (13.6)$$

where λ is the constant of proportionality and is called **decay constant**. It is the characteristic of a radioactive element or decay process. In terms of $\lambda (= 1/\tau)$, the law of radioactive decay [Eq. (13.3)] is expressed as:

$$N = N_0 \exp(-\lambda t) \quad (13.7)$$

Activity of a radioactive substance

The *activity A of a radioactive substance is defined as the number of atoms disintegrating per unit time*. Mathematically, it is expressed as

$$A = \left| \frac{dN}{dt} \right| \quad (13.8)$$

From Eqs. (13.6) and (13.8), we can write

$$A = \lambda N \quad (13.9)$$

So, the activity of a given radioactive substance at any time is directly proportional to the number of radioactive atoms present at that instant. If A_0 is the initial activity of the source at $t = 0$, then from Eq. (13.7), we get

$$\frac{A}{A_0} = \frac{N}{N_0} = e^{-\lambda t} \quad (13.10)$$

The quantity $\frac{A}{A_0}$ is termed as **relative activity** and is a *measure of the radioactivity of a given source*. From this result you will note that even the relative activity of a given radioactive substance decays exponentially with time.

The most natural way to express decay rate is in *disintegrations per second*. But the activities encountered in practice are usually so high that a larger unit, the **curie**, abbreviated as Ci, is more often used. Initially the curie was defined as the activity of 1g of radium but its value changed with improvements in measuring techniques. The curie is now defined as

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ disintegrations per second}$$

In SI system, decay rate is expressed in becquerel:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ disintegration per second}$$

You may have seen persons of all ages with swollen thyroid gland. Do you know that to scan the thyroid, radioisotope ^{131}I is used? Let us now calculate its decay rate.

The number of nuclei in 1g of ^{131}I is

$$N = 1\text{g} \times \frac{1\text{mol}}{131\text{g}} \times 6.023 \times 10^{23} \text{ nuclei mol}^{-1} = 4.6 \times 10^{21}$$

Hence, the decay rate

$$-\frac{dN}{dt} = \frac{N}{\tau} = \frac{\ln 2}{T_{1/2}} N$$

Note from Table 13.2 you would note that $T_{1/2}(^{131}\text{I}) = 8.05$ days.

$$\begin{aligned}\therefore -\frac{dN}{dt} &= \frac{0.693}{8.05 \text{ days}} (4.6 \times 10^{21} \text{ nuclei}) \\ &= \frac{3.188 \times 10^{21} \text{ atoms}}{6.955 \times 10^5 \text{ s}} \\ &= 4.58 \times 10^{15} \text{ disintegrations per second}\end{aligned}$$

Expressing this in curies, we find that

$$\begin{aligned}\left| \frac{dN}{dt} \right| &= (4.58 \times 10^{15} \text{ disintegrations per second}) \\ &\times \frac{1 \text{ Ci}}{(3.7 \times 10^{10} \text{ disintegrations per second})} \\ &= 1.24 \times 10^5 \text{ Ci}\end{aligned}$$

That is, $1 \text{ g } ^{131}\text{I}$ is equivalent to $= 1.24 \times 10^5 \text{ Ci}$. This is an extremely large disintegration rate. It is controlled by adjusting the dose. The amount of ^{131}I injected into human body is only 10^{-9} g so that the decay rate is nearly 10^{-4} Ci , which is well within the safely limit.

You must have read about carbon dating to determine the age of a specimen. It is a common technique used to determine the age of formation of the rocks, asteroids discovered in celestial objects as also fossils on the fossil fuels on the earth. Do you know that knowledge of decay rate enables us to estimate the age of specimen using the carbon dating technique? We will now illustrate this with an example.

EXAMPLE 13.2: RADIOACTIVE DATING

^{14}C isotope of carbon is used for radioactive dating of organic materials. Samples of fresh carbon from trees in equilibrium with the CO_2 of the atmosphere have an abundance of 98.89% ^{12}C , 1.11% ^{13}C and $1.3 \times 10^{-12}\%$ ^{14}C . After a tree dies, the abundance of ^{12}C and ^{13}C in the wood does not change but the abundance of ^{14}C decreases because of radioactive decay. A piece of wood is taken from an Egyptian tomb. Each gram of carbon exhibits an activity of $3.9 \times 10^{-12} \text{ Ci}$. Estimate the age of the wood.

SOLUTION ■ The number of nuclei in 1g of carbon is

$$1 \text{ g} \times \frac{1 \text{ mol}}{12 \text{ g}} \times 6.02 \times 10^{23} \text{ nuclei mol}^{-1} = 5.02 \times 10^{22} \text{ nuclei. Thus, 1g}$$

of fresh carbon should contain $5.02 \times 10^{22} \times 1.3 \times 10^{-12} = 6.53 \times 10^{10}$ nuclei of ^{14}C . From Eq. (13.3), (13.6) and (13.8) the activity at $t = 0$ is given by:

$$A_0 = \left| \frac{dN}{dt} \right|_{t=0} = \frac{\ln 2}{T_{1/2}} N_0$$

From Table 13.2 we note that $T_{1/2}$ (^{14}C) is 5730 yr. We convert the number of years into seconds. Hence,

$$A_0 = \frac{0.693}{(5730 \times 365 \times 24 \times 3600 \text{ s})} \times (6.53 \times 10^{10} \text{ nuclei})$$

$$= \frac{4.53 \times 10^{10} \text{ nuclei}}{1.807 \times 10^{11} \text{ s}} = 0.251 \text{ disintegrations per second}$$

To express it in curies we note that

$$A_0 = \frac{(0.251 \text{ disintegrations per second}) \times 1 \text{ Ci}}{3.7 \times 10^{10} \text{ disintegrations per second}}$$

$$= 6.78 \times 10^{-12} \text{ Ci}$$

We are told that the measured activity of the sample is $3.9 \times 10^{-12} \text{ Ci}$. This is smaller than the initial activity by a factor of $\frac{3.9 \times 10^{-12} \text{ Ci}}{6.78 \times 10^{-12} \text{ Ci}} = 0.575$

Since the activity is proportional to the amount of radioactive material, from Eq. (13.10) we get,

$$\frac{A}{A_0} = \frac{N}{N_0} = e^{-(\ln 2)t / T_{1/2}} = 0.575$$

So taking logarithm of both sides, we find that

$$t = -\frac{T_{1/2}}{\ln 2} \ln(0.575)$$

$$= -\frac{5730 \text{ yr}}{0.693} \times (-0.553)$$

$$= 4572.4 \text{ yr}$$

SAQ 3 – Activity of a radioactive substance

The half-life of ^{238}U is known to be $4.51 \times 10^9 \text{ yr}$. Calculate the decay constant (in s^{-1}). Also, calculate the rate of disintegrations for 1 g of uranium. Take: Avogadro's number = 6.03×10^{23} .

13.4 RADIOACTIVE EQUILIBRIA

You now know that naturally occurring radioactive elements disintegrate continuously and their decay is governed by a characteristic decay constant. Physically, it means that the number of atoms (of parent nuclei) in a radioactive sample will decrease steadily [according to Eqs. (13.3) and (13.7)]. However, if the daughter nuclei happen to be radioactive, their (daughter nuclei's) activity will also start to build up with time (and may compensate for the loss of parent nuclei). But this cannot go on indefinitely. Let us now learn more about the observed growth and decay of the activities of daughter nuclei.

13.4.1 Growth and Decay of Radioactivity

Let us reconsider the decay of ^{238}U . We now know that it decays by emitting a α -particle with a half-life of $4.5 \times 10^9 \text{ yr}$ into thorium. But ^{234}Th atoms disintegrate by emitting β -particles with a half-life of 24.1 days. This means

that in the original sample of uranium, ^{234}Th atoms will disintegrate significantly faster. In fact, the entire activity of the sample will be practically due to ^{234}Th . And, if you separate ^{234}Th from ^{238}U , its activity will decay exponentially and reach half its initial value in 24.1 days.

To know how ^{234}Th grows in a freshly separated sample of ^{238}U , we note that if at any instant of time t , the number of atoms of ^{238}U are N_U , the rate of disintegration of the parent element dN_U is given by

$$\frac{dN_U}{dt} = -\lambda_U N_U$$

Physically, it implies that ^{234}Th atoms are produced at the rate $\lambda_U N_U$, where λ_U is the decay constant of ^{238}U . However, ^{234}Th atoms will, in turn, disintegrate at a rate $\lambda_{\text{Th}} N_{\text{Th}}$, where N_{Th} is the number of ^{234}Th atoms present at time t and λ_{Th} is their characteristic decay constant. Hence, the net rate of increase of ^{234}Th atoms in uranium sample will be given by

$$\frac{dN_{\text{Th}}}{dt} = \lambda_U N_U - \lambda_{\text{Th}} N_{\text{Th}}$$

$$\text{or } \frac{dN_{\text{Th}}}{dt} + \lambda_{\text{Th}} N_{\text{Th}} = \lambda_U N_U \quad (13.11)$$

To solve this first order ODE for N_{Th} , we have to convert its left hand side into an exact differential. We can do so by multiplying Eq. (13.11) throughout by $\exp(\lambda_{\text{Th}} t)$, which is as integrating factor (see margin remark). This gives

$$\exp(\lambda_{\text{Th}} t) \frac{dN_{\text{Th}}}{dt} + \lambda_{\text{Th}} N_{\text{Th}} \exp(\lambda_{\text{Th}} t) = \lambda_U N_U \exp(\lambda_{\text{Th}} t)$$

We can rewrite this equation as:

$$\frac{d}{dt} [N_{\text{Th}} \exp(\lambda_{\text{Th}} t)] = \lambda_U N_U \exp(\lambda_{\text{Th}} t)$$

Verify the result before studying further. On integrating it, we obtain

$$N_{\text{Th}} \exp(\lambda_{\text{Th}} t) = \frac{\lambda_U}{\lambda_{\text{Th}}} N_U \exp(\lambda_{\text{Th}} t) + K$$

where K is constant of integration. Therefore,

$$\therefore N_{\text{Th}} = \frac{\lambda_U}{\lambda_{\text{Th}}} N_U + K \exp(-\lambda_{\text{Th}} t) \quad (13.12)$$

To determine K , we note that at $t = 0$, i.e., in a freshly prepared uranium sample, $N_{\text{Th}} = 0$. Hence, $K = -\frac{\lambda_U}{\lambda_{\text{Th}}} N_U$. Therefore, Eq. (13.12) takes the

form:

$$N_{\text{Th}} = \frac{\lambda_U}{\lambda_{\text{Th}}} N_U [1 - \exp(-\lambda_{\text{Th}} t)] \quad (13.13a)$$

This result shows that in a freshly prepared uranium sample, as $t \rightarrow \infty$ and $[\exp(-\lambda_{\text{Th}} t)] \rightarrow 0$, the number of ^{234}Th atoms ultimately reach a *constant equilibrium value* of $N_{\text{Th}}^\infty = (\lambda_U / \lambda_{\text{Th}}) N_U$. On combining this result with Eq. (13.13a), we can write

$$N_{\text{Th}} = N_{\text{Th}}^\infty [1 - \exp(-\lambda_{\text{Th}} t)] \quad (13.13b)$$

You have learnt the method of solving such ODEs in Unit 3 of BPHCT-131 (Mechanics).

The growth of ^{234}Th nuclei is shown in Fig. 13.3.

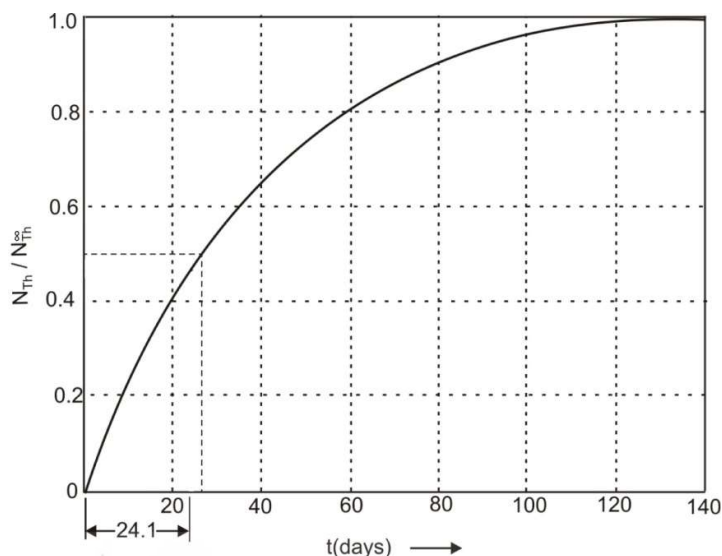


Fig. 13.3: Growth of ^{234}Th in a freshly prepared Uranium sample.

13.4.2 Successive Radioactive Transformations

In Sec. 13.4.1, you have learnt about the growth and decay of radioactivity when parent nuclei decay into daughter nuclei, which decay, in turn. You now know that naturally occurring radioactive elements undergo successive decays till a stable element is produced. While studying a particular radioactive series, we often find it necessary to work out the number of atoms of each member of the series at a specified time. We can express the problem as follows:

Suppose that initially we have N_{A0} nuclei of a parent element A . It decays to an element B , which, in turn, decays to C , and so on. If $\lambda_A, \lambda_B, \lambda_C, \dots$ are the respective disintegration constants of $A, B, C, \dots$ then we would like to derive expressions for the number of nuclei N_A, N_B, N_C at time t .

To derive expressions for the number of nuclei N_A, N_B, N_C at time t , we start with Eq. (13.7) and note that the number of nuclei of A present at time t is given by

$$N_A = N_{A0} \exp(-\lambda_A t) \quad (13.14)$$

The atoms of element B are produced at the rate $\lambda_A N_A$ and decay at the rate $\lambda_B N_B$. Hence, the net rate at which the number of atoms of B grow is given by:

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad (13.15)$$

For element C , we can similarly write

$$\frac{dN_C}{dt} = \lambda_B N_B - \lambda_C N_C \quad (13.16)$$

On substituting for N_A from Eq. (13.14) in Eq. (13.15) and rearranging terms in the resultant expression, we get

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_{A0} \exp(-\lambda_A t) \quad (13.17)$$

Multiplying both sides by $\exp(\lambda_B t)$ and following the steps outlined in the previous section, you can show that the expression for number of daughter nuclei produced and present at time t is (SAQ 4):

$$N_B = \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \quad (13.18)$$

SAQ 4 – Successive radioactive transformations

Starting from Eq. (13.17), derive Eq. (13.18) using the condition that initially only the parent element was present, i.e., $N_B = 0$ at $t = 0$.

Let us now obtain the expression for the element C produced as second stage nuclei in the decay series. To achieve this, you now have to substitute the expression given in Eq. (13.18) for N_B in Eq. (13.16) and multiply throughout by $\exp(\lambda_C t)$, and rearrange the resultant expression as

$$\frac{d}{dt} \exp(\lambda_C t) N_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0} [e^{(\lambda_C - \lambda_A)t} - e^{(\lambda_C - \lambda_B)t}]$$

This can be integrated to yield:

$$\exp(\lambda_C t) N_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0} \left[\frac{e^{(\lambda_C - \lambda_A)t}}{(\lambda_C - \lambda_A)} - \frac{e^{(\lambda_C - \lambda_B)t}}{(\lambda_C - \lambda_B)} \right] + K$$

so that

$$N_C = \frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0} \left[\frac{e^{-\lambda_A t}}{(\lambda_C - \lambda_A)} - \frac{e^{-\lambda_B t}}{(\lambda_C - \lambda_B)} \right] + K \exp(-\lambda_C t) \quad (13.19)$$

where K is constant of integration. To evaluate it, we use the fact that at $t = 0$, $N_C = 0$. This gives

$$K = -\frac{\lambda_A \lambda_B}{\lambda_B - \lambda_A} N_{A0} \left[\frac{1}{(\lambda_C - \lambda_A)} - \frac{1}{(\lambda_C - \lambda_B)} \right]$$

On simplification, we get

$$K = \frac{\lambda_A \lambda_B N_{A0}}{(\lambda_C - \lambda_A)(\lambda_C - \lambda_B)} = \frac{\lambda_A \lambda_B N_{A0}}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)}$$

Hence, the expression for the number of nuclei C at time t is given by

$$N_C = \lambda_A \lambda_B N_{A0} \left[\frac{\exp(-\lambda_A t)}{(\lambda_B - \lambda_A)(\lambda_C - \lambda_A)} + \frac{\exp(-\lambda_B t)}{(\lambda_C - \lambda_B)(\lambda_A - \lambda_B)} + \frac{\exp(-\lambda_C t)}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)} \right] \quad (13.20)$$

In a compact form, we can rewrite it as

$$N_C = N_{A0} (a_1 e^{-\lambda_A t} + a_2 e^{-\lambda_B t} + a_3 e^{-\lambda_C t}) \quad (13.21)$$

where

$$a_1 = \frac{\lambda_A \lambda_B}{(\lambda_B - \lambda_A)(\lambda_C - \lambda_A)}$$

$$a_2 = \frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_B)(\lambda_C - \lambda_B)}$$

$$\text{and } a_3 = \frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)} \quad (13.22)$$

You can now extend the procedure outlined above to a chain of radioactive elements. We leave this exercise for you to fix your ideas.

Let us now apply these equations to the specific case of successive radioactive transformations. Let us assume that a metal wire is exposed for a few seconds to radioactive radon gas. We obtain a deposit of RaA, which is a decay product of radon. RaA has a half-life of 3.05 min and decays to RaB, which has a half-life of 27 min. RaC, produced due to decay of RaB, decays to RaD with a half-life of 20 min. RaD has a half-life of 22 years and therefore, for all practical purposes during the experiment, we can assume that the number of RaD atoms will be constant.

The plots of the number of RaA, RaB, RaC and RaD nuclei as a function of time are shown in Fig. 13.4. Note that the number of RaA atoms, initially assumed to be 100, decreases exponentially with time and reduces to 50 after 3.05 min. At time $t = 0$, there were no atoms of RaB, RaC and RaD present. However, the number of RaB nuclei increases with time, becomes maximum about 11 min later and then decreases with time. Similarly, the number of RaC nuclei becomes maximum after about 35 min. However, the number of RaD nuclei increases continuously and reaches a maximum when RaB and RaC have practically disappeared. Do you know the reason? It is because of the comparatively longer half-life of RaD nuclei. In fact, RaD will decay exponentially with a half-life of 22 years.

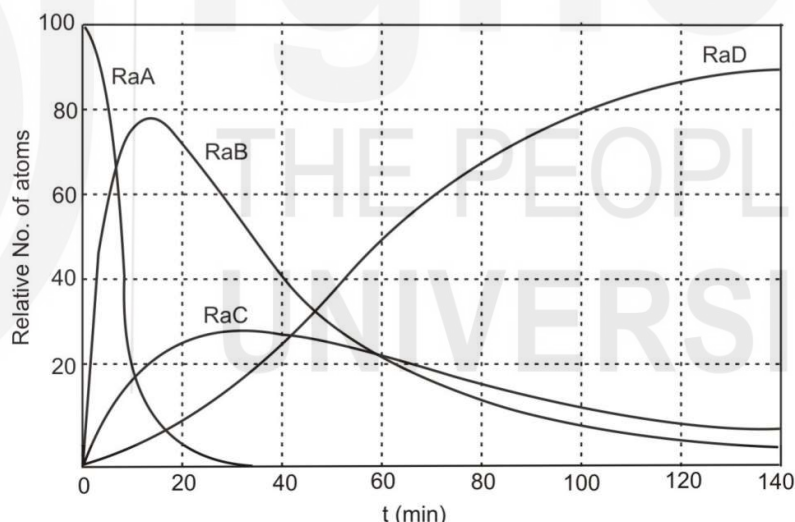


Fig. 13.4: Variation of the relative numbers of RaA, RaB, RaC and RaD nuclei with time in a radioactive sample of radon.

Before proceeding further, we would like you to answer an SAQ.

SAQ 5 – Successive radioactive transformations

In the decay chain of radon, calculate the time at which the number of RaB nuclei will be maximum. It is given that, $\lambda_A = 3.8 \times 10^{-3} \text{ s}^{-1}$ and $\lambda_B = 1.3 \times 10^{-4} \text{ s}^{-1}$.

13.4.3 Radioactive Equilibrium

You now know that different radioactive elements, occurring either naturally or produced in a decay chain, have their characteristic half-lives. The law

governing successive disintegrations deals with the quantity of any given radioactive isotope present at any time. Depending on the relative magnitude of the half lives of various nuclei in a decay chain, we may obtain a situation where the number of parent and/or daughter nuclei either remain constant or bear a constant ratio. This situation is referred to as **radioactive equilibrium**. There are two possibilities. We will discuss these in turn.

