UNIT 3 THE SOLOW MODEL

Structure

3.0 OBJECTIVES

After going through this unit you should be in a position to

- explain the basic premises on which the neoclassical Solow model is based;
- describe the properties of neoclassical production function;
- identify the implications of Solow model; and
- explain the steady state growth of an economy.

3.1 INTRODUCTION

In the previous block we observed that when saving is translated into investments, accumulation of capital stock takes place. Such accumulation in capital stock implies an increase in the level of capital input. A basic feature of capital input is that it does not get exhausted in a single use, that is, it is durable in nature. However, capital input undergoes wear and tear, generally termed depreciation, during the course of production. Thus when we deduct depreciation from gross investment we obtain net investment, which tantamount to addition to capital stock.

As you know, capital and labour are two primary inputs used in production process and with the aid of technology transform intermediate inputs into output. If we assume no shortage of intermediate inputs the production capacity of an economy depends upon the quantity, quality and utilisation of the primary inputs. We should note that most of the growth models are built upon the premise of assured availability of raw materials.

The classical economists in general did not pay much attention to the long-run impact of investment. In the classical framework the economy passed through the same cycle of production period after period. The post-Keynesian economists, particularly Harrod and Domar, emphasized the increase in capital stock due to investment (see Block 1 of the course MEC-004: Economics of Growth and Development). Later on neoclassical economist R. M. Solow presented a model of economic growth in 1956 which continues to remain as a landmark in growth theory. Subsequent growth models have brought in refinements and innovations into the Solow model. The Solow model explains the effect of saving, population growth and technological progress on the economy's output growth. We discuss below the basic features and implications of the Solow model.

3.2 FEATURES OF THE SOLOW MODEL

The Solow model considers an aggregate production function for the economy as a whole such that a composite good is produced through a common technology by utilizing homogenous inputs, labour and capital. The first step to understand the model is to study what determines the supply of and demand for goods in the economy.

3.2.1 Supply of Goods

The supply of goods in the Solow model is determined by the production function. The production function has three inputs (K, L, A) and one output (Y) variables and takes the form

$$Y(t) = F(K(t), A(t), L(t)) \quad \ldots(3.1)$$

Y = output

K = capital

L = labour

A = technology of production

The variable t in equation (3.1) denotes time that does not enter the model directly. It implies that over time change in Y will take place only due to change in inputs K, L and A.

It is worthwhile to note that change in A occurs as a consequence of technological progress. Labour and effectiveness of labour are introduced in the model multiplicatively such that AL means effective labour. It means that technological progress is labour-augmenting, i.e., it increases the productivity or efficiency of labour. Thus even if quantity of L remains unchanged, technological progress increases the quantity of effective labour (AL) in the economy.

The production function described at (3.1) presents constant returns to scale (CRS). This assumption greatly simplifies the analysis and is often considered realistic. The presence of CRS implies that if we double all the inputs output will be doubled.

$$F(aK, aAL) = aF(K, AL) \quad \ldots(3.2)$$

where a can be any non-negative constant.

The CRS assumption can be utilized to convert the production function specified in equation (3.1) to per effective labour terms. If $a = \frac{1}{AL}$ we can represent (3.1) as

$$F\left(\frac{K}{AL}, 1\right) = \frac{1}{AL} F(K, AL) \quad \text{...(3.3)}$$

This form of the production function given at (3.3) is known as the intensive form. It allows us to analyse all variables in the economy relative to the size of effective labour force.

Thus $\frac{K}{AL}$ is capital expressed relative to effective labour, i.e., it is capital per unit effective labour. Moreover, $F\left(\frac{K}{AL}, 1\right) = \frac{Y}{AL}$ which is output per unit effective labour. We designate these in lowercase letters such as $\frac{K}{AL} = k$ and $\frac{Y}{AL} = y$ and $f(k) = F(k, 1)$ so that the production function in (3.3) can be written as

$$y = f(k) \quad \text{...(3.4)}$$

Fig. 3. 1 illustrates the above production function

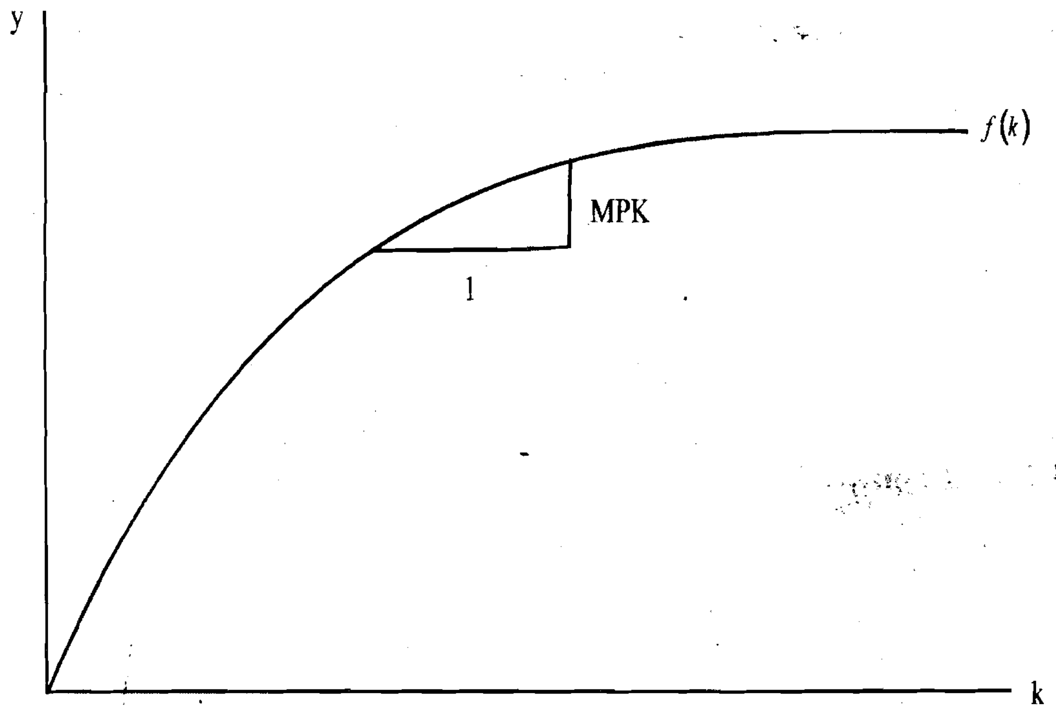

Fig. 3.1: Neoclassical Production Function

In Fig. 3.1 we measure k on the x-axis and y on the y-axis. Since k represents capital labour ratio, as we move along x-axis the amount of capital available per unit of labour increases. A basic feature of (3.1) is that while CRS prevails, there is diminishing returns to capital input. It implies that when both K and L are increased proportionately there is

CRS. On the other hand, if there is an increase in K while keeping AL constant, we obtain diminishing returns to K.

The slope of the production function gives the marginal productivity of capital (MPK) that shows the extra output per effective labour produced when $\frac{K}{AL}$ is increased by 1 unit. MPK can be mathematically written as

$$\text{MPK} = f(k+1) - f(k) \qquad \ldots(3.5)$$

The intensive form of production function given at (3.4) and depicted through Fig. 3.1 is assumed to satisfy the following conditions:

1) a) at $k = 0$, $f(k) = f(0) = 0$

   b) MPK is positive, that is, $f'(k) > 0$

   c) MPK declines as k rises, that is, $f''(k) < 0$

2) The intensive form in addition is also assumed to satisfying the *Inada* conditions.

   (a) $\lim_{k \to 0} f'(k) = \infty$, which implies that when capital stock is too small the MPK is very large.

   (b) $\lim_{k \to \infty} f'(k) = 0$, which implies that MPK is very small when the capital stock is too large.

These conditions are much stronger than are needed to derive the results of the model and have been introduced to ensure that the growth path of the economy does not diverge.

It is evident from Fig. 3.1 that a tangent drawn at any point on the curve has a positive slope, but the slope of tangents is declining as we go towards the right (as k rises). Hence MPK is positive but declining. Moreover at $k = 0$ the tangent to the production function is a vertical line (which implies that MPK $= \infty$). As k rises the production function becomes flatter and when $k \to \infty$ we have MPK $= 0$. It is important to note here that Solow model assumes other inputs beside K, L and A not to be important and hence are not included in the production function.

3.2.2 Demand for Goods

In a Solow economy goods are demanded for consumption and investment purposes. Thus

$$Y = C + I \qquad \ldots(3.6)$$

If we express (3.6) in per effective labour terms (that is, divide by $\frac{1}{AL}$) we obtain

$$y = c + i \qquad \ldots(3.7)$$

We omit government expenditure and net exports in (3.6) which are present in the national income accounts identity (see Unit 1). While government expenditure is ignored for simplification in presentation, net exports (that is, exports *minus* imports) is excluded because the economy is assumed to be closed.

