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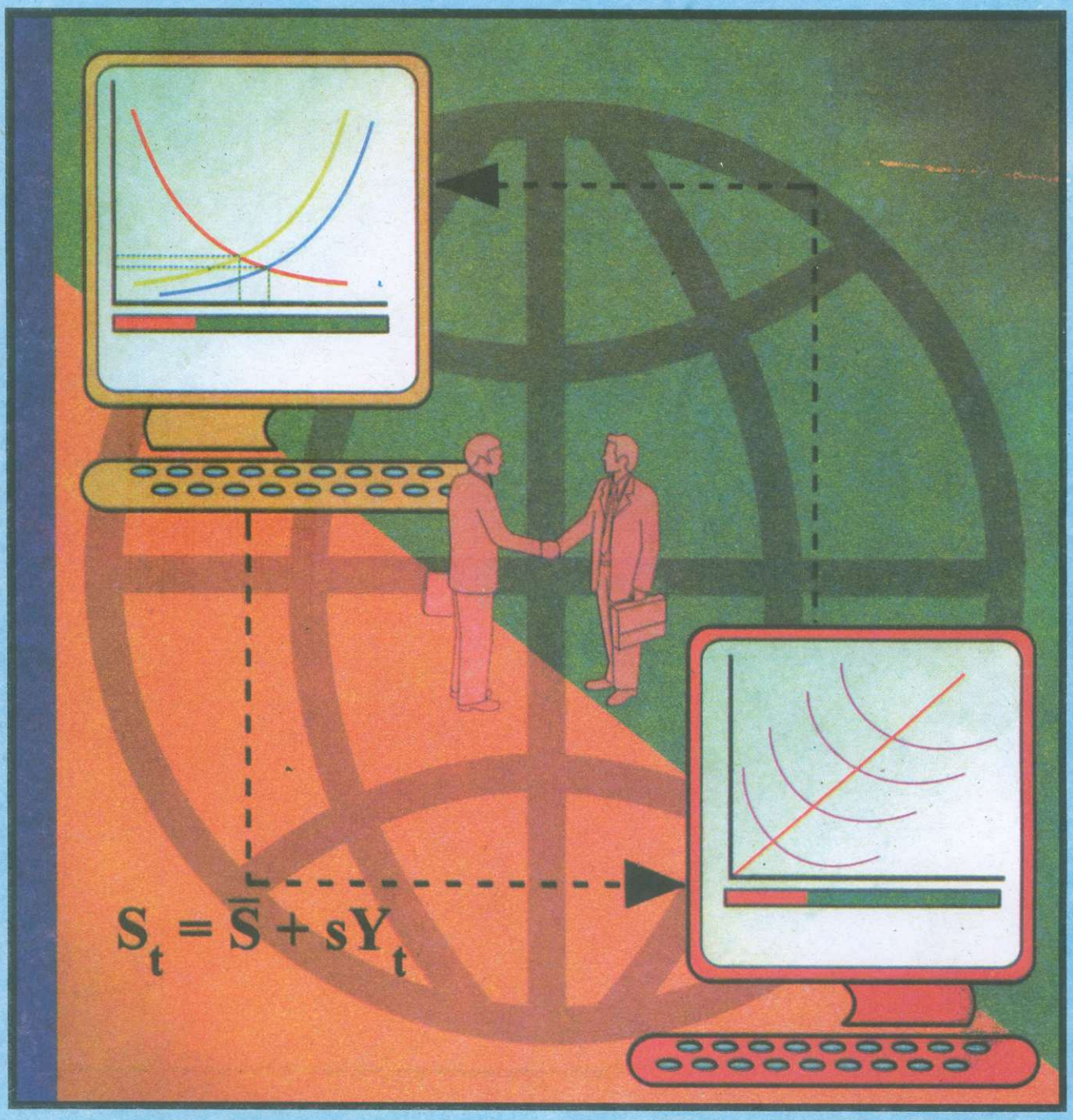
MEC - 101
MICROECONOMIC ANALYSIS



Consumer Behaviour

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MEC-101
Microeconomic Analysis

Block

1

CONSUMER BEHAVIOUR

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BLOCK 1 CONSUMER BEHAVIOUR

Introduction

Consumer's behaviour is analysed in the block. Building on the assumption that consumers behave rationally and maximise utility of consumption, **Unit 1** attempts to document the method of measuring utility either cordially or ordinary and seeks to discuss the demand for commodity based on the premise of utility maximisation in the presence of budget constraints.

Unit 2 extends the consumer choice and deals with demand curve. Approaches to optimal choice of consumption and basic duality relations are discussed. In the process, it deals with analytical refinement in the utility and demand derivations with the aid of indifference curve approach.

The last unit of block, (**Unit 3**), covers the recent developments in demand analysis by including linear expenditure system, treatment of consumer's surplus and inter-temporal consumption. After looking into the theory of price formation through demand-supply analysis, it discusses the cobweb model and lagged adjustment methods.

UNIT 1 THEORY OF COSUMER BEHAVIOUR: BASIC THEMES

Structure

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1.0 OBJECTIVES

The objective of this unit is to relate how individual consumers take decisions of consumption in a situation where market prices are given to them and they can't influence the market prices by altering their consumption. This unit will enable you to:

- determine the optimum choice of a consumer;
- explain how the price effect can be decompose into income effect and substitution effect; and
- determine the individual demand curve.

1.1 INTRODUCTION

It is generally observed that market aggregate demand curve for a commodity is downward sloping, given other things. Our problem is to investigate economic rationality behind this for a commodity of all individual consumers. The market demand basically depends on the characteristics of demand for a commodity by individual consumers, and the demand for a commodity of an individual consumer depends upon the behaviour of the consumer. Clearly, to

investigate economic rationality behind the law of demand, we shall start with the analysis of consumer behaviour.

1.2 THE BASIC THEMES

There are different approaches to analyse the consumer behaviour. But in all approaches, it is assumed that the consumer is rational. This means that the consumer's objective is to maximise her utility by choosing one commodity bundle from among all the commodity bundles (money income and the prices of the commodities are given to the consumer).

1.3 CONSUMER CHOICE CONCERNING UTILITY

Consumers can't maximise her utility unless she can measure it. Hence, utility must be a measurable concept. The measurement is undertaken differently in different approaches. In traditional frame, we have two types of measurement of utility,

1) Cardinal analysis

2) Ordinal analysis

1.3.1 Cardinal Theory: An Introduction

In cardinal approach, utility is measured cardinally or numerically in terms of money. The consumer not only knows which one is preferred but also by what amount. The assumptions of this approach is given below:

1) Consumer is rational.

   Implication: The consumer's objective is to maximise her utility by choosing one of the commodity bundle from all other available commodity bundles at given prices of commodities and money income.

2) If the taste and preferences are given, the total utility of the consumer depends on the quantity of consumption.

3) Goods are good.

   Implication: Let 'U' denote utility level of the consumer and let 'x' be the consumption bundle. As 'x' increases (decreases) 'U' increases (decreases). Therefore, marginal utility is positive.

4) Marginal utility of 'x' is diminishing.

   Implication: As 'x' increases (decreases) MU_x decreases (increases). Therefore, MU_x curve is downward sloping

5) Utility is measured cardinally or numerically in terms of money.

   Implication: Since it is measured numerically consumer not only knows which commodity bundle is preferred but also by how much amount.

6) Marginal utility of money is constant.

Implication: $MU_m = \lambda$ where λ is positive and constant. That means as money income increases (decreases) by one unit, utility increases (decreases) by λ unit.

Consumer Equilibrium

According to our assumption for 'x' units consumption of the commodity, gross utility obtained by the consumer is U(x).But for this, the consumer must spend $p_x.x$ units of money income if p_x be the price of the commodity 'x', which is given to the consumer. Since from assumption 6, λ represents fall in utility due to one unit fall in money income, the net utility of the consumer is given by $N(x) = U(x)-\lambda p_x.x$, where λ and p_x are given to the consumer. So consumer's objective is to maximise N(x) by choosing 'x'. For that we take the first derivative of N(x) and set that equal to zero, $\frac{dN(x)}{dx}=0$. Or, we get $\frac{dU(x)}{dx}-\lambda p_x=0$. From this first order condition, we can derive the optimum value of 'x' which is (say) $x^* = x^*(p_x,\lambda)$. The second order condition for utility maximisation requires $\frac{\partial^2 N(x)}{\partial x^2}=\frac{\partial^2 U(x)}{\partial x^2}<0$, which is ensured by the assumption of falling MU_x.

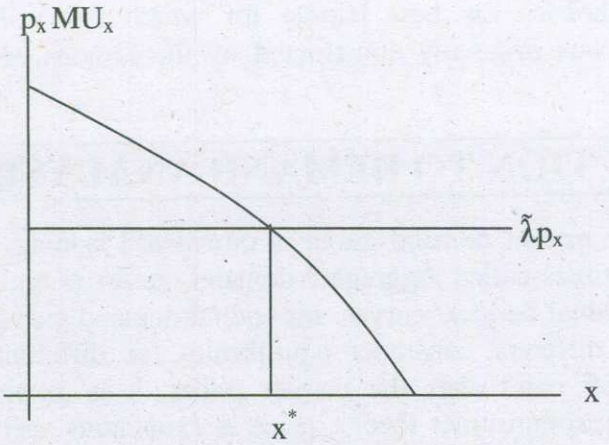

Fig. 1.1: Consumer Equilibrium in Cardinal Theory

Check Your Progress 1

1) What are the assumptions of cardinal utility theory?

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2) Consider the utility function U (x) = log (x), let $p_x = 2$ and $\lambda = 5$. Derive the consumer equilibrium and check the second order condition.

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1.3.2 Ordinal Theory: A Short Note

In ordinal approach, utility is measured ordinally i.e., qualitatively (not numerically or quantitatively). Alternatively, consumer can rank her preferences according to the order she wants to compare but not in terms of the different amount. It's a qualitative measure and therefore more realistic measurement of utility or satisfaction.

There are two different approaches of ordinal theory, viz.,

1) Indifferent curve approach
2) Revealed preference approach

1.3.2.1 Indifferent Curve Approach

Indifference curve is constructed by taking utility level constant, so different indifferent curves imply different level of utility for same consumer. The equilibrium is achieved when indifferent curve become tangent to the budget line.

1.3.2.2 Revealed Preference Approach

In revealed preference approach, consumer equilibrium can be found by ranking different bundle of goods in the commodity space. Given the budget constraint, consumer chooses the best bundle for which her utility will maximise. This theory was originally constructed by the famous economist Paul. A. Samuelson.

1.4 INTRODUCTION TO DEMAND ANALYSIS

It is generally seen that market demand curve is downward sloping. Market demand curve (or sometimes called Aggregate demand curve) is nothing but the aggregation of individual demand curves. Individual demand curve can be constructed by joining different consumer equilibrium for different prices (remember that consumer can't alter the market prices, it is given to the consumer). In neo-classical consumer theory, price is exogenous variable, so demand curve can be obtain only if we change the price exogenously and join all the equilibrium points. From next on our objective is to find out the consumer demand curve, for which we will adopt ordinal theory and in that, we will take indifferent curve approach.

1.5 ORDINAL THEORY: INDIFFERENCE CURVE APPROACH

In indifference curve approach consumer is assumed to be rational, so that consumer's objective is to maximise her utility by choosing a commodity bundle among all other available commodity bundles (under budget constraint) where total utility ('U') depends on quantity consumption given her taste and preferences. Therefore, in a two-commodity world (say x_1 and x_2) utility function is given by $U = U(x_1,x_2)$ and it depends on taste and preferences of the consumer, which is specified by axioms given below:

1) **Axiom of reflexiveness:** Consumer's choice is reflexive.

   Implication: Weak preference relation is denoted by 'R'. Suppose there are two goods x_1 and x_2 and suppose x_1 is weakly preferred to x_2 i.e., $x_1 R x_2$

which implies that either x_1 is strictly preferred over x_2 (it is denoted by x_1Px_2) or x_1 is indifferent to x_2 (it is denoted by x_1Ix_2), where 'P' and 'I' implies strict preference relation and indifference respectively.

The set constituted by all commodity bundles or vector is known as commodity set (X). Any one commodity bundle is denoted by 'x' is weakly preferred (i.e., either strictly preferred or indifferent) over any other commodity bundle (i.e., in respect to 'x'). Therefore, we have xRx.

Clearly, any one commodity bundle may be indifferent to another commodity bundle i.e., there is a possibility of indifference or same level of utility between the commodity bundles.

None of the commodity bundles are not preferred i.e., consumer can choose any commodity bundle. So choice set of this consumer is specified by the commodity set 'X'.

2) **Axiom of completeness:** Consumer's choice is complete.

   Implication: Since consumer is rational, she must have a unique preference relation. That means the consumer choice is either x_1Rx_2 or x_2Rx_1. Alternatively, consumer's choice is consistent or comparable. For unique preference relation, consumer choice must be transitive, where transitivity implies that if x_1Rx_2 and x_2Rx_3 then x_1Rx_3, where x_3 is another commodity.

3) **Axiom of continuity:** Consumer's preference relation (R) is continuous.

4) **Axiom of non-satiation:** Consumer's choice is non-satiated in all goods.

   Implication: Non-satiation means larger the consumption of a good leads to larger satisfaction or utility or lower the consumption lower is the satisfaction or utility. Non-satiation of all goods (which means "goods are good" or "more is better") means any commodity bundle 'A' is preferred over another commodity bundle 'B' only if bundle 'A' consists larger quantity of at least one good and no less quantity of any other goods. Notationaly if A>B, then A is preferred over B or APB where B is any other commodity bundle.

5) **Axiom of convexity:** Consumer choice is such that indifference curve is strictly convex to the origin (i.e., utility function is quassi-concave).

6) **Axiom of selfishness:** Consumer choice is selfish.

   Implication: Consumer's choice is self-guided. It is not influenced by any other consumer.

1.5.1 Concept of Preference, Utility Function and Indifference Curve

Consumer preference ('R') specified by the above axioms can be represented by a function where total utility ('U') depends on quantity consumption (x_1, x_2), which satisfied all other axioms. The function $U = U(x_1, x_2)$ is known as utility function. Since consumer is rational, her objective is to maximise the utility specified by the utility function $U = U(x_1, x_2)$ subject to her budget constraint. To solve the consumer utility maximisation problem, we use a graphical tool, which is known as Indifference curve.

Meaning and definition of indifference curve: Different combination of goods x_1 and x_2 along which consumer is indifferent (or consumer has same level of utility) give a curve in commodity-commodity plane known as indifference curve. Therefore, along the indifference curve utility or satisfaction remains unchanged.

Existence of indifference curve: Because of axiom of reflexiveness consumer can choose a commodity bundle over another commodity bundle i.e., consumer may be indifferent between any commodity bundle and such a choice might be continuous. So, indifference curve may exits anywhere in the commodity space.

1.5.2 Derivation of Indifference Curve and It's Properties

Graphical Presentation

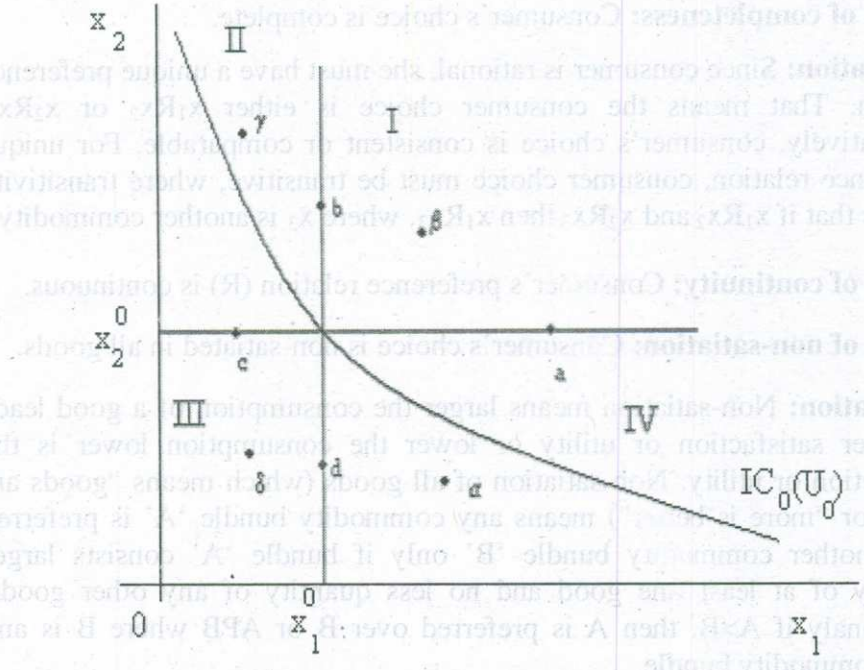

Fig. 1.2: A Typical Indifference Curve

Consider any commodity bundle denoted by point A in the above figure which consist x^0_1 and x^0_2 amount of good I and good II respectively and from which consumer obtains particular level of utility, say U_0. We compare the commodity bundle 'A' with other commodity bundle in the commodity space. For that we divide the entire commodity plane into four phases from 'A'.

Consider any point in phase I say β, where we have large quantity of both x_1 and x_2 compared to point 'A'. Again, if we consider any point say 'a' in horizontal line in phase I, we have larger quantity of x_1 with same quantity of x_2 compared to point 'A'. Similarly, for any point 'b' in vertical axis, we have larger x_2 with same x_1. That means in phase I including the borderlines, we have larger quantity of at least one commodity and no less quantity of any other commodity compared to 'A'. Thus, we have larger utility in phase I including the borderlines compared to 'A'.

By similar logic, we have lower consumption of at least one good and no larger consumption of any other good in phase III including the borderlines compared

to point 'A'. Hence, we have lower level of utility in phase III including the borderlines compared to 'A' by the axiom of non-satiation for all goods.

Clearly, in phases I and III, including borderlines, utility is not constant between the commodity bundles compared to point 'A'. So, indifference curve (along which utility is constant) can't pass through phases I and III including their borderlines.

Consider any point in phase IV excluding borderlines, say α. We have larger x_1 (for which utility is larger) and lower x_2 (for which utility is lower) compared to 'A'. Since both goods are non-satiated, utility of point α may be larger, lower or equal compared to point 'A'. Similarly, for any point in phase II excluding the borderlines, say δ, we have larger consumption of x_2 but lower of x_1 compared to point 'A'. Therefore, by axiom of non-satiation in all goods, utility at point δ may be larger, lower or equal compared to 'A'.

Clearly, only in phases II and IV excluding the borderlines, there is a possibility of the same level of utility between the bundles compared to point 'A'. So, indifference curve, along which utility remains unchanged, must pass through the phase II and phase IV, excluding their lines. Thus, indifference curve is necessarily downward sloping where all goods are non-satiated given that a consumer choice is continuous, reflexive and complete.