We begin again by referring to the decay of ^{238}U . You may recall that ^{238}U is an extremely long-lived element compared to its daughter nuclei $^{234}\text{Th} (\lambda_{\text{U}} \ll \lambda_{\text{Th}})$. Let us assume that the initial sample is pure, i.e., has only ^{238}U . Then from Eqs. (13.7) and (13.13), we can write

$$N_{\text{U}} \cong N_{\text{U}_0} \quad (13.23)$$

and

$$N_{\text{Th}} = N_{\text{U}_0} \frac{\lambda_{\text{U}}}{\lambda_{\text{Th}}} [1 - \exp(-\lambda_{\text{Th}} t)] \quad (13.24)$$

where we have assumed $\exp(-\lambda_{\text{U}} t) \cong 1$.

Let us pause for a while and reflect on what we have achieved so far. Eqs. (13.23) and (13.24) suggest that though the number of uranium (parent) nuclei remain essentially constant for relatively shorter times, the number of daughter nuclei changes with time.

After a time t much greater than the half-life of daughter nuclei, $\exp(-\lambda_{\text{Th}} t)$ becomes negligibly small and N_{Th} reaches an equilibrium value of

$$N_{\text{Th}} = N_{\text{U}} \frac{\lambda_{\text{U}}}{\lambda_{\text{Th}}}$$

or

$$N_{\text{Th}} \lambda_{\text{Th}} = N_{\text{U}} \lambda_{\text{U}} \quad (13.25)$$

This result tells us that in the equilibrium state the rate of decay of daughter nuclei is equal to their rate of production implying that *the number of parent and daughter nuclei remains constant*. Such a **long-term equilibrium** between the parent and daughter nuclei is known as **secular equilibrium**. Such equilibrium can be observed in the formation of radon ($T_{1/2} = 5.5$ days) from radium ($T_{1/2} = 2300$ yr).

SAQ 6 – Secular equilibrium

Uranium minerals in which secular equilibrium has been obtained contain one atom of radium for every 2.8×10^6 atoms of uranium. If the half-life of radium is 1620 years, calculate the half-life of uranium.

You now know that in the case of successive radioactive decays, *secular equilibrium is obtained when parent nuclei are longer lived compared to the daughter nuclei*. You may now like to know: What will happen when the parent is longer-lived than the daughter ($\lambda_{\text{A}} < \lambda_{\text{B}}$), but the half-life of the parent is not very long. That is, the half-life of the parent is slightly greater than that of the daughter nuclei. You may like to identify such a situation in the uranium as well as actinium series. In such a situation, we cannot use the approximation $\exp(-\lambda_{\text{A}} t) = 1$. If the parent and daughter atoms are initially

separated, the number of their nuclei are respectively given by Eqs. (13.14) and (13.18). Moreover, if $\lambda_B t > 1$, then the term involving $e^{-\lambda_B t}$ in Eq. (13.18), becomes negligible compared with the term involving $e^{-\lambda_A t}$. Then the number of daughter nuclei is given by

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} N_{A0} \exp(-\lambda_A t) \quad (13.26)$$

That is, the daughter nuclei eventually decay with the half-life of the parent. On combining this result with Eq. (12.14) we find that

$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} = \text{constant} \quad (13.27)$$

In words, the ratio of the number of parent nuclei and the number of daughter nuclei attains a constant value. This constitutes what we term as **transient equilibrium**.

When the parent has a shorter half-life than the daughter ($\lambda_A > \lambda_B$), no equilibrium is attained. If the parent and daughter are separated initially, then as the parent decays, the number of daughter nuclei is given by

$$N_B = N_0 \frac{\lambda_A}{\lambda_B - \lambda_A} \exp(-\lambda_B t) \quad (13.28)$$

That is, the parent substance disappears completely and the daughter nuclei eventually decay with their own half-life.

SAQ 7 – Secular equilibrium

For two radioactive elements A and B in transient equilibrium, show that the activity of daughter nuclei is greater than that of the parent by a factor

$$\frac{\lambda_A}{\lambda_B - \lambda_A}.$$

As you know stable nuclei do not decay into different nuclei. On the other hand, certain nuclei having a set of Z and N values disintegrate spontaneously and become stable. In general, for each value of A , there is one or at best two stable nuclei. All other nuclei with same A value are unstable and undergo some decay process, until stability is reached. Experiments have shown that unstable nuclei transform into other nuclear species by two different decay processes. These processes are *alpha decay* and *beta decay*. Nuclei in excited states emit photons, called *gamma rays* and make transitions that eventually lead them to the ground state. However, no change of Z or N occurs in this case.

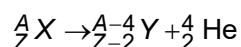
13.5 ALPHA DECAY, BETA DECAY AND GAMMA DECAY

Let us learn about the processes of alpha, beta and gamma decays in this section.

Alpha decay

From Fig.13.1, note that nuclei with 210 or more nucleons are large and unstable. In these nuclei, short range nuclear forces that hold them together are not able to counter balance mutual repulsion between protons. In alpha

decay, an unstable nucleus disintegrates into an alpha particle (${}^4\text{He}$), a somewhat lighter nucleus:



where X and Y are different nuclear species.

Alpha decay process is accompanied by release of energy. This liberated energy appears as kinetic energy of the alpha particle and the daughter nucleus Y .

Let $m(X)$ be the mass of the initial nucleus (parent nucleus), $m(Y)$ represent the mass of the daughter nucleus, and $m({}^4\text{He})$, the mass of the alpha particle. Then the expression for energy (Q) released in alpha decay is given by

$$Q = [m(X) - m(Y) - m({}^4_2\text{He})]c^2 \quad (13.29)$$

This energy appears as kinetic energy of the decay products:

$$Q = K_Y + K_{4\text{He}}$$

An alpha particle can travel relatively farther through matter than a heavy nucleus (as we don't observe the daughter nucleus Y in the laboratory). Typical α -decay energies are a few MeV; thus the kinetic energy of the α -particle and the daughter nucleus are significantly lower than the corresponding rest energies. We state here the relation between kinetic energy of an α -particle (KE_α), Q and the mass number A of the original nucleus as:

$$KE_\alpha = \frac{A-4}{A} Q \quad (13.30)$$

Now we solve an example to help you grasp the basic concepts of alpha decay.

EXAMPLE 13.3: ALPHA DECAY

Calculate the kinetic energy of an alpha particle emitted in the alpha decay of polonium isotope ${}^{210}_{84}\text{Po}$. It is given that $m({}^{206}_{82}\text{Pb}) = 205.974449 \text{ u}$, $m({}^4_2\text{He}) = 4.002603 \text{ u}$, $m({}^{210}_{84}\text{Po}) = 209.982857 \text{ u}$.

SOLUTION ■ Note that in this case, the equation of alpha decay can be expressed as ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + \alpha$ - particle

The energy released in alpha decay is given by:

$$Q = [m(\text{Po}) - m(\text{Pb}) - m(\alpha - \text{particle})]c^2$$

$$Q = 0.005805 \text{ u} \times 931.5 \frac{\text{MeV}}{\text{u}} = 5.4074 \text{ MeV}$$

Therefore, the kinetic energy of alpha particle is:

$$KE_\alpha = \frac{A-4}{A} Q$$

$$KE_\alpha = \frac{206}{210} \times 5.4074 \text{ MeV} = 5.304 \text{ MeV}$$

Now we would like you to answer an SAQ.

SAQ 8 – Disintegration energy and daughter nuclei

The radium isotope ($^{226}_{88}\text{Ra}$) emits a 4.7848 MeV alpha particle. Write the equation for the decay process. Calculate the disintegration energy, Q , and hence calculate the atomic mass of the daughter nuclei. It is given that atomic mass $^{226}_{88}\text{Ra}$ is 226.0254 u, and mass of α -particle is 4.0026 u.

After working out Example 13.3 and answering SAQ 8, you must have noted that the kinetic energy of the emitted alpha particle is not equal to the disintegration energy, Q . This is because the nucleus recoils and a small amount of kinetic energy is used when an alpha particle emerges. However, linear momentum is conserved in this process.

Beta (β) decay

Like alpha decay, beta decay is a process in which an atomic nucleus of the original nuclide changes to an isobar of that nuclide and becomes stable. For example, beta decay of a neutron changes it into a proton. Conversely, beta decay of a proton leads to a neutron. When a radioactive nucleus undergoes beta decay, Z and N each change by one unit, but A doesn't change.

As mentioned in the previous paragraph, in beta decay, neutron changes into a proton, and vice versa. Let us consider the transformation $n \rightarrow p + e^-$. Initial experiments showed that the principles of conservation of energy and linear momentum were not satisfied in this transformation.

- a) When energy of the electrons (β^- particles) emitted in the decay process was observed, it was found that it varied continuously from 0 to some maximum value KE_{max} characteristic of the nuclei.

Refer to Fig. 13.5, which shows the energy distribution of electrons in the β^- decay of ^{210}Bi . Note that the energy of electrons varies from 0 to 1.17 MeV (maximum kinetic energy). Moreover, the maximum number of electrons have energy of 0.15 MeV and very few emitted electrons had

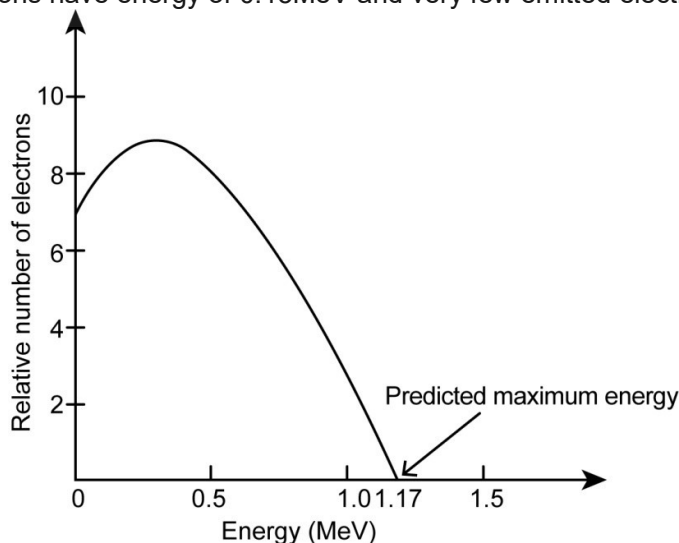


Fig. 13.5: Energy spectrum of electrons from Beta decay of ^{210}Bi .

energy 1.17MeV. It may be mentioned here that β -decay of ^{210}Bi was associated with loss of certain mass and the mass-energy relation was not satisfied implying violation of the law of conservation of energy.

- b) In this decay process, the law of conservation of linear momentum was also violated. When the directions of the emitted electron and the recoiling nuclei were observed, they were not exactly opposite as needed for conservation of linear momentum.
- c) We know that the electron, proton and neutron are spin $1/2$ particles and for the relation $n \rightarrow p + e^-$, spin is not conserved implying violation of the principle of conservation of angular momentum.

You may now ask: How did physicists solve this puzzle? The answer was provided by Pauli in 1930 when he proposed that in β -decay, an additional particle with the following properties should also be emitted:

- i) zero charge ii) small or zero rest mass, and iii) spin equal to $1/2$.

This low or no mass neutral particle was named **neutrino** (little neutron) by Enrico Fermi.

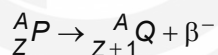
So, we can now say that in β -decay of a free neutron, a proton (p), an electron (e^-) and the antiparticle of the neutrino, called antineutrino ($\bar{\nu}$) is emitted.

The decay process is thus



Similarly, β -decay of a proton (p) results in a neutron (n), an electron (e^-) and neutrino.

You now know that β -decay of a nucleus having Z protons and N neutrons transforms it to a nucleus with $Z+1$ protons and $N-1$ neutrons:



The energy released in the decay process appears as the energy $E_{\bar{\nu}}$ of the antineutrino, the kinetic energy of the electron, and a small (usually negligible) recoil kinetic energy of the nucleus X' :

$$Q = E_{\bar{\nu}} + K_e$$

Gamma decay

So far, you have learnt about alpha and beta decays. You may like to know: Can daughter nucleus be left in an excited state after alpha or beta decay? And how does an excited nucleus return to the ground state? The answers to these questions are:

- It is possible that the daughter nuclei may be left in an excited state.
- The excited nucleus returns to the ground state by emitting one or more photons. The energy of the photon accounts for the energy difference between the initial and final states in the transition involved. These photons are called **gamma rays** (represented as γ -rays). The energy of the photons emitted by the nuclei are typically in the range 100 keV -

Pauli's letter

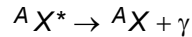
An invisible particle to explain the missing beta energy puzzle.

(Source:

https://www.radioactivity.eu.com/site/pages/Pauli_Letter.htm)

The neutrino does not appear in the calculation of Q because its mass is either zero or negligibly small.

several MeV. An excited nucleus is denoted by an asterisk (*) after its usual symbol. For example, If X is any nucleus, then X^* indicates a nucleus in an excited state. Gamma decay represented as:



Note that γ -rays associated with β -decay may be emitted only after the β particle has been ejected. This means that the nucleus in the excited state attains the normal state by the emission of γ -rays. The energy spectrum of γ -rays consists of several discrete lines.

Now you may be interested in knowing some examples of γ -decay. Refer to Fig. 13.6 which shows a typical energy level diagram for ^{27}Mg .

- i) Note that gamma rays follow successive beta emissions before attaining the stable nuclei of ^{27}Al . Note that $^{27}\text{Al}^*$ nucleus undergoes two gamma decays to reach the ground state.

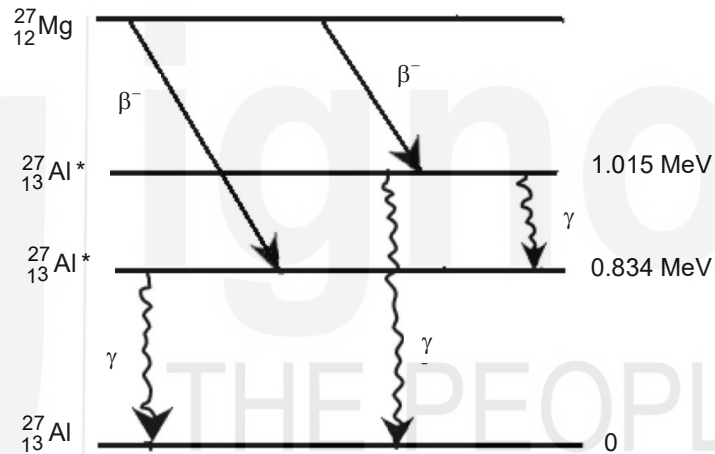


Fig. 13.6: Gamma emission in the decay of ^{27}Mg to ^{27}Al .

- ii) Refer to Fig. 13.7, which shows emission of γ -rays in β -decay of ^{60}Co .

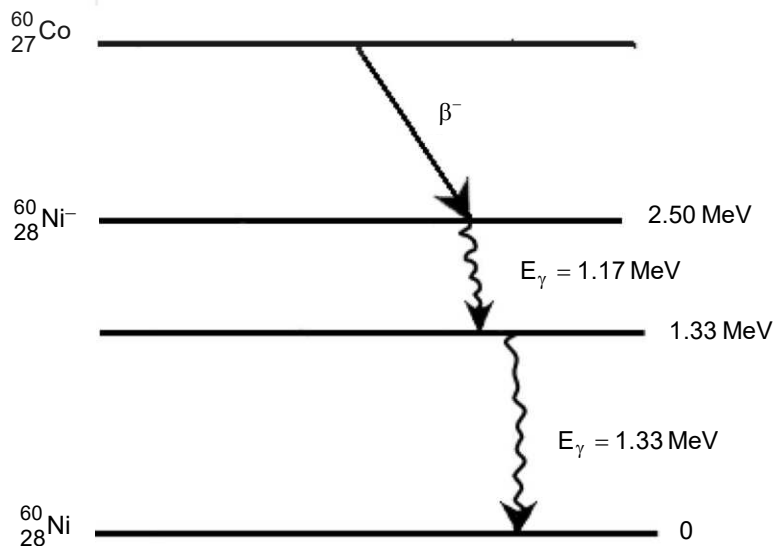


Fig. 13.7: γ -emission in the decay of ^{60}Co to ^{60}Ni .

You should note that nuclei can be excited from the ground state to an excited state by absorption of a photon of appropriate energy.

The study of nuclear gamma emission is an important tool of the nuclear physicist; the energies of the gamma rays can be measured with great precision, and these provide a powerful means of determining the energies of the excited states of nuclei.

Applications of radioactivity

In Sec. 13.3.3, you have learnt that radiocarbon dating is used to determine the age of many biological and geological specimens like mummies, minerals in asteroids, dating of wooden implements, clothes, leather, charcoals etc.

You have studied one example. Here we give another example on carbon dating.

EXAMPLE 13.5: CARBON DATING

A piece of wood from the ruins of an ancient dwelling was found to have a ^{14}C activity of 13 disintegrations per minute per gram of its carbon content. The ^{14}C activity of living wood is 16 disintegrations per minute per gram. How long ago did the tree die from which the wood sample came? Take half-life of radiocarbon as 5760 years.

SOLUTION ■ If the activity of a certain mass of carbon from a living plant or animal is R_0 and the activity of the same mass of carbon from the sample to be dated is R , then we use the equation

$$R = R_0 e^{-\lambda t}$$

$$\therefore e^{\lambda t} = \frac{R_0}{R} \Rightarrow \lambda t = \ln \frac{R_0}{R}$$

or
$$t = \frac{1}{\lambda} \ln \frac{R_0}{R}$$

The decay constant λ of ^{14}C is:

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5760} \text{ yr}^{-1}$$

and
$$\frac{R_0}{R} = \frac{16}{13}$$

$$\therefore t = \frac{1}{\lambda} \ln \frac{R_0}{R} = \frac{5760 \text{ y}}{0.693} \ln \frac{16}{13} = 1.7 \times 10^3 \text{ yr}$$

The β -active carbon isotope ^{14}C called radiocarbon, is used to determine the age of objects: More the proportion of daughter nuclei than the normal value, the older is the specimen. When plants and animals die, their radiocarbon content keeps decaying to ^{14}N . Radiocarbon beta decays to ^{14}N with a half-life of 5760 years. It means that after 5760 years, only one-half ^{14}C will be left relative to the total carbon content that they had as living matter. So, by determining the proportion of radiocarbon (^{14}C) to ordinary carbon, we can determine the age of the remains and ancient objects of organic origin.

Let us now sum up what you have learnt in this unit.

13.6 SUMMARY

Concept	Description
Nuclear stability	■ Certain nuclei transform spontaneously from one value of Z and N to another by emitting an α or a β-particle and attain stability.
Radioactive series	■ The naturally occurring radioactive elements conform to three radioactive series $(4n+2)$, $(4n+3)$ and $(4n)$, known as the uranium, actinium and thorium series. Each series starts with an element having an extremely long half-life and terminates in a stable isotope of lead. With the discovery of transuranic elements, a fourth radioactive series $(4n+1)$ known as the neptunium series has been identified; this starts with plutonium and terminates in a stable isotope of bismuth.
Units of radioactivity	■ The standard unit of radioactivity, the curie, is defined as the quantity of any radioactive material giving 3.7×10^{10} disintegrations per second. The SI unit of radioactivity is Rutherford. It is defined as the amount of a radioactive substance giving 10^6 ds^{-1}.
Half-life of a radioactive element	■ The half-life of a radioactive element is the time in which half of the radioactive atoms disintegrate; this is related to the disintegration constant λ and mean life τ through the relations $T_{1/2} = 0.693/\lambda = 0.693\tau$.
Radioactive equilibrium	■ Depending on the relative magnitudes of the half-lives of various nuclei in a decay chain, a situation is obtained where the number of parent and/or daughter atoms either remains constant or bears a constant ratio. This is known as radioactive equilibrium.
Law of radioactive decay	■ The number of atoms disintegrating per unit time is given by $N = N_0 \exp(-\lambda t)$ where N_0 is the number of atoms present initially and λ is the disintegration constant of the radioactive element.
Secular equilibrium	■ If the half-life of parent atom is very long compared to any of its decay products, we get a long-term equilibrium, known as secular equilibrium, between the parent and the daughter nuclei; each member decays at the same rate at which it is produced, i.e., we have $\lambda_A N_A = \lambda_B N_B = \lambda_C N_C$
Transient equilibrium	■ If the parent A is longer lived than the daughter B, but half-life is not very long, we obtain transient equilibrium, in which the ratio of the members of nuclei A and B at any instant remains constant.
Radioactive decay	■ A radioactive nucleus disintegrates spontaneously by emitting either an α or a β-particle, usually accompanied by γ-radiation.
Alpha decay	■ In alpha decay, an unstable nucleus disintegrates as follows: ${}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 \text{He}$

Beta decay

- In beta decay, a negatively charged particle called a β -particle, later identified with the electrons, is emitted from the nucleus and the original nuclide transforms to an isobar of that nuclide.
- In order to conserve energy and momentum in β -decay, Wolfgang Pauli proposed that in β -decay, another particle is emitted which is neutral, has small or zero rest mass and spin equal to $1/2$. This neutral particle is named **neutrino** (little neutron).