Each year people save a fraction s of their income. Thus s is the saving rate and it assumes a value between 0 and 1. The relationship between output and saving is depicted in Fig. 3.2.

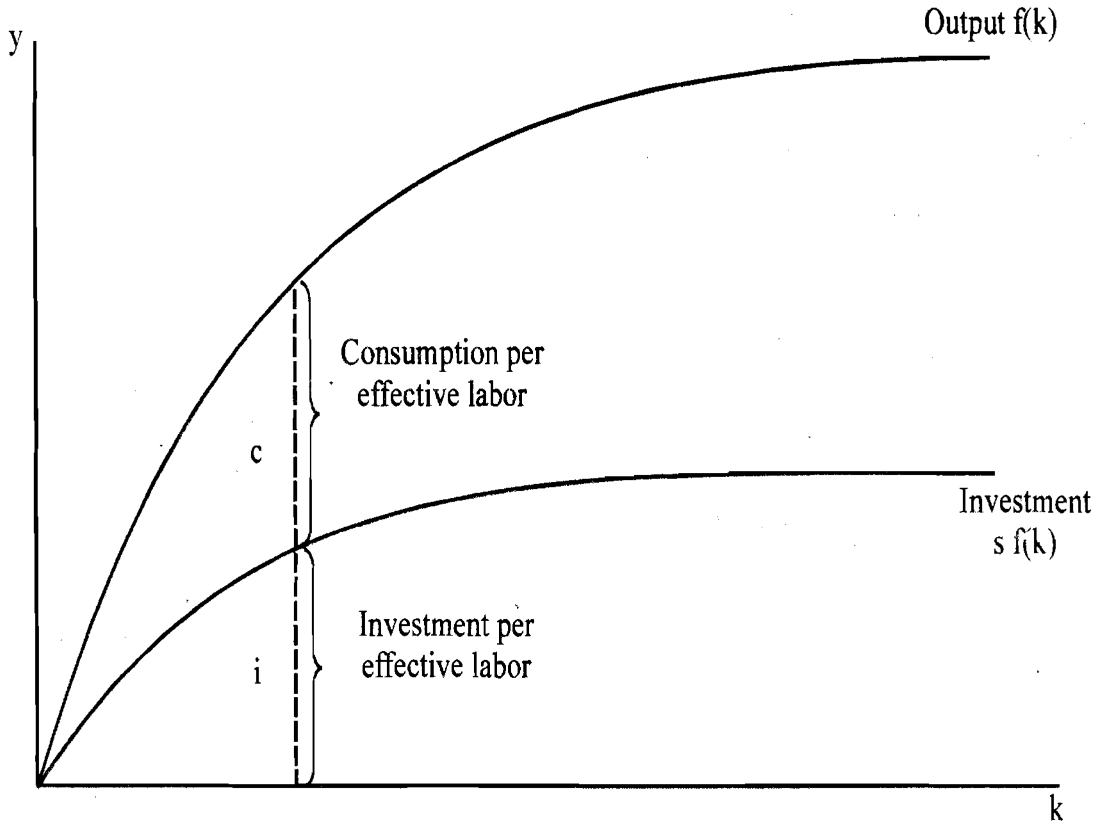

Fig. 3.2: Output and Saving

Saving per effective labour = sy

Therefore, consumption $c = (1 - s)\,y$

$$\Rightarrow y = (1 - s)\,y + i \qquad \ldots(3.8)$$

$$\Rightarrow y - (1 - s)\,y = i$$

$$\Rightarrow y - y + sy = i$$

$$\Rightarrow sy = i \qquad \ldots(3.9)$$

Equation (3.9) shows that saving equal investment, a condition necessary for equilibrium in the economy.

Check Your Progress 1

1) What are the basic assumptions on which the Solow model is based?

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2) What is the basic equilibrium condition depicted by the Solow model?

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3.3 THE STEADY STATE

As seen in equation (3.9) investment per unit of effective labour equals saving per unit of effective labour.

$i = sy$

Since $y = f(k)$ we can write equation (3.9) as

$$i = s\,f(k) \qquad \ldots(3.10)$$

The above shows the relationship between existing capital stock (k) and accumulation of new capital (i) expressed in 'per effective labour' term.

As you know, increase in capital stock is due to investment. Thus the difference between capital stock in two successive years is equal to investment that has taken place during the year. In other words, the rate of growth capital stock is equal to the rate of investment. In per effective labour term, we can express (3.10) as

$$\dot{k} = sf(k) \qquad \ldots(3.11)$$

where $\dot{k}$ refers to the growth rate in k (In general we put a dot over a variable to represent its growth rate).

The above equilibrium condition is true for an economy where there is no depreciation to capital stock, there is no population growth and technological progress does not take place. Let us assume for the time being that there is no population growth and technological progress in the economy. We will relax these unrealistic assumptions later on.

3.3.1 Growth of Capital and Steady State

The Solow model assumes that existing capital depreciates at the rate δ. Thus each year δK amount of capital is depreciated.

Investment and depreciation act in opposite directions and the growth in capital stock is net of the two quantities.

$$\dot{k}(t) = i(t) - \delta k(t)$$

since $i = sf(k(t))$

$$\dot{k}(t) = sf(k(t)) - \delta k(t) \qquad \ldots(3.12)$$

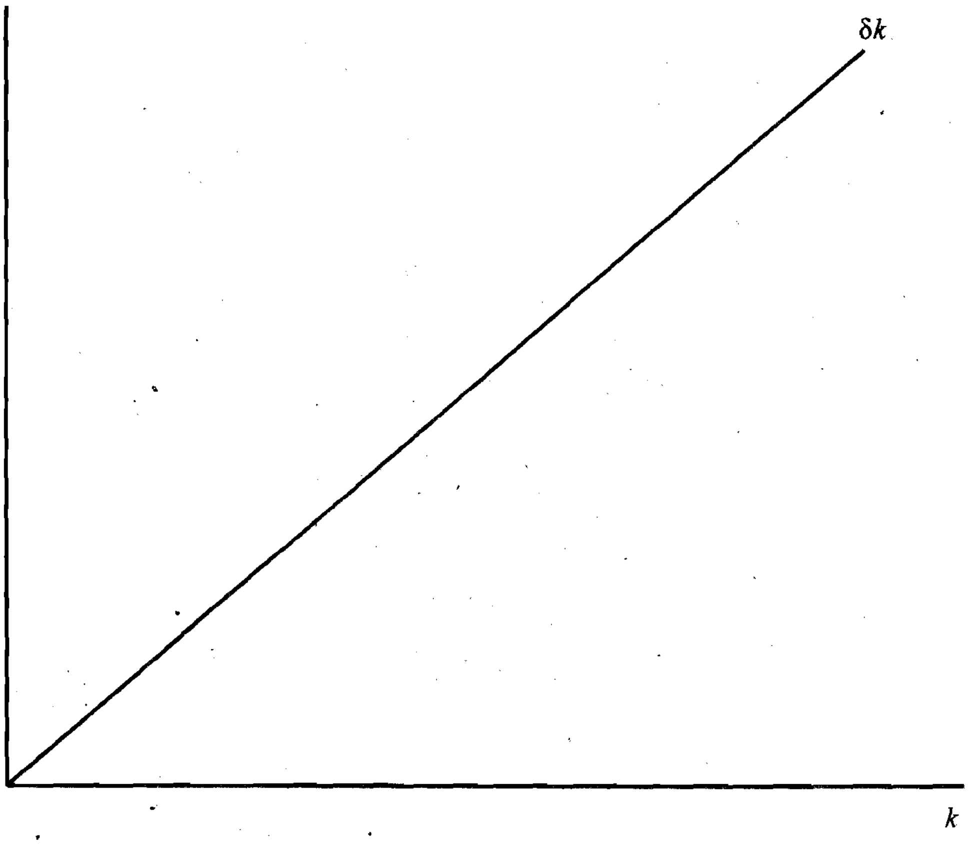

Fig. 3.3: Depreciation Rate

From (3.12) we infer that capital stock rises when $sf\,k(t)) > \delta k(t)$; falls when $s\,f(k(t)) < \delta k(t)$ and remains constant when $sf(k(t)) = \delta k(t)$.

Fig. 3.4 depicts equation (3.12) graphically.