Mathematical Presentation

Consider the utility function $U = U(x_1, x_2)$. Differentiating totally, we get the following:

$dU = U_1dx_1 + U_2dx_2 = 0$ (as along the indifference curve utility is constant, $dU = 0$). Therefore,

$\frac{dx_2}{dx_1} = -\frac{U_1(x_1, x_2)}{U_2(x_1, x_2)}$, which is the slope of the indifference curve. It is negative since $U_1(x_1, x_2) > 0$ and $U_2(x_1, x_2) > 0$ by assumption of non-satiation of all goods. Thus, indifference curve is downward sloping because all goods are non-satiated, choice is continuous, reflexive and complete.

Economic meaning

All goods are non-satiated i.e., larger (lower) consumption leads to lager (lower) utility. Hence, for given x_2, as x_1 increases, utility increases. Thus, to maintain same level, utility must be reduced, which is possible by reducing x_2. Hence, as x_1 increases, x_2 must decreases in order to maintain same level of utility. That is why indifference curve is downward sloping.

Properties of indifference curve

Property I: Higher indifference curve gives higher utility.

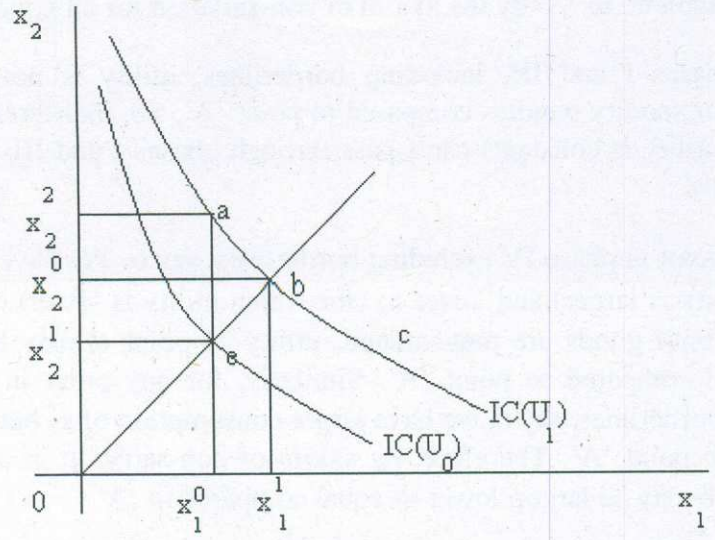

Fig. 1.3: Higher Indifference Curve gives Higher Level of Utility

Explanation: Since all goods are non-satiated, larger consumption of any good leads to larger utility. Thus, a commodity bundle, which consists of larger quantity of at least one good and no less consumption of any other goods, gives larger utility compared to any other commodity bundle. Consequently, higher indifference curve represents higher consumption of at least one commodity and no less consumption of any other commodity.

Property II: Indifference curves can't intersect with each other.

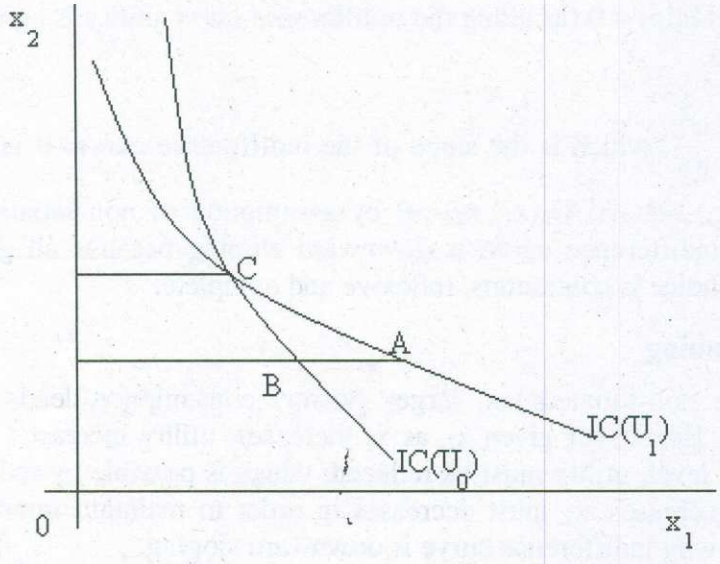

Fig. 1.4: Indifference Curves Can't Intersect Each Other

Explanation: Suppose two indifference curves intersect each other. By definition, along the indifference curve, utility is constant. So, consumer is indifferent between points 'A' and 'C' that lie on the same indifference curve. Similarly, consumer is indifferent between points 'B' and 'C', as they also lie on the same indifference curve. So, AIC and BIC, where 'I' denotes indifference. Now, from transitivity we have AIB i.e., point 'A' and point 'B' give the same utility to the consumer. But for given x_2, x_1 is larger in point 'A' compared to point 'B'. So, by the assumption of non-satiation, we have point 'A' that gives lager utility to consumer as compared to point 'B'. This contradicts the fact that point 'A' and 'B' gives the same level of utility to the

consumer (as we have proved above). Therefore, when all goods are non-satiated and transitivity holds, indifference curves can't intersect.

1.5.3 Utility Maximisation

Graphical Presentation

Let consider a two-commodity world, x_1 and x_2 representing good I and good II respectively. p_1 and p_2 are the prices of good I and good II respectively, where the prices are given to the consumer, i.e., prices are exogenously given and consumer can't change them. Money income of the consumer is M, which is also exogenously given to the consumer. Note that $p_1x_1+p_2x_2$ is the total expenditure of the consumer when she consumes $x_1$units of good I and x_2 units of good two. The total expenditure of the consumer can't exceed her money income, therefore $p_1x_1 + p_2x_2 \leq M$ ------- (a)

Equation (a) is known as consumer budget constraint. Let $U = U(x_1, x_2)$ is the utility function of the consumer. Therefore, consumer must solve the following maximisation problem(UMP):

Problem UMP: Max $U(x_1, x_2)$

subject to $x_1 > 0$

$x_2 > 0$

and $p_1x_1 + p_2x_2 \leq M$

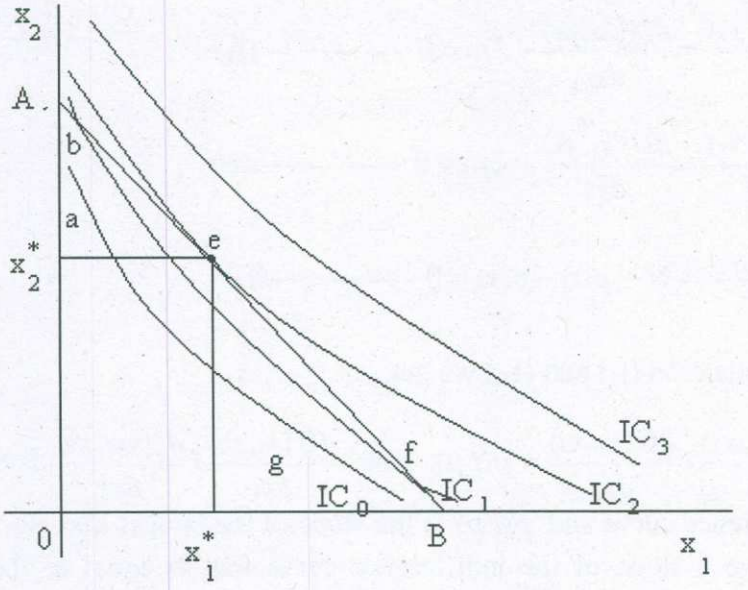

Fig. 1.5: Derivation of Consumer Equilibrium

As consumer objective is to maximise her utility and as larger consumption leads to larger utility, she always wants to consume more of any goods. But she also has to spend some amount of her income to consume larger amount of goods. So ultimately in equilibrium she will spend all her income and $M = p_1x_1+p_2x_2$.

Now suppose that the line segment AB represents the budget line. Along AB $p_1x_1+p_2x_2=M$ holds. Let initial indifference curve of the consumer is IC_0. In

IC_0, there are many points along that indifference curve such that $p_1x_1 + p_2x_2 \leq M$ holds. Therefore, utility maximising consumer will spend more as she moves to higher indifference curve (say IC_1). In IC_1 there are still such points along the indifference curve such that $p_1x_1 + p_2x_2 \leq M$ holds, so again consumer spends more. This process will continue as long as consumer reaches an indifference curve where for no point along the indifference curve $p_1x_1 + p_2x_2 \leq M$ holds and at least one point of the indifference curve is on the budget line. At that point, we have consumer equilibrium, $C(x_1, x_2) = (x_1^*(M,p_1,p_2), x_2^*(M,p_1,p_2))$(in Figure 1.5.3 point 'e' is the equilibrium point). Not that at equilibrium, slope of the indifference curve is equal to the slope of the budget line. Therefore, at equilibrium we have

1) Budget constraint holds with equality sign.

2) Slope of the indifference curve is equal to the slope of the budget line.

Mathematical Presentation

Consumer's objective is to maximise her utility by solving UMP. To solve UMP, we set the Lagrange function of the corresponding problem, which is,

$$L(x_1, x_2) = U(x_1, x_2) + \lambda (M - p_1x_1 - p_2x_2)$$

Our objective is to maximise this Lagrange function by choosing x_1, x_2 and λ. For that we differentiate the Lagrange function by x_1, x_2 and λ, and set all equal to zero.

$$\frac{dL(x_1,x_2)}{dx_1} = \frac{dU(x_1,x_2)}{dx_1} - \lambda p_1 = 0 \text{ ---------- } (f_1)$$

$$\frac{dL(x_1,x_2)}{dx_2} = \frac{dU(x_1,x_2)}{dx_2} - \lambda p_2 = 0 \text{ ---------- } (f_2)$$

$$\frac{dL(x_1,x_2)}{d\lambda} = M - p_1x_1 - p_2x_2 = 0 \text{ ---------- } (f_3)$$

From equation (f_1) and (f_2), we get,

$\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2} = p_1 / p_2$. Note $\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2}$ is the slope of the indifference curve and p_1 / p_2 is the slope of the budget line. So, at equilibrium we have a slope of the indifference curve that is equal to the slope of the budget line. Again, from equation (f_3) we get $M = p_1x_1 + p_2x_2$, so budget equation holds with equality sign.

Check Your Progress 2

1) Define indifference curve in one sentence. What are measured in the axes of the figure to draw an indifference curve?

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2) If $U(x_1, x_2) = 10x_1^{0.3}x_2^{0.7}$, M=200, p_1=5 and p_2=2, set up the Lagrange function and derive the simplest form of $\frac{dU(x_1,x_2)}{dx_1} / \frac{dU(x_1,x_2)}{dx_2} = p_1 / p_2$.

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3) If $U(x_1, x_2) = 10x_1^{0.5}x_2^{0.5}$, M=100, p_1=2 and p_2=4 calculate the consumer equilibrium.

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1.5.4 Concepts of Income and Substitution Effects

Change in demand for a good due to one unit change in price of that good for given prices and money income is known as own price effect for that good. Thus, own price effect $= \frac{dx_1}{dp_1}$ and it consists of own substitution effect and income effect for a price change.

Own ***Substitution Effect***: Change in demand quantity for a good (say x_1) due to change in its own price under constant real income (in terms of utility) is called substitution effect for that good and can be written as $(\frac{dx_i}{dp_i})_{\bar{U}, p_j}$.

Income Effect: Income effect for a good (say x_1) represents change in demand quantity for that good for a change in real income. So income effect = $(\frac{dx_i}{dM})_{\bar{p}}$, which is positive for a normal good, negative for inferior good and zero for neutral good.

Income Effect For A Price Change: For given money income, as price of any one good change one unit then real income (M/p_i) changes for which demand for the good changes by income effect. It is known as income effect for a price change. Thus, income effect for a price change $= -x_i(\frac{dx_i}{dM})$. Note that income effect and income effect for a price change have opposite sign and different magnitude.

1.5.5 Slutsky's Theorem

Graphical Presentation

We prove here that own price effect is the sum of own substitution effect and income effect for a price change, which is known as Slutsky's theorem. Before doing that we give the following approach by given by Hicks.

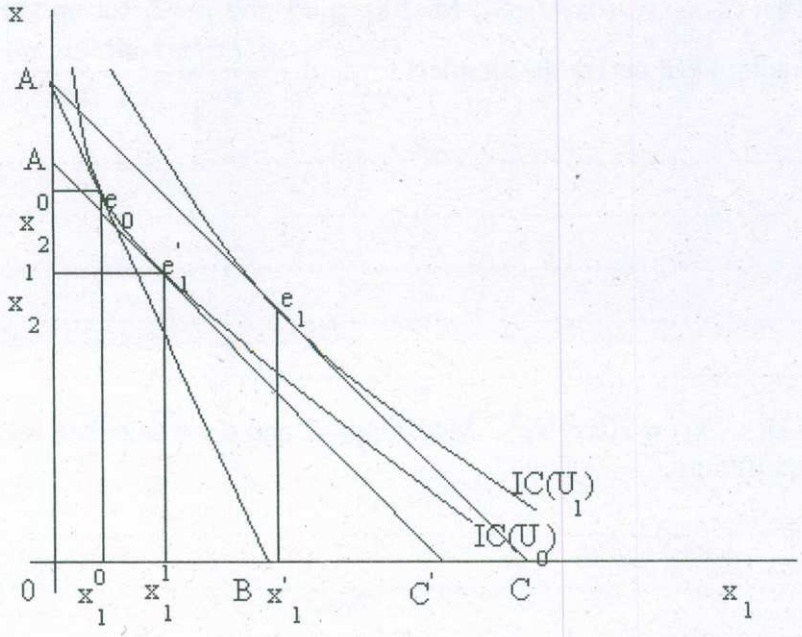

Fig. 1.6: Substitution and Income Effects of a Price Change

At initial prices and money income, budget line is AB and according to the condition of the equilibrium e_0 is the initial equilibrium point. The consumer gets U_0 level of utility. Suppose at constant income and p_2, p_1 decreases (say by one unit). Consequently, the intercept of the budget line (M/p_2) remains unchanged but absolute slope of the budget line (p_1/p_2) decreases. The new budget line becomes flatter with the same intercept. It is denoted by AC line. New equilibrium can be achieved at any point on the new budget line AC (and therefore own price effect can take any algebraic sign). Suppose the equilibrium takes place at point e_1. Hence, as p_1 decreases, for given p_2 and M, demand for good I increases from x_1^0 to x_1^1. This is the own price effect for x_1 and here it is negative. A part of this change is due to change in real income (since for given p_2 and M as p_1 decreases, real income increases) and another part is originated at constant real income. To decompose these effects, we reduce money income (M) of the consumer in such a way that real income helps her in procuring the optimal consumption bundle before the price change. After such a readjustment of M, intercept of the new budget line AC, i.e., (M/p_2) decreases with the same slope (p_1/p_2) for given $p_1$and p_2. Hence the new budget line shifts parallely downwards. Note that now consumer is to purchase the original bundle with new prices. After the shift, she can move to a new indifference curve as well.

If we allow the consumer to have the income so as to spend on the commodity bundles such that the original level utility is retained, (it implies that she take into account the new prices along with new commodity combinations. In such a readjustment process she moves along the old indifference curve to arrive at another equilibrium position. Here the parallel budget line $A'C'$ is tangent to the previous indifference curve. Thus, the consumer can attain the same level of utility and the real income remains constant in terms of utility after adjusting money income and utility is also maximised). After adjustment of money income, budget line is $A'C'$ along which real income in terms of utility remains constant after change in p_1 for given p_2. This budget line is known as compensated budget line. Under such budget line equilibrium will necessarily take place at point e_1'. Hence under constant real income in terms of utility, as p_1 decreases for given p_2, x_1 increases (from x_1^0 to x_1') by substituting x_2 (from

x_2^0 to x_2^1). This is known as own price substitution effect for x_1 which is negative and indifference curve is downward sloping strictly convex to the origin. But as x_1 increases from x_1^0 to x_1^1 and real income also increases, the demand for good I increases from x_1^0 to x_1' through a rise in real income. This would indicate that by income effect for a price change, x_1 is a normal good. Clearly, we have own price effect consists of own substitution effect and income effect for a price change, where own substitution effect in negative but income effect for a price change can take any algebrical sign depending on the good is normal, superior or inferior.

Mathematical Presentation (Slutsky Equation)

We already know from the first order conditions of utility maximisation that,

$$\frac{dL(x_1, x_2)}{dx_1} = \frac{dU(x_1, x_2)}{dx_1} - \lambda p_1 = 0 \text{ ---------- (a)}$$

$$\frac{dL(x_1, x_2)}{dx_2} = \frac{dU(x_1, x_2)}{dx_2} - \lambda p_2 = 0 \text{ ---------- (b)}$$

$$\frac{dL(x_1, x_2)}{d\lambda} = M - p_1 x_1 - p_2 x_2 = 0 \text{ ---------- (c)}$$

We then totally differentiate these equations and get:

$U_{11}\, dx_1 + U_{12}\, dx_2 - p_1\, d\lambda = \lambda\, dp_1$ ---------- (e)

$U_{21}\, dx_1 + U_{22}\, dx_2 - p_2\, d\lambda = \lambda\, dp_2$ ---------- (f)

$-p_1\, dx_1 - p_2\, dx_2 + 0\, .d\lambda = -dM + x_1\, dp_1 + x_2\, dp_2$ ---------- (g)

By using Cramer's rule we have,

$$dx_1 = \begin{pmatrix} \lambda dp_1 & U_{12} & -p_1 \\ \lambda dp_2 & U_{22} & -p_2 \\ -dM + dp_1 + dp_2 & -p_2 & 0 \end{pmatrix} / |D|$$

where, $|D| = \begin{pmatrix} U_{11} & U_{12} & -p_1 \\ U_{21} & U_{22} & -p_2 \\ -p_1 & -p_2 & 0 \end{pmatrix}$ and $U_{ij} = \dfrac{\partial^2 U(x_i, x_j)}{\partial x_i \partial x_j}$.

Or, we can write,

$$dx_1 = \frac{\lambda dp_1 D_{11} + \lambda dp_2 D_{21} + (-dM + x_1 dp_1 + x_2 dp_2) D_{31}}{|D|} \text{ ---------- (h)},$$

where D_{ij} is the co-factor of the i^{th} row and j^{th} column of the determinant |D|. For income effect we know $dp_1 = dp_2 = 0$, therefore we have from equation (h),

$$\partial x_1 = \frac{-D_{31} \partial M}{|D|}, \text{ or } \left(\frac{\partial x_1}{\partial M}\right)_{\bar{p}} = \frac{p_2 U_{12} - p_1 U_{22}}{|D|} \text{ ---------- (i)}$$

Now for own price effect we have $dM = dp_2 = 0$. So from equation (h) we get,

$$dx_1 = \frac{\lambda D_{11} \partial p_1 + x_1 D_{31} \partial p_1}{|D|} \text{ or, } \left(\frac{dx_1}{\partial p_1}\right)_{\bar{M}, \bar{p}_2} = \frac{\lambda D_{11}}{|D|} + x_1 \frac{\lambda D_{31}}{|D|} \text{ ---------- (j)}$$

Lastly, to find out own substitution effect we consider utility is constant in terms of income so, $-dM+x_1dp_1+x_2dp_2=0$ and $dp_2=0$. We have from equation (h), $(\frac{dx_1}{\partial p_1})_{\bar{U},\bar{p}_2} = \frac{\lambda D_{11}}{|D|}$ ---------- (k).