Gamma decay

- After alpha or beta decay, the daughter nucleus may be left in the excited state. Such nuclei return to their ground state by the emission of one or more photons known as nuclear **gamma rays**.

13.7 TERMINAL QUESTIONS

1. The half-lives of Radium and Radon are 1620 yr and 3.82 days respectively. Calculate the volume of Radon gas at N.T.P. equivalent to one curie.
2. A sample containing 0.1 mg of ^{230}Th undergoes 4.32×10^6 disintegrations per minute. What is the half-life of this nuclide?
3. Due to accident in a nuclear reactor, a radioactive element got spread inside the operator room and the level of radiation became 50 times the permissible level for normal occupancy of the room. After how many days would the room be safe for occupation? The half-life of the radioactive substance is 30 days.
4. A sample of pitchblende contains 40 kg uranium. The half-life of uranium is 4.5×10^9 yr. The atomic weight of uranium is 238.4. The total number of uranium atoms in the beginning is 1.269×10^{26} atoms. Calculate the age of the sample.
5. The half life of an element (^{131}I) is 8.05 days. After how many days will only 10% of the element (^{131}I) be left over?

13.8 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. Loss of 6 α and 4 β -particles implies loss of 6×4 nucleons and $6 \times 2 - 4 = 8$ units of positive charge from the $^{232}_{90}\text{Th}$ nucleus giving rise to an isotope of mass number $N = 232 - 24 = 208$ and charge number $Z = 90 - 8 = 82$.

The end product is thus $^{208}_{82}\text{X} = ^{208}_{82}\text{Pb}$.

2. Here $\tau = 14.43$ months

$$\therefore T_{1/2} = \tau \ln 2$$

$$= (14.43 \text{ months}) \times 0.693$$

$$= 10 \text{ months}$$

Since 75% of the substance decays, only 25% remains.

$$\therefore N = \frac{25N_0}{100} = \frac{N_0}{4} = \left(\frac{1}{2}\right)^2 N_0$$

Using Eq. (13.2) we can write

$$\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

so that

$$\frac{t}{T_{1/2}} = 2$$

$$t = 10 \times 2 \text{ months}$$

$$= 20 \text{ months}$$

$$3. \text{ Half-life } T_{1/2} = 4.51 \times 10^9 \text{ yr} = 4.51 \times 10^9 \times 365 \times 86400 \text{ s}$$

$$\text{Decay constant } \lambda = \frac{0.693}{T_{1/2}}$$

$$= \frac{0.693}{4.51 \times 10^9 \times 365 \times 86400} \text{ s}^{-1} = 4.87 \times 10^{-18} \text{ s}^{-1}$$

Number of atoms per g of uranium is

$$N = \frac{6.03 \times 10^{23}}{238}$$

$$\therefore \text{Rate of disintegration } \left| \frac{dN}{dt} \right| = \lambda N$$

$$= \frac{6.03 \times 10^{23}}{238} \times 4.87 \times 10^{-18} \text{ s}^{-1}$$

$$= 1.234 \times 10^4 \text{ s}^{-1}$$

4. From Eq. (13.17) we know that

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_0 \exp(-\lambda_A t)$$

Multiplying throughout by $\exp(\lambda_B t)$ and re-arranging terms, you will obtain

$$\frac{d}{dt} [N_B \exp(\lambda_B t)] = \lambda_A N_0 \exp[(\lambda_B - \lambda_A)t]$$

This can readily be integrated to give

$$N_B e^{\lambda_B t} = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 \exp[(\lambda_B - \lambda_A)t] + K$$

where K is constant of integration. To evaluate it, we use the condition

$$N_B = 0 \text{ at } t = 0.$$

This gives

$$K = -\frac{\lambda_A}{\lambda_B - \lambda_A} N_0 = \frac{\lambda_A}{\lambda_A - \lambda_B} N_0$$

On inserting this value of K in the above expression we obtain the required result:

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_{A0} [\exp(-\lambda_A t) - \exp(-\lambda_B t)]$$

5. We know from Eq. (13.18) that the number of RaB atoms is given by

$$N_B = \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

If N_B attains maximum value at

$$t = t_m, \text{ we must have } \frac{dN_B}{dt} = 0 \text{ at } t = t_m \text{ and } \frac{d^2 N_B}{dt^2} < 0.$$

Therefore, on differentiating the expression for N_B once with respect to t and equating it to zero, we obtain

$$\therefore -\lambda_A e^{-\lambda_A t_m} + \lambda_B e^{-\lambda_B t_m} = 0$$

$$\text{or } \lambda_B e^{-\lambda_B t_m} = \lambda_A e^{-\lambda_A t_m}$$

$$\therefore e^{(\lambda_A - \lambda_B)t_m} = \frac{\lambda_A}{\lambda_B}$$

$$\text{and } t_m = \frac{\ln(\lambda_A / \lambda_B)}{\lambda_A - \lambda_B}$$

Substituting $\lambda_A = 3.8 \times 10^{-3} \text{ s}^{-1}$ and $\lambda_B = 1.3 \times 10^{-4} \text{ s}^{-1}$, we get

$$t_m = \frac{\ln\left(\frac{3.8 \times 10^{-3} \text{ s}^{-1}}{4.3 \times 10^{-4} \text{ s}^{-1}}\right)}{(3.8 \times 10^{-3} - 4.3 \times 10^{-4}) \text{ s}^{-1}} = \frac{\ln(8.837)}{33.7 \times 10^{-4} \text{ s}^{-1}} = \frac{2.179 \text{ s}}{33.7 \times 10^{-4}}$$

$$= 647 \text{ s} = 10 \text{ m } 47 \text{ s}$$

6. Since uranium is in secular equilibrium with radium, we have

$$\frac{N_1}{\tau_1} = \frac{N_2}{\tau_2}$$

$$\text{or } \tau_1 = \frac{N_1}{N_2} \tau_2 = 2.8 \times 10^6 \times 1620 \text{ yr} = 4.5 \times 10^9 \text{ yr}$$

7. For elements A and B in transient equilibrium, we have from Eq. (13.27),

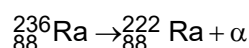
$$\frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A}$$

Hence the ratio of the measured activities at equilibrium is given by

$$\frac{A_B}{A_A} = \frac{\lambda_B N_B}{\lambda_A N_A} = \frac{\lambda_B}{\lambda_A} \cdot \frac{\lambda_A}{\lambda_B - \lambda_A} = \frac{\lambda_B}{\lambda_B - \lambda_A}$$

$$\therefore A_B = \frac{\lambda_B}{\lambda_B - \lambda_A} A_A$$

8. The decay process of ${}^{236}_{88}\text{Ra}$ is



Using Eq. (2), we can calculate the value of Q as

$$Q = \frac{A}{A-4} KE_{\alpha} = \frac{226}{222} \times 4.7848$$

$$= 4.8710 \text{ MeV}$$

The mass equivalent of this Q value is

$$m_Q = \frac{4.8710 \text{ MeV}}{931.5 \text{ MeV/u}} = 0.0052 \text{ u}$$

Therefore,

$$Q = (m_i - m_f - m_{\alpha}) C^2$$

$$m_Q = \frac{Q}{C^2} = m_i - m_f - m_{\alpha}$$

$$m_f = m_i - m_{\alpha} - m_Q$$

$$m_f = 226.0254 \text{ u} - 4.0026 \text{ u} - 0.0052 \text{ u}$$

$$m_f = 222.0176 \text{ u}$$

Terminal Questions

1. One curie is equivalent to the amount of radon in equilibrium with 1 g of radium. Hence if N_{Rn} is the number of Rn atoms in equilibrium with 1 g of Ra and N_{Ra} is the number of atoms in 1 g of radium, then we have

$$\lambda_{\text{Rn}} N_{\text{Rn}} = \lambda_{\text{Ra}} N_{\text{Ra}}$$

$$\therefore N_{\text{Rn}} = N_{\text{Ra}} \frac{\lambda_{\text{Ra}}}{\lambda_{\text{Rn}}} = N_{\text{Ra}} \frac{\lambda_{\text{Rn}}}{\lambda_{\text{Ra}}} = \frac{3.82 \text{ d}}{1620 \text{ yr}} \times N_{\text{Ra}}$$

$$= \frac{3.82}{1620 \times 365} \times \frac{N}{226}$$

where N is Avogadro's number.

$\therefore$ Volume occupied by N_{Rn} atoms at NTP

$$= \frac{N_{\text{Rn}}}{N} \times 22.4 \times 10^3 \text{ cm}^3$$

$$= \frac{3.82}{1620 \times 365 \times 226} \times 22.4 \times 10^3 \text{ cm}^3$$

$$= 6.4 \times 10^{-4} \text{ cm}^3$$

2. Number of atoms in 0.1 mg of ^{230}Th

$$N = \frac{6.02 \times 10^{23}}{230 \text{ g}} \times (10^{-4} \text{ g}) = 2.62 \times 10^{17}$$

$$\text{Rate of disintegration } \frac{dN}{dt} = 4.32 \times 10^6 \text{ s}^{-1}$$

$$= 7.2 \times 10^4 \text{ s}^{-1}$$

$$\therefore \text{Decay constant } \lambda = \frac{1}{N} \frac{dN}{dt}$$

$$= \frac{7.2 \times 10^4 \text{ s}^{-1}}{2.62 \times 10^{17}} = 2.75 \times 10^{-13} \text{ s}^{-1}$$

$$\therefore T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{2.75 \times 10^{-13} \text{ s}}$$

$$= 7.99 \times 10^4 \text{ yr}$$

We have from Eq. (13.13b)

$$\frac{N_2(t)}{N_2(\infty)} = 1 - e^{-\lambda_2 t} = \frac{90}{100} \text{ (given)}$$

$$\therefore e^{-\lambda_2 t} = \frac{1}{10} \text{ or } t = \frac{\log_e 10}{\lambda_2} = \tau_2 \frac{\log_e 10}{\log_e 2}$$

$$\text{Hence, } \frac{t}{\tau_2} = \frac{\log_e 10}{\log_e 2} = 3.32$$

3. Here

$$\frac{N}{N_0} = \frac{1}{50} \text{ and } T_{1/2} = 30 \text{ days}$$

$$\text{so that } \tau = \frac{T_{1/2}}{0.693} = \frac{30 \text{ days}}{0.693} = 43.3 \text{ days}$$

$$\text{Hence } \frac{N}{N_0} = \frac{1}{50} = \exp(-t / 43.3 \text{ days}).$$

After rearranging, we get

$$\frac{t}{43.3 \text{ days}} = \ln 50 = 3.912$$

and

$$t = 3.912 \times 43.3 \text{ days} = 169.4 \text{ days}.$$

4. Since the amount of uranium is 40 kg. The number of atoms in 40 kg uranium is

$$N = \frac{40}{238.4} \times 6 \times 10^{26} = 1.007 \times 10^{26}$$

It is given that total number of uranium atoms in the beginning

$$\therefore N_0 = 1.269 \times 10^{26} \text{ atoms}$$

$$\text{Since } T_{1/2} = 4.5 \times 10^9 \text{ yr, } \tau = \frac{T_{1/2}}{\ln 2} = \frac{4.5 \times 10^9 \text{ yr}}{0.693} = 6.494 \times 10^9 \text{ yr}$$

From Eq. (13.3) we know that

$$N = N_0 e^{-t/\tau}$$

$$\frac{N_0}{N} = \frac{1.269 \times 10^{26} \text{ atm}}{1.007 \times 10^{26}}$$

$$= 1.26$$

so that

$$\frac{t}{\tau} = \ln\left(\frac{N_0}{N}\right)$$

$$\begin{aligned}\text{or } t &= \tau \ln\left(\frac{N_0}{N}\right) = 6.494 \times 10^9 \text{ yr} \ln(1.26) \\ &= 6.494 \times 10^9 \text{ yr} \times 2.303 \log(1.26) \\ &= (6.494 \times 10^9 \text{ yr}) \times 0.2312 \\ &= 1.50 \times 10^9 \text{ yr}\end{aligned}$$

5. The half-life of the element is = 8.05 days

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{8.05 \text{ days}}{0.693} = 11.62 \text{ days}$$

So, no. of days in which only 10% element (I^{131}) is left is given by

$$\frac{N}{N_0} = \frac{1Q}{10Q} = 0.1$$

Therefore,

$$0.1 = \exp\left(-\frac{t}{11.62 \text{ days}}\right)$$

On rearranging terms

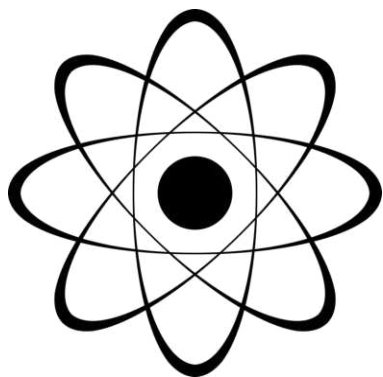
$$\exp\left(\frac{t}{11.62 \text{ days}}\right) = 10$$

or taking logarithm both sides, we get

$$\frac{t}{11.62 \text{ days}} = \ln 10 = 2.303 \log 10$$

$$\frac{t}{11.62 \text{ days}} = 2.303$$

$$t = 26.76 \text{ days}$$



UNIT 14

Picture of an atom. The nucleons stay in the nucleus, electrons revolve around it in elliptical orbits.

(Source: <https://pixabay.com/vectors/nucleus-physics-atom-protons-35000/>)

THE ATOMIC NUCLEUS

Structure

- | | | | |
|------|-------------------------------|------|--|
| 14.1 | Introduction | 14.4 | How Do Nucleons Stay Together: Nuclear Force |
| | Expected Learning Outcomes | | |
| 14.2 | Constituents of the Nucleus | 14.5 | Summary |
| | Isotopes | 14.6 | Terminal Questions |
| | Nuclear Density | 14.7 | Solutions and Answers |
| 14.3 | Binding Energy of the Nucleus | | |
| | Semi-empirical Mass Formula | | |

STUDY GUIDE

In this unit, you will learn about the atomic nucleus; its constituents and the nature of the forces that keep the nucleons (protons and neutrons) confined within the nucleus. You will also learn Heisenberg's uncertainty principle, which tells us why negatively charged electrons cannot reside inside the nucleus. You would be able to discuss the stability of the nuclei based on the binding energy curve. This will enable you to estimate the size and density of the nucleus. You will know why some naturally occurring elements are more abundant and stable than others. We have included several examples in the unit to enhance your grasp of various concepts discussed here. Several SAQs and TQs are included in the unit to help you gain the essence of various concepts. You should answer SAQs and work out problems yourself to enhance your conceptual clarity. We are aware that some of the concepts discussed here are known to you. But we advise you to refresh your memory and revise these.

"Imagination is more important than knowledge."

Albert Einstein

14.1 INTRODUCTION

In Unit 13, you have learnt about radioactivity exhibited by heavier elements in the form of spontaneous emission of alpha and beta particles and gamma rays to attain stability. From your senior school physics classes, you will recall that existence of atoms was postulated by Greek philosopher Democritus around 450 BC. (Indian scientist Acharya Kanad for the concept of parmanu.) However, Aristotle, an influential Greek philosopher rejected Democritus' idea. But in 1808, John Dalton, a British chemist, revived the brilliant idea of the atom as the smallest indivisible entity of all matter to explain the laws of chemical composition. Through his subsequent researches, Dalton provided evidence for the existence of atoms. As we now know, atoms are too small to be weighed even with most sensitive balance available now or observed even by the most powerful microscope using visible light. (The wavelength of visible light is in the range 400 to 700 μm , which is too large compared to the size of the atom).

Dalton's conjecture was substantiated by later researches of Thomson, Rutherford and Bohr. While Thomson discovered electrons and proposed that these were fundamental constituents of all matter, Rutherford gave the model of atom as consisting of a hard central core –the nucleus--surrounded by electrons. Thereafter, using the ideas of quantum mechanics proposed by Planck, Bohr argued that electrons revolve around the nucleus in well defined orbits. Einstein's theory of Brownian motion by assuming that solute particles were continuously being knocked around by water molecules was experimentally confirmed by Jean Perrin. Using Avogadro's hypothesis, he calculated the size of the atom to be around 10^{-8}cm . These studies opened a very fertile field of research in nuclear physics in the first half of the 20th century. Some of the questions that engaged the leading physicists of that time were:

- How are electrons distributed in an atom?
- What is the nucleus made of?
- How do nucleons stay together in the nucleus?

Answers to these and such other questions led to a pool of brilliant knowledge. In Sec. 14.2, you will learn about constituents of the nucleus and how electrons are distributed in a nucleus. In Sec. 14.3, you will learn that stability of a nucleus is governed by its binding energy per nucleon (BE/A). An important consequence of BE/A is that very light and very heavy elements show tendency for fusion or fission under suitable conditions. (These concepts have been used to generate electricity and you will learn about these in the next unit.) In Sec. 14.4 you will learn about nuclear forces which hold the nucleons (protons and neutrons) together while overcoming Coulomb repulsion between protons.

In the next unit, you will learn about a few applications of nuclear physics.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ list the constituents of the nucleus;
- ❖ explain why electrons cannot residing inside the nucleus;
- ❖ explain the nature of nuclear forces;
- ❖ discuss the stability of nuclei based on the binding energy curve; and
- ❖ calculate the binding energy using Weizsäcker's semi-empirical mass formula.

We begin our journey of the study of the nucleus by discussing what constitutes it.

14.2 CONSTITUENTS OF THE NUCLEUS

Let us estimate the size of the atom.

We know that iron is one of the most abundant elements found on earth. The density of pure iron is 7.874 gcc^{-1} and its molar mass of 56 g. On the basis of Avogadro's hypothesis, we can say that the number of atoms in 56 g iron is equal to Avogadro's number; 6×10^{23}). To get an estimate of the volume occupied by an iron atom, we will take density of iron as 8 gcc^{-1} . It means

that 6×10^{23} atoms of iron occupy a volume $\frac{56}{8} = 7 \text{ cm}^3$.

Therefore, the volume occupied by one iron atom is $\frac{7}{6 \times 10^{23}} \text{ cc}$ which can be taken to be approximately as 10^{-23} cc .

If we assume that iron is packed as hard iron spherical atoms, then its volume will be $\frac{4}{3} \pi r_0^3$ where r_0 is the radius of the iron atom. That is volume of an atom of iron is directly proportional to the third power of its radius. Hence, the order of magnitude of the radius of an iron atom would be $(10^{-23})^{1/3} \sim 10^{-8} \text{ cm}$.

When light of sufficiently high frequency is made to fall on a metal plate, negatively charged particles are emitted due to photo-electric effect.

J.J. Thompson identified the negatively charged particles as electrons and measured their charge to mass ratio, $\frac{e}{m}$. He argued that since electrons are

constituents of atoms, electrical neutrality of an atom implies that there must be an equal positive charge in the atom. On the basis of these results, Thomson proposed his 'plum-pudding model' of atom. He suggested that the atom consisted of a uniform positive sphere of radius nearly 10^{-10} m . with electrons embedded in such a way as to give it a stable configuration. (That is, electrons were embedded in a spherical cloud of positive charges like seeds in a water melon.),

On the basis of his experimental results, Rutherford proposed that the total positive charge of the atom was not distributed uniformly. It was, in fact,

concentrated in an extremely small region of size $\sim 10^{-14}$ m inside the atom. Atomic size was found to be roughly ten thousand times the size of the inner region ($\sim 10^{-10}$ m) where most of the mass and positive charges resided.

Rutherford thus proposed that

- all the mass and positive charge of an atom is concentrated in a tiny nucleus;
- the radius of the nucleus is of the order of a few fermi (1 fermi = 10^{-15} m) and
- electrons reside outside the nucleus.