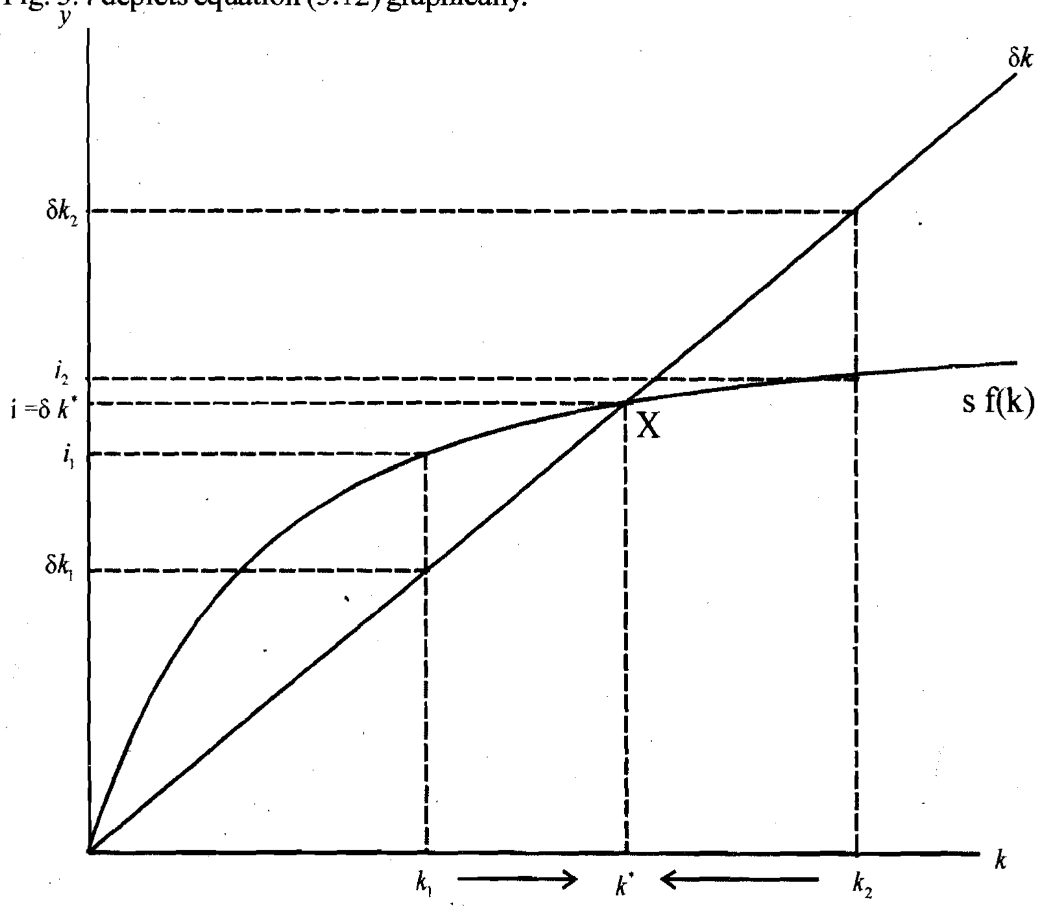

Fig. 3.4: Steady State of an Economy

The two curves, saving and depreciation curves, intersect at point X, where capital stock is k^* and depreciation equals investment. At this point there is no growth in capital stock hence output also remains steady. k^* is therefore known as the steady state level of capital. There are two unique features of the steady state: (i) economy in steady state will remain there till there is change in any other variable, and (ii) economy will always move towards the steady state. For example, if the economy starts at k_1 level of capital, where $k_1 < k^*$ (see Fig. 3.4), investment exceeds depreciation. As a result, capital stock k_1 along with output $f(k)$ rises till k reaches k^*. On the other hand, if the economy starts at k_2 level of capital, which is greater than k^*, investment is less than depreciation. Consequently there is a decline in capital stock and output in the economy until steady state capital, that is, k^* is reached.

Once the economy attains the steady state there is no pressure on k to increase or decrease hence the economy stays there. Thus the Solow model does not explain sustained economic growth. An economy with a high saving rate will have higher level of output and capital as compared to a country with low rate of saving. Thus saving rate is an important determinant of an economy's output and capital. This to some extent explains the disparity in output across countries. Saving rate may vary across countries due to plethora of reasons like development of financial markets, tax policy, cultural differences, retirement policies, political stability and political institutions. Empirical evidence given by Mankiw comparing income per person with investment as percentage of output of 84 countries confirms the association between high saving/investment rate and high level of income per person. However he cites the case of Mexico and Zimbabwe that have similar investment rates but income per person in Mexico is thrice of that in Zimbabwe. This leads us to the question that besides saving and investment there are other potential factors that determine living standards. We study two of them below, viz., population growth and technical change.

3.3.2 Population Growth and Steady State

To analyse the effect of population growth we expand the Solow model. We now consider the case where population and the labour force grow at a constant rate n. When labour force grows, additional capital is required to maintain the same level of k. Hence the economy should have adequate investments to take care of depreciation (δk) as well as population growth (nk). In order to introduce n we modify equation (3.12) as

$$\dot{k}(t) = sf(k(t)) - (n+\delta)k(t) \qquad \ldots(3.13)$$

For steady state the amount of investment required must not only cover depreciation (δk) but also provide new workers with capital (nk). Break-even investment now would be $(n+\delta)k$. The steady sate is achieved at the point of intersection of investment and $(n+\delta)k$ curves. The line δk in Fig. 3.4 accordingly is adjusted to represent $(\delta+n)k$.

The steady state is reached in a similar manner as discussed in earlier section. If $k_1 < k^*$ investment is greater than break even investment so k and y rise. On the other hand, if $k_2 > k^*$, k declines till it reaches k^*.

Population growth succeeds in explaining sustained economic growth in an economy. In this framework, however, output per effective labour remains unchanged. Remember that at the steady state we witness constant k and y. But capital stock (K) and output (Y) keeps on growing at the rate n to keep k and y constant. Such a feature explains the cross-country disparity in incomes. Let us assume that country I has population growth of n_1 and country II has n_2 such that $n_1 > n_2$. Saving rate in both countries is assumed to

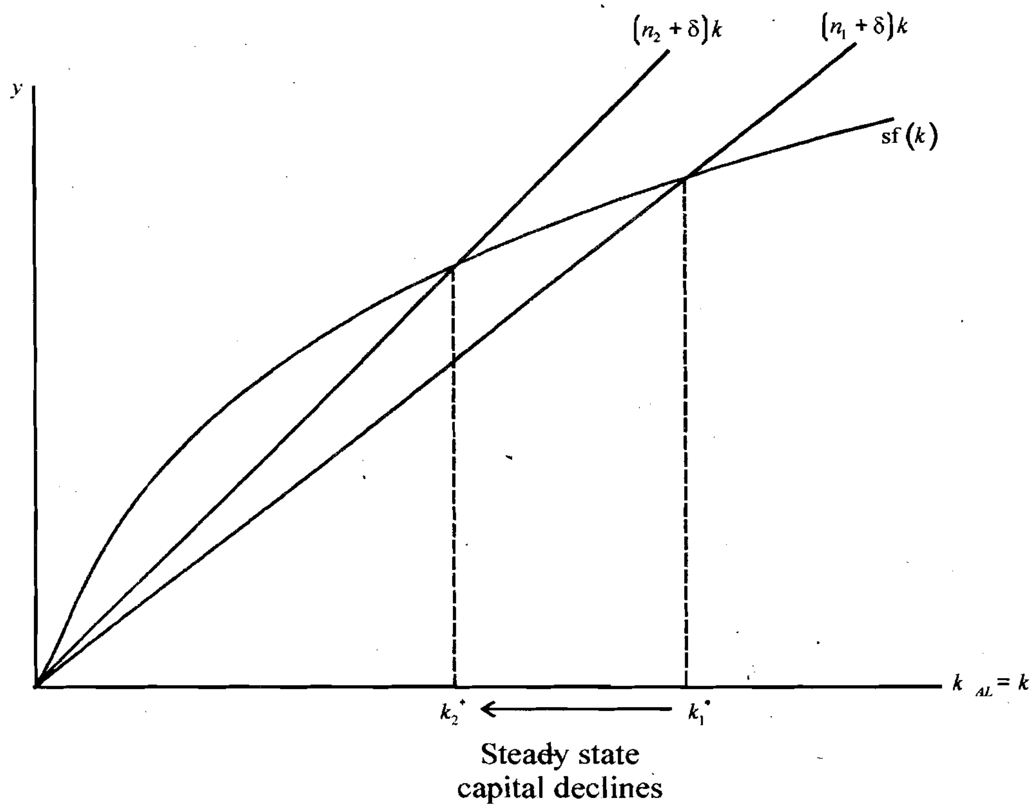

Fig. 3.5: Steady State in Two Countries

be the same. Accordingly, in Fig. 3.5 we draw a line $(\delta+n_2)k$ with a slope greater than that of $(\delta+n_1)k$.

We can easily comprehend from Fig. 3.5 that the country with high growth rate of population n_2 has lower level of k^* (the steady state level of k) and thus lower y. Empirical evidence cited by Mankiw supports the above model. Thus it is often emphasized by policy makers in India to control population growth in order to attain higher living standards.