Therefore, from equation (i), (j) and (k), we get,

$(\frac{\partial x_1}{\partial x_1})_{\bar{M},\bar{p}_2} = (\frac{\partial x_1}{\partial p_1})_{\bar{U},\bar{p}_2} - x_1(\frac{\partial x_1}{\partial M})_{\bar{p}}$, which is the Slutsky's equation.

1.5.6 Compensated Demand Curve

Compensated demand function for a commodity (say x_1) of an individual consumer represents demand quantity for that good (which is purchased by the consumer) as a function of price of that good and prices of other goods under constant real income and constant other things.

Notationaly, it is given by $x_1=x_1(p_1, p_2, y)$, where y is the real income.

Demand curve for a good showing the relationship between demand quantity for that good and its own price given other things and given real income is known as compensated demand curve along which real income is constant (real income is defined by the ratio between money income and price level). Along the demand curve price of that good changes, so money income should be proportionately adjusted or compensated such that real income is constant. That is why the corresponding demand function and demand curve is known as compensated demand function and compensated demand curve.

There are two different approaches to the measurement of real income, viz.,

- **Hicksian Approach:** In Hicksian approach, real income is measured in forms of utility. A constant real income means a constant utility. Thus, demand quantity for a good purchased by a consumer as a function of prices of all goods under constant utility and constant other things is known as compensated Hicksian demand function.

  Demand curve for a commodity showing the relationship between quantity demand for that commodity and it's own price under constant other things and constant real income in terms of utility is known as compensated Hicksian demand curve.

- **Slutsky's Approach:** In this approach, real income is measured in terms of purchasing power. A constant real income means a constant purchasing power (it is denoted by y_p). Demand quantity for a good purchased by a consumer as a function of prices of all goods under constant other things and constant purchasing power is known as compensated Slutsky's demand function and corresponding demand curve is known as compensated Slutsky's demand curve.

  Below we discuss the Hicksian approach graphically.

Effect of a change in price of a good (say p_1) on compensated demand of that good (i.e., on x_1):

Derivation of compensated demand curve

Hicksian compensated demand function for x_1 is given by $x_1=x_1(p_1, p_2, U)$, where Hicksian compensated demand curve for a good represent the relationship between price of that good with its own demand quantity for given prices of other goods and real income in terms of utility.

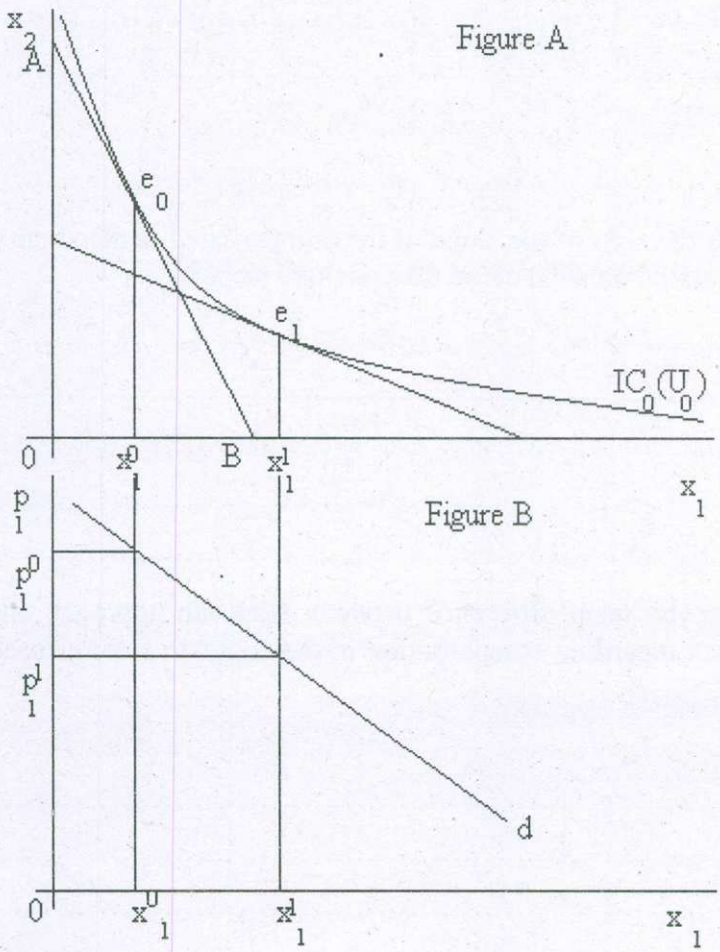

Fig. 1.7A and B

We now derive this graphically. Suppose, initial equilibrium is attained at e_0 in Figure 1.7A where price of good on is p_1^0 and price of good two is p_2^0 respectively and utility is fixed at U_0. Corresponding indifference curve is IC_0. Compensated Hicksian demand for x_1 is at x_1^0. Expenditure line is AB at initial equilibrium with absolute slope p_1^0/p_2^0.

Plot this x_1^0 and p_1^0 in Figure 1.7B. Suppose, for given utility and p_2, p_1 decreases to p_1^1. Therefore, absolute slope of the budget line decreases, i.e., expenditure line become flatter. Since utility is constant, the indifference curve remains the same as before. Therefore, expenditure is minimised for given utility at point e_1 in Figure 1.7A, as indifference curve is downward sloping strictly convex to the origin. So compensated Hicksian demand for good I increases to x_1^1 plot p_1^1 and x_1^1 in Figure 1.7B. By joining all such pair of p_1 and x_1 in Figure B, we have a downward sloping curve in p_1-x_1 plane, for given p_2 and utility. This downward sloping demand curve is the Hicksian compensated demand curve. This is shown in the above Figure 1.7B.

Check Your Progress 3

1) Define compensated demand curve in one sentence. What are measured in the axes of the figure to draw a compensated demand curve in Hicksian approach?

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2) What is the sign of the slope of the compensated demand curve? Can the compensated demand curve take positive slope?

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3) What is the main difference between Hicksian approach and Slutsky's approach regarding compensation in the context of compensated demand curve?

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1.6 LET US SUM UP

In this unit, we discussed various aspect of consumer behaviour theory. We elaborated two classical theories (viz. Cardinal Approach and Ordinal Approach). In ordinal approach discussing the indifference curve theory we show that indifference curve in general is downward sloping and strictly convex to the origin. Consumer equilibrium in ordinal approach was found out both graphically and algebrically. In the ordinal approach at equilibrium two condition must satisfied. The first condition is the equality between slope of the indifference curve and slope of the budget line, which indicates that at equilibrium slope of the budget line must be equal to slope of the indifference curve. The second condition shows that at equilibrium budget constraint must satisfy with equality sign, i.e., consumer spends all her income in consumption. This condition is derived from the assumption of non-satiation of all goods. The income effect and substitution effect have been presented and meanings explained. An income effect is the change in consumer demand due to unit change in income when other things are held constant and substitution effect is the change in consumer demand due to change in prices of any one good, the utility and other things remaining unchanged. In the next section, we discussed the Slutsky's theorem, which is the relationship between price effect, income

effect and substitution effect. It shows that price effect is the sum of substitution effect and income effect. Finally, we discussed the compensated demand curve analysis derived by Hicks.

1.7 KEY WORDS

Axiom : A statement of universally recognized truth.

Budget Line : A graphical depiction of the various combinations of two selected products that a consumer can afford at specified prices for the products given their particular income level.

Cardinal Utility : Utility analysis based on the cardinal measurement of utility which assumes utility to be measurable and additive.

Compensated Demand Curve : A demand curve that ignores the income effect of a price change, only taking into account the substitution effect.

Consumer Equilibrium : The state of balance achieved by an end user of products that refers to the amount of goods and services they can purchase given their present level of income and the current level of prices.

Convex Indifference Curve : Shape that captures the idea of diminishing marginal utility.

Diminishing Marginal Utility : Law of economics stating that as a person increases consumption of a product, while keeping consumption of other products constant, there is a decline in the marginal utility.

Equilibrium : Point where supply equals demand for a product.

Hicksian Approach : Measurement of real income in forms of utility

Hicksian Demand : Correspondence to the demand of a consumer over a bundle of goods that minimises their expenditure while delivering a fixed level of utility.

Income Effect : The change in consumption resulting from a change in real income.

Indifference Curve : A curve on a graph (the axes of which represent quantities of two commodities) linking those combinations of quantities which the consumer regards as of equal value.

A curve showing the combinations of two goods that leave the consumer with the same level of utility.

Inferior Good : A good for which demand decreases when income rises and increases when income falls.

Ordinal Utility : A numerical indicator of preference which characterised by the entities amongst which one may choose are simply placed ordinal scale.

Slutsky Equation : Equation that relates changes in Marshallian (uncompensated) demand to changes in Hicksian (compensated) demand.

Slutsk's Approach : Measurement of real income in terms of purchasing power.

Slutsk's Equation : Equation used to break down a change in demand due to price change into the substitution effect and the income effect.

Substitution Effect : Effect due only to the relative price changes, controlling for the change in real income.

Utility Maximisation : When making a consumption decision, a consumer attempts to get the greatest value possible from expenditure of least amount of money. Her objective is to maximise the total value derived from the available money.

Utility : A measure of preferences over some set of goods and services.

1.7 SOME USEFUL BOOKS

J. Henderson & Richard E. Quandt (2003), *Microeconomic Theory: Mathematical Approach*, Tata McGraw-Hill Publishing Company Limited, New Delhi.

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillian.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York.

1.8 ANSWER OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) See Sub-section 1.3.1

2) At consumer equilibrium $\frac{dU(x)}{dx} - \lambda p_x = 0$ must hold, we have $\frac{dU(x)}{dx} = 1/x$, $\lambda = 5$ and $p_x = 2$, therefore at equilibrium $1/x = 10$ or, $x^* = 1/10$. Second order condition is satisfied because $\frac{\partial^2 U(x)}{\partial x^2} = -1/x^2 < 0$ at equilibrium.

Check Your Progress 2

1) See Sub-section 1.5.1

2) Lagrange function is given by

$L(x_1, x_2) = 10x_1^{0.3}x_2^{0.7} + \lambda (200 - 5x_1 - 2x_2)$

$\frac{dU(x_1, x_2)}{dx_1} = 3x_1^{-0.7} x_2^{0.7}$ and $\frac{dU(x_1, x_2)}{dx_2} = 7x_1^{0.3}x_2^{-0.3}$, therefore

$\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1 / p_2 \Rightarrow 3/7x_1^{-0.4}x_2^{0.4} = 5/2$, or

$x_1 = (6/35)^4 x_2$.

3) Lagrange function is given by

$L(x_1, x_2) = 10x_1^{0.5}x_2^{0.5} + \lambda (100 - 2x_1 - 4x_2)$,

$\frac{dU(x_1, x_2)}{dx_1} = 5x_1^{-0.5} x_2^{0.5}$ and $\frac{dU(x_1, x_2)}{dx_2} = 5x_1^{0.5}x_2^{-0.5}$, therefore

$\frac{dU(x_1, x_2)}{dx_1} / \frac{dU(x_1, x_2)}{dx_2} = p_1 / p_2 \Rightarrow x_1^{-1}x_2 = 1$, or $x_1 = x_2$ ------ (I).

$M = p_1x_1 + p_2x_2 \Rightarrow 100 = 2x_1 + 4x_2$ ---------- (II).

Solving equation (I) and (II), we get $(x_1^*, x_2^*) = (50/3, 50/3)$.

Check Your Progress 3

1) See Sub-section 1.5.6

2) Compensated demand curve is always downward sloping, it can't have positive slope in any occasion.

3) See Sub-section 1.5.6

UNIT 2 THEORY OF DEMAND

Structure

2.0 Objectives

2.1 Introduction

2.2 Preference and Utility

- 2.2.1 Three Building Blocks
- 2.2.2 Basic Axioms
- 2.2.3 Lexicographic Preference Relation
- 2.2.4 Quasi-concavity of Utility Function
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2.3 Indifference Curve and Budget Set

- 2.3.1 Slope and Curvature of Indifference Curve: –Mathematical Derivation
- 2.3.2 Properties of Indifference Curve
- 2.3.3 Properties of Budget Set

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- 2.6.1 Basic Duality Propositions
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2.7 Let Us Sum Up

2.8 Key Words

2.9 Some Useful Books

2.10 Answer or Hints to Check Your Progress

2.11 Exercises and Answers/Hints

2.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the basic axioms of preference relation;
- determine the optimal choice of consumption through two different approaches (Utility Maximisation Problem (UMP) and Expenditure Minimisation Problem (EMP);

- learn to derive four different functions of consumer choice i.e., Indirect utility function, Ordinary Demand functions (through UMP) and Expenditure Function, Compensated demand functions (through EMP) and understand their properties;
- differentiate between the two kinds of individual demand functions (Ordinary demand function and Compensated demand functions);
- undertake the decomposition of price effect into income effect and substitution effect; and
- appreciate the basic duality relations.

2.1 INTRODUCTION

There are different approaches to analyse the way a consumer behaves while making a choice. To understand it, we assume that the consumer is rational. This means the consumer's objective is to maximise her utility by choosing a specific bundle from many that gives her maximum possible satisfaction

In the traditional theory of consumer behaviour, utility is assumed to be measurable numerically (i.e., Cardinal Approach), but in the modern approach we assume utility is measured ordinally (i.e., Ordinal Approach). Under the modern approach, it is assumed that a consumer can rank her preferences in accordance with the order she wants to compare different commodities. In the present unit we will focus on the ordinal approach and build up the theory of demand.

Whenever we try to explain human behaviour in Economics, we need a framework on which our analysis can be based. We use a framework based on two simple principles, viz.,

i) optimisation principle – people try to choose the best possible consumption basket that they can afford to choose; and

ii) equilibrium principle – prices keep on adjusting till the point where the demand of something is exactly equal to the amount that is supplied.

2.2 PREFERENCE AND UTILITY

An individual who reports 'A is preferred to B' ($A \succsim B$) implies she feels better off under situation A than under B. Such preference is based on the *three building blocks:*

2.2.1 Three Building Blocks

1) **Consumption Set (X):** Set of all alternative commodity vectors which she consumer can conceive of. Thus, if $\in R_+^2$, where R_+ is the set of all positive real numbers and the consumer's choice is based on only two commodity vectors. So the consumption set is $x_1,\ x_2$, where $x_1, x_2 \in X$, and $x_1, x_2 \in R_+^2$. The consumption set can be represented in the positive quadrant of the cartesian plane, where x axis and the y axis can be denoted for commodity 1 and commodity 2 respectively. So each point on the positive quadrant of the Cartesian plane denotes a particular commodity bundle, consisting of a specified quantity of commodity 1 and 2 (see Figure 2.1). If $X \in R_+^n$, consumer's choice is based on the n number of commodities. If $\hat{x}$ represents a particular commodity bundle, then $\hat{x} =$

$(\widehat{x_1}, \widehat{x_2}, \widehat{x_3}, \ldots\ldots \widehat{x_n}) \in R_+^n$. Therefore, $\hat{x}$ is the n commodity vector containing a unique combination of each of the n commodities. When we write $\hat{x} \in X$, it represents a point in the n consumption space.

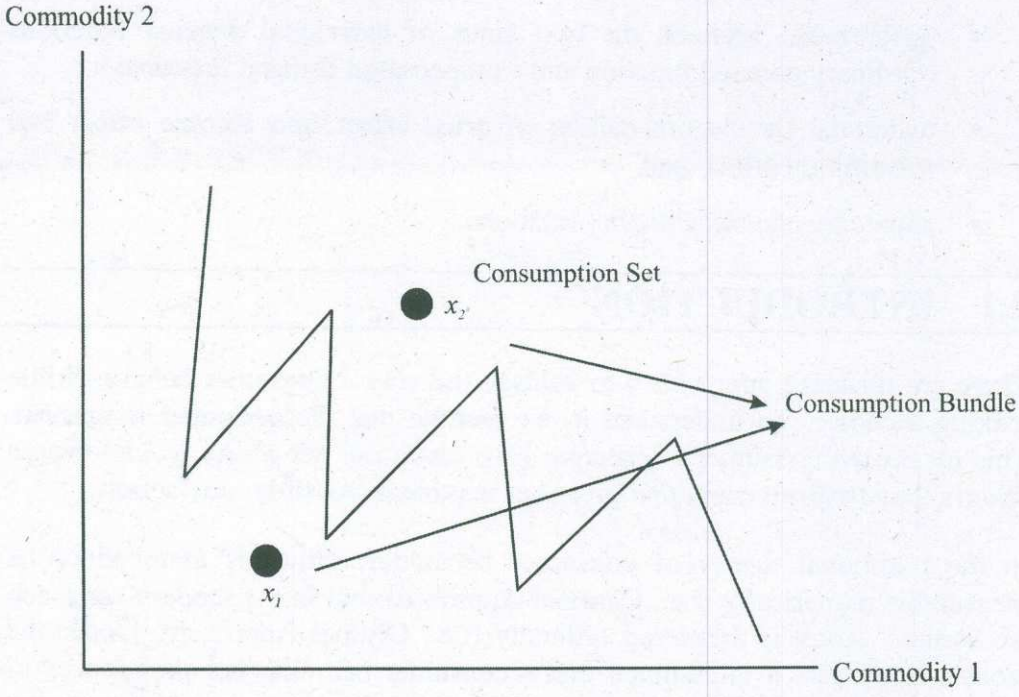

Fig. 2.1: Consumption Set

2) **Feasible Set:** Suppose that a consumer has a fixed amount of money to spend, M. There are two goods X and Y, with associated prices p_x and p_y. The feasible choices that the consumer can make satisfy $p_x x + p_y y \leq M$. That is, the total expenditure must be such that it cannot exceed income. So a feasible set may be some subset of consumption set.

3) **Preference Relation** ($\succsim$): Let us consider $\succsim$ to represent a binary relation defined over the consumption set X. If we consider a two commodity bundle x_1, x_2, such that $x_1, x_2 \in X$, then the relation is basically captured through:

   1) Weak preference: $x_1 \succsim x_2 \Longrightarrow x_1$ is as good as x_2 for that particular consumer which implies $x_1 \succ x_2$ or $x_1 \sim x_2$ (i.e., x_1 is preferred to x_2 or x_1 is indifferent to x_2).