In a series of experiments carried out by Chadwick in 1920, it was established that the charge on the nucleus was exactly equal to the total electronic charge of the atom, thus rendering the atom neutral. Each atom can be uniquely characterised by a single number, known as the atomic number. It is denoted by the symbol Z . The nucleus thus consists of Z positively charged particles called protons and their mass is roughly equal to the mass of atomic hydrogen. You may now ask: Do all nuclei consist of only protons? To answer this question, we consider three forms of hydrogen namely, Hydrogen, Deuterium and Tritium. All three forms of hydrogen have identical chemical properties corresponding to one atomic electron each. Their masses however, differ

$$m_H : m_D : m_T :: 1 : 2 : 3$$

Thus, if the mass of the atom is mainly contributed by the positively charged protons, Hydrogen, Deuterium and Tritium should contain 1, 2 and 3 protons, respectively, in their nuclei. The charge neutrality would then demand that the nucleus must also contain the electrons to neutralize the excess positive charge. Thus, the composition of these three forms of hydrogen should be as follows:

Element	No. of Atomic Electron	Particles in the Nucleus
Hydrogen	1	1 proton
Deuterium	1	2 protons + 1 electron
Tritium	1	3 protons + 2 electrons

But there is a flaw in this reasoning. This brings us to the question: Do electrons reside inside the nucleus? The answer is not affirmative. To know the correct response, go through the following paragraph.

Let us find out whether the electrons stay inside the nucleus or are the constituents of the nucleus. (You have studied about Heisenberg's uncertainty principle in Sec. 6.2, Unit 6 of Block 2.)

If electrons were trapped in the nucleus of size 10^{-14} m, then according to Heisenberg's uncertainty principle, there would be uncertainty in its position $\Delta x \sim 10^{-14}$ m. The corresponding uncertainty in its momentum (according to Heisenberg's Uncertainty Principle ($\Delta x \Delta p_x \geq \hbar$)) would then be

$$\Delta p_x \sim \frac{\hbar}{\Delta x} = \frac{\hbar c}{\Delta x c} = \frac{1}{c} \frac{1.97 \text{ MeV}}{0.1} \sim 19.7 \frac{\text{MeV}}{c}$$

The kinetic energy of the electron would thus to

$$K = \sqrt{p^2 c^4 + m_e^2 c^4} - m_e c^2 \sim 19 \text{ MeV}$$

Thus, electrons confined inside the nucleus would have an uncertainty in their kinetic energy of the order of 19 MeV where $m_e c^2$ is the rest mass energy of the electron.

However, the electrons emitted from the nucleus in nuclear beta decay typically had kinetic energy of the order of few MeV. This therefore suggested that electrons could not possibly reside inside the nucleus. You may now ask: If the number of protons inside the nucleus equals the number of atomic electrons (required for charge neutrality), what is the source of missing mass in a nucleus? To know answer to this question, go through the following example.

EXAMPLE 14.1 : MISSING MASS IN THE NUCLEUS

Iron has 26 electrons but its mass is roughly 56 times the mass of the hydrogen atom. Charge neutrality implies that the nucleus should have 26 positively charged protons, which would account for roughly 26 times the hydrogen mass to its overall mass value. Where then has the missing mass (equal to $56 - 26 = 30$ proton mass) gone?

SOLUTION ■ This discrepancy in the atomic mass pointed to the possibility of the existence of new particles in the nucleus that contributed to its mass but not to its charge.

Rutherford conjectured the existence of a particle devoid of all charge but slightly heavier than the proton in the nucleus. The particle was named neutron by him. The experimental evidence for the existence of neutrons came in 1932 through the research of Chadwick. It then became clear that all nuclei were composed of protons and neutrons with the exception of hydrogen nucleus. The number of protons and neutrons together define the mass number, denoted by A . Neutrons and protons are collectively referred to as nucleons.

Table 13.1: Constituents of the Nucleus

Name	Symbol	Charge	Mass
Proton	p	+ve	1.007276 u = 938.28 MeV
Neutron	n	0	1.008666 u = 939.57 MeV

14.2.1 Isotopes

We often come across elements whose atoms have the same number of electrons but different number of neutrons. So, their nuclear masses differ. Such atoms are called **Isotopes**. For example, deuterium nucleus has one proton and one neutron and tritium nucleus consists of one proton and two neutrons. Since they have only one electron like hydrogen, these are three

different isotopes of hydrogen and are denoted by the symbols ^1H , ^2H , ^3H . Uranium for example has three isotopes ^{233}U , ^{235}U and ^{238}U and so on. You should write out isotopes of other elements known to you.

14.2.2 Nuclear Density

The alpha-particle experiment performed by Geiger and Marsden under the guidance of Rutherford provided the first evidence that nuclei are of extremely small size ($\sim 10^{-15}\text{m}$). They bombarded gold foil by alpha particles emitted by radioactive Bismuth. Since then, various experiments have been performed using high energy electrons and neutrons to determine nuclear dimensions. These experiments have revealed that

- nuclei do not have sharp boundaries; and
- the density of nuclear matter is maximum at the centre of the nucleus and decreases gradually to zero as the distance increases.

You would now like to know the order of magnitude of the density of nuclear matter. Let us consider the lightest nucleus of hydrogen of mass $1.673 \times 10^{-27}\text{kg}$ and radius $1.2 \times 10^{-15}\text{m}$. If we assume it to be spherical in shape, the density of nuclear matter can be calculated as follows:

$$d_{\text{H}} = \frac{M_{\text{H}}}{\frac{4\pi}{3}R_{\text{H}}^3} = \frac{1.673 \times 10^{-27}\text{kg}}{\frac{4\pi}{3}(1.2 \times 10^{-15}\text{m})^3} = 2.3 \times 10^{17}\text{kgm}^{-3}$$

This value is extremely high. Recall the densities of water ($= 10^3\text{kgm}^{-3}$) and mercury ($13.6 \times 10^3\text{kgm}^{-3}$). It means that nuclear matter is extremely densely packed. The mass of our Earth ($= 6 \times 10^{24}\text{kg}$), if packed to such high density, would lead to a sphere of radius only $\sim 184\text{m}$! You should convince yourself by doing this calculation. Also, calculate the radius of the nuclear sphere whose mass will be equal to the mass of our sun. Your answer should be nearly 10km !

Next, let us calculate the density of nuclear matter for oxygen. It is reliably known that the radius of oxygen nucleus $R_0 = 3 \times 10^{-15}\text{m}$ and its mass $M_0 = 2.7 \times 10^{-26}\text{kg}$. Therefore, the density of oxygen nucleus is given by

$$d_{\text{O}} = \frac{2.68 \times 10^{-26}\text{kg}}{\frac{4\pi}{3}(3 \times 10^{-15}\text{m})^3} = 2.39 \times 10^{17}\text{kgm}^{-3}$$

Note that the densities of hydrogen and oxygen nuclei are nearly the same. Is it a mere coincidence? To discover the answer, solve the following SAQ.

SAQ 1 – Density of nuclei

Calculate the density of carbon and lead nuclei using the following data:

$$M_{\text{C}} = 19.92 \times 10^{-27}\text{kg}, \quad R_{\text{C}} = 2.7 \times 10^{-15}\text{m}$$

$$M_{\text{Pb}} = 3.4 \times 10^{-25}\text{kg} \quad \text{and} \quad R_{\text{Pb}} = 7 \times 10^{-15}\text{m}$$

On solving SAQ 1, you will come to the conclusion that nuclei of all elements have roughly the same density. This suggests that the

- nucleus is analogous to a drop of liquid, and
- there is an empirical relation between the radius of a nucleus, R , and its mass number A . Indeed, experimental evidences suggests that R and A are connected through the relation $R = 1.12 \times 10^{-15} A^{1/3} \text{m}$.

To get a quick idea about this, let us calculate the radii of carbon and lead nuclei:

$$M_C = M_H A_C = \frac{4\pi}{3} R_C^3 d_C$$

$$R_C = \left[\frac{3}{4\pi} \left(\frac{M_H}{d_C} \right) \right]^{1/3} A_C^{1/3}$$

Similarly, $R_{\text{Pb}} = \left[\frac{3}{4\pi} \left(\frac{M_H}{d_{\text{Pb}}} \right) \right]^{1/3} A_{\text{Pb}}^{1/3}$

On substituting the values given and/or calculated in SAQ 1 for carbon and lead, you will get

$$\begin{aligned} R_C &= \left[\frac{3}{4 \times 3.1417} \left(\frac{1.673 \times 10^{-27} \text{ kg}}{2.42 \times 10^{17} \text{ kg m}^{-3}} \right) \right]^{1/3} A_C^{1/3} \\ &= \left(\frac{1.673 \times 0.2387}{2.42} \right)^{1/3} \times 10^{-15} A_C^{1/3} \text{ m} \\ &= \left(\frac{3.993}{2.42} \right)^{1/3} \times 10^{-15} A_C^{1/3} \text{ m} \\ &= (1.6502)^{1/3} \times 10^{-15} A_C^{1/3} \text{ m} \\ &= 1.1181 \times 10^{-15} A_C^{1/3} \text{ m} \\ &= 1.12 \times 10^{-15} A_C^{1/3} \text{ m} \end{aligned}$$

Similarly, for lead, we have

$$\begin{aligned} R_{\text{Pb}} &= \left[\frac{3}{4 \times 3.1416} \left(\frac{1.673 \times 10^{-27} \text{ kg}}{2.37 \times 10^{17} \text{ kg m}^{-3}} \right) \right]^{1/3} A_{\text{Pb}}^{1/3} \\ &= \left(\frac{16.73 \times 0.2387}{2.37} \right)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3} \text{ m} \\ &= \left(\frac{3.993}{2.37} \right)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3} \text{ m} \\ &= (1.685)^{1/3} \times 10^{-15} A_{\text{Pb}}^{1/3} \text{ m} \\ &= 1.1190 \times 10^{-15} A_{\text{Pb}}^{1/3} \text{ m} \\ &= 1.12 \times 10^{-15} A_{\text{Pb}}^{1/3} \text{ m} \end{aligned}$$

Now-a-days, atomic masses are expressed in terms of the actual mass of ^{12}C isotope of carbon. The unit of atomic mass, abbreviated as u , is (1/12)th of the actual mass of the ^{12}C . This is equal to $1.66 \times 10^{-27} \text{ kg}$. The energy equivalent of $1u$ is

$$1u = (1.66 \times 10^{-27} \text{ kg}) \times$$

$$(2.998 \times 10^8 \text{ ms}^{-1})^2$$

$$= 14.92 \times 10^{-11} \text{ J}$$

$$= 931.3 \times 10^6 \text{ eV}$$

$$= 931.3 \text{ MeV}$$

since

These calculations show that the relation between the radius of a nucleus, R , and its mass number A is the same, irrespective of the size of the nuclei.

Let us now discuss about binding energy of a nucleus, which determines its stability.

14.3 BINDING ENERGY OF THE NUCLEUS

You now know that the nucleus of deuterium contains one proton and one neutron. The measured (rest) masses of the proton and the neutron are $1.6723 \times 10^{-27} \text{ kg}$ and $1.6747 \times 10^{-27} \text{ kg}$, respectively. This means that the total rest mass of a neutron plus a proton is $3.34709 \times 10^{-27} \text{ kg}$. But the rest mass of a deuterium nucleus is $3.34313 \times 10^{-27} \text{ kg}$. This means that measured mass of a deuterium nucleus is $(3.34709 \times 10^{-27} \text{ kg} - 3.34313 \times 10^{-27} \text{ kg}) = 3.96242 \times 10^{-30} \text{ kg}$ less than the measured masses of a neutron and a proton. In fact, it is quite well known now that mass of any nucleus is always less than the sum of the rest masses of its constituent nucleons. This difference is termed the **mass defect**. We denote it by Δm . Mathematically, we can write

$$\begin{aligned}\Delta m &= (Z m_p + N m_n) - (M - Z m_e) \\ &= Z m_H + N m_n - M\end{aligned}\quad (14.1)$$

where M is the actual mass of the neutral atom containing Z protons and N neutrons. $m_H = (m_p + m_e)$, m_p , m_n and m_e are the masses of the hydrogen atom, the proton, the neutron and the electron respectively. It is often convenient to express the mass defect by its equivalent energy through Einstein's mass-energy equivalence relation (about which you have learnt in Unit 3 of this course):

$$\text{BE} = \Delta m c^2$$

For deuterium, we get

$$\begin{aligned}\text{BE} &= (3.96242 \times 10^{-30} \text{ kg}) \times (2.998 \times 10^8 \text{ ms}^{-1})^2 \\ &= 35.614 \times 10^{-14} \text{ kgm}^2 \text{ s}^{-2} \\ &= 3.5614 \times 10^{-13} \text{ J} \\ &= 2.223 \times 10^6 \text{ eV} = 2.223 \text{ MeV}\end{aligned}$$

since $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$.

This means that we will have to provide at least 2.223 MeV energy to free the constituent nucleons – protons and neutrons – of deuterium nucleus. We can generalise this result to say that **mass defect appears as the energy which binds the nucleons together**. This is essentially used up in doing work against the forces which bind the nucleons together.

If we provide more energy than 2.223 MeV, the extra energy appears as kinetic energy of free nucleons. This result is confirmed by observations of the photo-disintegration of a deuteron. When deuterium is bombarded by gamma ray photons, it breaks up into a proton and a neutron on absorbing a photon of energy at least equal to the binding energy.

$$E_\gamma = (m_p + m_n - m_d) c^2$$

This is shown in Fig. 14.1. Inverting the above argument we can say that when a neutron and a proton combine to form a deuteron, a small mass is found missing. Does this not suggest that the binding energy can be looked upon as a direct measure of nuclear stability? To convince yourself with the answer to this question, you may like you to solve SAQ 2.

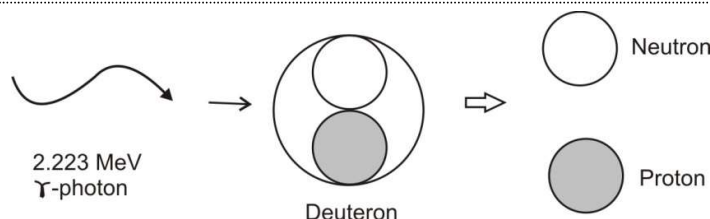


Fig. 14.1: When deuterium is bombarded by a 2.223 MeV gamma ray photon, it breaks up into a proton and a neutron.

SAQ 2 – Binding energy of nuclei

Calculate the binding energy of (i) ${}^4_2\text{He}$, (ii) ${}^{35}_{17}\text{Cl}$, (iii) ${}^{56}_{26}\text{Fe}$ and (iv) ${}^{235}_{92}\text{U}$. Given

$$m_n = 1.008665 \text{ u}, M({}^1_1\text{H}) = 1.007825 \text{ u}, M({}^4_2\text{He}) = 4.002604 \text{ u},$$

$$M({}^{35}_{17}\text{Cl}) = 34.96885 \text{ u}, M({}^{56}_{26}\text{Fe}) = 55.934932 \text{ u} \text{ and } M({}^{235}_{92}\text{U}) = 235.043933 \text{ u}.$$

On solving this SAQ you will find that binding energy of a nucleus is an increasing function of the mass number (28.3 MeV for ${}^4_2\text{He}$, 298 MeV for ${}^{35}_{17}\text{Cl}$, 492 MeV for ${}^{56}_{26}\text{Fe}$ and 1784 MeV for ${}^{235}_{92}\text{U}$. But if we divide these binding energies by the mass numbers of particular nuclei, the binding energies per nucleon are found to be 7.1 MeV, 8.5 MeV, 8.8 MeV and 7.6 MeV. Fig. 14.2 shows an explicit plot of binding energy per nucleon as a function of mass number. You will note that

- binding energy curve shows sharp peaks, particularly for ${}^4_2\text{He}$, ${}^9_4\text{Be}$, ${}^{12}_6\text{C}$, ${}^{16}_8\text{O}$ and ${}^{20}_{10}\text{Ne}$;
- with the exception of light nuclei ($A \leq 20$), the values of binding energy per nucleon fall on a smooth curve;
- binding energy per nucleon increases monotonically with a maximum in the vicinity of $A = 56$ (corresponding to the iron nucleus) with a maximum value of 8.8 MeV; and
- For $A > 56$, BE/A decreases steadily and drops to 7.6 MeV for $A = 238$.

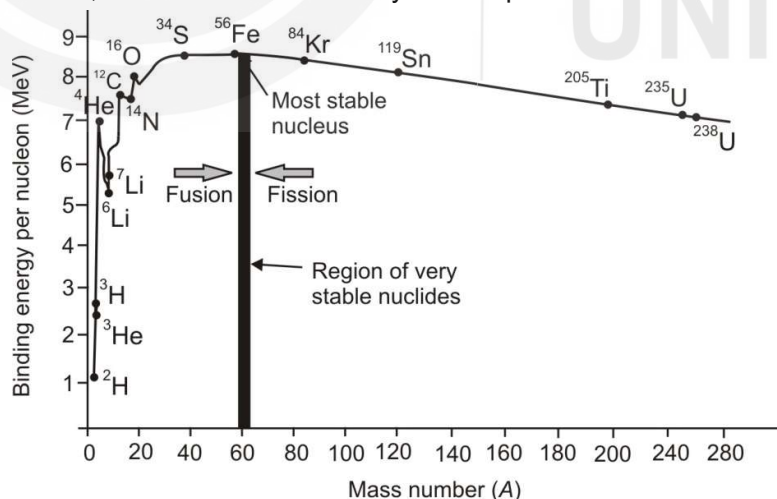


Fig. 14.2: Binding energy per nucleon as a function of mass number.

From this discussion, we can conclude that nuclei at either extreme of the periodic table are less stable as compared to the nuclei in the middle. It means that BE/A , rather than BE , is a better measure of the stability of a nucleus.

The variation of the binding energy per nucleon with mass number hints at the possibility of tapping the energy of the nucleus. For instance, when two light nuclei fuse to produce a more stable nucleus, energy is released. Such reactions are called **fusion reactions** and are responsible for the release of energy in the Sun. Efforts are now on in France and China to use controlled fusion reaction and create “artificial sun” on the earth. It is hoped that the world would get commercial fusion energy by 2050, which, once achieved, should meet our all future energy needs.

Similarly, when a very heavy nucleus breaks into two parts, the binding energy per nucleon increases leading to liberation of energy. This process is called **nuclear fission**. The amount of energy released in fission is equal to the number of nucleons times the difference in binding energy per nucleon of the reactants and the products. For example, the binding energy per nucleon in ^{235}U is nearly 7.6 MeV, whereas it is about 8.5 MeV for nuclei with mass number around 120. Thus, if a ^{235}U nucleus splits into two nearly equal fragments, there would be a gain in binding energy of the system of 0.9 MeV per nucleon. The total energy released in one fission event would therefore be nearly equal to $235 \times 0.9 \cong 212$ MeV. Do you know that the heat of combustion of a carbon atom is only about 4eV. It means that when 1 kg of uranium undergoes fission, we get 2.4 million kWh of power which is huge compared to the power (8kWh) obtained by burning 1kg coal.

14.3.1 Semi-empirical Mass Formula

A semi-empirical formula for calculating the binding energy of nuclei was given by **Weizsäcker** by considering the similarity that exists between forces which make nucleons cling together in a nucleus and the forces that bind molecules in a liquid drop. Following Weizsäcker, we quote the result for the binding energy of a nucleus containing Z protons and A nucleons:

$$\text{BE (MeV)} = \alpha A - \beta A^{2/3} - \gamma \frac{(A - 2Z)^2}{A} - \delta \frac{Z(Z-1)}{A^{1/3}} \pm \frac{\epsilon}{A^{3/4}} \quad (14.2)$$

where $\alpha = 15.8$, $\beta = 17.8$, $\gamma = 23.7$, $\delta = 0.71$ and

$$\epsilon = \begin{cases} 34, & \text{for even - even or odd - odd nuclei} \\ 0, & \text{otherwise} \end{cases}$$

Before proceeding further, let us understand the meanings of various terms in Eq. (14.2).

The first term in Eq. (14.2) represents the attractive energy of nucleons (volume energy since A is related to the cube root of radius of a nucleus); the second term corrects for the over-estimation due to the weaker binding of nucleons near the surface and is proportional to the surface area (surface effect), the third term is a negative correction due to excess neutrons (asymmetry effect) and the fourth term takes account of the electrostatic energy of protons (Coulomb effect). Since each charged particle in a nucleus repels all the other charged particles, this energy is negative and proportional to the number of pairs of protons, which is equal to $Z(Z-1)/2$. The last term is the spin term. This term is positive for even-even nuclei (i.e., both Z and N even), negative for odd-odd nuclei and zero for odd-even or even-odd nuclei. The relative contributions of the various terms in Weizsäcker's semi-empirical

formula to binding energy per nucleon as a function of mass number are plotted in Fig. 14.3.