3.3.3 Technological Progress and Steady State

To explain growth in output per effective labour, we need to introduce technological progress in the Solow model. As stated earlier technological progress is assumed to be

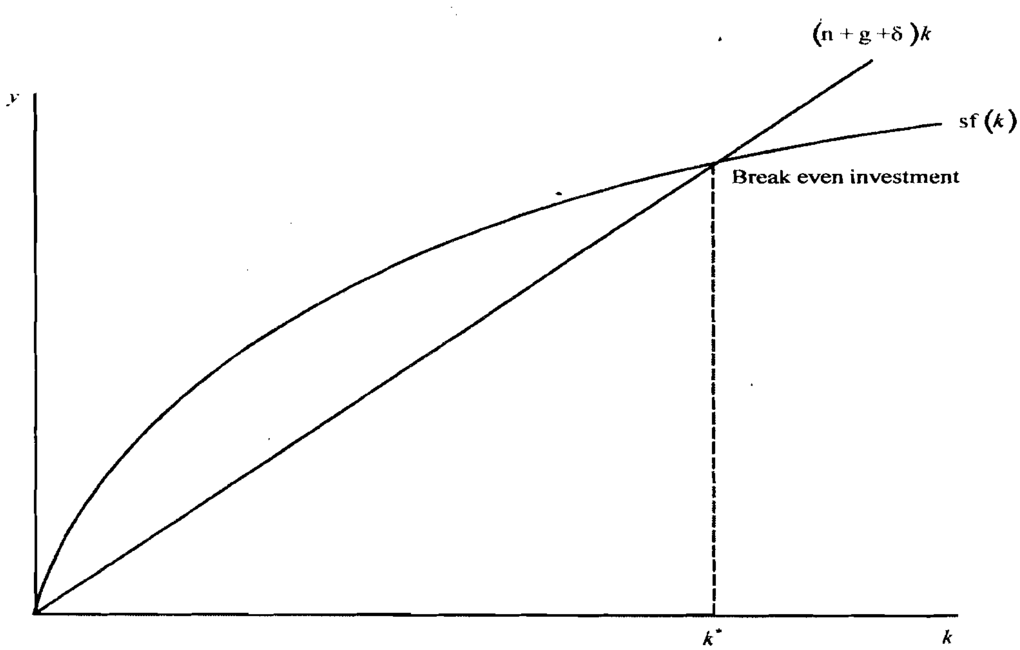

Fig. 3.6: Steady State

labour-augmenting. Thus technological progress increases the quantity of effective labour (AL). Let us assume that the rate of technological progress is g.

The change in k over time is now modified as

$$\dot{k}(t) = sf(k(t)) - (n + g + \delta)k(t) \qquad \ldots(3.14)$$

In fact, equation (3.14) is the key equation of the Solow model of growth. Fig. 3.6 explains the equation through a diagram.

As is evident, the analysis of steady state does not change with the inclusion of technological progress. But the break-even investment now is $(n+g+\delta)k$. Out of total investment $sf(k)$, δk is needed to cover depreciation and nk to maintain capital per effective labour constant. However, as a result of technological progress y grows at a rate of g. Although n, g and δ are not individually restricted, the sum is assumed to be positive in the model, i.e., $n + g + \delta > 0$. As mentioned earlier, in the steady state capital per effective labour and output per effective labour remains unchanged. There is a growth in output per effective labour at a rate g and total output grows at the rate $(n+g)$. We see that introduction of technological progress enables us to account for increase in output per worker.

We conclude that according to Solow model persistently rising living standards (Y/L) can be explained only through technological progress.

3.4 THE GOLDEN RULE

Let us see the impact of variation in saving rate on steady state. Let us assume that saving rate rises while n, g, and δ remain unchanged (see Fig. 3.7). Since $i = sf(k)$ there will be higher investment which in turn will lead to capital accumulation and output growth and the economy will eventually reach to a new steady state with higher capital and output.

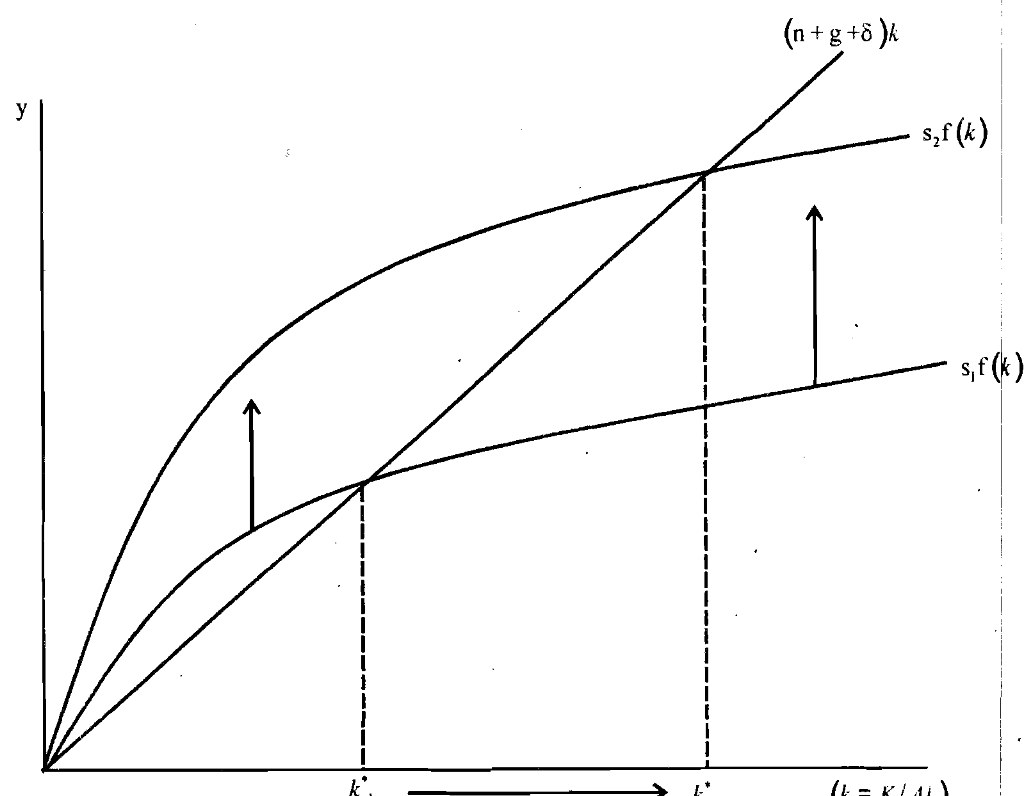

Fig. 3.7: Impact of Saving Rate

When saving rate rises from s_1 to s_2 the investment curve shifts up from $s_1f(k)$ to $s_2f(k)$. Thus the economy reaches a new steady state k_2^* after going through the process of capital accumulation as described above.

It may lead us to think that a high rate of saving is always desirable since higher saving results in higher capital stock and output. You may think that if saving is 100% there will be largest possible output and capital stock in the economy. At various levels of s different steady state are achieved with different levels of capital accumulation. However, there is an optimum level of capital accumulation which is called the golden rule level of capital.

At the golden rule level of capital the level of s is such that the consumption per effective labour is maximum at the steady state. Why is consumption per effective labour maximized? This is because individuals, who make up the economy, are not concerned about the capital stock or total output of the economy. For them what is important is the amount of output they consume. Among the various steady states one that maximizes consumption per effective labour is thus the most desirable one and hence called the 'golden rule level'.

You know that income (which is equal to output) is allocated on consumption and saving (which is equal to investment), that is, $Y = C + I$. Steady state consumption is output net of investment. Thus

$c^* = y^* - i^*$

We can write the above as

$$c^* = f(k^*) - (n+g+\delta)k^* \qquad \text{...(3.15)}$$

As increase in steady state capital has contrasting effect on steady state consumption - more capital leads to more output which contributes positively to consumption, but it also means higher break even investment $(n+g+\delta)k$.

Fig. 3. 8 illustrates steady state y and break-even investment as a function of steady state capital, k^*.