   2) Strict Preference ($\succ$): $x_1 \succ x_2 \Leftrightarrow x_1 \succsim x_2$ and not $x_2 \succsim x_1$. $x_1 \succ x_2$ is read as x_1 is preferred to x_2 or x_1 is strictly preferred to x_2.

   3) **Indifference** ($\sim$): $x_1 \sim x_2 \Longrightarrow x_1 \succsim x_2$ and $x_2 \succsim x_1$.

2.2.2 Basic Axioms

The preference relation ($\succsim$) follows these basic axioms:

i) **Completeness**: A consumer can distinguish between two consumption bundles and express preference. Thus for all pairs of consumption bundles like x_1 and x_2 ($x_1, x_2 \in X$, X is the consumption set) either $x_1 \succsim x_2$ or $x_2 \succsim x_1$, or both.

ii) **Transitivity**: For any three bundles x_1, x_2 and $x_3 \in X$, if $x_1 \succsim x_2$, $x_2 \succsim x_3 \Longrightarrow x_1 \succsim x_3$. Hence although the preference is a binary relation,

the consumer can always rank finite number of consumption bundle in consumption set X from best to worst.

iii) **Reflexive:** Any bundle is at least as good as itself. $x \gtrsim x, \forall$ (for all)$x \in X$.

A binary $' \gtrsim '$ relation on the consumption set X is said to be rational if it satisfies Axiom 1, 2, 3.

iv) **Continuity: Essentially says preferences cannot have jumps.** $\forall x_1, x_2 \in X$, the set $\{x \in X: x_1 \gtrsim x_2\}$ is called upper contour set (UCS) of x. Similarly, $\forall x_1, x_2 \in X$, the set $\{x \in X: x_2 \gtrsim x_1\}$ is called the lower contour set (LCS) of x. For the continuity of preference relation $(\gtrsim)$, UCS and LCS must be closed sets (i.e., the set which includes the boundary points). Thus, $UCS \cap LCS$ is non-empty. Continuity of preference relation guarantees certain drastic reversal of preference does not occur (see Figure 2.2).

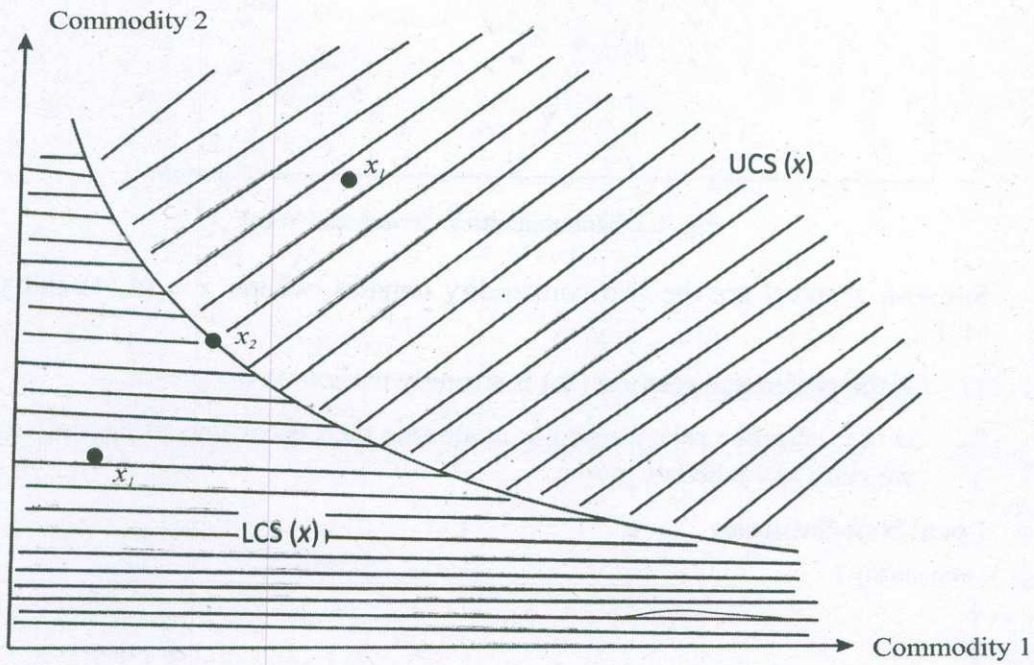

Fig. 2.2: Upper Contour Set and Lower Contour Set

v) **Monotonicity:** If bundle $X_1 = (2,3)$ and bundle $X_2 = (4,5)$, we can represent it mathematically as: $X_2 >> X_1$. This implies that: 4 is preferred to 2 (4>2) and 5 is preferred to 3 (5>3). Generically, it can be represented as if $X = (X_1, X_2)$ and $Y = (Y_1, Y_2)$, $X >> Y \Rightarrow X_1 > Y_1$ and $X_2 > Y_2 \Rightarrow X > Y$. Thus, more is always better.

A preference relation $(\gtrsim)$ on X is **monotone** if for all x,$y \in$ X, and $y \gg x$ and $y \neq x$ (means y and x bundles are distinctively different)that is $\Rightarrow y \succ x$

Note: Since x and y represents two vectors, $y \gg x \Rightarrow (y_1 > x_1 \text{ or}, y_2 > x_2)$

A preference relation $(\gtrsim)$ on X is **strongly monotone** if for all $x, y \in X$, and $y \geq x$ and $y \neq x \Rightarrow y \succ x$

Note: Since x and y represents two vectors, $y \geq x \Rightarrow$

i)$(y_1 > x_1 \text{ or}, y_2 > x_2)$;

ii) $(y_1 > x_1 \text{ or}, y_2 = x_2)$; and

iii) $(y_1 = x_1 \text{ or}, y_2 > x_2)$.

Monotonicity is a more restrictive assumption whereas strong monotonicity is a weaker assumption, which is used to derive more robust results. Thus, strong monotonicity implies monotonicity but not the other way round. Monotonicity of preference relation eliminates the possibility of indifference curves to bend upward (see Figure 2.3).

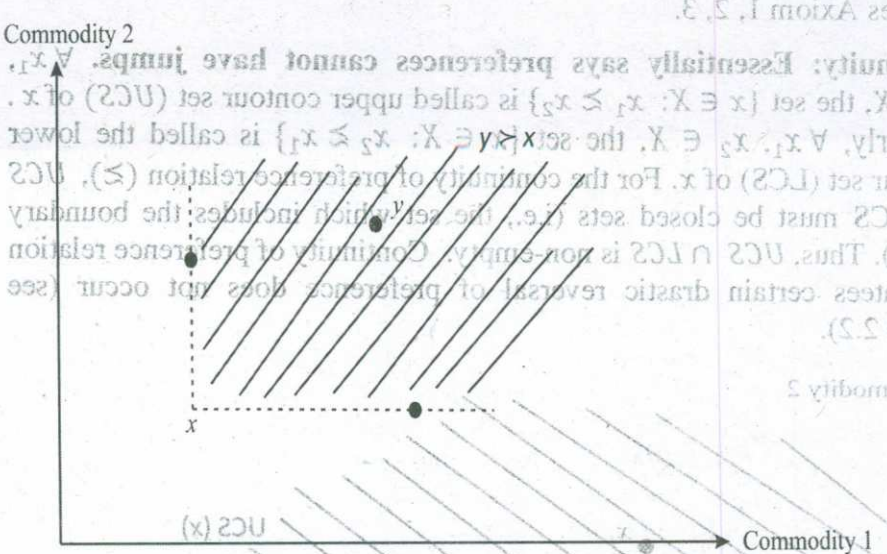

Fig. 2.3: Monotonicities: Strong and Weak

Suppose x and y are the two commodity bundles, where, $x = (4, 4)$ and $y = (4, 8)$.

1) If the preference relation ($\succsim$) is strongly monotone $\Rightarrow y \succ x$.

2) If the reference relation ($\succsim$) is monotone then according to the definition we can't say whether $y \succ x$.

Local Non-Satiation

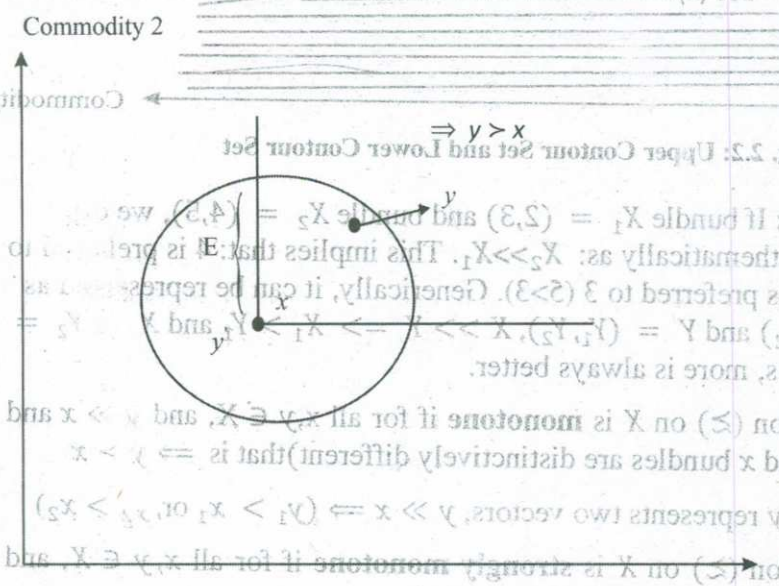

Fig. 2.4: Local Non-Satiation

A preference relation ($\succsim$) satisfies local non-satiation if and only if for every x and every $\varepsilon, \varepsilon > 0, \exists$ a point (consumption bundle) y, such that $\|x - y\| \leq \varepsilon$ *and* $y \succsim x$. Essentially, local non-satiation allows for the fact that some commodities may be bad and in that case consumer would prefer less of them (like garbage or noise). Thus local non-satiation is a weaker assumption (than strong monotonicity and monotonicity) that also allows consumer to prefer less to more (see Figure 2.4).

Non-satiation says that consumers are never fully satisfied. For any bundle x^0 and any $\epsilon > 0$ there exists another bundle $x^1 \in X$ where $|x^0 - x^1| < \varepsilon$ and $x^1 \succ x^0$. This, with continuity, ensures that indifferent sets are indifference curves -they cannot have any "thick" regions to them.

Convexity: Preference Relation ($\succsim$) should be strictly convex. Suppose x, y and z are the three consumption bundles. Further suppose, $x \succsim y, z \succsim y$ and $x \neq z$ (x and z are the two disctinctly different bundles). Then convex combination *of x and* z $\Longrightarrow \lambda x + (1-\lambda)z \succ y$, where, λ is a fraction $0 \leq \lambda \leq 1$. Also, $\hat{x} = \lambda x + (1-\lambda)z$, defines a new bundle and $\hat{x} \succ y$. Convexity of preference relation implies indifference curve should be convex to the origin. (see Figures 2.5a – 2.5c).

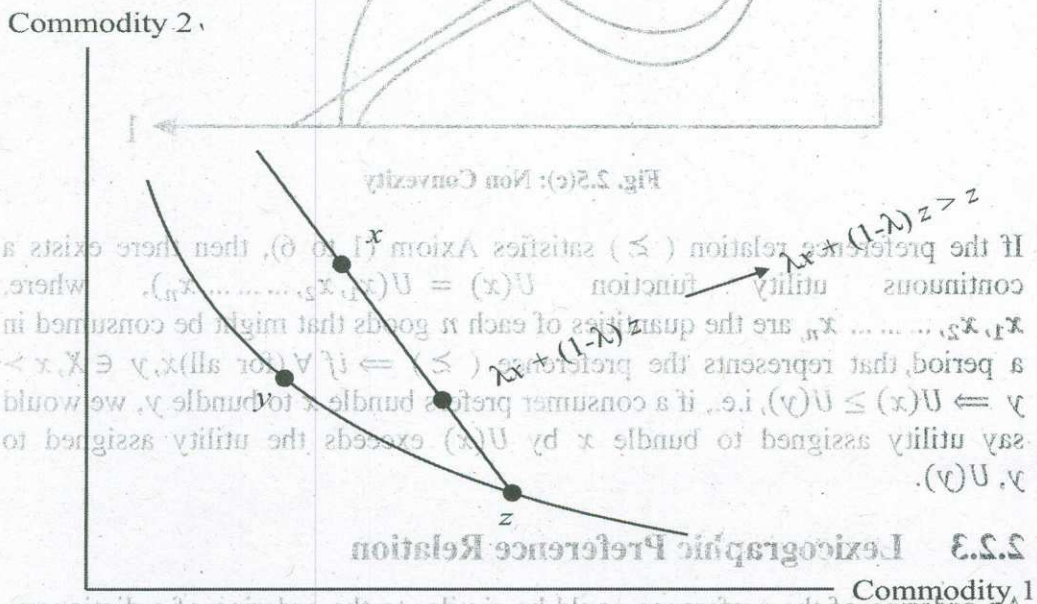

Fig. 2.5: Convexity

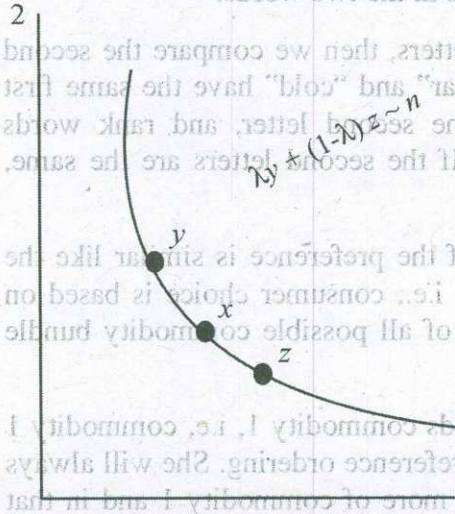

Fig. 2.5(a): Weak Convexity

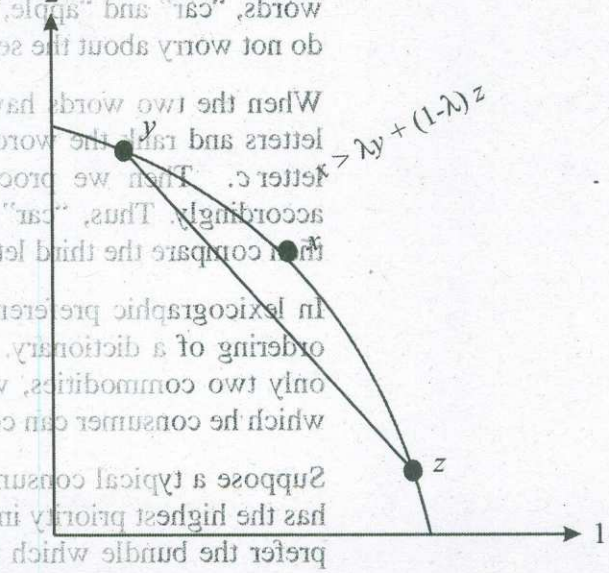

Fig. 2.5(b): Non Convexity

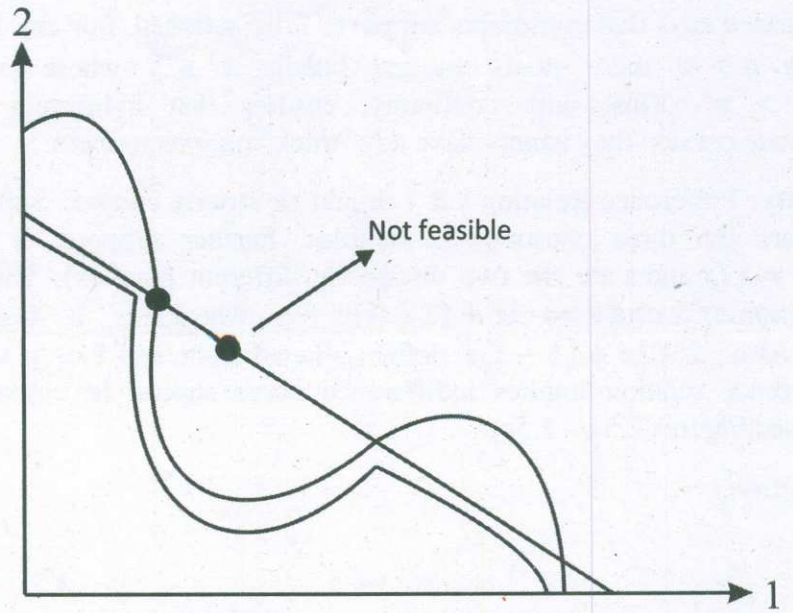

Fig. 2.5(c): Non Convexity

If the preference relation ($\gtrsim$) satisfies Axiom (1 to 6), then there exists a continuous utility function $U(x) = U(x_1, x_2, \ldots\ldots\ldots x_n)$, where, $x_1, x_2, \ldots\ldots\ldots x_n$, are the quantities of each n goods that might be consumed in a period, that represents the preference ($\gtrsim$) $\Longrightarrow if \forall$ (for all)$x, y \in X, x \succ y \Longrightarrow U(x) \geq U(y)$, i.e., if a consumer prefers bundle x to bundle y, we would say utility assigned to bundle x by $U(x)$ exceeds the utility assigned to y, $U(y)$.

2.2.3 Lexicographic Preference Relation

An ordering of the preference could be similar to the ordering of a dictionary. When arranging words alphabetically, we first compare their first letters, not paying attention to all other letters. When comparing two words, we first compare the first letters of two words and rank them. For instance, given two words, "car" and "apple," apple precedes car because a precedes c. In this we do not worry about the second or third letters in the two words.

When the two words have the same first letters, then we compare the second letters and rank the words. For instance, "car" and "cold" have the same first letter c. Then we proceed to compare the second letter, and rank words accordingly. Thus, "car" precedes "cold." If the second letters are the same, then compare the third letters, and so on.

In lexicographic preference, the ordering of the preference is similar like the ordering of a dictionary. Assume$X \in R_+^2$, i.e., consumer choice is based on only two commodities, where X is the set of all possible commodity bundle which he consumer can conceive of.

Suppose a typical consumer is biased towards commodity 1, i.e, commodity 1 has the highest priority in determining the preference ordering. She will always prefer the bundle which would provide her more of commodity 1 and in that case whether she is getting more or less of commodity 2 will not affect her preference. But, if both the bundle, x and y offer her same amount of commodity 1, she will in that case prefer the bundle which offer her more of commodity 2.

Thus define the commodity bundle x is as good as bundle y i.e., $x \gtrsim y \Longrightarrow$ if

1) Either, $x_1 > y_1$ (i.e, bundle x has more of commodity 1 as compared to bundle y

2) $Or, x_1 = y_1$ and $x_2 \geq y_2$

Otherwise, $y \gtrsim x$.