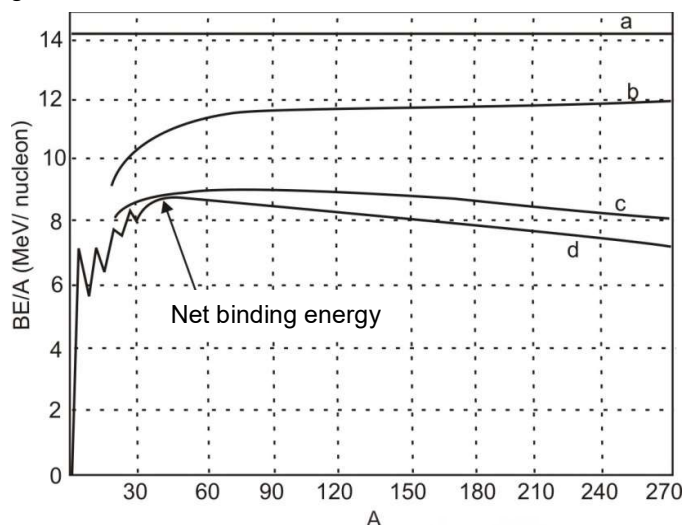


Fig. 14.3: Plot of binding energy per nucleon as a function of mass number. Curve *a* represents the volume energy. Curve *b* represents the combined effect of volume and surface energies. Curve *c* represents the cumulative effect of the first, second and fourth terms in Weizsäcker's formula. When the contribution of asymmetric and spin terms is added, we get curve *d* for net binding energy per nucleon.

Now we illustrate how Weizsäcker's formula can be used to calculate BE/A .

EXAMPLE 14.2: WEIZSÄCKER'S SEMI-EMPIRICAL FORMULA

Using Weizsäcker's semi-empirical formula, calculate the binding energy per nucleon for ^{235}U . When it is made to undergo fission, suppose that $^{149}_{60}\text{Nd}$ and $^{85}_{32}\text{Ge}$ are the two fission products. Calculate BE/A for these nuclei as well.

SOLUTION ■ In the case of ^{235}U , $A = 235$, $Z = 92$ and $N = 143$. Also since it is an even-odd nucleus, the contribution of spin term would be zero. Hence, using Eq. (14.2) we get

$$\begin{aligned} BE \text{ (MeV)} &= 15.8 \times 235 - 17.8 \times (235)^{2/3} - \frac{23.7 \times (51)^2}{235} - \frac{0.7 \times 92 \times 91}{235^{1/3}} \\ &= 3713.00 - 677.85 - 262.3 - 963.25 = 1808.6 \end{aligned}$$

You will note that the Coulomb term (fourth) dominates the surface term (second). This is due to the large number of protons in the nucleus of ^{235}U .

$$\text{The binding energy per nucleon is } = \frac{1808.6}{235} \cong 7.7 \text{ MeV.}$$

For $^{149}_{60}\text{Nd}$, we have $A = 149$, $Z = 60$ and $N = 89$. As before, the contribution of the spin term is zero. Therefore,

$$\begin{aligned} BE \text{ (MeV)} &= 15.8 \times 149 - 17.8 \times (149)^{2/3} - \frac{23.7 \times (29)^2}{149} - \frac{0.71 \times 60 \times 59}{(149)^{1/3}} \\ &= 2354.20 - 500.28 - 133.77 - 474.18 \\ &= 1246.0 \end{aligned}$$

The binding energy for nucleon is $= \frac{1246}{149} \approx 8.4 \text{ MeV}$

What do you observe? We note that for $^{149}_{60}\text{Nd}$, the Coulomb term and the surface term are nearly equal. In this case BE per nucleon comes out to be 8.4 MeV showing that fission product nuclei ^{149}Nd is more stable than ^{235}U .

For the case of $^{85}_{32}\text{Ge}$, $A = 85, Z = 32$ and $N = 53$. The contribution of spin term is zero even here. Hence,

$$\begin{aligned} \text{BE (MeV)} &= 15.8 \times 85 - 17.8 \times (85)^{2/3} - \frac{23.7 \times (21)^2}{85} - \frac{0.71 \times 32 \times 31}{(85)^{1/3}} \\ &= 1343.00 - 344.11 - 122.96 - 160.19 = 715.7 \end{aligned}$$

Therefore, binding energy per nucleon is 8.4 MeV.

You will note that in this case surface energy is more than the Coulomb energy and, of the two product nuclei, $^{85}_{32}\text{Ge}$ is more stable.

In the above example you have seen that stability of nuclei decreases as the difference between the number of neutrons and protons increases. It is now well known that in lighter stable nuclei, the number of neutrons is nearly equal to the number of protons. As we move towards higher A , the neutron number increases relative to the proton number and the excess gradually increases with increasing A . And only certain combinations of protons and neutrons form stable nuclei. This can be well understood from Fig. 13.1 of Unit 13, where we had plotted the number of neutrons (y-axis) versus the number of protons (x-axis) for stable nuclei (shown by solid circles). We noted that for nuclei with $Z \leq 20$, the stability curve was a straight line with $Z = N$. For $Z > 20$, the stability curve bent towards $N > Z$. This can be understood on the basis of semi-empirical formula by answering the following SAQ.

SAQ 3 – Semi-empirical formula

Show that for light nuclei, the fact that $Z \cong N$ is explained by the semi-empirical formula.

It is an observed fact that nuclei like $^{58}_{28}\text{Ni}$, $^{50}_{22}\text{Ti}$, $^{40}_{20}\text{Ca}$, etc. in which either N or Z , or both, are equal to 2, 8, 20, 28, 50, 82 and 126 have some very special properties, which are markedly different from those of other nuclei:

- these are more abundant in nature; and
- these are more stable than others.

These numbers are called **magic numbers**. These have proved very helpful in speculating the structure of atomic nuclei.

It is instructive to calculate the amount of energy required to remove a neutron from a nucleus. Consider a nucleus of the atom $^A_Z X_N$ where $N = (A - Z)$ is the number of neutrons in the atom which has Z atomic electrons and A is its mass number. After the removal of one neutron from the nucleus the resulting atom will have one less neutron and change into the isotope $^{A-1}_Z X_{N-1}$. Thus, the amount of energy required to remove one neutron from the atom would be

$$E_N = \left[m \left({}^A_{Z-1}X_{N-1} \right) + m_N - m \left({}^A_ZX_N \right) \right] c^2 \quad (14.3)$$

where m_N is the mass of the neutron.

You should now solve an SAQ.

SAQ 4 – Neutron separation energy

Calculate the neutron separation energy of ^{17}O and ^{57}Fe . It is given that

$$m(^{17}\text{O}) = 16.999132 \text{ u}; m(^{16}\text{O}) = 15.994916 \text{ u}$$

$$m(^{57}\text{Fe}) = 56.935398 \text{ u}; m(^{56}\text{Fe}) = 55.934942 \text{ u}$$

Take $m_N = 1.008665 \text{ u}$; $1\text{u} = 931.5 \text{ MeV}/c^2$

14.4 HOW DO NUCLEONS STAY TOGETHER: NUCLEAR FORCE

Once physicists accepted the neutron-proton model of nucleus, an important question arose: How do nucleons stay together? In other words: What is the nature of force that is responsible for the binding of nucleons in a nucleus?

Since gravitation and electromagnetic interactions explain most of the observed facts, you may be tempted to identify one of these forces as the likely force. However, the extremely small size of the nucleus, where all the protons and neutrons are closely packed suggests the existence of strong short-range attractive forces to hold them together. Do you know that these attractive forces cannot have electrostatic origin?

This is because electrostatic forces between protons are repulsive and if only these were acting, the nucleons would have been blown apart. Instead, the forces between nucleons are responsible for the large binding energy per nucleon (nearly 8 MeV) in a nucleus. Let us consider the other alternative. The force between nucleons may be gravitational since it is an attractive force between every pair of nucleons. However, it is far too weak to account for the powerful attractive forces between nucleons. If the nucleon – nucleon force is taken to be unity, the gravitational force would be of the order of 10^{-39} . We may, therefore, conclude that the purely attractive forces between nucleons are of a new type with no analogy whatsoever with other known forces in the realm of classical physics. This new attractive force is called **nuclear force**.

The gravitational as well as electrostatic forces obey the inverse square law. The situation in the case of the nucleus is entirely different. All the nucleons are closely packed in the tiny nucleus like a set of marbles in a box. The force that holds the nucleons together must exist between the individual neighbouring nucleons in the nucleus. The nuclear force between nucleons should therefore be a short range force acting over very short distances ($\sim 10^{-15} \text{ m}$). The nuclear force is negligible at large distances. It suggests that each nucleon interacts only with its nearest neighbours.

There are two major pieces of evidence for the short range nature of nuclear forces. The first evidence comes from the constancy of the nuclear density irrespective of the number of nucleons present in the nucleus (SAQ 1). The second evidence for the short range nature of nuclear forces follows from the fact that total binding energy is proportional to A , a long range force will have the total binding energy proportional to A^2 .

Recent experimental evidences suggest that nuclear forces have a charge dependent part. However, it is quite small ($< 1\%$).

We now note that nuclear forces must account for the attractive force between

- a proton and a neutron
- two protons, and
- two neutrons.

Since BE/A is roughly constant, irrespective of the mix of neutrons and protons in the nucleus, we are quite justified in considering the force between them as equivalent. That is, we consider **nuclear forces as charge independent**.

If nuclear forces were only attractive in character, nucleons should coalesce under their influence. But we know that the average separation between nucleons is constant, resulting in a nuclear volume proportional to the total number of nucleons. The possible explanation is that nuclear forces exhibit attractive character only so long as nucleons are separated through a certain critical distance. For distances less than this critical value, the character of nuclear forces changes abruptly; attraction changes to repulsion. (You should not confuse this repulsion with electrostatic repulsion.) These qualitative aspects of nuclear forces are shown in Fig. 14.4.

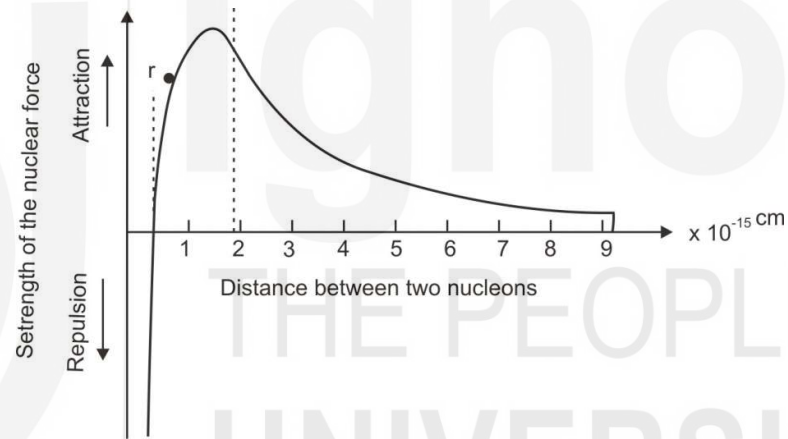


Fig. 14.4: Typical variation of nuclear forces with distance.

Let us pause for a while and ask: How do nuclear forces act between nucleons? In 1932, Heisenberg suggested that electrons and positrons shift back and forth between nucleons. A neutron, for instance, might emit an electron and become a proton, while a proton on absorbing the electron would become a neutron. However, theoretical considerations showed that the forces resulting from electron and positron exchange by nucleons would be too small (by a factor of 10^{14}) to be significant. In 1935 Japanese physicist Hideki Yukawa proposed that particles of mass in-between the masses of electrons and nucleons are responsible for nuclear forces. Now these particles are called **pions**. Pions may be charged (π^+ , π^-) or neutral (π^0); the word pion is contraction of the original name **pi-meson**.

According to Yukawa's theory, every nucleon continually emits and reabsorbs pions. An emitted pion can also be absorbed by another nucleon. The associated transfer of momentum is equivalent to the action of a force. One of the strengths of Yukawa's theory of nuclear forces is that it can account for their attractive as well as repulsive characters. There is no simple way to demonstrate this aspect formally. But as a rough analogy, let us imagine two boys exchanging volleyballs (Fig. 14.5).

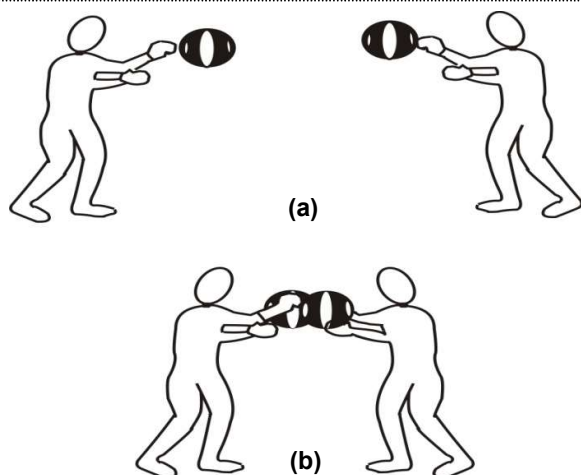


Fig. 14.5: Attractive and repulsive forces can both arise from particle exchange
a) Repulsive force due to particle exchange; b) Attractive force due to particle exchange.

You may now ask: If nucleons constantly emit and absorb pions, why are they not found with other than their usual masses? The answer is provided by Heisenberg's uncertainty principle. We know that the laws of physics refer only to measurable quantities and the accuracy with which certain combinations of measurements can be made is limited by the uncertainty principle. The emission of a pion by a nucleon, which does not change in mass – a clear violation of the law of conservation of energy – is possible if the nucleon absorbs the same or another pion so soon afterward that even in principle it is not possible to measure change in mass. The uncertainty principle does not bar an event in which energy is not conserved for the time less than $\hbar / (2\Delta E)$. This condition enables us to estimate the pion mass. This is illustrated in the following example.

EXAMPLE 14.3 : PION MASS

Assume that a pion travels between nucleons at a speed of $v \sim c$. The emission of a pion of mass m_π , represents a temporary energy discrepancy of $E - mc^2$. Calculate m_π .

SOLUTION ■ Nuclear forces have a maximum range of about 1.7 fm and the time t needed for the pion to travel this far is given by

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

Hence $(m_\pi c^2) \times \frac{r}{c} \sim \hbar$

so that $m_\pi \sim \frac{\hbar}{rc}$.

On substituting the values of $\hbar$, r and c , we get

$$m_\pi = \frac{1.05 \times 10^{-34} \text{ J.s}}{(1.7 \times 10^{-15} \text{ m}) \times (3 \times 10^8 \text{ ms})} = 2.1 \times 10^{-28} \text{ kg}$$

This is about 230 times the rest mass of the electron. Yukawa's mesons were actually discovered in 1946 in cosmic rays by Powell. The rest mass of charged pions is $273 m_e$ and that of neutral pions is $264 m_e$.

Let us now sum up what you have learnt in this unit.

14.4 SUMMARY

Concept	Description
Constituents of the Nucleus	- The atom consists of a tiny nucleus where roughly all the mass of the atom is concentrated. The nucleus consists of positively charged protons and neutral neutrons. - The size of the nucleus is roughly $\sim 1 \text{ fm} = 10^{-15} \text{ m}$, which is appreciably less than the size of the atom $\sim 10^{-10} \text{ m}$. - Nuclei of all atoms have roughly the same density of the order of $\sim 10^{17} \text{ kg m}^{-3}$.
Binding Energy	The binding energy of the nucleus is equivalent to mass defect, which, in turn, equals the difference between the observed mass of the nucleus and its constituents: $\text{BE} = \Delta mc^2 = (Z m_H + N m_N - M) c^2$
Nature of Nuclear Force	- Nucleons (protons and neutrons) are held together by strong nuclear force. - The nuclear forces are attractive at short range ($< 1.7 \text{ fm}$) and charge independent.

14.5 TERMINAL QUESTIONS

- Calculate the energy released when two ${}^2_1\text{H}$ nuclei are fused together to form a ${}^4_2\text{He}$ nucleus. The binding energy per nucleon of ${}^2\text{H}$ is 1.1 MeV and that of ${}^4\text{He}$ is 7.0 MeV.
- Calculate the binding energy of a ${}^4\text{He}$ nucleus on the basis of semi-empirical mass formula and compare it with the value obtained on the basis of mass defect (SAQ 2).
- From the semi-empirical mass formula given by Eq. (14.2), calculate the value of the atomic number (Z) for the most stable nucleus at a given mass number. Calculate Z_0 for $A = 56$.
- Calculate the radius of a neutron star of 1.4 Solar mass ($\sim 2.8 \times 10^{30} \text{ kg}$). In the neutron star, the nucleons are assumed to be squeezed to nuclear density ($\sim 10^{17} \text{ kg/m}^3$).
- Given $m_p = 1.005 \text{ u}$, $m_n = 1.009 \text{ u}$ and $m({}^4_2\text{He})$ which is an α -particle = 4.003 u, calculate the binding energy of the α -particle.
- Calculate the total binding energy of the isotopes ${}^3\text{He}$ and ${}^3\text{H}$. Also, calculate the Coulomb energy for ${}^3\text{He}$ using Weizsäcker's formula. How this difference is accounted for?

14.6 SOLUTIONS AND ANSWERS

Self-Assessment Questions

$$1. \quad d_C = \frac{M_c}{\frac{4\pi}{3} R_c^3} = \frac{19.92 \times 10^{-27} \text{ kg}}{\frac{4}{3} \times 3.1416 \times (2.7 \times 10^{-15} \text{ m})^3}$$

$$\begin{aligned}
 &= \frac{19.92}{82.45} \times 10^{18} \text{ kg m}^{-3} \\
 &= 2.42 \times 10^{17} \text{ kg m}^{-3} \\
 d_{\text{Pb}} &= \frac{M_{\text{Pb}}}{\frac{4\pi}{3} R_{\text{Pb}}^3} = \frac{3.4 \times 10^{-25} \text{ kg}}{\frac{4}{3} (7 \times 10^{-15} \text{ m})^3} \\
 &= \frac{3.4 \times 10^{20} \text{ kg m}^{-3}}{\frac{4\pi}{3} \times (7)^3} \\
 &= 2.37 \times 10^{17} \text{ kg m}^{-3}
 \end{aligned}$$

2. From Eq. (14.1) we know that

$$\text{BE} = \Delta m c^2$$

where $\Delta m = Zm_{\text{H}} + Nm_{\text{n}} - M$.

i) For ${}^4_2\text{He}$, we have $Z = 2$ and $N = 2$. On substituting the given values, we get

$$\begin{aligned}
 \Delta m({}^4\text{He}) &= 2 \times (1.007825 \text{ u}) + 2 \times (1.008665 \text{ u}) - 4.002604 \text{ u} \\
 &= 14.03298 \text{ u} - 4.002604 \text{ u} = 0.030376 \text{ u}
 \end{aligned}$$

and

$$\begin{aligned}
 \text{BE} &= 0.030376 \times (1.66 \times 10^{-27} \text{ kg}) \times (2.998 \times 10^8 \text{ ms}^{-1})^2 \\
 &= 0.030376 \times 14.94 \times 10^{-11} \text{ J} \\
 &= 0.030376 \times (14.94 \times 10^{-11} \text{ J}) / (1.602 \times 10^{-19} \text{ J eV}^{-1}) \\
 &= 0.030376 \times 9.313 \times 10^8 \text{ eV} \\
 &= 2.829 \times 10^7 \text{ eV} \\
 &= 28.3 \text{ MeV}
 \end{aligned}$$

ii) For ${}^{35}_{17}\text{Cl}$, we have $Z = 17$ and $N = 18$. Therefore,

$$\begin{aligned}
 \Delta m({}^{35}\text{Cl}) &= 17 \times (1.007825 \text{ u}) + 18 \times (1.008665 \text{ u}) - 34.96885 \text{ u} \\
 &= 17.133025 \text{ u} + 18.15597 \text{ u} - 34.96885 \text{ u} \\
 &= 0.320145 \text{ u}
 \end{aligned}$$