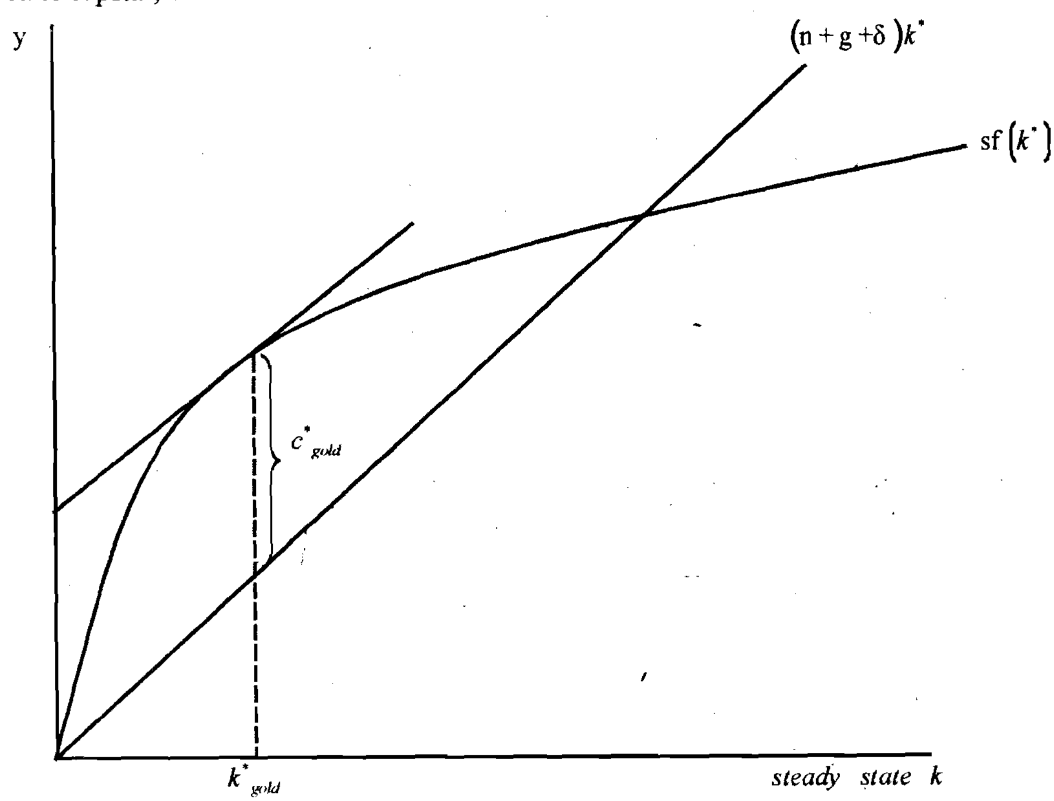

Fig. 3.8: Golden Rule Steady State

Steady state consumption is the gap between the steady state output and steady state break-even investment, which is maximized at k^*_{gold} level of capital per effective labour. Recall that the slope of the production function is the marginal product of capital MPK. We notice in Fig. 3.8 that at c^*_{gold} level of consumption (golden rule level) the slope of production function equals the slope of break-even investment, that is,

$$\text{MPK} = (n + g + \delta)$$

Or

$$(\text{MPK} - \delta) = (n + g) \qquad \ldots(3.16)$$

At the golden rule level of capital, the MPK net of depreciation is equal to the rate of growth of total output $(n + g)$.

The golden rule steady state is not achieved automatically. It requires a particular rate of saving s_{gold} as shown in Fig. 3.9.

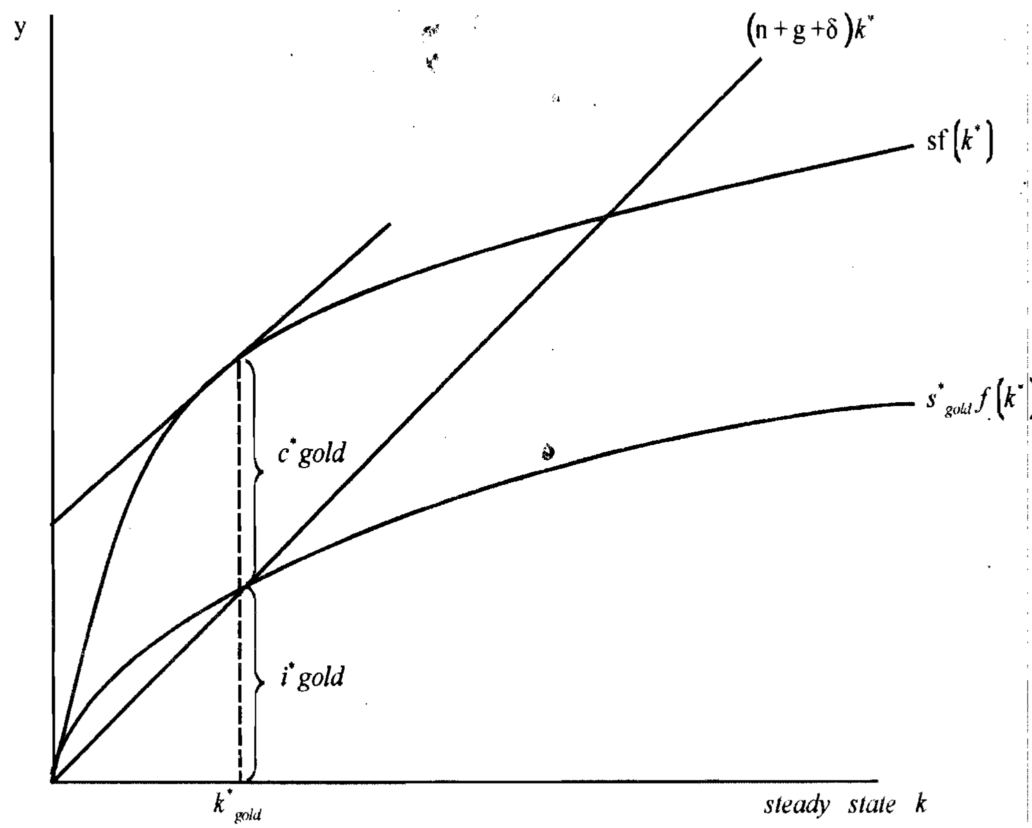

Fig. 3.9: Golden Rule Saving

To achieve the golden rule steady state level of capital k^*_{gold} a saving rate of s_{gold} is required such that consumption is maximized at c^*_{gold}.

If $s > s_{gold}$ (equivalently, $k^* > k^*_{gold}$) the economy is said to be dynamically inefficient. In this case a fall in saving from s to s_{gold} will increase consumption. On the other hand, if $s < s_{gold}$ (equivalently, $k^* < k^*_{gold}$) a rise in saving will decrease consumption in the short run but will eventually lead to higher consumption in the long run. How the transition to golden rule steady sate can be achieved by changing the saving rate from s to s_{gold} is dealt with in the next Section.

3.5 TRANSITION TO THE GOLDEN RULE STEADY STATE

Let us discuss the impact of change in saving rate to achieve golden rule level of consumption, investment and capital. there can be two situations: (i) when $s > s_{gold}$ or steady state capital per effective labour is more than required for golden rule level, and (ii) when $s < s_{gold}$ or steady state capital per effective labour is less than the golden rule level of k^*_{gold}.

Case 1: when $s > s_{gold}$

In this case to achieve the golden rule level the saving rate needs to be decreased to s_{gold}. When saving rate falls consumption per effective labour immediately rises and investment declines. The economy is no longer in a steady state as investment is less than break-even investment of $(n + g + s)\, k^*$. This leads to a decline in capital stock per effective labour. Output per effective labour is a function of k so that Y/AL also declines. Since $c = y - i$ we observe that consumption per effective labour also declines. These variables y, c and i fall till the economy reaches a new steady state which is the golden rule steady state.

At this level consumption is higher than the earlier steady state although the levels of output and investment are lower. Consumption per effective labour is higher in the entire period of transition to the golden rule steady state from the earlier level.

Case 2: when $s < s_{gold}$

When at a steady state $s < s_{gold}$ we have $k^* < k^*_{gold}$. In this case s needs to be increased to achieve the golden rule steady state. When s rises there is a rise in investment which now exceeds break-even investment $(n + g + \delta)$ and the economy is in a state of transition. There is accumulation of capital leading to rise in output per effective labour (Y/AL). So when saving is increased consumption per effective labour declines immediately but with rise in output per effective labour it rises to a level higher than the level which initially prevailed.

The dilemma of whether to try to reach the golden rule steady state or not is a question of choice between current and future consumers. It is a trade off between present and future levels of consumption. We will discuss more about the inter-temporal consumption decisions in Block 4.

Check Your Progress 2

1) What do you mean by steady State?

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2) Under what condition does an economy realise steady state?

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3) What is the golden rule steady state for an economy?

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3.6 LET US SUM UP

The Solow model explains long-term growth in per capita output experienced by economies world over. It assumes a constant returns to scale production function which allows for diminishing returns to scale to capital input. The model depicts that the economy has a tendency to converge to a steady state where availability of capital per worker remains unchanged. In its simplest form the Solow model is built upon the equality between saving and investment.

Investment results in increase in capital input which is durable in nature and undergoes depreciation during the course of production. In steady state as capital stock available per unit of labour remains unchanged, the economy should have just enough investment to take care of, i) depreciation, ii) population growth, and iii) technical progress.

The Solow model concludes that saving has only a 'level effect' and no 'growth effect'. It means that higher saving in an economy increases per capita availability of capital and output, but no sustained output growth. Sustained growth in output is explained in the model neither by higher saving nor by population growth. Rather high population growth reduces the per capita availability of output and capital. According to the Solow model sustained output growth is possible only through technological progress.