In a stricter sense, $x \succ y \Longrightarrow 1. x_1 > y_1$ or, $2. x_1 = y_1, x_2 > y_2$

Lexicographic preference ordering does not have the utility function. Since the consumer preference exhibits a drastic jump, the preference ordering violates the assumption of continuity as it rejects the principle substitution. With this kind of preference ordering, no two distinct bundles are indifferent (see Figure 2.6).

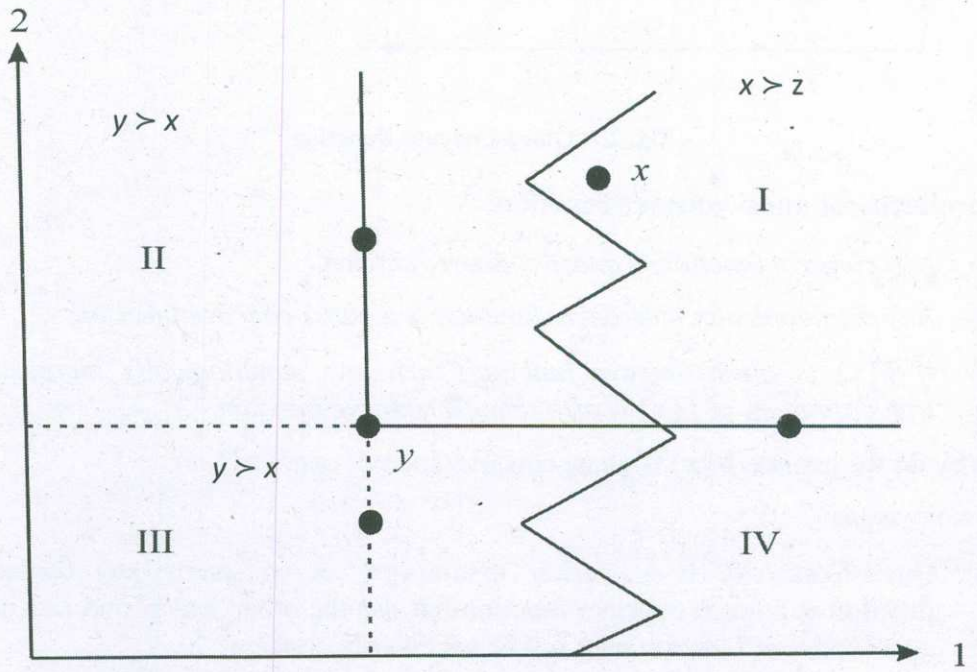

Fig. 2.6: Lexicographic Preference (consumer biased toward commodity 1)

Given the preference is lexicographic and the consumer is biased towards commodity 1, then $x \succ y$, in the Region I and Region IV only. The dark vertical line is the line of compartmentalisation where a minute reduction in commodity 1 in any of the bundles located in the lines, will cause a drastic reversal of preference i.e., $y \succ x$ (in the Region II and III). The dotted vertical line is again a line of compartmentalisation where along the line (and to its left) $y \succ x$, but a tiny change of bundle to the right (implies more of commodity 1) will cause a reversal of preference towards x (i.e., $x \succ y$).

2.2.4 Quasi-Concavity of the Utility Function

For the *uniqueness* of a solution from the utility maximisation exercise we need the property of **strict quasi-concavity** of the utility function. For, if the utility function $U(x)$ quasi-concave then the indifference curves are convex to the origin and if $U(x)$ is strictly quasi-concave, then (x^*) is unique.

A function f(x) is **quasi-concave** (see Fig. 2.7), if for all x' and x'',

$f(\lambda x' + (1-\lambda)x'') \geq \min\{ f(x'), f(x'')\}, \ 0 \leq \lambda \leq 1$ ..... (2.1)

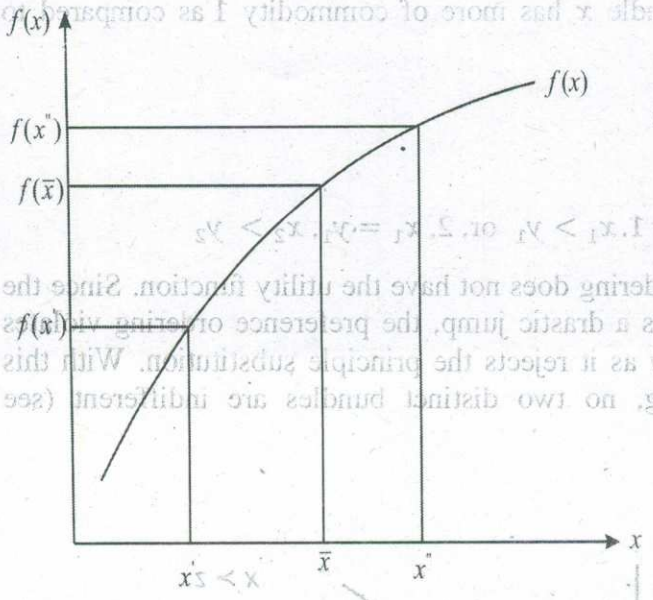

Fig. 2.7: Quasi-Concave Function

Properties of quasi-concave function:

1) *Any concave function is quasi-concave function.*
2) *Any monotonically increasing function is a quasi-concave function.*
3) *If $f(x)$ is quasi-concave function, then any monotonically increasing transformation of $f(x)$ is also a quasi concave function.*

Why do we assume $U(x)$ is quasi concave and not concave?

Two reasons:

i) Quasi-Concavity is a weaker assumption as we know any concave function is a quasi concave function but not the other way round i.e., any quasi concave function may not be necessarily concave.

ii) Quasi-Concavity is an ordinal property, which is invariant to any monotonic transformation.

2.2.5 A Note on Ordinality of Utility Function

We say utility is an ordinal concept because the order of number assigned to reflect the preference of various bundles do matter but not the number itself. The number assigned to represent the utility is just an arbitrary assignment. If we multiply all the number by 2 or add 6 to them, or take a square root, the number assigned to each of the indifference curves after the transformation would still represent the same preference. This has an important implication for demand analysis. To see why, see the following:

1) If $U(x)$ represents the preference ($\succsim$) and f(.) is monotonically increasing function ($\Rightarrow f'(x) \geq 0$ for all x and x is increasing) then $v(x) = f\{u(x)\}$ also represents the same preference ($\succsim$). Suppose the utility function is a Cobb-Douglas function $U(x) = x_1^{\alpha} x_2^{\beta}$. If we take a monotonically increasing transformation of $U(x)$, then $\log(x) = v(x)$ where $v(x) = \log\{x_1^{\alpha} x_2^{\beta}\}$. Thus, $v(x) = \alpha \log x_1 + \beta \log x_2$ also represents the same preference as $U(x)$.

   Because of such an ordinal property of the utility function is monotonically increasing transformation of a utility function that does not

change the equilibrium solutions for a given prices of the commodities and income.

2) Another implication of the ordinal nature of utility function is that the difference between the utility of two bundles does not conveys any meaning. For example, if $U(x) - U(y) = 7$ and $U(z) - U(v) = 14$, where x, y, z, v are the four consumption bundles then it does not mean that going from consuming v to consuming x is twice as much of an improvement than going from y to x.

Check Your Progress 1

1) Write down the basic axioms of preference.

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2) List the three building blocks of preference relation. Consumption set, feasible set and preference relation.

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3) Why is it necessary to assume $U(x)$ to be quasi-concave and not concave?

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4) What is significance of a monotonically increasing of a utility function?

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2.3 INDIFFERENCE CURVE AND BUDGET SET

We have already seen in the previous unit that indifference curve is a locus of all alternative combinations of x and y, for which an individual is equally well off. Its properties are as stated to be downward sloping, non–intersecting, higher indifference curve implying higher utility level and strictly convex to the origin. In the following discussion we will work on these ideas to generate more results on consumer maximisation problem.

2.3.1 Slope and Curvature of Indifference Curve: Mathematical Derivation

Consider the consumer's utility function in a two commodity framework, $U = U(x, y)$. We can write the total differential of this function as,

$$du = \frac{\partial U}{\partial x}x + \frac{\partial U}{\partial y}y \quad \ldots\ldots\ldots (2.2)$$

Along the particular indifference curve, utility is constant so, $du = 0$. A simple manipulation of the equation yields the slope of the indifference curve as

$$\frac{dy}{dx}\Big|_{u\,=\,constant} = -\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = -\frac{MU_x}{MU_y} < 0 \quad \ldots\ldots\ldots (2.3)$$

where, $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ are the marginal utilities of x (MU_x) and y (MU_y) respectively and marginal rate of substitution (MRS) is the ratio of the two marginal utilities (MU_x) and (MU_y) . Since we have assumed that x and y are both goods, so their marginal utilities are positive, and hence the indifference curve is **downward sloping** or slope of the indifference curve is **negative**.

To show the convexity of the indifference curve, we have to see its curvature. That is, the second order derivative of the equation concerned with the indifference curve is positive, i.e., $\frac{d^2y}{dx^2} > 0$.

Since we have $U = U(x, y)$, let $\frac{\partial u}{\partial x} = f_1 = MU_x$ and $\frac{\partial u}{\partial y} = f_1 = MU_y$, such that, $\frac{dy}{dx}|(u = _{\text{constant}}) = -\frac{f_1}{f_2}$. Differentiating equation (2.2) with respect to (w.r.t.) x, we get,

$$\frac{d^2y}{dx^2} = \frac{-f_2\left(f_{11} + f_{12}\frac{dy}{dx}\right) - f_1(f_{21} + f_{22}\frac{dy}{dx})}{{f_2}^2}$$

Since, second order cross partial derivatives are equal by Young's theorem, $f_{12} = f_{21}$. Substituting $\frac{dy}{dx} = -\frac{f_1}{f_2}$, in the above equation, we get

$$\frac{d^2y}{dx^2} = \frac{-f_2\{\left(f_{11} + f_{12}(-\frac{f_1}{f_2})\right)\} - f_1\{(f_{21} + f_{22}(-\frac{f_1}{f_2})\}}{{f_2}^2}$$

$$\frac{d^2y}{dx^2} = \frac{-[(f_{11}{f_2}^2 - 2f_1f_{12}) + f_{22}{f_1}^2]}{{f_2}^3} \quad \ldots\ldots\ldots (2.4)$$

Given that $f_{11} < 0, f_{22} < 0$, because of the diminishing marginal utility and $f_{12} = f_{21} > 0$, along with that we are considering both x and y commodities as goods only (not bads), their marginal utilities are positive ($f_1 > 0$ and $f_2 > 0$), it shows from equation (2.4) that $\frac{d^2y}{dx^2} > 0$. Hence the indifference curve is **convex to the origin**.

As, MRS $= \frac{(MU_x)}{(MU_y)} \Rightarrow MRS = -\frac{dy}{dx}\Big|_{u\,=\,constant}$ or, $dMRS/_{dx} = -\frac{d^2y}{dx^2} < 0$.

Thus, it shows the assumption of diminishing MRS i.e., $dMRS/_{dx} < 0$. If the consumption of x increases (along the indifference curve) the consumer has to substitute y for x (to keep the utility constant along the indifference curve) and that makes y relatively attractive at the margin, such that for each additional unit of x she will be willing to substitute lesser units of y.

2.3.2 Properties of Budget Set

Budget Set is a set of all possible consumption bundles (like x and y) which for a given income (M) and prices of the commodities (p_1, p_2) defines the feasible region of consumption i.e., $B = \{x, y : p_1x + p_2y \leq M\}$, $x_i \geq 0, i = 1,2$. Thus, if AB is the budget line, then diagrammatically the budget set includes the area of the triangle along with the line AB. Any point on the budget set represents a commodity bundle which is feasible for the consumer (i.e., total expenditure for the two commodities is less than equal to the given income). See Figure 2.8.

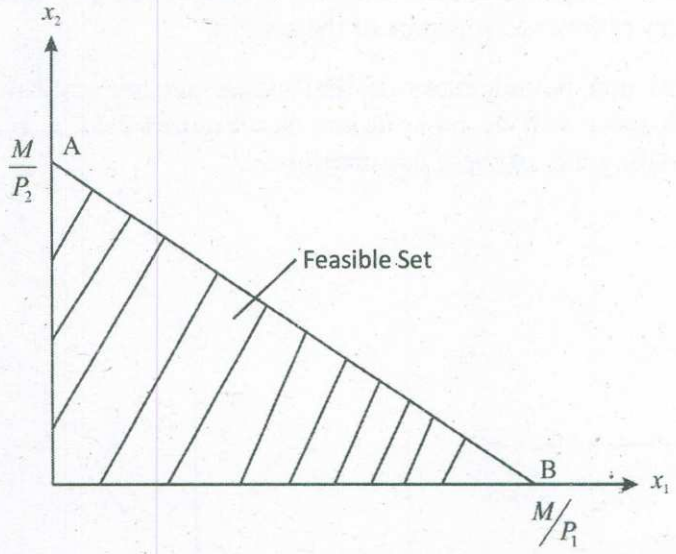

Fig. 2.8: Budget Set

Following are the properties of a budget set:

- **Bounded Set:** Consider the budget set $B = \{x, y : p_1x + p_2y \leq M\}$, $x_i \geq 0, i = 1,2$, and define a upper fixed points $(\bar{x}(\bar{y}))$ and a lower fixed point $(\underline{x}, \underline{y})$, such that for all, x and $y \in B \in R^2$: (i) $\underline{x} \leq x \leq \bar{x}$ and (ii) $\underline{y} \leq y \leq (\bar{y})$. (See Figure 2.9)

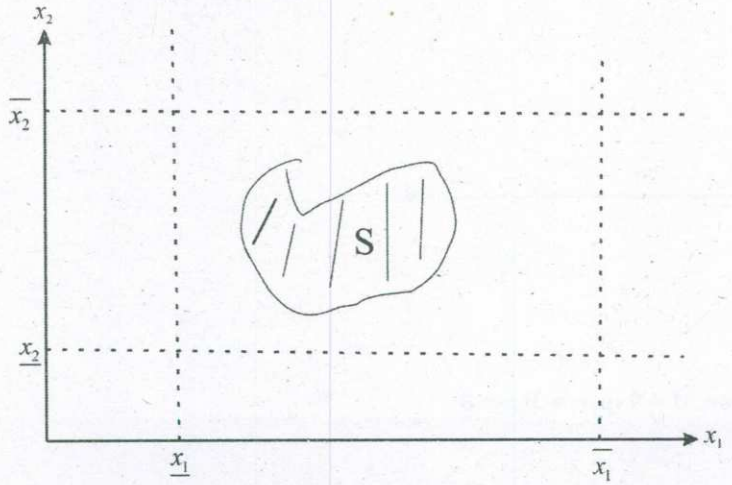

Fig. 2.9: Bounded Set

- **Closed Set** – the budget set B is a closed set since it includes all its boundary points. If we take any point on the boundary of budget set and draw a circle (however small) some points will lie on the set itself and some outside, then those points are called the boundary points. A closed set

is a set which includes all the boundary points. A boundary point is a point such that if a circle is drawn centering that point some points will lie inside the set and some points outside the set (see Figure 2.10).

Fig. 2.10: Closed Set

(You may check on yourself after drawing a budget set. The line AB includes all the boundary points and it is part of the set B.).

The closedness and boundedness of the budget set are critical assumptions, without which there will be no solution. See Figures 2.11 a, b and c for the budget set violating any of these assumption.

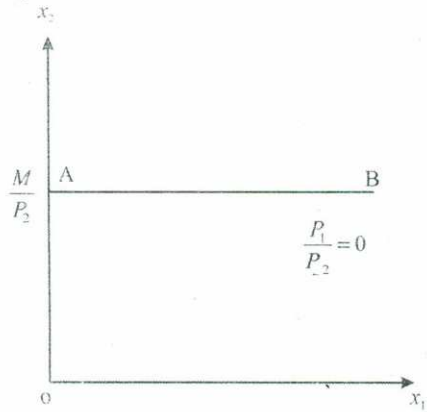

Fig. 2.11a: Budget Line when x_1 is a free goods $p_1 = 0$

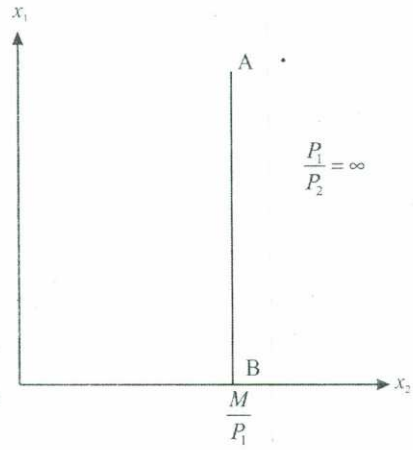

Fig. 2.11b: Budget Line when x_2 is a free goods $p_2 = 0$

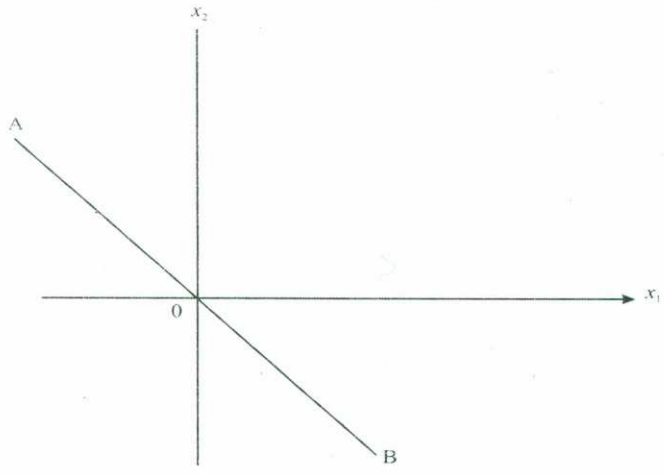

Fig. 2.11c: Budget Line when M = 0 ($p_1x_1 + P_2x_2 = 0$)

Suppose the budget set is not closed. Then B = $\{x,y:p_1x + p_2y < M\}$, $x_i \geq 0$, $i - 1,2$. Let us assume x^* is an interior point represents a bundle, which is an equilibrium solution. Then $(x^* + \varepsilon)$ represents another bundle ($\varepsilon > 0$, but very small) which yields a greater utility (due to strong monotonicity). Thus, x^* can't be equilibrium solution. Since $(x^* + \varepsilon)$ is also not a boundary points (it can't be equilibrium solution either). One can define another interior

bundle $(x^* + \varepsilon + \delta)$ (where δ is negligibly small but positive), which is still a part of the budge set, which gives the consumer higher utility (due to strong monotonicity). Hence equilibrium solution cannot be defined if the boundary points are not included (i.e., B should be a closed set).

If B is not bounded ($i.e.$, if p_1 or $p_2 = 0$), solution cannot be defined.

- **Convex Set** – The budget set is a convex set. If you take any two arbitrary points on the budget set and join them together, the corresponding points on the line will also part of the set. (See Figures 2.12a,b,c).