Since $1 \text{ u} = 931.5 \text{ MeV}$, we find that

$$\text{BE}({}^{35}\text{Cl}) = 298.2 \text{ MeV}$$

iii) For ${}^{56}_{26}\text{Fe}$, $Z = 26$ and $N = 30$. Therefore

$$\begin{aligned}
 \Delta m({}^{56}\text{Fe}) &= 26 \times (1.007825 \text{ u}) + 30 \times (1.008665 \text{ u}) - 55.934932 \text{ u} \\
 &= 26.20345 \text{ u} + 30.25995 \text{ u} - 55.934932 \text{ u} \\
 &= 0.528468 \text{ u}
 \end{aligned}$$

and

$$\text{BE}({}^{56}\text{Fe}) = 492.2 \text{ MeV}$$

iv) For ^{235}U , $Z = 92$, $N = 143$ so that,

$$\begin{aligned}\Delta m &= 92 \times (1.007825 \text{ u}) + 143 \times (1.008665 \text{ u}) - 235.043933 \text{ u} \\ &= 92.7199 \text{ u} + 144.239095 \text{ u} - 235.043933 \text{ u} \\ &= 1.915062 \text{ u}\end{aligned}$$

so that $\text{BE}(^{235}\text{U}) = 1783.8 \text{ MeV}$

3. For a given A , the ratio $\frac{N}{Z}$ will be such that the total energy E tends to a minimum. Since $m_p = m_n$, the only terms which we have to consider in discussing this minimum for a particular value of A are $\frac{(N-Z)^2}{A}$ and $\frac{Z^2}{A^{1/3}}$. The first term demands $N = Z$ while the second term demands Z to be as small as possible. This is consistent with the fact that light nuclei have $Z \cong N$.
4. Using the formula given in Eq. (14.3), separation energy of the neutron is given by

$$\begin{aligned}E_n(^{17}\text{O}) &= (15.994915 + 1.008665 - 16.999132)c^2 \\ &= 4.143 \text{ MeV}\end{aligned}$$

$$\begin{aligned}E_n(^{57}\text{Fe}) &= (55.934942 + 1.008665 - 56.935398)c^2 \\ &= 7.65 \text{ MeV}\end{aligned}$$

Terminal Questions

1. B.E. of ^4_2He nucleus $= 7.0 \times 4 = 28 \text{ MeV}$

$$\text{B.E. of } ^2_1\text{He nucleus} = 1.1 \times 2 = 2.2 \text{ MeV}$$

$$\therefore \text{Mass of } ^4_2\text{He nucleus} = 2m_p + 2m_n - 28.0 \text{ MeV}$$

$$\text{Mass of } ^2_1\text{He nucleus} = m_p + m_n - 2.2 \text{ MeV}$$

Energy released in the fusion reaction

$$\begin{aligned}E &= 2 \times \text{Mass of } ^2_1\text{He} - \text{mass of } ^4_2\text{He} \\ &= 2(m_p + m_n - 2.2) - (2m_p + 2m_n - 28.0) = 23.6 \text{ MeV}\end{aligned}$$

2. For ^4_2He , $A = 4$ and $Z = 2$ and $\varepsilon = 34$

Hence from Eq. (14.2) we get

$$\begin{aligned}\text{BE (MeV)} &= 15.8 \times 4 - 17.8 \times 4^{2/3} - 0.71 \times \frac{2}{4^{1/3}} + \frac{34}{4^{3/4}} \\ &= 63.2 - 44.9 - 0.895 + 12.02 = 29.43\end{aligned}$$

This is slightly greater than the value calculated on the basis of mass difference of helium nucleus.

3. For the most stable nucleus at a given mass number A , we have

$$\left(\frac{dBE}{dZ} \right)_A = 0 \text{ for } Z = Z_0$$

Hence we have from Eq. (14.2)

$$-2\gamma \frac{(A - 2Z_0)}{A}(-2) - \frac{\delta}{A^{1/3}}(2Z_0 - 1) = 0$$

$$\Rightarrow 4\gamma(A - 2Z_0) - \delta A^{2/3}(2Z_0 - 1) = 0$$

$$\text{or } 4\gamma A - 8\gamma Z_0 - 2\delta A^{2/3}Z_0 + \delta A^{2/3} = 0$$

$$\text{or } Z_0(8\gamma + 2\delta A^{2/3}) = 4\gamma A + \delta A^{2/3}$$

$$\Rightarrow Z_0 = \frac{4\gamma A + \delta A^{2/3}}{8\gamma + 2\delta A^{2/3}}$$

$$= \frac{23.7 \times 4A + \delta A^{2/3}}{8 \times 23.7 + 2 \times 0.71 A^{2/3}}$$

$$= \frac{94.8A + \delta A^{2/3}}{1.42 A^{2/3} + 189.6}$$

For

$$A = 56$$

$$Z_0 = \frac{94.8 \times 56 + 0.71 \times 56^{2/3}}{1.42 \times 56^{2/3} + 189.6} = \frac{5319.2}{210.4}$$

$$= 25.3 \Rightarrow 26$$

4. We know that

$$\frac{4\pi}{3} r^3 \rho = 2.8 \times 10^{30} \text{ kg}$$

Substituting the value of the density, we get

$$r = \{0.212 \times 10^{13}\}^{1/3} \text{ m} \approx (6.68)^{1/3} \times 10 \text{ Km}$$

$$\approx 18.8 \text{ Km}$$

5. Using the Eq. of Binding energy, we can write

$$BE(\alpha) = 2m_p + 2m_n - m\left({}_2^4\text{He}\right) \quad (i)$$

On substituting, the values in Eq. (i), we get

$$= 2 \times 1.005 u + 2 \times 1.009 u - 4.003 u = 0.025 u$$

$$\text{or } = 23.28 \text{ MeV}$$

6. B.E. of ${}^3\text{He} = 7.718 \text{ MeV}$

$${}^3\text{H} = 8.482$$

$\therefore$ Difference in B.E. of ${}^3\text{H}$ and ${}^3\text{He}$ is $= 0.764 \text{ MeV}$

Electrostatic B.E. from the Weizsacker's mass formula is

$$\text{Electrostatic BE (MeV)} = 0.71 \frac{Z(Z-1)}{A^{1/3}}$$

for ${}^3\text{He}$, $Z = 2$, $A = 3$

$$\therefore \text{Electrostatic BE} = \frac{-0.71 \times 2}{3^{1/3}} \approx -0.98 \text{ MeV}$$

As this accounts for the bulk of energy due to Coulomb's repulsion.





A nuclear power plant for generating electricity.

(Source: <https://pixabay.com/photos/power-plant-industry-chimney-2411932/>)

UNIT 15

APPLIED NUCLEAR SCIENCE

Structure

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STUDY GUIDE

This unit deals with applied aspects of nuclear physics. In particular, we will focus on nuclear fission and fusion. Here you will learn how energy released in a nuclear reactor based on nuclear fission is harnessed safely. You will learn about the general features of a nuclear reactor and how safe energy is produced. Experience shows that nuclear fuel such as $^{233}_{92}\text{U}$, ^{238}U and $^{232}_{90}\text{Th}$ used in a nuclear fission reactor form ^{239}Pu and $^{233}_{92}\text{U}$ by absorbing fast neutrons produced in the fission chain reaction. These are more efficient fuels and if the reactor is operated in high energy region, these even breed fuel leading to the possibility of generating unlimited energy. In Unit 14, you have learnt that fusion of two light nuclei also leads to release of large amount of energy. This is the guiding principle governing energy generation in a nuclear fusion reactor. You will learn about these in this unit. The concepts discussed in this unit are simple and It will be a good idea to master these topics one at a time. You should work out numerical problems and answer SAQs and TQs yourself to develop conceptual clarity.

“Success can come to you by courageous devotion to the task lying in front of you.”

C.V. Raman

15.1 INTRODUCTION

Energy is fundamental for our existence. We need energy to meet all our requirements: to cook food, run machines, travel from one place to another. We also need energy for agriculture, industry, communication, safer living conditions and economic growth. With continuous growth of population and our unending quest for comfort and security, the demand for energy has been growing continuously.

Today most of the energy we use is supplied by fossil fuels (coal, oil, gas etc.) which are limited resources whose supply is limited. At the projected levels of consumption, the known reserves of fossil fuels are bound to dwindle in about 50 years from now. Moreover, increased reliance on fossil fuel may further disturb the ecological balance. It can threaten all life forms and lead to catastrophic changes in climate air and water pollution, ozone depletion, acid rain, melting of glaciers, and rising sea level. Thus, we need to move away from fossil fuels by harnessing renewable sources of energy.

Another viable alternate is nuclear energy. Presently most of the nuclear energy comes from fission reactors in which fissile isotopes such as ^{235}U , ^{233}U and ^{239}Pu are used. But public acceptability of these reactors has been rather low. It has now been realised that the growing electrical power requirements of the world in the 21st century can be met either by fusion energy or renewable sources. Experience shows that nuclear fuel such as U-233 , ^{238}U and Th-232 can be used to form ^{239}Pu and U-233 by absorbing fast neutrons produced in the chain reaction in a fission reactor. These are more efficient fuels and if the reactor is operated in high energy region, it is possible to breed greater amounts of fuel. Such reactors lead to the possibility of generating unlimited energy. When two light nuclei colliding in plasma coalesce forming a heavier nucleus, a large amount of energy is released. This process is referred to as nuclear fusion. The device in which such thermonuclear reactions occur in a controlled manner is called the controlled thermonuclear reactor (CTR) or a fusion reactor. While nuclear fuel used in a fission reactor is limited, the likely ingredients of a fusion reactor – deuterium, lithium, ^3He etc. – occur in nature in enormous amounts. Though generation of commercial energy from a fusion reactor continues to be in the realm of possibilities, the world would acquire a practically inexhaustible source of energy with successful operation of a CTR. Further nuclear fusion seems attractive as it by-passes many of the safety problems associated with nuclear fission. Also, it is 'clean' in the sense that it does not leave behind harmful radioactive waste. Moreover, the energy released per unit mass of the reacting material is much greater than the corresponding quantity in nuclear fission involving heavy nuclei. For instance, energy released per nucleon in D-T fusion is nearly five times larger than the energy released per nucleon in a fission event (though the energy released per fusion 17.6 MeV is much less than the available energy of about 200 MeV in a fission reaction). However, the choice between nuclear fusion and fission as the likely energy source will probably hinge upon the public perception about the safety and eco-friendliness.

There are reports that France and China are working to produce **Artificial Sun** on the Earth.

In Sec. 15.2 to 15.4 we have discussed the fission process in some detail. You will note that since the number of neutrons emitted in each fission event

is between 2 and 3, there exists an exciting possibility of maintaining a self-sustained controlled chain reaction. Such a reaction is in fact used in a modern nuclear power reactor to generate electricity. We have discussed the functioning of nuclear fission reactors, after presenting its general features. In Sec. 15.5 we have briefly discussed how a breeder reactor can solve all energy related issues. This is followed by discussion of nuclear fusion, energy generation in stars and how a Fusion Reactor is being conceived and the progress made so far in Sec. 15.6.

Expected Learning Outcomes

After studying this unit, you should be able to:

- ❖ explain how energy is released in a fission and a fusion reaction;
- ❖ explain how a self-sustained fission chain reaction is achieved in a nuclear reactor;
- ❖ describe the functions of various components of a nuclear fission reactor; and
- ❖ discuss the working of fast breeder reactors and fusion reactors as sources of unlimited energy.

15.2 NUCLEAR FISSION

Recall from Fig. 14.2 which shows Binding Energy curve per nucleon, that intermediate mass nuclei such as barium, krypton and iron have somewhat higher binding energy per nucleon as compared to heavy nuclei such as uranium, plutonium and thorium. This is mainly because of electrostatic repulsion among protons in heavier nuclei. Thus, when a heavy nucleus splits into more tightly bound intermediate mass nuclei, a large amount of energy is released (Fig. 15.1). This process was termed **nuclear fission** by Lise Meitner and Otto Frisch following an analogy from biological cell division. Initially, Lise Meitner and Otto Frisch provided qualitative analysis of the results of Otto Hahn and Fritz Strassmann on the basis of liquid drop model of

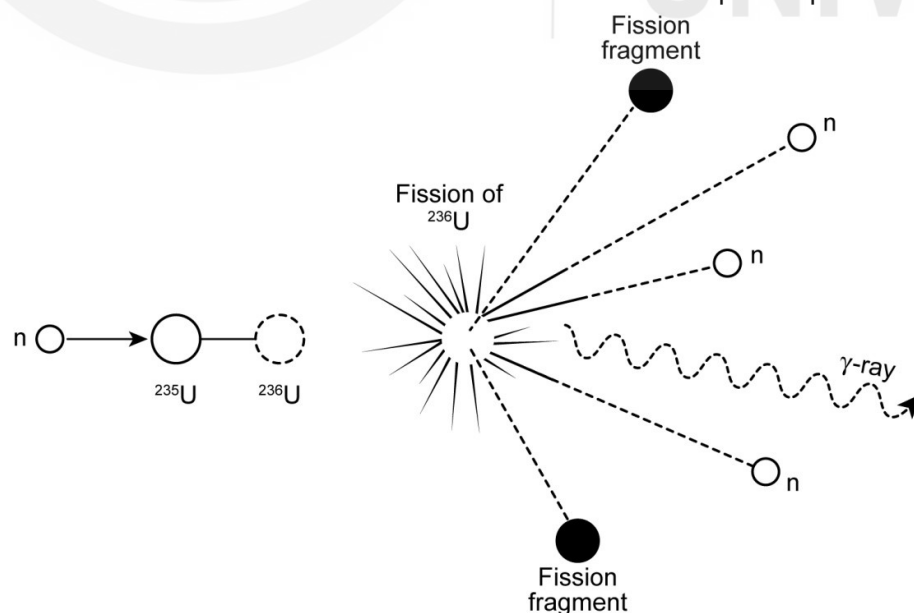
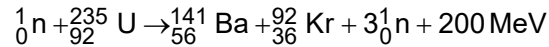


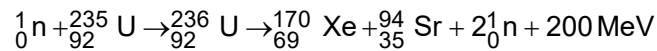
Fig. 15.1: Schematic representation of nuclear fission of ^{235}U .

atomic nucleus. They showed that when the uranium isotope $^{235}_{92}\text{U}$ absorbed a neutron and split into two lighter nuclei with the emission of two or three neutrons, nearly 200 MeV energy was released.

Otto Hahn and Fritz Strassmann established that barium and krypton or xenon and strontium were produced when ^{235}U was made to fission:



or



The detailed theory of fission was developed by Bohr and Wheeler using the analogy between nuclear forces and the forces which bind molecules together in a liquid. Using liquid drop model, they explained the existence of spontaneous fission and predicted the ability of ^{235}U to undergo fission more readily than ^{238}U . They also showed that when the energy of neutrons produced in fission was reduced to about 0.025 eV, their effectiveness in causing fission of $^{235}_{92}\text{U}$, increased almost hundred-fold. Note that the fission fragments are usually of unequal mass; one being much heavier than the other. Such fission is said to be **asymmetric**.

We now know that ^{235}U can fission in more than 40 different ways. This means that about 80 different nuclei are directly produced in fission. The heavier fragments normally lie in the mass range 125-150 with a well-defined maximum around 140, whereas the lighter fragments lie in the mass range 80-110 with a maximum around 95 as shown in Fig. 15.2.

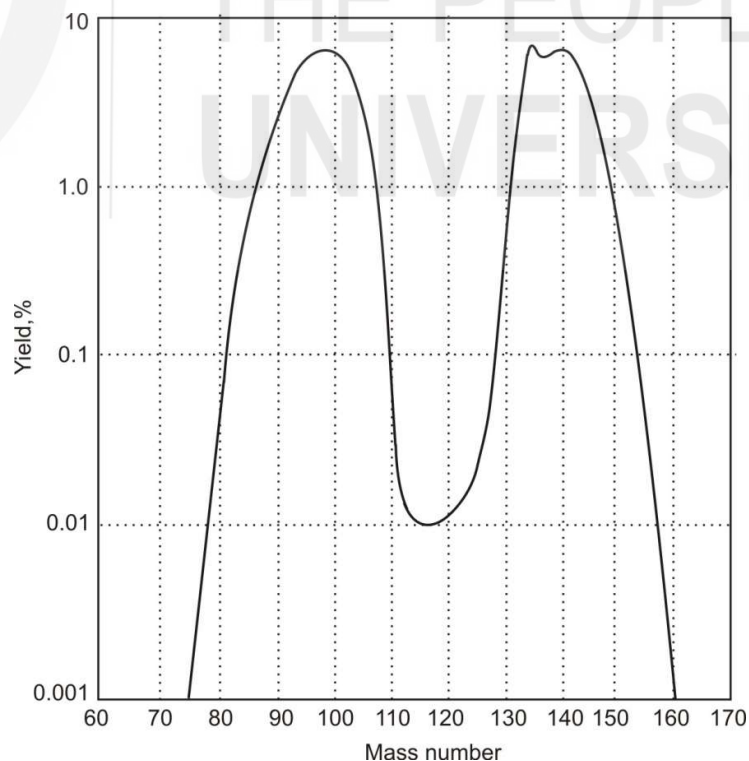


Fig. 15.2: The distribution of mass number in the fragments from the fission of ^{235}U .

Nuclear Chain Reaction

Soon after the discovery of nuclear fission, it was realised that, since each fission reaction is accompanied by emission of two or three neutrons (the average being 2.53), it should be possible to induce a sequence of subsequent fissions through absorption of the emitted neutrons by the fissionable nuclei. This may lead to a chain-reaction (Fig. 15.3).

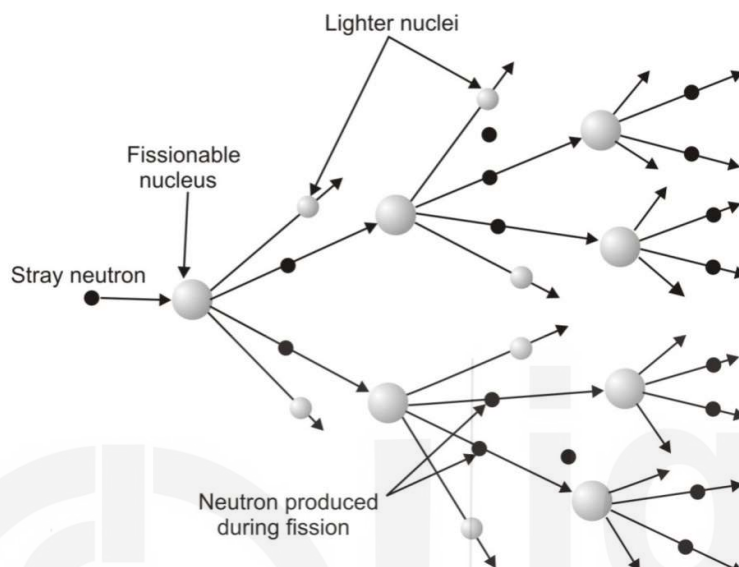


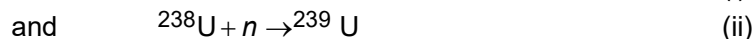
Fig. 15.3: A nuclear chain reaction.

Let us consider an example to calculate energy released in nuclear fission.

EXAMPLE 15.1: NUCLEAR ENERGY

Calculate the energy of the neutron required to fission the isotopes of ^{235}U and ^{238}U . Given: $m_n = 1.008665 \text{ u}$, $m(^{235}\text{U}) = 235.043933 \text{ u}$, $m(^{236}\text{U}) = 236.04556 \text{ u}$, $m(^{238}\text{U}) = 238.05078 \text{ u}$, $m(^{239}\text{U}) = 239.05429 \text{ u}$.

SOLUTION ■ ^{235}U and ^{238}U absorb one neutron each to form ^{236}U and ^{239}U isotopes through the reactions



The energy difference in reaction (i) is

$$\begin{aligned} &= m(^{235}\text{U}) + m(n) - m(^{236}\text{U}) \\ &= .00702 \text{ u} = 6.54 \text{ MeV} \end{aligned}$$

The corresponding energy difference in reaction (ii) is

$$\begin{aligned} &= m(^{238}\text{U}) + m(n) - m(^{239}\text{U}) \\ &= .00515 \text{ u} \approx 4.8 \text{ MeV} \end{aligned}$$

Thus ^{235}U after absorbing one neutron produces ^{236}U with an excess energy of 6.54 MeV which can be considered as excitation energy of ^{236}U . It is more than the excitation energy of ^{238}U . Thus, ^{235}U can fission with low energy neutrons while ^{239}U will require relatively faster neutrons to fission.

$$1 \text{ u} = 931.5 \frac{\text{MeV}}{c^2}$$

15.3 SELF-SUSTAINED CHAIN REACTION

According to Laura Fermi, the news of having attained a self-sustained nuclear chain reaction to the world outside was conveyed by Prof. A. Compton through Mr. Conant of the Office of Scientific Research and Development at Harvard. To quote her: "The Italian Navigator has reached the New World", said Compton as soon as he got Conant on the line. "And how did he find the natives?" "Very friendly."