Higher saving rate, although it increases per capita output, is not always desirable. The objective of an economy is to maximize consumption, not saving. The golden rule suggests that capital should be at that level where MPK net of depreciation is equal to output growth.

3.7 KEY WORDS

Break-even Investment	The amount of investment just enough to compensate for the depreciation to the existing capital stock.
Capital Acumulation	Capital input does not get exhausted in a single use and remains in use for a long period of time. Thus due to investment in successive years, capital input gets accumulated.
Depreciation	Natural weal and tear undergone by capital inputs (such as mechineries, buildig, etc.) when production of goods and services take place.
Golden Rule	The condition for obtaining the steady state which maximizes consumption. It is given by the equality betwen MPK net of depreciation and output growth rate, that is, $(MPK - \delta = n + g)$.
Inada Conditions	Two conditions required for the production function to be well behaved, formulated by Ken-Ichi Inada in 1963.
Labour-augmenting Technical Change	Technical change that results in increasing the productivity or efficiency of labour.
Steady State	A condition when the economy does not have to change its capital-labour ratio.
Sustained Growth	Growth in output on a sustained basis, over a longer period of time.

3.8 SOME USEFUL BOOKS

Solow, R. M., 1970, *Growth Theory*, Oxford University Press, Oxford.

Romer, D., 1996, *Advanced Macroeconomics*, McGraw Hill Company Ltd., New York., Chapters 1, 2, 3.

Mankiw, N.G., 2003, *Macroeconomics* (Fifth Edition), Worth Publishers, New York, Chapters 7 & 8.

3.9 ANSWERS/HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) The basic assumptions pertain to properties of the neoclassical production function. Mention these properties. Also mention that the model applies to a closed economy.

2) The supply demand equilibrium condition needs to be mentioned. Point out that equilibrium is realized when saving = investment.

Check Your Progress 2

1) It refers to a state or condition when the economy does not need to change its capital stock relative to its labour force.

2) Go through Section 3.3 and explain the condition $sf(k) = n + g + \delta$ through a diagram (similar to Fig. 3.4). Explain the process how the economy reverts back to the above equality in case there is deviation.

3) Go through Section 3.4 and answer.

UNIT 4 ENDOGENOUS GROWTH MODEL

Structure

4.0 OBJECTIVES

After going through this Unit you will be in a position to

- explain the implications and predictions of the Solow model;
- evaluate the Solow neoclassical model; and
- explain the new or endogenous growth models.

4.1 INTRODUCTION

Economic growth is a critical factor in determining living standards in the long term. As we observed in the previous unit, the standard tool for analysing long run growth in macroeconomic literature has been the neoclassical model of economic growth developed by Robert Solow. Using a few basic assumptions Solow demonstrated that in the long run an economy would tend toward an equilibrium marked by continual growth of output. This equilibrium, known as steady state, is characterized by constant levels of capital per worker and output per worker. In case there is a deviation from the equilibrium level of capital labour ratio the economy bounces back to the steady state. The Solow model, however, is not free from limitations, which has motivated new research. In recent years economists have attempted to explain long-term growth through the endogenous growth model, which we discuss in the present unit.

4.2 SOLOW MODEL AS A THEORY OF GROWTH

The neoclassical model as developed by Robert Solow (1956) and subsequently applied by others constituted a giant step forward in the process of constructing a formal model of growth. The main features of the Solow model can be described as follows:

a) The assumption of diminishing marginal returns to capital leads the growth process within the economy to reach the steady state.

b) In the long run, once the economies reach the steady state, output, capital and labour grow at the same rate equaling the exogenously given rate of technological progress. This rate of growth is independent of the underlying parameters of the economies (such as saving rateand population growth) and also independent of their initial position.

c) The parameters of the economies determine the steady state level per capita income, which is positively related with saving rate and negatively related with population growth and depreciation rate.

Thus saving and other parameters in the model have no effect on the rate of growth in the long run but they have only the level effect. For example, a higher saving ratio implies higher per capita income but does not influence long term growth rate of output. Now we will discuss some important issues related to the Solow model.

4.3 ABSOLUTE AND CONDITIONAL CONVERGENCE

The major focus of recent empirical studies on economic growth has been on the issue of convergence in growth rates across countries. We discuss below the convergence hypothesis implied by the Solow model. There are two versions of the convergence hypothesis: absolute convergence and conditional convergence.

4.3.1 Absolute Convergence

Let us have a group of countries having access to the same technology [f(.)], the same population growth rate (n) and the same saving propensity (s) and they only differ in terms of their initial capital-labour ratio (k). Then according to absolute convergence hypothesis in the Solow model in the long run, all countries converge to the same steady state capital-labour ratio, output per capita and consumption per capita and growth rate (n). That is, there must be a convergence in income per capita in the long run.

Absolute convergence is shown in Fig. 4.1. In the diagram subscript 1 refers to poor countries while subscript 2 refers to rich countries. Thus,

y_1, k_1 = income per head and capital labour ratio of poor countries.

y_2, k_2 = income per head and capital labour ratio of rich countries.

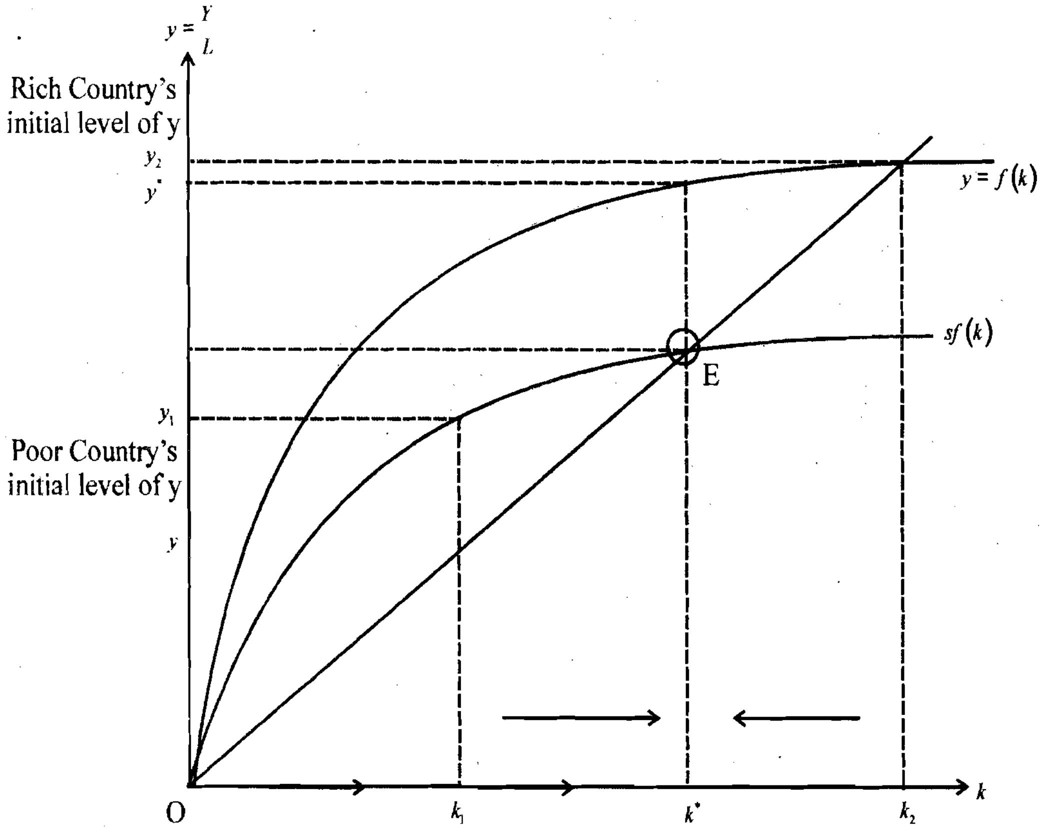

Fig. 4.1: Absolute Convergence

The two countries are assumed to be otherwise identical. The stability of the Solow model predicts that both the countries will reach the long run equilibrium point E with the same capital –labour ratio k^* as shown in Fig.4.1.

The obvious implication of this analysis is that the poor countries will grow relatively fast (i.e., capital and output grow faster than population growth) while rich countries will grow relatively slower (i.e., capital and output grow slower than population growth).

4.3.2 Conditional Convergence

The structural parameters like technological progress, rate of savings, population growth rate, and depreciation rate differ across countries. Given the assumption of diminishing returns to capital conditional convergence states that each country will converge to its own steady state rather than to a common steady state across countries. Conditional convergence also is a prediction of neoclassical theory of growth.

4.4 CONVERGENCE AND GROWTH REGRESSION

Using the data from Maddison (1982), Baumol (1986) examines convergence from 1870 to 1979 among 16 industrialized countries. Over this period of time Baumol regressed output growth on a constant and initial income.

$$\log [(Y/N)_{i,1979}] - \log [(Y/N)_{i,1870}] = a + b \log [(Y/N)_{i,1876}] + e_i$$

where

log Y/N	=	log income per person
e	=	an error term

i = index of countries

If convergence prevails the value of b will be negative implying that countries with higher initial income have lower growth.