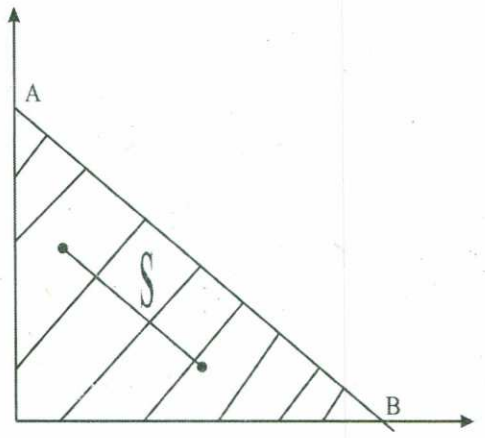

Fig. 2.12a: Convex Set

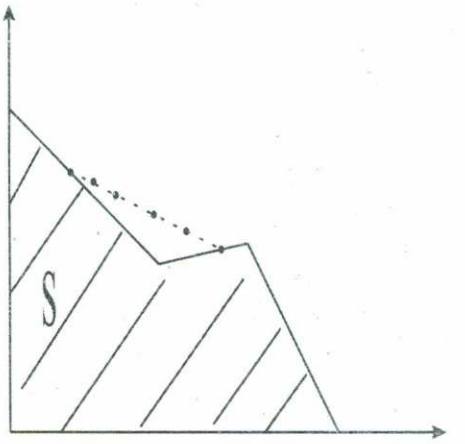

Fig. 2.12b: Non Convex set

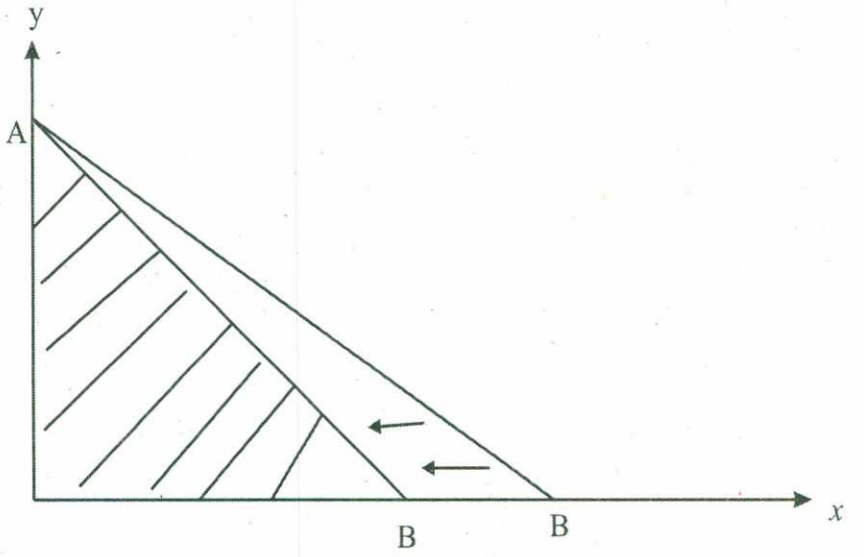

Fig. 2.12c: Convex Set

Note that convexity of the budget set ensures the existence of solution.

Check Your Progress 2

1) It is correct to say, strong monotonicity of preferences implies that if all prices fall and income y stays constant then the consumer achieves higher utility.

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2) Suppose two distinct consumption bundles x^1 and x^2 achieve the exact same maximum utility level for a consumer, given prices and income (p, y). Does it imply that this consumer does not have a strictly quasiconcave utility function?

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3) If a person consumes only two goods then can both of them be inferior goods.

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4) A person with income y facing prices p uses the choice rule: $x_1(p; y) = 0$ and $x_2(p, y) = \frac{y}{p_2}$ to choose consumption bundles in R_+^2. For any $p \in R_{++}^2$ and $y \in R_+$, is this choice rule a valid Marshallian demand of a utility maximising consumer?

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5) Suppose two distinct consumption bundles x^1 and x^2 achieve the exact same maximum utility level for a consumer, given prices and income (p, y). Does it imply that this consumer does not have a strictly quasi-concave utility function?

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2.4 UTILITY MAXIMISATION PROBLEM (UMP)

Let us consider a two commodity framework (x and y) in which the representative consumer is entitled to consume; p_1 *and* p_2 are the prices of the two commodities, exogenously given and M is the given income of the consumer. Thus, the total expenditure of the consumer is $[p_1x + p_2y]$. When she consumes x units and y units of the two commodities, the budget constraint implies that her total expenditure can't exceed her money income. Let the consumer be rational, as implied by well behaved preference relation. Her utility function exists. Let us also assume that $U(x, y)$ be the utility function of the consumer which represents her preference, and which is a quasi concave function so that the uniqueness of the solution is ensured.

2.4.1 Mathematical Derivation of UMP

The problem of the consumer is to maximise utility function $U(x, y)$ subject to the constraint of the budget set. That is: $p_1x + p_2y \leq M$ and non-negativity constraints $x > 0$ and $y > 0$ are satisfied.

We set the Lagrangian expression for the constraint utility maximisation problem. Our objective is to maximise the Lagrangian expression by optimally choosing the$x$, y and λ (λ is the unknown lagrangian multiplier to be determined).

$$\text{Max } \mathcal{L} = Max\{U(x,y) + \lambda[M - p_1x - p_2y]\}, \lambda > 0 \ldots\ldots\ldots (2.5)$$

$$\text{Max } \mathcal{L}_{(\text{w.r.t } x, y \text{ and } \lambda)} = Max[U(x,y) + \lambda[M - p_1x - p_2y]$$

So we differentiate the Lagrangian expression with respect to$x$, y and λ. The first order conditions (FOCs) are :

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial U(.)}{\partial x} - \lambda p_1 = 0 \ldots\ldots\ldots\ldots (2.6)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\partial U(.)}{\partial y} - \lambda p_2 = 0 \ldots\ldots\ldots\ldots (2.7)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = M - p_1x - p_2y = 0 \ldots\ldots\ldots (2.8)$$

From (2.6) and (2.7) we get: $\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{p_1}{p_2} \ldots\ldots\ldots (2.9)$,

where $\frac{\partial U(.)}{\partial x} = (MU_x)$ and $\frac{\partial U(.)}{\partial y} = (MU_y) \Rightarrow \text{MRS}_{x,y} = \frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{(MU_x)}{(MU_y)} =$ $\frac{p_1}{p_2} \Rightarrow \text{MRS}_{x,y}$ =slope of the indifference curve= $\frac{p_1}{p_2}$ =Slope of the budget constraint= law of equimarginal utility. [Note that since the utility function $U(x,y)$ is a function of (x,y), the marginal functions $\frac{\partial U(.)}{\partial x}$ and $\frac{\partial U(.)}{\partial y}$ are also the functions of (x,y)].

Solving equation (2.8) and (2.9) we get the equilibrium values of x, y and λ as a function of (p_1, p_2, M). Thus,

$$x^* = x^*(p_1, p_2, M) \ldots\ldots\ldots (2.10)$$

$$y^* = y^*(p_1, p_2, M) \ldots\ldots\ldots\ldots (2.11)$$

$$\lambda^* = \lambda^*(p_1, p_2, M) \ldots\ldots\ldots (2.12)$$

Equation (2.10) and (2.11) are called the **Marshallian** or the **Walrasian** or the **Ordinary demand functions**. Marshallian or the ordinary demand functions captures the entire **price effect** (which can be decomposed into substitution effect and income effect). For a normal commodity the ordinary demand functions are **negatively sloped** or downward sloping implying the inverse relationship between own price and quantity demanded. So $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} < 0, \frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2} < 0$. But for a **Giffen Good** (which actually exhibits the violation of law of demand, the **price effect is positive** that is demand functions are

positively sloped or upward sloping viz., $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} > 0, \frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2} > 0$ (implying direct relationship between price and quantity demanded).

Plugging the equilibrium values of x^*, y^* in the utility function $U(x,y)$ we get:

$$U = \mathrm{U}(x^*, y^*) = \mathrm{U}(x^*(p_1,p_2,M), y^*(p_1,p_2,M)) = v(p_1,p_2,M) \ldots\ldots\ldots\ldots (2.13)$$

The function $\mathrm{v}(p_1,p_2,M)$ in equation (2.12) is called the **Indirect Utility Function,** which is the maximum utility the consumer can attain at the equilibrium for a given prices and income (p_1,p_2,M). So $\mathrm{v}(p_1,p_2,M)$ is the maximum utility to be obtained after optimisation only.

Interpretation of the Lagrangian Multiplier:

From the given $U = \mathrm{U}(x,y)$, differentiating both sides w.r.t. M

$$\frac{\partial U(.)}{\partial M} = \frac{\partial U(.)}{\partial x}\frac{\partial x}{\partial M} + \frac{\partial U(.)}{\partial y}\frac{\partial y}{\partial M}$$

$$\frac{\partial U(.)}{\partial M} = \lambda p_1 \frac{\partial x}{\partial M} + \lambda p_2 \frac{\partial y}{\partial M} \implies \frac{\partial U(.)}{\partial M} = \lambda\left[p_1 \frac{\partial x}{\partial M} + p_2 \frac{\partial y}{\partial M}\right] \implies \frac{\partial U(.)}{\partial M} = \lambda\,[1]$$

[Note that from the FOCs (equations (2.5) and (2.6) we get $\frac{\partial U(.)}{\partial x} = \lambda p_1$ and $\frac{\partial U(.)}{\partial y} = \lambda p_2$. Again differentiating the budget constraint $[p_1x + p_2y = M]$, w.r.t. M we get: $[p_1 \frac{\partial x}{\partial M} + p_2 \frac{\partial y}{\partial M}] = 1]$

Thus, the Lagrangian multiplier λ can be interpreted as the **marginal utility of income**.

From the budget equation $[p_1x + p_2y = M]$, we get the important condition

$[p_1 \frac{\partial x}{\partial M} + p_2 \frac{\partial y}{\partial M}] = 1$. This condition is called the **Engel Aggregation**. So with some minor manipulation we get

$$p_1 \frac{\partial x}{\partial M}\frac{M}{x}\frac{x}{M} + p_2 \frac{\partial y}{\partial M}\frac{M}{y}\frac{y}{M} = 1 \implies \left(\frac{\partial x}{\partial M}\frac{M}{x}\right)\frac{p_1x}{M} + \left(\frac{\partial y}{\partial M}\frac{M}{y}\right)\frac{p_2y}{M} = 1$$

$$\implies \eta_1 S_1 + \eta_2 S_2 = 1$$

where, $\left(\frac{\partial x}{\partial M}\frac{M}{x}\right) = \eta_1$ = Income elasticity of demand for x, $\left(\frac{\partial y}{\partial M}\frac{M}{y}\right) = \eta_2$= Income elasticity of demand for y. $\frac{p_1x}{M} = S_1$ = Share of the expenditure on commodity x, $\frac{p_2y}{M} = S_2$ = Share of the expenditure on commodity y.

Thus, the weighted average of the income elasticity of demand for all commodities is unity, where the weights are the respective proportion of income spent on commodities.

2.4.2 Diagramatic Representation of UMP

Let us now explain the UMP diagrammatically. Here the consumer's objective is to maximise the utility. So a typical consumer will always want to achieve the highest possible indifference curve (because by strong monotonicity more is always better, i.e., higher is the consumption higher is the utility). However she cannot do so as she is constraint by the size of her budget. Consider the Figure 2.13. Let AB is the budget line. The triangle OAB captures the budget set, which defines feasible region of consumption, as given by the budget constraint and non negativity constraints. The indifference map is given by the level curves U_0, U_1, U_2, U_3 and so on. Let us consider the indifference curve

U_0. There are many points along the indifference curve for which $p_1x + p_2y < M$, where the consumer has an incentive to spend more and increase consumption and hence attain higher utility. There are two points at C_1 and C_2 points, where $p_1x + p_2y = M$. However neither C_1 and C_2 is equilibrium. At C_1 we see slope of the indifference curve is steeper than the slope of the budget constraint i.e., $\text{MRS}_{x,y} = ((\partial U(.))/\partial x)/((\partial U(.))/\partial y) > \frac{p_1}{p_2} \Rightarrow \frac{(\partial U(.))/\partial x)}{p_1} > \frac{(\partial U(.))/\partial y)}{p_2}$.

That is marginal utility of per rupee spent on commodity x is greater that the marginal utility of per rupee spent on commodity y. So a rational consumer at the point C_1 will increase her purchase of x in order to increase her utility. Similarly you can check at C_2, exactly the opposite happen i.e., $\frac{(\partial U(.))/\partial x)}{p_1} < \frac{(\partial U(.))/\partial y)}{p_2}$ and is just the reverse of the earlier result. So both C_1 and C_2 are not the equilibrium. Now let us consider the next higher indifference curve U_1 which is just tangent to the budget constraint AB at the point ε_0. Thus at ε_0, $\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{p_1}{p_2} \Rightarrow \frac{(\partial U(.))/\partial x)}{p_1} = \frac{(\partial U(.))/\partial y)}{p_2}$. If one rupee is given to a consumer and is asked to consume commodity x or y then marginal utility for per rupee spent on commodity x is exactly equal to the marginal utility for per rupee spent on commodity y. That means E is the point of equilibrium where the law of equi-marginal utility holds and the rational consumer has no incentive to deviate from the point E and at the equilibrium

i) The budget constraint holds with equality.

ii) The slope of the indifference curve is equal to the slope of the budget constraint.

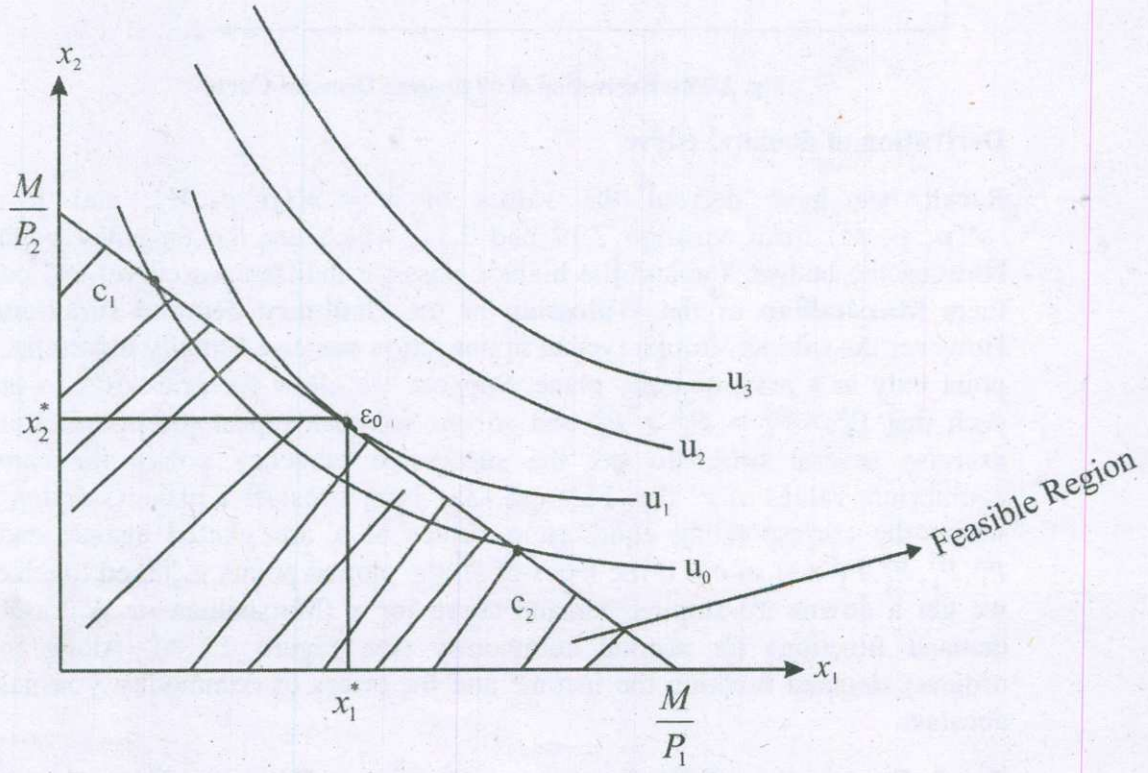

Fig. 2.13a: Price Effect for Commodity

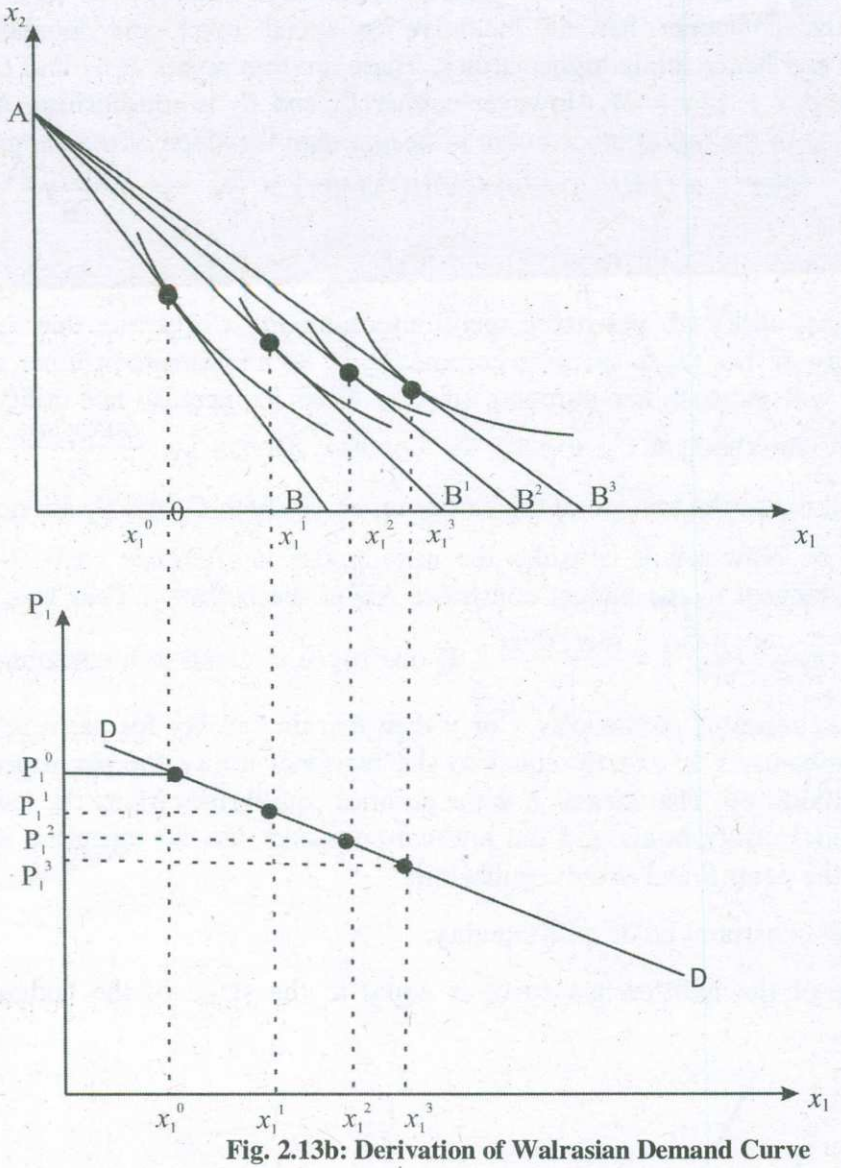

Fig. 2.13b: Derivation of Walrasian Demand Curve

Derivation of demand curve

Recall, we have derived the values of $x^* = x^*(p_1, p_2, M)$, and $y^* = y^*(p_1, p_2, M)$ from equation 2.10 and 2.11, which are the tangency points between the budget line and the highest possible indifference curve, and call them **Marshallian** or the **Walrasian** or the **Ordinary demand functions**. However the solution from a typical optimisation exercise actually represents a point only in a price-quantity plane. Suppose we allow the price of x to fall such that $P_1^o > P_1^1 > P_1^2 > P_1^3$ and so on. We then repeat the optimisation exercise several times to get the successive tangency points for some equilibrium values of x^* (see Figure 2.13a). Now consider a plane (x and p_1), where the corresponding equilibrium values of x are plotted against each $P_1^o, P_1^1, P_1^2, P_1^3$ and so on. If the locus of all the plotted points is joined together we get a downward sloping demand curve for x (Marshallian or Walrasian demand function) for normal commodity (see Figure 2.13b). Along the ordinary demand function the income and the prices of commodity y remain constant.