A chain reaction will not be self-sustaining if neutrons are lost at a faster rate than they are produced by fission. A self-sustaining chain reaction requires a critical mass. Critical mass is determined by a combination of factors depending on the production of neutrons in the interior of the material and the leakage of neutrons from the surface. Critical mass is roughly given by

$$M_c \approx \frac{1}{\rho^2} \left(\frac{m\sqrt{s}}{\sigma} \right)^3$$

where ρ is the density and m is the nuclear mass of the fissionable material, s is the number of scattering events per fission event and σ is the interaction cross-section.)

When the question of establishing a fission chain reaction was examined for the first time by Fermi, he took natural uranium as fuel. We know that natural uranium has 99.7% ^{238}U and 0.3% ^{235}U . He immediately ran into difficulties because ^{238}U absorbed neutrons and did not necessarily undergo fission (^{238}U fissions by absorbing high energy neutrons). It was found that, on an average, more neutrons were removed from the system than were created in fission. Hence, even if a fission reaction was started using an external neutron source, no sooner was the source removed than the number of neutrons in the system began to decrease rapidly and the reaction stopped. On the other hand, the probability of ^{235}U to undergo fission was large for low energy neutrons. This led Fermi to suggest that effective use of this isotope to produce further fissions could be made if the energy of fission neutrons was reduced through collisions in a moderating medium before they had a chance to interact with other uranium nuclei. Fermi and his co-workers suggested that

- graphite should be used as a moderator to slow down neutrons produced in fission; and
- uranium rods should be distributed uniformly in the moderator.

When these two suggestions were incorporated, Fermi succeeded in achieving the first self-sustained chain reaction on December 2, 1942 at the University of Chicago, USA.

For a fission chain reaction to be self-sustained, the rate of production of neutrons must be equal to the rate of loss of neutrons. Can you name the processes responsible for loss of neutrons from a fission chain? This happens by basically two processes: Either absorption by nuclei of the medium or leakage from the system. The rate of loss by leakage depends on the shape and size of the assembly. The most efficient shape is a sphere. For a spherical assembly, the leakage rate is proportional to the surface area and the rate of production is proportional to the volume (if uranium is distributed uniformly throughout the assembly). Therefore, the proportion of neutrons that leak out will decrease as the size of the sphere increases.

Thus, if we start with a small assembly, the rate of loss will exceed the rate of production and a self-sustained chain reaction will not be possible. As the size is gradually increased, we will reach a size when a self-sustained chain reaction just becomes possible. This size of the sphere of fissile material is said to be the **critical size**. The corresponding mass of the fuel is termed as the **critical mass**. Do you know that for ^{235}U the critical sphere has a diameter of 18 cm and the corresponding critical mass is 53 kg? However, for ^{239}Pu , the critical mass is about one-third of this. Do you know the reason? It is because of the larger cross section of ^{239}Pu .

The behaviour of a neutron chain reaction is described quantitatively in terms of a constant called the **multiplication factor**. It is denoted by k and is defined as the ratio of the number of neutrons in any one generation to the corresponding number in the immediately preceding generation, i.e.

$$\text{Multiplication factor, } k = \frac{\text{Number of neutrons in } (n+1)\text{th generation}}{\text{Number of neutrons in } n\text{th generation}}$$

If $k > 1$, the number of neutrons will increase indefinitely with time and the chain reaction will get out of control. If $k < 1$, the number of neutrons will decrease with time and the chain reaction will stop. In the special case when $k = 1$, the number of neutrons in any two successive generations remains the same and the chain reaction proceeds at a constant rate, independent of time. Depending on whether $k > 1$, $k = 1$ or $k < 1$, the reactor is said to be **super-critical**, **critical** or **sub-critical**, respectively. For this reason, k is also very frequently referred to as the **criticality factor**. Note that a precise knowledge of the multiplication factor for a given reactor is essential for reactor control.

An approximate expression for the behaviour of a neutron chain reaction with time can be derived as follows:

Let us suppose that at any instant of time, there are N_1 neutrons in the first generation. Then, in the n th generation, the number of neutrons will be

$$N_n = N_1 k^{n-1} \quad (15.1)$$

Let $\bar{t}$ be the average time between the creation of a neutron and its final removal either by leakage or absorption. Then, the time elapsed between the start of the first generation (which is taken to be at zero time) and the birth of n th generation neutrons will be $t = (n - 1)\bar{t}$. On substituting for $(n - 1)$ from this expression in Eq. (15.1), we get

$$N(t) = N(0)k^{t/\bar{t}} = N(0)\exp\left(\frac{t}{\bar{t}}\ln k\right) \quad (15.2)$$

where we have replaced N_n by $N(t)$ and N_1 by $N(0)$. Assuming k to be close to unity, $\ln k$ can be approximated by noting that $\ln(1 + (k-1)) \cong k - 1$ (see margin remark). Hence, we can write

$$N(t) \cong N(0)\exp\left[(k - 1)\frac{t}{\bar{t}}\right] \quad (15.3)$$

This tells us that neutron number increases exponentially with time for $k > 1$ and decreases exponentially with time for $k < 1$. For $k = 1$ we have a steady state.

In any fission chain, two types of neutrons are emitted: prompt and delayed. For prompt neutrons, the mean life time is nearly 10^{-3} s. For $k = 1.01$, the increase in the number of neutrons per second is $N(t)/N(0) = \exp(0.01/0.001) = e^{10} \cong 22000$. Such a reactor would be impossible to control by any mechanical means. Thus, if the above considerations were the only ones operative, even an excess of 0.01 (1%) over unity in the value of k would make a fission chain reaction uncontrolled and the reactor would explode. Fortunately the presence of delayed neutrons appreciably increases the neutron life time (to about 0.1 s) and it becomes possible to control a nuclear reactor.

Maclaurin expansion
of $\ln(1 + x) = x + \dots$
For $-1 < x < 1$.

Neutrons emitted within 10^{-14} s of the fission event are said to be prompt. However, neutrons emitted by fission product nuclei considerably after the fission event are known as **delayed neutrons**.

15.4 NUCLEAR FISSION REACTORS

Ever since the world's first nuclear reactor was constructed by Fermi and his co-workers at the University of Chicago in 1942, a variety of nuclear reactors have been built, primarily to meet our increasing demand of energy. Nuclear reactors are highly complex installations and great care has to be exercised in designing them, in spite of the fact that they work on a very simple principle.

The heat generated in fission is removed by circulating a fluid, called **coolant**, around the fuel and this heat is subsequently used to generate high pressure and high temperature steam. This is fed to a turbine-generator system to produce electricity. Some part of the heat produced in a power reactor is discharged into a lake, river, sea or the environment using cooling towers. For experimental nuclear reactors, the heat is invariably discharged into the sea.

Nuclear reactors are classified variously but it is customary to classify them depending on the mean energy of neutrons causing most of the fissions and the purpose for which a reactor is meant. Before proceeding further, we present the general features of a fission reactor.

15.4.1 General Features of a Fission Reactor

All nuclear reactors consist of the following basic components: reactor core, reflector, reactor vessel, radiation shield, structural materials, coolant loops and heat exchangers, etc. These are schematically illustrated in Fig. 15.4. In a fast reactor, a blanket is placed between the core and the reflector. We discuss these features in some detail now.

Core

The central region of a reactor where fission takes place, resulting in release of energy, is known as the core. In fast reactors, it contains a nuclear fuel, a coolant, control rods and structural materials. In thermal reactors, a moderator is also present. An **ideal fuel** should have high thermal conductivity and high melting point, high resistance to radiation damage, and be chemically inert. Moreover, it should be easy to fabricate and be corrosion resistant. Usually the fuel is in the form of a ceramic, i.e., either an oxide or carbide or a nitride. (oxy-carbides and nitrides of uranium have also been proposed as nuclear fuels.)

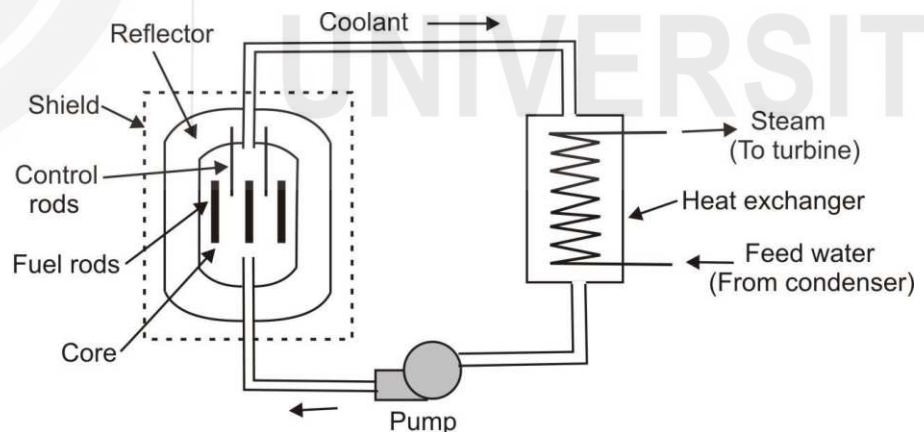


Fig. 15.4: Schematic diagram of a nuclear reactor depicting its general features.

To prevent escape of fission fragments into the coolant or the moderator (if present), apart from providing protection to the fissionable material from corrosion, the fuel rods are **cladded** or canned. An ideal fuel clad material is highly resistant to corrosion. It is a poor neutron absorber, cheap and readily available. It should also have good mechanical strength and high melting point. Zirconium, steel, aluminium, magnesium, nickel and such other materials are used for this purpose. Of these, zirconium is the best and

generally a zirconium alloy, known as Zircalloy-2 (Zr-2), is used in thermal power reactors. In fast reactors, stainless steel is used. Aluminium is used mainly in research reactors.

A single clad unit of fuel is known as the fuel element. (In a fast reactor, the fuel element is thinner than that used in a thermal reactor and is called a **fuel pin**.) Several such fuel elements when put together constitute a **fuel assembly**. (It is this single unit which is inserted or taken out of the reactor as a whole to control chain reaction.) A reactor core contains a large number of fuel assemblies arranged in the form of a regular square or hexagonal lattice. To remove heat from the core (as also from any other part of the reactor), it is necessary to circulate a fluid – a liquid or a gas – through the reactor. This fluid is referred to as the **coolant**. A reactor coolant should have high thermal capacity, low cost, low neutron absorption cross section, good radiation and thermal stability, and compatibility with fuel and clad. In fast reactors, the coolant should further be a poor moderator. In these reactors, liquid sodium or helium are used as coolant. In thermal reactors, ordinary water, steam, heavy water, carbon dioxide, helium and some organic gases are used as coolants.

To slow down neutrons born in fission, a **moderator** is also present in the core of a thermal reactor. It is a material of low mass number, large scattering cross section and small absorption cross section. (Other requirements for a moderator are almost the same as those for a coolant.) The best moderators are heavy water, light water and graphite. Of these, heavy water and graphite are so good that in their presence even natural uranium can sustain a chain reaction. (Beryllium can also be used as a moderator. But it is costly.)

To ensure safe operation of a reactor at a desired power level, to start up and to shut down a reactor (during routine operation or in an emergency), provision has to be made in the reactor core to control the multiplication factor. The reactor engineers also have to compensate for the gradual production of poisons and fuel burn up, etc. This is achieved by having some **control rods** in the core. The control rods are made of a highly neutron absorbing material such as boron, cadmium, hafnium, gadolinium or their alloys. For example, boron is used in the form of boron carbide (B_4C) powder stuffed in stainless steel rods. These move up and down between fuel assemblies.

Sometimes, empty tubes are built-in inside the core and the absorber in the form of a liquid, say gadolinium nitrate. These can be pumped in and out. Similarly, in H_2O or D_2O moderated reactors, boric acid (H_3BO_3) is frequently added to the moderator. This is known as chemical shim. Many a times, a small amount of ^{10}B or ^{155}Gd is added to the fuel. A significant feature of these isotopes is that on neutron capture, they change into isotopes (^{11}B or ^{156}Gd), which have negligible affinity for absorption of neutrons. Thus, as the fuel depletes, the poison also burns out itself resulting in compensation for the loss of multiplication factor due to fuel burn up. Such a poison is termed as **burnable poison**. Use of gadolinium oxide (Gd_2O_3) powder with UO_2 powder is a common example of burnable poison.

In fast reactors, boron with increased fraction of ^{10}B is used. In experimental fast reactors, periodic removal and installation of the reflector as well as the

fuel (or core) have also been used to control the multiplication factor or to shut down the reactor.

Control rods may be sub-grouped as **shim rods**, **regulating rods** or **safety rods**, depending on their function. **Shim control rods** are used to bring the reactor to the desired power level when the system is started and to make occasional coarse adjustments in the power level. Before the reactor is started, these rods are completely inserted inside the reactor core. They are then gradually raised until the reactor attains criticality; they are normally fully out of the core when the reactor is operating at its peak power. **Regulating rods** are used to make fine adjustments in the power level and to compensate for the effects of changes in temperature and pressure, fuel depletion and build up of poisons. **Safety rods** are provided to stop the chain reactor rapidly and are used for an emergency shut down of a reactor in case of failure of the system. When the reactor is critical, these rods are kept in a cocked position outside the core and to shut down the reactor they are moved rapidly into the core. They are distributed uniformly inside the core.

To support fuel elements, for making coolant channels and for various other purposes, it is necessary to use some structural materials inside the core. The desirable properties for such materials are the same as those of the clad and generally the same material is used for both purposes. For example, stainless steel is used for spacer grids — the structure which maintains spacing between fuel rods in a fuel assembly when the reactor is operating in normal conditions. Similarly, for coolant tubes and fuel channels, Zr-2 is used.

Blanket

In fast reactors, which are generally compact, a significant fraction of neutrons in the core leaks out of the system. To reduce this leakage and also to make proper use of the leaking neutrons, core in these reactors is surrounded by a region of fertile material (^{232}Th or ^{238}U). This region is referred to as the **blanket**. (This also serves as an additional neutron reflector as well as a shield.) Neutrons absorbed in the blanket eventually lead to the production of fissile nuclei, ^{233}U or ^{239}Pu .

Reflector

A region of non-moderating, non-absorbing material put next to the core (or the blanket, if it is present) to inhibit neutron leakage from the core (or blanket) is known as the **reflector**. In fast reactors, high mass number material is used so that the mean energy of neutrons returned back from this region is not much different from that of neutrons entering it. Ni, Cu and Mo (Molybdenum) are frequently used as reflectors. In thermal reactors, any good moderating material can be used as a reflector.

Reactor Vessel

The reactor core, blanket and reflector, etc are placed inside a vessel, called the 'pressure vessel'. Usually a few inches thick stainless steel is used to make the reactor vessel.

Shielding

To protect the scientists and other personnel working around the reactor as well as the equipment placed around it from radiations emanating from the

reactor core, the reactor vessel is encased inside thick concrete walls. In some cases, alternate layers of heavy and light elements such as concrete and polyethylene or concrete and water are also used. This is referred to as the **biological shield** or the **radiation shield**.

The continuous absorption of leaking radiations by the walls of the reactor vessel as well as the biological shield results in generation of heat there, causing thermal stresses in them. To reduce this heating effect of nuclear radiation and hence prevent radiation damage of the pressure vessel, a **thermal shield** usually made of stainless steel, is placed next to the reflector.

Reactor Building

The entire structure discussed above is placed inside a reactor building. It is air tight and is maintained at a pressure slightly lower than the atmospheric pressure so that no air leaks out of the building, except through the ventilation channels. In the event of an accident, the building may implode and help to contain the radioactive materials and prevent their dispersal into the surroundings.

Coolant Loops, Heat Exchangers and Electric Generators

The heat generated due to fission inside the reactor core is removed by circulating a coolant through it. Usually, the coolant circulates in a closed loop, called the primary (or reactor core) loop. The heated coolant carrying fission heat can become intensely radioactive when it comes out of the core. To prevent this radioactivity from spreading, it becomes necessary to introduce a secondary (or steam generating) loop, which may be closed or open. The primary fluid is made to give up its heat to the secondary fluid, usually water, in a heat exchanger. This results in production of steam and for this reason, it is also called a **steam generator**. These are shown in Fig. 15.5.

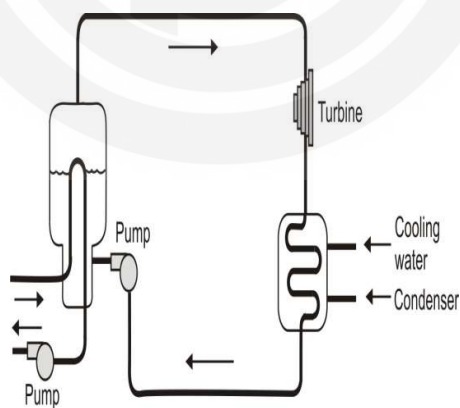


Fig. 15.5: Coolant loop and heat exchanger.

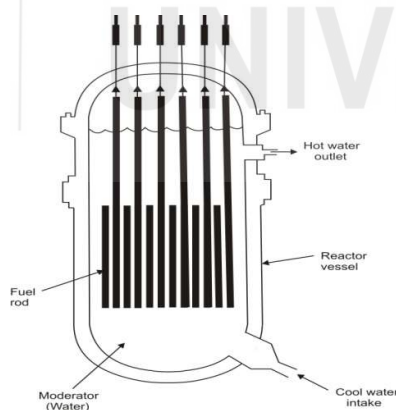


Fig. 15.6: A schematic diagram of a boiling water reactor.

In research reactors, secondary loop is usually open so that the heat produced inside the core is just discharged into a river or the sea. In power reactors, the secondary loop provides energy to an electric generator. However, in some reactors, high pressure steam is produced in the core itself and is directly fed to the generator so that no heat exchangers are needed. Such reactors are called boiling water reactors (BWRs). A schematic diagram

India produces roughly 3.2% of its electricity at an installed capacity of 6780 MW in 7 nuclear power plants. The rest of the energy is produced in coal, gas, hydro-electric, solar and wind power plants. There were a total of 440 nuclear power plants in the world as of 2017 with a combined electric capacity of 390 GW. Nuclear power provided 10% of the world's electricity. United States of America had maximum no. of plants and about 20% of countries electric needs were being met from nuclear power. Tokyo has the world's largest nuclear power plant with a capacity of 7965 MW. Some countries notably, Denmark, Norway, Australia and New Zealand among others have no nuclear power plants.

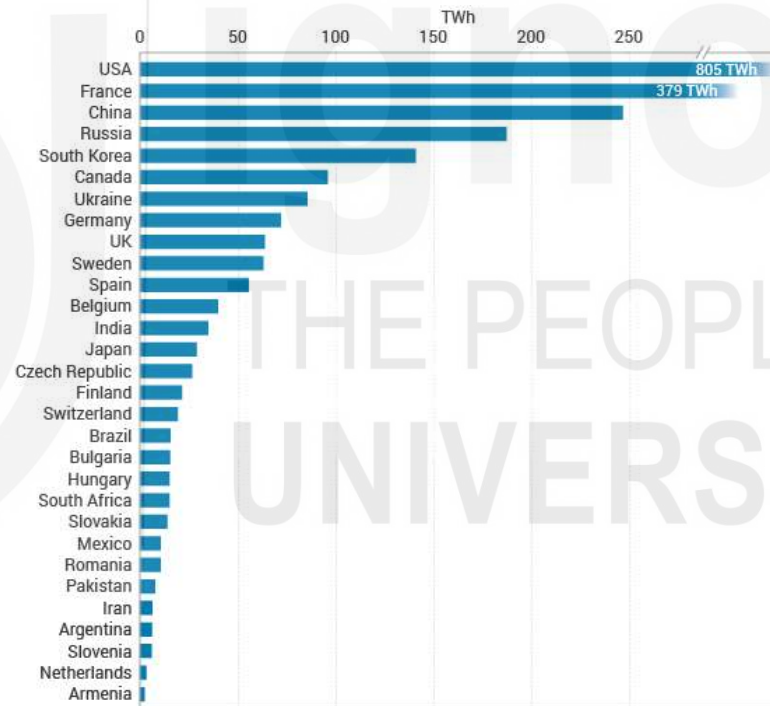
Nuclear energy is a low- carbon dioxide emitting source. Nuclear power plants however, are a source of radio-active waste which remains dangerous to human health for thousands of years. In addition nuclear power plants even though less hazardous in comparison to the traditional coal or gas based plants present catastrophic consequences at the time of natural disaster or accidental malfunction.

of a boiling water reactor is shown in Fig. 15.6. On the other hand, in fast reactors, cooled by liquid sodium, an intermediate cooling loop (and hence heat exchanger) is also required. This is used to transfer heat from the primary sodium loop to the secondary loop.