For perfect convergence the value of b will be -1 and for no convergence the value will be 0.

The results of the estimation as obtained by Baumol are

$$\log[(Y/N)_{i,1979}] - \log[(Y/N)_{i,1870}] = 8.457 - 0.995 \log[(Y/N)_{i,1870}],$$

The estimated value of b is very close to -1 and the standard error of b is very low so that it is statistically significant.

However, results such as the above have been criticized by economists like De Long (1988). According to De Long there are two problems in Baumol's analysis, (i) sample selection, and (ii) error of measurement. If we consider these two limitations then it will undermine most of the support Baumol's estimates provide for convergence.

4.5 QUANTITATIVE IMPLICATIONS OF THE SOLOW MODEL

In the Solow model discussed in the previous unit we observed that the economy will reach a steady state in the long run in order to understand the quantitative implications of the Solow model we need to describe the specific functional forms and values of the parameters.

Let, $y = f(k)$, ...(4.1)

We see that (4.1) is the intensive form production function in the Solow model which states output per unit of effective labour as a function of capital per unit of effective labour where

$$f(k) = 0$$

$$f'(k) > 0$$

$$f''(k) < 0$$

The production function as specified by (4.1) also satisfies the 'Inada' conditions, viz.,

$$\lim_{k \to 0} f'(k) = \infty$$

$$\lim_{k \to \infty} f'(k) = 0$$

Let us take

$y^* = f(k^*)$ = the level of output per unit of effective labour along the balanced growth path.

k^* = the value of k where actual investment and break-even investment $[(n+g+\delta)k]$ are equal, i.e., $\dot{k}(t) = 0$.

From the Solow model we know,

$$\dot{k}(t) = sf(k(t)) - (n+g+\delta)k(t) \qquad \ldots(4.2)$$

If we take partial derivative of (4.2) with respect to *s*, we obtain the long run effect of a rise in saving on output. Thus we can write

$$\frac{\partial y^*}{\partial s} = f'(k^*).\frac{\partial k^*(s,n,g,\delta)}{\partial s} \qquad \ldots(4.3)$$

Therefore, in order to get $\frac{\partial y^*}{\partial s}$ we need to know the second term, i.e., $\frac{\partial k^*}{\partial s}$ term.

From the definition of k^* we find that at k^*, $\dot{k}(t) = 0$.

This implies, at k^*,

$$sf[k^*(s,n,g,\delta)] = (n+g+\delta)k^*(s,n,g,\delta) \qquad \ldots(4.4)$$

Equation (4.4) holds for all values of s, n, g and δ.

Taking derivative of both sides of the equation (4.4) with respect to s we get (by using multiplication law of derivatives),

$$sf'(k^*).\frac{\partial k^*}{\partial s} + f(k^*) = (n+g+\delta)\frac{\partial k^*}{\partial s} \qquad \ldots(4.5)$$

Rearranging the terms in (4.5) we get,

$$\frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)} \qquad \ldots(4.6)$$

Substituting (4.6) into (4.3) we obtain,

$$\frac{\partial y^*}{\partial s} = \frac{f'(k^*)\, f(k^*)}{(n+g+\delta) - sf'(k^*)} \qquad \ldots(4.7)$$

Now multiplying both sides of (4.7) by s/y^* (to convert it to an elasticity) and using

$sf(k^*) = (n+g+\delta)k^*$ for substituting s, we get

$$\frac{s}{y^*}.\frac{\partial y}{\partial s} = \frac{s}{f(k^*)}.\frac{f'(k^*)f(k^*)}{(n+g+\delta) - \partial f'(k)} = \frac{(n+g+\delta)k^* f'(k^*)}{f(k^*).(n+g+\delta) - (n+g+\delta)} \qquad \ldots(4.8)$$

$$= \frac{k^* f'(k^*)/f(k^*)}{1 - [k^* f'(k^*)/f(k^*)]} \qquad \ldots(4.9)$$

In (4.9)

$\frac{k^* f'(k^*)}{f(k^*)}$ is the elasticity of output with respect to capital at $k = k^*$.

Let us take this elasticity as α_k

Then equation (4.9) becomes,

$$\frac{s}{y^*} = \frac{\partial y^*}{\partial s} = \frac{\alpha_k(k^*)}{1 - \alpha_k(k^*)} \qquad \ldots(4.10)$$

We know from microeconomic theory that if there exists perfect competition in the market with no externalities, capital will earn its marginal product. So the total amount received by capital (per unit of effective labour) along the balanced growth path is $\frac{k^*f(k^*)}{f(k^*)}$ or $\alpha_k\ (k^*)$.

Empirical studies show that the share of income paid to capital is about one-third. Using this as an estimate of $\alpha_k(k^*)$, we arrive at the result on the basis of (4.10) that the elasticity of output with respect to saving rate in the long run is about one-half. So we can easily conclude that significant changes in saving only have a moderate effect on the level of output on the balanced growth path.

4.6 SPEED OF CONVERGENCE

Sometimes in addition to the information about the eventual effects of some changes (e.g., change in saving rate) we are also interested in knowing the speed with which those effects occur. In particular, we are interested in knowing how rapidly k approaches to k^* in the Solow model framework.

For this we again recall equation (4.2) that

$$\dot{k}(t) = sf(k(t)) - (n+g+\delta)k(t)$$

So $\quad \dot{k} = \dot{k}(k)$.

And

at $k = k^*$, $\quad \dot{k}(t) = 0$

Taking a first order Taylor series approximation of $\dot{k}(k)$ around $k = k^*$ we get,

$$\dot{k} \simeq \left(\frac{\partial \dot{k}(k)}{\partial k}\bigg|_{k=k^*} \right)(k - k^*) \qquad \text{...(4.11)}$$

i.e., $\dot{k}$ is approximately equal to the product of the difference between k and k^* and the derivative of k with respect to *k* at k^*.

Taking a differentiation of equation (4.2) with respect to *k* and evaluating the result at $k = k^*$ we get

$$\frac{\partial \dot{k}(k)}{\partial k}\bigg|_{k=k^*} = sf'(k^*) - (n+g+\delta)$$

$$= \frac{(n+g+\delta)k^*f'(k^*)}{f(k^*)} - (n+g+\delta)$$

$$\text{or, } s = \frac{(n+g+\delta)}{f(k^*)}$$

$$= [\alpha_k(k^*) - 1]\,(n+g+\delta) \qquad \text{...(4.12)}$$

and, $\quad \alpha_k = \dfrac{k^* f(k^*)}{f(k^*)}$

Substituting (4.12) in (4.11) we get,

$$\dot{k}(t) \simeq -[1 - \alpha_k(k^*)](n+g+\delta)\quad [k(t) - k^*] \qquad \text{...(4.13)}$$

Equation (4.13) has an important implication that, along the balanced growth path, capital per unit of effective labour will converge toward k^* at a speed proportional to its distance from k^*.

If we assume

$$x(t) = k(t) - k^*, \quad \text{and}$$

$$\lambda = (1-\alpha_k)\,(n+g+\delta)$$

Then from (4.13) we can write

$$\dot{x}(t) \simeq \lambda x(t)$$

The growth rate of x is constant and equals –l.

So the path of x is given by,

$$x(t) \simeq x(0)e^{-\lambda t} \qquad \text{...(4.14)}$$

Where x(0) = the initial value of x. Substituting the values of x = k(t)-k* back again we get the path in terms of k,

$$k(t) - k^* \simeq e^{-(1-\alpha_k)(n+g+\delta)t}k(0) \qquad \text{...(4.15)}$$

We can also show that, y approaches y^* at the same rate at which k approaches k^*

i.e. $$y(t) - y^* \simeq e^{-\lambda t}[y(0) - y^*] \qquad \text{...(4.16)}$$

Empirical finding has shown that $(n + g + \delta)$ is generally 6%. If we take capital share to be roughly one-third, $(1-\alpha_k)\,(n + g + \delta)$ is roughly 4%.

This means *k* and *y* will travel 4% of the remaining distance toward k^* and y^* each year and approximately it will take 18 years to reach half way to their balanced growth path values. So changes in saving rate do not have any significant effect on the speed of convergence.

4.7 CRITICAL ASSESSMENT OF THE SOLOW MODEL

The neoclassical growth theory, as developed by Solow (1956) and his followers has come under the fire of criticism since the late 1980s. The main objections to the traditional neoclassical approach can be discussed under a few headings.

i) **Technology**

In the neoclassical Solow model technology plays the most dominant role since the equilibrium rates of growth of the relevant variables depend on the rate of technological progress. But the model treats it as an exogenous variable and provides no explanation of it.