2.4.3 Properties of Walrasian or Ordinary Demand Function

1) The Walrasian (or Ordinary) demand functions are **homogenous function of degree zero** in (p_1, p_2, M).

If the demand function $x^*(p_1, p_2, M)$ *is* homogenous function of degree zero in prices and income (p_1, p_2, M) then if the $p_1, p_2,$ *and* M are allowed to increase in the same direction and same proportion, there will be no change in the equilibrium quantity demanded.

Given the equilibrium demand is $x^* = x^*(p_1, p_2, M)$, let the $(p_1, p_2, \text{and } M)$ are all increased by λ proportion $(\lambda > 0)$, then the homogeneity of degree zero of demand function implies $x\text{^}*(\lambda p_1, \lambda p_2, M) = x\text{^}*(p_1, p_2, M), \forall\, p_1, p_2, M \text{ and } \lambda$.

The reason being that the changes in (p_1, p_2, M) will have no impact on the slope of the indifference curve at the initial point of equilibrium (which is just $\text{MRS}_{x,y} = \frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{(MU_x)}{(MU_y)}$). The slope of the budget constraint which is now $\Rightarrow \frac{\lambda p_1}{\lambda p_2} = \frac{p_1}{p_2}$. So there is no change in the slope of the budget constraint at the initial point of equilibrium. So the FOCs of UMP will not change. So the equilibrium condition $(\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{p_1}{p_2})$ remain unchanged even though (p_1, p_2, M) changes by λ proportion.

From the homogeneity property of Walrasian demand function we get an important condition. Consider the Walrasian demand function $x = x(p_1, p_2, M)$, since it is homogenous of degree zero in all prices and income, we can use Euler's Theorem:

$$\frac{\partial x(p_1,p_2,M)}{\partial p_1} p_1 + \frac{\partial x(p_1,p_2,M)}{\partial p_2} p_2 + \frac{\partial x(p_1,p_2,M)}{\partial M} M = 0.\, x(p_1, p_2, M) = 0$$

Dividing both sides by $x(p_1, p_2, M)$ we get:

$$\frac{\partial x(p_1,p_2,M)}{\partial p_1} \frac{p_1}{x(p_1,p_2,M)} + \frac{\partial x(p_1,p_2,M)}{\partial p_2} \frac{p_2}{x(p_1,p_2,M)} + \frac{\partial x(p_1,p_2,M)}{\partial M} \frac{M}{x(p_1,p_2,M)} = 0$$

$\Rightarrow \varepsilon_{11} + \varepsilon_{12} + \eta_1 = 0$, where ε_{11} = Own price elasticity of demand, ε_{12} = Cross price elasticity of demand and η_1 = Income elasticity of demand.

Similarly, for $y = y(p_1, p_2, M) \Rightarrow \varepsilon_{22} + \varepsilon_{21} + \eta_2 = 0$

So the sum of the own price elasticity of demand, cross price elasticity of demand and income elasticity of demand is always equal to zero, which follows from the homogeneity property of the ordinary demand functions.

2) The demand functions $(x^* = x^*(p_1, p_2, M), y^* = y^*(p_1, p_2, M))$ follow **Walras Law** that is, $p_1 x^*(p_1, p_2, M) + p_2 y^*(p_1, p_2, M) \equiv \text{M}$, $\forall\, p_1, p_2 \text{ and } M$ which follows from the strong monotonicity. Recall the definition that a preference relation $(\gtrsim)$ on X is **strongly monotone** if for all $x, y \in \text{X}$, and $z \geq x$ with $z \neq x \Rightarrow y \succ x$. The only way y to be most preferred bundle is that any neighbouring point which is better than the optimal bundle y, is not in the feasible region (or in the budget set). But that can only happen if and only if $p_1 x^*(p_1, p_2, M) + p_2 y^*(p_1, p_2, M) \equiv \text{M}$.

2.4.4 Properties of Indirect Utility Function

Note that the indirect utility function is defined as the *maximum* utility that can be attained given money income and goods prices. Such a function says how

much utility consumers are getting when they face prices (p_1, p_2) and have income M. From the utility function we have learnt that utility depends directly upon the quantities of x and y the consumer chooses. The quantity demanded however is determined by the income and prices of the commodities which is given in the indirect utility function. The properties of the indirect utility function are as follows:

1) $v(p_1, p_2, M) = \mathrm{U}\big(x^*(p_1, p_2, M), y^*(p_1, p_2, M)\big)$ is a continuous function.

   Take the indirect utility function $v(p_1, p_2, M)$ and find that it is continuous function in P and M. That is, small change in p_1, p_2 *and* M will cause a small change in utility. It follows from the fact the utility function $\mathrm{U}(x^*, y^*)$ and ordinary demand functions $(x^*(p_1, p_2, M), y^*(p_1, p_2, M))$ are continuous functions and preferences are convex.

2) $v(p_1, p_2, M)$ is decreasing in prices and increasing in income M.

   i) take $v(p_1, p_2, M)$ is decreasing in prices (p_1, p_2). Since an increase in p_i (with constant M), shrinks the feasible region (as $\frac{M}{p_i}$ falls, there is a fall in the intercept value of the budget line). Further, the budget line will shrink inside the initial budget line. Since the optimal bundle of consumption will now be on the new budget line, any interior equilibrium that is optimal will lie on the lower indifference curve. Thus, $v(p_1, p_2, M)$ will fall as seen in Figure 2.

   ii) Similar argument holds for $v(p_1, p_2, M)$ is increasing in prices (p_1, p_2). Here an increase in M (with constant prices (p_1, p_2)), will lead to an expansion of the feasible region, that is $\frac{M}{p_i}$ rises and there will be a parallel outward shift of the budget line. So for any interior equilibrium the optimal point will lie on the higher indifference curve. Consequently, $v(p_1, p_2, M)$ will increase.

3) $v(p_1, p_2, M)$ is **homogenous function in degree zero** in p_1, p_2 *and* M

   Proof: Let p_1, p_2 *and* M are increased by λ proportion $(\lambda > 0)$. Then $v(p_1, p_2, M)$ is homogenous function in degree zero implies $v(\lambda p_1, \lambda p_2, \lambda M) = \lambda^0 v(p_1, p_2, M) = v(p_1, p_2, M)$, $\forall\, \lambda > 0$ which follows from the homogeneity of degree zero of the demand functions (shown earlier) i.e.,

   $x^*(\lambda p_1, \lambda p_2, \lambda M) = x^*(p_1, p_2, M)$, $y^*(\lambda p_1, \lambda p_2, \lambda M) = y^*(p_1, p_2, M)$.
   Thus,

   $v(\lambda p_1, \lambda p_2, \lambda M) = U(x^*(\lambda p_1, \lambda p_2, \lambda M), y^*(\lambda p_1, \lambda p_2, \lambda M)) =$
   $U(x^*(p_1, p_2, M), y^*(p_1, p_2, M)) = v(p_1, p_2, M)$.

   In other words, since the optimal bundles consumed do not changed due to the change with the scaling up of all prices and income $(p_1, p_2$ *and* $M)$ by the same proportion λ. The utility at the optimum remains unchanged.

4) **Roy's Identity**: Roy's identity provides a way of obtaining a demand function from an indirect utility function. Differentiating each argument of the indirect utility function $v(p_1, p_2, M)$ with respect to p_i and M, we get

   i) $\dfrac{\frac{\partial v(p_1, p_2, M)}{\partial p_1}}{\frac{\partial v(p_1, p_2, M)}{\partial M}} = -\, x^*(p_1, p_2, M)$ ..... (2.14)

ii) $\frac{\frac{\partial v(p_1,p_2,M)}{\partial p_2}}{\frac{\partial v(p_1,p_2,M)}{\partial M}} = -y^*(p_1,p_2,M)$ ..... (2.15)

From Duality (see the Section 2.6) we know $U^* = v\{(p_1,p_2,e(p_1,p_2,U^*)\}$,

Now differentiating both sides of the above equation w.r.t. p_1 and get

$$0 = \frac{\partial v(p_1,p_2,M)}{\partial p_1} + \frac{\partial v(p_1,p_2,M)}{\partial M}\frac{\partial e(p_1,p_2,U^*)}{\partial p_1}$$

$$0 = \frac{\partial v(p_1,p_2,M)}{\partial p_1} + \frac{\partial v(p_1,p_2,M)}{\partial M} h_1(p_1,p_2,U^*)$$

$$0 = \frac{\partial v(p_1,p_2,M)}{\partial p_1} + \frac{\partial v(p_1,p_2,M)}{\partial M} x^*(p_1,p_2,M)$$

$$-x^*(p_1,p_2,M) = \frac{\frac{\partial v(p_1,p_2,M)}{\partial p_1}}{\frac{\partial v(p_1,p_2,M)}{\partial M}}$$

[By *Shephard's Lemma*: We can show $\frac{\partial e(p_1,p_2,U^*)}{\partial p_1} = h_1(p_1,p_2,U^*)$, (see Section 2.6), where $h_1(.)$ is the compensated demand function for commodity 1, (to be done in the next section under EMP)]. Again by duality relation $h_1(p_1,p_2,U^*) = x^*(p_1,p_2,M)$

Check Your Progress 3

1) What is homogenous function of degree zero?

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.....

2) Suppose that a consumer's preferences are given by $U = \ln x_1 + x_2$, and his budget constraint is given by $p_1x_1 + p_2x_2 = m$ where x_1 and x_2 are goods, p_1 is the price of x_1, and m is the consumer's income. Find the demand functions for x_1 and x_2. Is x_1 an ordinary good? Why or why not?

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3) If a consumer purchases a combination of commodities x and y such that MUx/Px = 20 and MUy/Py = 10, what would consumer do to maximise utility?

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4) If a consumer has utility function $(x_1, x_2) = \frac{1}{\frac{1}{x_1}+\frac{4}{x_2}}$, what would be consumer's Marshallian demand function for each good.

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5) If a consumer has utility function $u(x_1, x_2) = \frac{1}{\frac{1}{x_1}+\frac{4}{x_2}}$, find consumer's indirect utility function.

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2.5 EXPENDITURE MINIMISATION PROBLEM (EMP)

Unlike the earlier optimisation exercise of UMP, here the problem of the consumer is to minimise the total expenditure (for two-commodity case of x and y) subject to the attainment of at least a stipulated level of utility. Hence the expenditure minimisation exercise aims to find the bundle that minimises the expenditure of achieving the stipulated utility $\overline{U}$, for the given prices $p_1 and\ p_2$.

2.5.1 Mathematical Derivation of EMP

The expenditure minimisation problem is:

Min $\mathcal{E} = p_1x + p_2y$, subject to the utility constraint $\mathrm{U}(x, y) \geq \overline{U}$.

Note that although the above is a minimisation problem with an inequality constraint, by the property of Hicksian demand function we rule out any equilibrium solution, for which $\mathrm{U}(x, y) > \overline{U}$ (see in details the justification added in Section 2.5.3). So EMP will be solved by the lagrangian technique with equality constraint (rather than the standard technique like Kuhn Tucker condition).

We assume that the preference relation is well behaved, continuous and quasi-concave, so that the utility function exists and the solution it offers is unique. Like the UMP we will construct the Lagrange for the minimisation exercise and set the first order condition with respect to x, y and μ equal to zero to solve for the compensated or the Hicksian demand functions. Thus,

$\mathcal{L} = p_1x + p_2y + \mu\left(\overline{U} - \mathrm{U}(x, y)\right)$, where μ is the unknown lagrangian multiplier, $\mu > 0$.

Min $\mathcal{L}$(w.r.t. x, y, μ) $= Min\ [p_1x + p_2y + \mu\left(\overline{U} - \mathrm{U}(x, y)\right)]$ ..... (2.16)

The first order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial x} = p_1 - \mu \frac{\partial U(.)}{\partial x} = 0 \ldots\ldots\ldots\ldots (2.17)$$

$$\frac{\partial \mathcal{L}}{\partial y} = p_2 - \mu \frac{\partial U(.)}{\partial y} = 0 \ldots\ldots\ldots\ldots (2.18)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \left(\bar{U} - \mathrm{U}(x, y)\right) = 0 \ldots\ldots\ldots\ldots (2.19)$$

From (2.17) and (2.18) we get, $\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{p_1}{p_2}$ ..... (2.20),

Again from (2.20) we get the same equilibrium condition (we got in the UMP), which satisfies the law of equi-marginal utility i.e.,

$\mathrm{MRS}_{x,y} = \frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{(MU_x)}{(MU_y)} = \frac{p_1}{p_2} \Rightarrow \mathrm{MRS}_{x,y}$ =slope of the indifference curve= $\frac{p_1}{p_2}$ =slope of the budget constraint. [As already stated the utility function $U(x, y)$ is a function of (x,y), the marginal functions $\frac{\partial U(.)}{\partial x}$ and $\frac{\partial U(.)}{\partial y}$ are also the functions of (x,y)].

Solving equations (2.19) and (2.20) we get the equilibrium values of x, y and μ as a function of $(p_1, p_2, \bar{U})$. With these equations we can solve for another category of demand function, called the Hicksian or the compensated demand function. Thus, from equations (2.19) and (2.20) we get the equilibrium values of x, y and λ as a function of $(p_1, p_2, \bar{U})$Thus, giving

$x^* = h_1^*(p_1, p_2, \bar{U})$..... (2.21)

$y^* = h_2^*(p_1, p_2, \bar{U})$ ..... (2.22)

$\mu^* = \mu^*(p_1, p_2, \bar{U})$ ..... (2.23)

Equations (2.21) and (2.22) are called the **Hicksian or the compensated demand functions**. If the utility function $U(x, y)$ is strictly quasi-concave then the optimal solutions $h_1^*$and h_2^* are unique. Hicksian or the compensated demand functions captures the substitution effect and it is always negative (i.e., the Hicksian demand functions are always downward sloping irrespective of the nature of commodity i.e., whether a normal commodity or a Giffen commodity) $\Rightarrow$ the slope is negative $\Rightarrow$

$$\frac{\partial h_1((p_1, p_2, \bar{U})}{\partial p_1} < 0, \frac{\partial h_2((p_1, p_2, \bar{U})}{\partial p_2} < 0$$

Plugging the equilibrium values of x^*, y^* in the expenditure equation

$\mathcal{E} = p_1 x + p_2 y$ we get : $\mathcal{E} = p_1 x^* + p_2 y^* = p_1 h_1^*(p_1, p_2, \bar{U}) + p_2 h_2^*(p_1, p_2, \bar{U}) = e(p_1, p_2, \bar{U})$..... (2.24)

$e(p_1, p_2, \bar{U})$ is called the **expenditure function**. $e(p_1, p_2, \bar{U})$ is the minimum expenditure in equilibrium that supports the $\bar{U}$ level of utility at the given prices (p_1, p_2).

2.5.2 Diagramatic Representation of EMP

Let us now explain the EMP diagrammatically (see Fig. 2.14). Here the consumer's objective is to minimise the expenditure. Let us have a family of iso-expenditure line represented by B_1, B_2, B_3 and so on. So a typical consumer will always want to achieve the lowest possible iso-expenditure line to minimise her expenditure. However she cannot do so as she is constrained by the utility constraint $(\overline{U})$, i.e., she has to attain at least $(\overline{U})$ level of utility while minimising her expenditure. The feasible region of consumption for the EMP thus consists of points above the indifference curve $(\overline{U})$including those along the curve.

Let us consider the iso-expenditure line represented by B_1. Although B_1minimises the expenditure the consumer cannot attain the optimum at B_1 because here the constraint is not satisfied as (B_1 lies below the stipulated indifference curve $(\overline{U})$). For minimisation we have to consider either B_2 *or* B_3, which lies in the feasible region, therefore satisfies the utility constraint $(\overline{U})$.

Let us consider the two points at C and D, where B_3 intersects the $(\overline{U})$. However neither C nor D is equilibrium. At C slope of the indifference curve is steeper than that of the iso-expenditure line, i.e., $\text{MRS}_{x,y}=\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} > \frac{p_1}{p_2}$

$\Rightarrow \frac{(\partial U(.))/\partial x)}{p_1} > \frac{(\partial U(.))/\partial y)}{p_2}$ $\Rightarrow$Marginal Utility for per rupee spent on commodity x is greater that the marginal utility for per rupee spent on commodity y. So a rational consumer at the point C will increase her purchase of x in order to increase her utility. Similarly, you can check at D, exactly the opposite happensi.e., $\frac{(\partial U(.))/\partial x)}{p_1} < \frac{(\partial U(.))/\partial y)}{p_2}$ and is just the reverse of the earlier result. So neither C not D is the equilibrium. Let us consider the next lower iso-expenditure line B_2, which is just tangent to the indifference curve $(\overline{U})$ at the point E. At E, $\frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{p_1}{p_2}$

$\Rightarrow \frac{(\partial U(.))/\partial x)}{p_1} = \frac{(\partial U(.))/\partial y)}{p_2}$. Thus, if one rupee is given to a consumer and is asked to consume commodity x or y the marginal Utility for per rupee spent on commodity x is exactly equal to the marginal utility per rupee spent on commodity y. That gives E as the point of equilibrium where the law of equi-marginal utility holds and the rational consumer has no incentive to deviate from the point E. At the equilibrium:

i) the utility constraint holds with equality.

ii) The slope of the indifference curve is equal to the slope of the iso-expenditure line.