The high temperature and pressure steam from the steam generator (or the core) expands in a turbine coupled to a large electric generator. The low pressure steam leaving the turbine is recondensed into liquid water in a steam condenser. The condensed water is then compressed and pumped back into the steam generator.

The efficiency of nuclear plants (33%) is lower than the efficiency of fossil fuel plants (40%). This is because the temperature of the nuclear fuel (and hence steam) is kept lower than that of the fossil fuel to avoid melt down. However, it is hoped that soon it will also be raised to about 40%.

Refer to Fig. 15.7, which shows nuclear power generated by different countries till 2018. Note that U.S. and France were leaders and India lagged behind China and Russia by a distance. We need to increase nuclear power production significantly to be self-sufficient in energy.



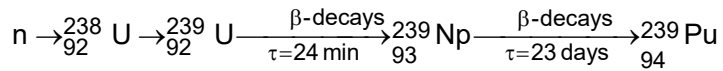
Source: IAEA PRIS Database

Fig. 15.7: Nuclear Power Generation Country-wise.

15.5 A FAST BREEDER REACTOR

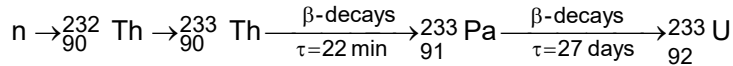
We now know that natural uranium consists of 99.3% ^{238}U which is non-fissionable and only a tiny fraction 0.7% consists of fissionable ^{235}U . The separation of ^{235}U from the naturally occurring uranium ore is an expensive and time-consuming process. However, some non-fissionable nuclide change into fissionable nuclide by absorption of a neutron. This happens through a series of reactions. Typical examples are:

1. ^{238}U captures a fast neutron and transforms into ^{239}U which through 2 step β -decay transforms into another isotope of plutonium:



The plutonium isotope α -decays with a half-life of 24,000 years.

2. Similarly, naturally occurring ${}_{90}^{232}\text{Th}$ absorbs a fast neutron and through a 2-step β -decay gets converted to an isotope of uranium:



Both isotopes plutonium (${}_{94}^{239}\text{Pu}$) and uranium (${}_{92}^{233}\text{U}$) thus produced from the naturally occurring non-fissionable abundant uranium and thorium ores are rendered fissile through capture of a fast neutron. The end point products (${}_{94}^{239}\text{Pu}$) or (${}_{92}^{233}\text{U}$) are fissile and can be used in a breeder reactor as fuel. Note that plutonium is chemically different from the original element uranium and so is uranium different from thorium. These fissile elements can be relatively easily separated from their parent elements.

A breeder reactor is so designed as to produce more fissionable plutonium than the fissile uranium isotope ${}^{235}\text{U}$ and this is achieved by converting the unproductive non-fissionable ${}^{238}\text{U}$ into fissile plutonium isotope. Currently two fast breeder reactors are operating in Russia. In India, a prototype 500 MW fast breeder reactor (FBR) is under construction at the Madras Atomic Power Station in Kalpakkam. The Indira Gandhi Centre for Atomic Research is responsible for the design of this reactor. The Kalpakkam FBR uses ${}^{238}\text{U}$ and not thorium to breed fissile plutonium and is yet to become operational. China and Japan are also reportedly operating FBRs. These reactors are expected to be safe, more efficient and pose lesser environmental hazards.

15.6 NUCLEAR FUSION

Refer again to Fig. 14.2 for binding energy per nucleon. As discussed earlier, when lighter nuclei are made to fuse in a nuclear reaction to form heavier nuclei, large amount of energy per nucleon is released. Can you estimate how much energy is released in such a reaction?

Let us consider a low energy nuclear reaction:



where x , y , X and Y are atomic nuclei. Let us assume that the target nucleus X is at rest. From the principle of conservation of energy, we can write

$$m(x)c^2 + KE_x + m(X)c^2 = m(y)c^2 + m(Y)c^2 + KE_y + KE_Y \quad (15.2)$$

where m 's represent the rest masses of the respective nuclei.

The difference in the rest mass energies of the initial and final nuclei is defined as **energy of reaction**. In the instant case, Q value of the reaction is given by

$$Q = [m(x) + m(X) - m(y) - m(Y)]c^2 \quad (15.3)$$

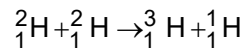
Note that Q is equal to the difference in kinetic energy between the final and initial particles and is the energy released in the nuclear reaction.

$$Q = KE(y) + KE(Y) - KE(x) - KE(X) \quad (15.4)$$

Before proceeding further, we would advise you to answer an SAQs.

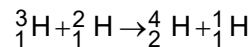
SAQ 1 – Q-value

- a) Calculate the Q value in the fusion reaction involving two deuterons to form a tritium and a proton



Take $m({}^2_1\text{H}) = 2.0141029 \text{ u}$; $m({}^3_1\text{H}) = 3.016049 \text{ u}$; and $m({}^1_1\text{H}) = 1.008665 \text{ u}$

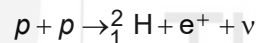
- b) Calculate the Q value in the fusion reaction:



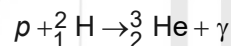
Take $m({}^4_2\text{He}) = 4.002603 \text{ u}$, masses of other nuclei are given in part (a).

15.6.1 Source of Energy in the Sun and Stars

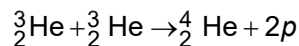
Hans A. Bethe in 1938 gave an account of the source of energy production in stars. Since the Sun is known to be composed mostly of hydrogen nuclei, he suggested that nuclear fusion processes power it. (This is also true of other similar stars.) In a fusion reaction, four hydrogen nuclei fuse into a helium nucleus through a sequence of processes now known as proton-proton cycle. Two hydrogen nuclei (proton) first fuse to form deuterium, which is accompanied by a positron and a neutrino:



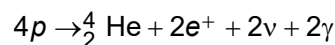
Next, deuterium absorbs a proton (hydrogen nucleus) to form an isotope of helium namely, ${}^3_2\text{He}$:



Then two ${}^3_2\text{He}$ isotopes fuse to form helium-4. This is accompanied with the emission of two protons:



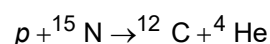
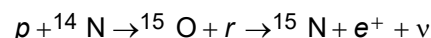
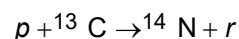
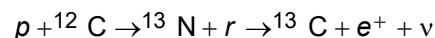
Thus, the net reaction is



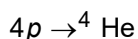
with a Q value

$$\begin{aligned} Q &= (m_i - m_f)c^2 \\ &= 26.7 \text{ MeV} \end{aligned}$$

A more efficient sequence of processes in heavier stars is known as carbon-nitrogen cycle. In this, the chain of processes proceeds as follows:



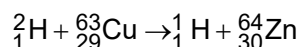
The net process is still



with the same Q value as in the proton-proton cycle.

SAQ 2 – Q -value

Calculate the Q value for the reaction:



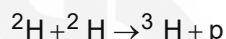
Take : $m({}^2_1\text{H}) = 2.014102 \text{ u}$; $m({}^1_1\text{H}) = 1.008665 \text{ u}$

$m({}^{63}_{29}\text{Cu}) = 62.929599 \text{ u}$; $m({}^{64}_{30}\text{Zn}) = 63.929146 \text{ u}$

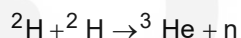
15.6.2 Fusion Reactors

We now know that huge amount of energy is released when hydrogen nuclei fuse to form helium (heavier ones). You will agree that it could be a source of energy production on the Earth, provided we can reproduce the conditions existing in the Sun and other stars. Just as fission of heavier elements into lighter nuclei has been harnessed in nuclear reactors to produce energy, it is tempting to explore the possibility of doing the same in a controlled thermonuclear fusion reactor. Some possible reactions that can be utilised in a possible fusion reactor are:

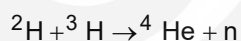
i) Deuterium fusion into tritium with a Q value of 4.0 MeV:



ii) Deuterium fusion into helium 3 isotope with a Q value of 3.3 MeV:



iii) Deuterium-Tritium fusion into helium 4 with a Q value of 17.6 MeV:



Note that the deuterium-tritium (D-T) reaction has the largest energy output. Deuterium is present in sea water with a concentration of 33 g/m^3 and is not expensive to extract. Tritium is not present in sufficient concentration to enable us to extract it economically. However, it can be produced in a nuclear reactor via neutron absorption by naturally occurring lithium isotopes.

For D-T reaction to take place, the fuel is taken in the form of a plasma (ionised gas). The temperature for the fusion to take place should be extremely high ~ 100 million K with a high density to ensure collisions between D-T nuclei to ignite fusion. The production and confinement of such high-density plasma at a temperature $\geq 10^8 \text{ K}$ and for a reasonable period of time is a challenge. This requires huge financial and human involvement.

An International Thermonuclear Experimental Reactor (ITER) is currently being constructed in France which is sponsored by European countries, Japan, China, India and South Korea. It is expected to take 10 to 15 years to build. The confinement of plasma in ITER is expected to be achieved through a combination of Poloidal and Toroidal magnetic fields in a doughnut shaped magnetic bottle in a Tokamak First Plasma in ITER (Fig. 15.8). The start of D-T operation does not seem possible before 2035 at an estimated expenditure of over Euro 13 billion, though China claims to have produced artificial sun.

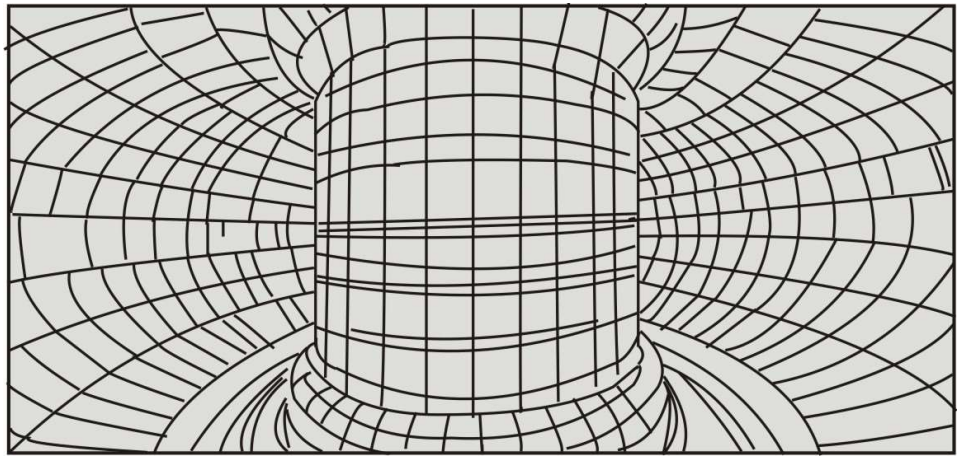


Fig. 15.8: An ITER Tokamak.

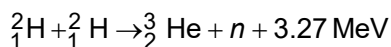
Let us now sum up what you have learnt in this unit.

15.7 SUMMARY

Concept	Description
Fission	A large amount of energy is released when a heavy nucleus is made to break up into more tightly bound lighter nuclei. This process is termed nuclear fission. A typical example of such a reaction is ${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + 3{}_0^1\text{n} + 200\text{MeV}$
Self- sustained chain reaction	- A fission chain reaction is said to be self-sustained when the rate of production of neutrons equals the rate of loss of neutrons. Multiplication factor (k) is defined as the ratio of the number of neutrons in any one generation to the corresponding number in the immediately preceding generation. - For a fission chain reaction, the time behaviour of neutrons in a chain reaction can be approximately known through the relation $N(t) = N(0)\exp[(k - 1)t/\bar{t}]$ Where k is a multiplication factor.
Nuclear reactors	Nuclear reactors are highly complex installations and the heat generated in fission is used to generate high pressure and high temperature steam which is fed into a turbine-generator system to produce electricity. Most of the nuclear energy generated so far has come from thermal reactors where fission is caused by 0.0253eV neutrons.
Fast breeder reactors	A fast breeder reactor is designed to produce more fissile plutonium than the uranium isotope ${}^{235}\text{U}$ which it uses. This is achieved by converting ${}^{238}\text{U}$ into the fissile isotope of plutonium. Plutonium fuelled fast breeder reactors can produce power and breed ${}^{233}\text{U}$ from ${}^{232}\text{Th}$.
Nuclear fusion	In nuclear fusion, a large amount of energy is released when lighter nuclei are made to fuse and heavier nuclei are formed.

15.8 TERMINAL QUESTIONS

1. Calculate the number of fissions of ^{235}U required per second to produce 1 kilowatt power. Given that a single fission produces 200 MeV of energy.
2. Calculate the amount of energy produced in the fission of 1 kg of ^{235}U . Given that energy produces in each fission is 215 MeV.
3. Two deuterium nuclei fuse to release 3.27 MeV energy,



Calculate the total energy released in the fusion of 1 kg of deuterium.

15.9 SOLUTIONS AND ANSWERS

Self-Assessment Questions

1. a) Using the Eq. (15.3), $Q = [m(x) + m(X) - m(y) - m(Y)]c^2$

On substituting the values, we can write

$$Q = \{2(2.0141024) - (3.016049 + 1.008665)\}c^2$$

On solving, we get

$$= 3.25 \text{ MeV}$$

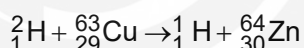
- b) Using Eq.(15.3) as given below, we can find the value of Q ,

$$Q = [m(x) + m(X) - m(y) - m(Y)]c^2$$

$$Q = (3.016049 + 2.0141029 - 4.002603 - 1.008665)c^2$$

$$= 17.6 \text{ MeV}$$

2. Using the equation $Q = (m_i - m_f)c^2$ and the given reaction is



On substituting the values in the above equation

$$m({}^2_1\text{H}) = 2.014102 \text{ u}; \quad m({}^1_1\text{H}) = 1.008665 \text{ u}$$

$$m({}^{63}_{29}\text{Cu}) = 62.929599 \text{ u}; \quad m({}^{69}_{30}\text{Zn}) = 63.929146 \text{ u}$$

On substituting the values, we get

$$= (2.014102u + 62.929599u - 1.008665u - 63.929146u)c^2$$

$$Q = 5.49 \text{ MeV}$$

Terminal Questions

1. Energy released in one fission = $200 \times 10^6 \times 1.6 \times 10^{-19}$

$$\text{Power} = \frac{nE}{t} \text{ where } n \text{ is the number of fissions in time } t.$$

$\therefore$ No. of fissions required per second is

$$\frac{n}{t} = \frac{\text{power}}{E} = \frac{10^3 \text{ watts}}{3.2 \times 10^{-11} \text{ Joule}} = 3.125 \times 10^{13}$$

2. Number of ^{235}U nuclei in 1 kg of ^{235}U

$$n = \frac{1000}{235} \times 6.023 \times 10^{23}$$

Total energy produced by the fission of 1 kg of ^{235}U is thus

$$\begin{aligned} E &= \frac{1000}{235} \times 6.023 \times 10^{23} \times 215 \times 10^6 \times 1.602 \times 10^{-19} \\ &= 8.8 \times 10^{13} \end{aligned}$$

3. Number of deuterium nuclei in 1 kg of deuterium is

$$n = \frac{6.02 \times 10^{23} \times 10^3}{2}$$

2 nuclei are needed in each fusion process and number of reactions

$$\text{available} = \frac{n}{2}.$$

$\therefore$ Total energy released

$$\begin{aligned} E &= \frac{6.012 \times 10^{23} \times 10^3}{2} \times \frac{1}{2} \times 3.27 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ &= 7.87 \times 10^{13} \text{ J} \end{aligned}$$

FURTHER READINGS

1. **Concepts of Modern Physics** by Beiser, Arthur; Mahajan, Shobhit and Choudhury, S. Rai, McGraw Hill Education (India) Pvt. Ltd., 7th Edition, SIE (2015).
2. **Introduction to Modern Physics** by Mani, H.S. and Mehta, G.K., East-West Press, India, (1990).



TABLE OF PHYSICAL CONSTANTS

Symbol	Quantity	Value
c	Speed of light in vacuum	$2.998 \times 10^8 \text{ ms}^{-1}$
μ_0	Permeability of free space	$1.257 \times 10^{-6} \text{ NA}^{-2}$
ϵ_0	Permittivity of free space	$8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
$1/4\pi\epsilon_0$		$8.988 \times 10^9 \text{ Nm}^2\text{C}^{-2}$
e	Charge of the proton	$1.602 \times 10^{-19} \text{ C}$
$-e$	Charge of the electron	$1.602 \times 10^{-19} \text{ C}$
h	Planck's constant	$6.626 \times 10^{-34} \text{ Js}$
$\hbar$	$h / 2\pi$	$1.054 \times 10^{-34} \text{ Js}$
m_e	Electron rest mass	$9.109 \times 10^{-31} \text{ kg}$
$-e/m_e$	Electron charge to mass ratio	$1.759 \times 10^{-19} \text{ Ckg}^{-1}$
m_p	Proton rest mass	$1.673 \times 10^{-27} \text{ kg}$
m_n	Neutron rest mass	$1.675 \times 10^{-27} \text{ kg}$
a_0	Bohr radius	$5.292 \times 10^{-11} \text{ m}$
N_A	Avogadro constant	$6.022 \times 10^{23} \text{ mol}^{-1}$
R	Universal gas constant	$8.315 \text{ J mol}^{-1}\text{K}^{-1}$
k_B	Boltzmann constant	$1.381 \times 10^{-23} \text{ JK}^{-1}$
G	Universal gravitational constant	$6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

LIST OF BLOCKS AND UNITS: BPHCT-141

BLOCK 1: THE SPECIAL THEORY OF RELATIVITY

- Unit 1 Postulates of Special Relativity
- Unit 2 Relativistic Kinematics
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BLOCK 4: NUCLEAR PHYSICS

- Unit 13 Radioactivity
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Special Theory of Relativity: Constancy of Speed of Light, Postulates of Special Theory of Relativity, Lorentz Transformation, Length Contraction, Time Dilation, Examples, Velocity Addition Theorem, Doppler Effect, Variation of Mass with Velocity, Energy-Mass Equivalence, Relativistic Energy and Momentum.

An Introduction to Quantum Mechanics: Birth of Quantum Physics, Blackbody Radiation, Planck's Quantum Hypothesis, Planck's constant and light as a collection of photons; Photo-electric effect and Compton scattering. Problems with Rutherford model – instability of atoms and observation of discrete atomic spectra; Bohr Model, Bohr's quantization rule and atomic stability; calculation of energy levels for hydrogen like atoms and their spectra.

de-Broglie hypothesis and matter waves; Davisson-Germer experiment, Wave-Particle Duality, Matter waves and wave amplitude, Wave Packet, Group Velocity.

The Uncertainty Principle and its Consequences, Thought Experiments: Position measurement – gamma ray microscope thought experiment; Two slit interference experiment with photons, atoms and particles; Complementarity; Estimating minimum energy of a confined particle using uncertainty principle; Energy-time uncertainty principle.

Postulates of Quantum Mechanics, Time Dependent and Time Independent Schrödinger Equation in One Dimension, Statistical Interpretation of Wave Function, Probability Current Density and Continuity Equation, Normalization of Wave Functions, Wave Function in Momentum Space; Observables and Operators, Linear Momentum and Energy Operators; Parity Operator and its Eigenvalues; Commutation Relations, Expectation Values.

Applications of Quantum Mechanics: One Dimensional Infinitely Rigid Box – energy eigenvalues and eigenfunctions, normalization, quantum dot as an example, Particle in a Box, Free Particle, Step Potential and Rectangular Potential Barrier, Quantum mechanical scattering and Tunneling; One Dimensional Potential Well and barrier.

Nuclear Physics: Radioactivity: Stability of nucleus; Law of Radioactive Decay, Mean life and half-life; Radioactive Equilibria, Natural Radioactive Series, Law of α decay, β decay – energy released, spectrum and Pauli's prediction of neutrino; γ -ray emission. General Properties of Nuclei: Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle. Nature of nuclear force, NZ graph, semi-empirical mass formula and binding energy; Stability of Nuclei: Binding Energy Curve. Fission and fusion – mass defect, generation of energy; Fission – nature of fragments and emission of neutrons. Nuclear reactor: slow neutrons and their interactions with Uranium 235; Fusion and thermonuclear reactions.