In reality it is observed that expenditure on Research and Development, attainments of education and good health have a positive impact on the technological improvement. The Solow model does not include the motivation to invent new goods or invest on human capital.

ii) Law of Diminishing Returns

The crucial assumption of the neoclassical model, i.e., the operation of the law of diminishing returns to capital has been questioned. Economists have argued that if we can broaden the concept of capital to include human capital (consisting of education, job training, health care, etc.) in addition to physical capital, then assumption of diminishing returns can be relaxed.

iii) Homogenous Inputs

The third problem in the classical growth model is that is assumes all labour and capital to be homogenous. It makes no room for differentiating skilled or unskilled labour and the capital in agriculture, industries or infrastructure. In reality, however, these differences have significant impact on economic growth.

iv) Constancy of Structural Parameters

The Solow model assumes saving rate, the rate of growth of labour supply, the skill of labour force and the pace of technological change to be constant. Thus it fails to take into account the intrinsic characteristics of economies that cause them to change.

v) Little Role for the Government

There is very little scope of Government or public policy in the Solow growth model to influence growth rate. This scenario is incompatible with the real life situation. The model is also built on the assumption of closed economy which is not realistic in today's world.

vi) Convergence Predictions

The model predicts that countries would converge around one another over time. But empirical evidence has shown that it has not been the case. Growth rates of the poor countries not only remain lower than the richer countries but also fail to hit their respective steady state per capita incomes.

All these inadequacies of the neoclassical model have inspired new research both theoretical and empirical, and as a consequence the concept of endogenous or new growth theory has emerged.

Check Your Progress 1

1) Define the concepts of absolute and conditional convergence.

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2) Suppose a production function satisfies constant returns to scale and a diminishing marginal product of the inputs capital K and labor L. Explain both properties and give an example of such a production function.

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3) Consider the Solow model. Derive the expression for the speed of convergence.

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4) Critically evaluate the Solow model of growth.

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4.8 ENDOGENOUS GROWTH THEORY

The new growth theory provides a theoretical framework for analyzing persistent growth of output that is determined within the system governing the production process. This is done either by avoiding diminishing returns to capital or by explaining technical change internally.

In the new growth theory models, there exist technological spillovers, externalities and increasing returns to scale. They do not expect convergence; they rather accept the fact that disparities among countries can persist or even enlarge. They emphasize on the importance of investments in human capital and potential gains from technology improvements – inventions and innovations.

We will start with the simplest and basic model of endogenous growth that is the AK model. This model is important since other endogenous growth models can be thought of as extensions or microfoundation of the basic one.

4.9 THE BASIC AK MODEL

The AK model is the simplest possible endogenous growth model. The production function is assumed to be linear in the only input capital.

$$Y_t = f(k) = AK \qquad \ldots(4.17)$$

where

A = an exogenous constant which reflects the level of technology

K = the aggregate capital broadly defined

The production of function given at (4.17) displays both constant returns to scale and constant returns to capital.

Recall that the Solow model assumed diminishing returns to capital. The inexistence of diminishing return to capital is the key difference between the AK model and the Solow model.

Now let us assume that a certain fraction of income is saved and invested, which remains constant (s).

The capital accumulation equation can be written as

$$\dot{K} = sY - \delta K \qquad \ldots(4.18)$$

where

i.e., change in capital stock equals investment (sY) minus depreciation (δK).

$$\therefore \quad \frac{\dot{K}}{K} = sA - \delta \qquad \ldots(4.19)$$

[From (4.17) we know that $\frac{Y}{K} = A$]

Using (4.19) along with (4.17), i.e., Y = AK we can write after a little re-arrangement of terms

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = sA - \delta \qquad \ldots(4.20)$$

Equation (4.20) shows the growth rate of output $\left(\frac{\Delta Y}{Y}\right)$ and states that as long as sA > δ, the economy's output / income grows forever, even without the assumption of exogenous technical progress.

The main results of the AK model can be summarized as follows:

1) The model allows for growth in output at a rate determined by (sA – δ).

2) Higher saving s, higher level of technology A and lower level of depreciation δ have positive effects on the growth rate of output.

3) The model does not exhibit any convergence; it rather accepts divergence.

4) There exist no transitional dynamics: the growth rate jumps instantaneously whenever there is a change in parameter value.

Before concluding we must say that, we cannot consider the AK model as a complete model since all the results are based on the assumption of constant returns to capital. It leaves unexplained why there are no diminishing returns to capital. However, we must acknowledge that the AK model highlights the key component required for any model of endogenous growth, i.e., there must be constant returns to the factor (or factors) that can be accumulated.

4.10 HUMAN CAPITAL AND R&D IN ENDOGENOUS GROWTH THEORY

The theories of endogenous growth can be broadly divided into two main parts

(i) where growth is driven by Research and Development (R&D), and

(ii) where growth is driven by Human Capital Accumulation.

R&D based models originate from the work of Romer (1990), and Grossman Helpman (1991). In all these models economic growth is the result of technological change that comes from purposive R&D activities by firms. Patents and blueprints are non-rival goods that can be accumulated without bounds and so the diminishing returns to capital accumulation can be avoided and growth continues.

Human Capital based growth models are derived from the work of Lucas (1988). These models place accumulation of human capital at the centre of the growth process. If we accumulate both physical and human capital we can think of constant returns to the broad concept of capital and as a result the economic growth does not diminish.

4.11 CRITICAL ASSESSMENT OF THE NEW GROWTH THEORY

The new growth theory is has been criticised mainly on two counts. First, it remains dependent on some of the traditional neoclassical assumption that are inappropriate for developing countries such as the existence of a single production function, i.e., all sectors are symmetrical. Secondly, economic growth in developing countries most of the times is impeded by inefficiencies arising from poor infrastructure, inadequate institutional structures, imperfect capital and goods markets. The new growth theory, however, does not consider these factors, and as a consequence its applicability to cross country development comparison is limited.

Check Your Progress 2

1) What are the key defining properties of Endogenous models?

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2) Why does the AK model of endogenous growth not exhibit any convergence?

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3) Identify the main difference of endogenous growth theory with that of neoclassical Solow model?

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4.12 LET US SUM UP

In this unit we explained the principal implications or predictions of the Solow model. We also made a critical assessment of the neoclassical model. Subsequently we shifted to the new paradigm of growth theory, i.e., endogenous or new growth theory. Under these theories we discussed one basic model in detail, that is the Ak model, to get an idea of these kinds of models.

The Solow model converges to a steady state primarily because of its assumption of diminishing returns to capital. A major conclusion of the Solow model that growth in an economy is determined by exogenously given technological change. Thus government policy, R&D, or human capital has no influence on growth. empirical evidence across countries, however, shows that these variables influence economic growth in an economy to a large extent. The endogenous growth theory takes into account these factors and explains growth as a function endogenous variables.

4.13 KEY WORDS

Returns to Scale: It is the rate at which output increases as all inputs are increased proportionately. If output more (less) than doubles when inputs are doubled there is increasing (decreasing) returns to scale.

Marginal Product: It measures the additional output as input increases by one unit.

Law of Diminishing Marginal Returns: The law states that when one or more inputs are fixed a variable input is likely to have a marginal product that eventually diminishes as the level of input increases.

Spillover — A spillover is an action taken by one person or firm that affects another person or firm.

Classical Convergence — The central idea is that diminishing returns to investment cause the growth rate of a country to decline as it approaches its steady state level of capital per unit of effective labour – implying that, *ceteris paribus*, richer economies grow slower than poorer economies.

Human Capital — Investments that improves the quality of labour and renders it more productive.

4.14 SOME USEFUL BOOKS

Solow, R. M., 1970, *Growth Theory*, Oxford University Press, Oxford.

Romer, D., 1996, *Advanced Macroeconomics*, McGraw Hill Company Ltd., New York., Chapters 1, 2, 3.

Mankiw, N.G., 2003, *Macroeconomics* (Fifth Edition), Worth Publishers, New York, Chapters 7 & 8.

4.15 ANSWER /HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) See Section 4.3 and answer.

2) See Section 4.15 and answer. Hint: CRS implies a situation where output doubles when all inputs are doubled. An example of this kind of production function is Cobb Douglas production function given by $F(K,L) = AK^a L^b$ where a,b < 1 and both are constants. When $a + b = 1 \Rightarrow$ C.R.S. On the other hand, if $a + b > (<) 1 \Rightarrow$ IRS (DRS).

3) See Section 4.5 and answer.

4) See Section 4.7and answer.

Check Your Progress 2

1) The defining characteristic of the new models is that they generate growth of per capita output endogenously, that is, without assuming that technological change occurs outside of the model's framework

2) Similar countries in terms of parameters A, s, n and δ grow at the same rate regardless of their level of income. Therefore factor differences in income per capita are constant over time and there is no tendency for countries to converge.

3) See Section 4.7, 4.8 and answer to question 1.