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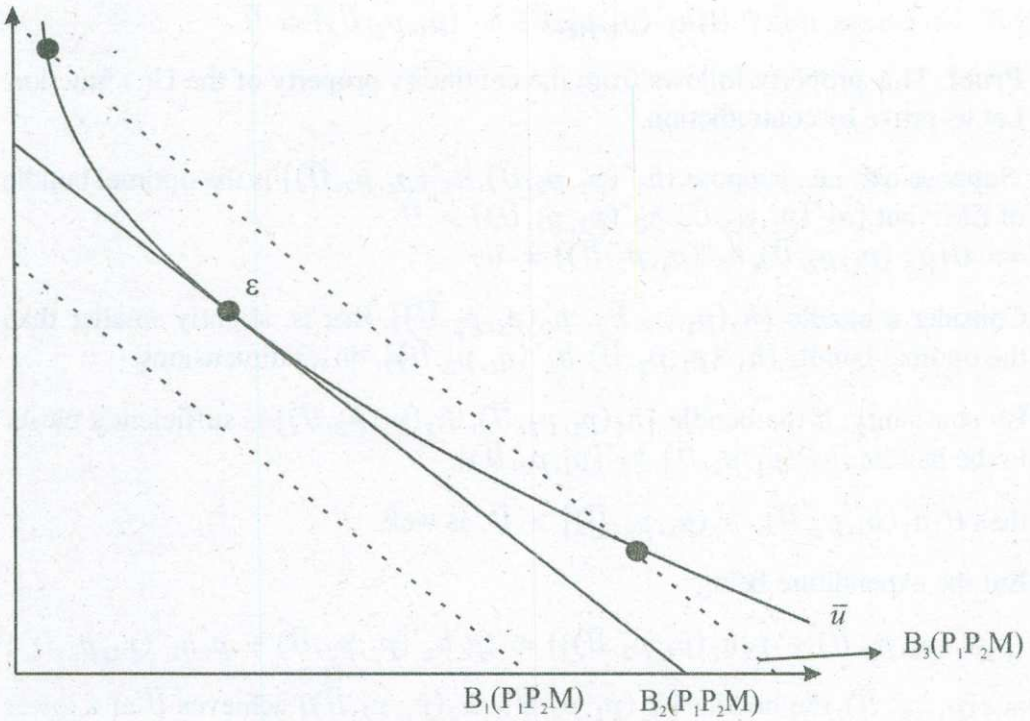

Fig. 2.14: Equilibrium under EMP

2.5.3 Properties of Hicksian Demand Function

Following are the properties of Hicksian demand function:

i) $h_i^*(p_1, p_2, \bar{U})$is continuous function in $p_1, p_2, and\ \bar{U}$.

Since the budget set is closed, bounded and convex set and $U(x, y)$ is continuous and quasi concave, the Hicksian demand functions $h_i^*(p_1, p_2, \bar{U}), i = 1, 2$ are continuous functions in their arguments $(p_1, p_2, and\ \bar{U})$.

ii) $h_i^*(p_1, p_2, \bar{U})$ is a homogenous function of degree zero in $(\boldsymbol{p_1, p_2})$

Proof: Homogeneity of degree zero of the Hicksian demand function means $h_i^*(\lambda p_1, \lambda p_2, \bar{U}) = \lambda^0 h_i^*(p_1, p_2, \bar{U}) = h_i^*(p_1, p_2, \bar{U}), \forall\ \lambda > 0.$

Suppose all the prices (p_1, p_2) are increased in the same direction and with the same proportion by $\lambda, \lambda > 0$. In EMP, as we have seen that the equilibrium is defined at the point of tangency between the lowest attainable budget iso-expenditure line and the indifference curve U(.)⟹

$$\frac{p_1}{p_2} = \frac{\frac{\partial U(.)}{\partial x}}{\frac{\partial U(.)}{\partial y}} = \frac{(MU_x)}{(MU_y)}$$

So following the same argument (for the homogeneity of degree zero for the Walrasian demand functions), that slope of the budget line remains unchanged when all prices increase at the same direction and same proportion, equilibrium point remains unchanged. So the optimal decision remains unchanged.

Therefore we can say $h_i^*(\lambda p_1, \lambda p_2, \bar{U}) = \lambda^0 h_i^*(p_1, p_2, \bar{U}) = h_i^*(p_1, p_2, \bar{U}), \forall\ \lambda > 0, \text{since } \lambda^0 = 1.$

iii) No excess utility: $U\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\} = \bar{U}$

Proof: This property follows from the continuity property of the U(.) function. Let us prove by contradiction.

Suppose not, i.e., suppose $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\}$ is the optimal bundle of EMP but $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\} > \bar{U}$
$\Rightarrow U\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\} \neq \bar{U}$.

Consider a bundle $\{h_1'(p_1, p_2, \bar{U}),\ h_2'(p_1, p_2, \bar{U})\}$, that is, slightly smaller than the optimal bundle $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\}$, on all dimensions.

By continuity, if the bundle $\{h_1'(p_1, p_2, \bar{U}),\ h_2'(p_1, p_2, \bar{U})\}$ is sufficiently closer to the bundle $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\}$,

then $U\{h_1'(p_1, p_2, \bar{U}),\ h_2'(p_1, p_2, \bar{U})\} > \bar{U}$, as well.

But the expenditure being

$\{p_1 h_1'(p_1, p_2, \bar{U}) + p_2 h_2'(p_1, p_2, \bar{U})\}\} < \{p_1 h_1^*(p_1, p_2, \bar{U}) + p_2 h_2^*(p_1, p_2, \bar{U})\}$

$= e(p_1, p_2, \bar{U})$, the bundle $\{h_1'(p_1, p_2, \bar{U}),\ h_2'(p_1, p_2, \bar{U})\}$ achieves $\bar{U}$ at a lower costs.

Hence it contradicts the assumption that $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\}$ is the expenditure minimising bundle.

Consequently, if $\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\}$ is the optimal bundle under EMP, the constraint always binds in the EMP and $U\{h_1^*(p_1, p_2, \bar{U}), h_2^*(p_1, p_2, \bar{U})\} = \bar{U}$.

2.5.4 Properties of Expenditure Function

i) $e(p_1, p_2, \bar{U})$ is continuous function in $p_1, p_2, and\ \bar{U}$.

Since, $e(p_1, p_2, \bar{U}) = \{p_1 h_1^*(p_1, p_2, \bar{U}) + p_2 h_2^*(p_1, p_2, \bar{U})\}$,

and $h_i^*(p_1, p_2, \bar{U})$ is continuous function in $(p_1, p_2, \bar{U})$, it follows that $e(p_1, p_2, \bar{U})$ is continuous function in $p_1, p_2, and\ \bar{U}$.

ii) $e(p_1, p_2, \bar{U})$ is strictly increasing in U and decreasing in price vector $p = p_1, p_2$.

(A) $e(p_1, p_2, \bar{U})$is strictly increasing in U

Proof: Suppose there exists two levels of utility U', U'', such that $U'' > U'$, for a given price vector $(p = p_1, p_2)$.

Let x' vector $(x' = x_1', x_2')$ solves EMP for the given prices and utility level (p_1, p_2, U') and x'' vector $(x'' = x_1'', x_2'')$ solves for the EMP for the given prices and utility level (p_1, p_2, U''), so that $e(p_1, p_2, U') = p\,x'$ and $e(p_1, p_2, U'') = p x''$.

We have to show $e(p_1, p_2, U'') > e(p_1, p_2, U') \Rightarrow p\ x'' > p\ x'$ which we will prove by contradiction. Suppose x' vector solves EMP for the given prices and utility level (p_1, p_2, U') and x'' vector solves for the EMP for the given prices and utility level (p_1, p_2, U'') but $p\ x'' \leq p\ x' > 0$.

Let us define another bundle $\tilde{x}$ (vector $\tilde{x} = \widetilde{x_1}, \widetilde{x_2}$), such that $\tilde{x} = \alpha x''$, and $0 < \alpha < 1$.

If we take α very close to 1,

$U(x'') > U(\tilde{x}) > U(x') \implies U(\tilde{x}) > U(x') \ldots\ldots (a)$,

since $U'' > U'$ (by assumption. α being a fraction, $\tilde{x}$bundle is closer to that of x'').

Thus, $p\tilde{x} = p(\alpha x'') < px'' \leq p\,x'$ (by assumption)$\implies p\tilde{x} \leq p\,x' \ldots\ldots\ldots$ (b)

Combining (a) and (b) we can claim $\tilde{x}$ is the optimal bundle which solves EMP. It contradicts the fact that x' solves EMP for the given prices and utility level (p_1, p_2, U').

Therefore we can claim that if x' vector solves EMP for the given prices and utility level (p_1, p_2, U') and x'' vector solves for the EMP for the given prices and utility level (p_1, p_2, U'') then $p\,x'' > p\,x'$ must hold. $e(p_1, p_2, U'') > e(p_1, p_2, U')$.

(B) To show $e(p_1, p_2, \bar{U})$ is increasing in price vector $p = p_1, p_2$.

Proof: Let the two price vectors be $p = (p_1, p_2)$ and $p' = (p_1', p_2')$ and let $p' > p$.

We have to show $e(p', U) > e(p, U) \implies e(p_1', p_2', U) > e(p_1, p_2, U)$.

Let x vector $(x = x_1, x_2)$ solve EMP for the given prices and utility level (p_1, p_2, U), such that $e(p_1, p_2, U) = px$.

Let $x'(x' = x_1', x_2')$ solves EMP for the given prices and utility level (p_1', p_2', U), such that $e(p_1', p_2', U) = p'x'$.

Following the definition $e(p', U) = e(p_1', p_2', U) = p'x' > px'$ (since $p' > p$)$> e(p, U) = e(p_1, p_2, U)$. This is because px' is not the expenditure function at the price vector p, rather px is.

Hence, $(p', U) > e(p, U) \implies e(p_1', p_2', U) > e(p_1, p_2, U)$.

iii) $e(p_1, p_2, \bar{U})$ is homogenous function of degree one in (p_1, p_2)

Proof: Homogeneity of degree one of the expenditure function in (p_1, p_2)means $e(\lambda p_1, \lambda p_2, \bar{U}) = \lambda\, e(p_1, p_2, \bar{U}), \forall\, \lambda > 0$.

Suppose all the prices (p_1, p_2) are increased by the same direction and same proportion by $\lambda, \lambda > 0$.

Since, $e(p_1, p_2, \bar{U}) = \{p_1 h_1^*(p_1, p_2, \bar{U}) + p_2 h_2^*(p_1, p_2, \bar{U})\} \implies$
$e(\lambda p_1, \lambda p_2, \bar{U}) = \{\lambda p_1 h_1^*(\lambda p_1, \lambda p_2, \bar{U}) + \lambda p_2 h_2^*(\lambda p_1, \lambda p_2, \bar{U})\} \implies$
$e(\lambda p_1, \lambda p_2, \bar{U}) =$
$\{\lambda p_1 h_1^*(\lambda p_1, \lambda p_2, \bar{U})\} + \{\lambda p_2 h_2^*(\lambda p_1, \lambda p_2, \bar{U})\} = \lambda\{p_1 \lambda^0 h_1^*(p_1, p_2, \bar{U}) +$
$p_2 \lambda^0 h_2^*(p_1, p_2, \bar{U})\}$

$= \lambda\{p_1 h_1^*(p_1, p_2, \bar{U}) + p_2 h_2^*(p_1, p_2, \bar{U})\}$

$= \lambda\, e(p_1, p_2, \bar{U})$, since Hicksian demands are of homogenous degree in prices. Thus, $(\lambda p_1, \lambda p_2, \bar{U}) = \lambda\, e(p_1, p_2, \bar{U})$.

iv) $e(p_1, p_2, \bar{U})$ is concave in (p_1, p_2)

Proof: Consider two price vectors: $\{p' = p_1', p_2'\}$ *and* $\{p'' = p_1'', p_2''\}$ at the given utility $\bar{U}$.

Let us consider another price vector $\bar{p} = (\overline{p_1}, \overline{p_2})$, where, $\bar{p} = \{\lambda(p') + (1-\lambda)(p'')\}$, where $0 \leq \lambda \leq 1$. Let $h(\bar{p}, \bar{U})$ be the vector of optimal solution from EMP with$(\bar{p}, \bar{U})$. $\{where, h(\bar{p}, \bar{U}) = h_1(\bar{p}, \bar{U}), h_2(\bar{p}, \bar{U})\}$.

We have to show that $e(\bar{p}, \bar{U}) = \bar{p}\, h(\bar{p}, \bar{U}) \geq \lambda(p')h(p', \overline{U}) + (1-\lambda)(p'')h(p'', \overline{U})$

Following the definition,

$e(\bar{p}, \bar{U}) = \bar{p}\, h(\bar{p}, \bar{U}) = \{\lambda(p') + (1-\lambda)(p'')\}h(\bar{p}, \bar{U})$, (by substituting $\bar{p}$)

or, $e(\bar{p}, \bar{U}) = \{\lambda(p')h(\bar{p}, \bar{U}) + (1-\lambda)(p'')h(\bar{p}, \bar{U})\}$

Thus, $e(\bar{p}, \bar{U}) \geq \lambda(p')h(p', \overline{U}) + (1-\lambda)(p'')h(p'', \overline{U})$.

This is because

a) $h(\bar{p}, \bar{U})$ is not the expenditure minimising bundle at price vectorp', rather $h(p', \overline{U})$ is the optimal in EMP for $(p'\overline{U}) \Longrightarrow \lambda(p')h(\bar{p}, \bar{U}) \geq \lambda(p')h(p', \overline{U})$.

b) $h(\bar{p}, \bar{U})$ is not the expenditure minimising bundle at price vector p'', rather $h(p'', \overline{U})$ is the optimal in EMP for $(p'', \overline{U}) \Longrightarrow (1-\lambda)(p'')h(\bar{p}, \bar{U}) \geq (1-\lambda)(p'')h(p'', \overline{U})$

Combining (a) and (b) we get the result.

Thus, $e(p_1, p_2, \bar{U})$ is concave in (p_1, p_2).

v) Derivative property of expenditure function

By **Shephard's Lemma**: $\frac{\partial e(p_1, p_2, \bar{U})}{\partial p_i} = h_i(p_1, p_2, \bar{U})$, $i = 1, 2$.....*(2.25)*

Proof: By definition $e(p_1, p_2, \bar{U}) = \sum p_i h_i(p_1, p_2, \bar{U}) = p_1 h_1(p_1, p_2, \bar{U}) + p_2 h_2(p_1, p_2, \bar{U})$

Differentiating both sides with respect to p_1:

$$\frac{\partial e(p_1, p_2, \bar{U})}{\partial p_1} = h_1(p_1, p_2, \bar{U}) + [p_1 \frac{\partial h_1(p_1, p_2, \bar{U})}{\partial p_1} + p_2 \frac{\partial h_2(p_1, p_2, \bar{U})}{\partial p_1}] \ldots\ldots\ldots (2.26)$$

The first term $h_1(p_1, p_2, \bar{U})$ is the direct effect and the second term$[p_1 \frac{\partial h_1(p_1, p_2, \bar{U})}{\partial p_1} + p_2 \frac{\partial h_2(p_1, p_2, \bar{U})}{\partial p_1}]$ is the indirect effect. We have to show that the indirect effect is 0.

From the duality $U\{h_1(p_1, p_2, \bar{U}), h_2(p_1, p_2, \bar{U})\} = \bar{U}$ (see Section 2.6 for duality relations)

Differentiating both sides with respect to p_1, we get

$$\frac{\partial U\{h_1(p_1, p_2, \bar{U}), h_2(p_1, p_2, \bar{U})\}}{\partial h_1} \frac{\partial h_1(p_1, p_2, \bar{U})}{\partial p_1} + \frac{\partial U\{h_1(p_1, p_2, \bar{U}), h_2(p_1, p_2, \bar{U})\}}{\partial h_2} \frac{\partial h_2(p_1, p_2, \bar{U})}{\partial p_1} = 0 \ldots\ldots\ldots (2.27)$$

From the FOCs (2.17 and 2.18) we get $\mu \frac{\partial U\{h_1(p_1, p_2, \bar{U}), h_2(p_1, p_2, \bar{U})\}}{\partial h_1} = p_1$ and

$$\mu \frac{\partial U\{h_1(p_1, p_2, \bar{U}), h_2(p_1, p_2, \bar{U})\}}{\partial h_2} = p_2$$

Substituting them in equation (2.27)

$$\frac{p_1}{\mu}\frac{\partial h_1(p_1,p_2,\bar{U})}{\partial p_1}+\frac{p_2}{\mu}\frac{\partial h_2(p_1,p_2,\bar{U})}{\partial p_1}=0 \Rightarrow \frac{1}{\mu}[p_1\frac{\partial h_1(p_1,p_2,\bar{U})}{\partial p_1}+p_2\frac{\partial h_2(p_1,p_2,\bar{U})}{\partial p_1}]=0$$

Now substituting it in the equation (2.6) we get $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_1}=h_1(p_1,p_2,\bar{U})$.

Similarly, we can show that $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_2}=h_2(p_1,p_2,\bar{U})$.

2.5.4.1 Rederiving Some Standard Results by using Shephard's Lemma

i) Substitution effect is negative, i.e., $\frac{\partial h_i((p_1,p_2,\bar{U})}{\partial p_i}\leq 0, i=1,2$ ..... (2.28)

ii) Cross substitution effects are same, i.e.,
$\frac{\partial h_i((p_1,p_2,\bar{U})}{\partial p_j}=\frac{\partial h_j((p_1,p_2,\bar{U})}{\partial p_i}$, i, j = 1,2 ..... (2.29)

i) **Proof**: Substitution effect is negative $\Rightarrow \frac{\partial h_i((p_1,p_2,\bar{U})}{\partial p_i}\leq 0, i=1,2$

From the Shephard's Lemma $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_i}=h_i((p_1,p_2,\bar{U}), i=1,2$

$\Rightarrow \frac{\partial e(p_1,p_2,\bar{U})}{\partial p_1}=h_1((p_1,p_2,\bar{U})$ for $i=1$.

Differentiating with respect to p_1, $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1^2}=\frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1}\leq 0$.

Since the expenditure function $e(p_1,p_2,\bar{U})$ is concave in prices (p_1,p_2).

Hence the substitution effect is negative.

ii) **Proof**: Cross substitution effects are same:

$\frac{\partial h_i((p_1,p_2,\bar{U})}{\partial p_j}=\frac{\partial h_j((p_1,p_2,\bar{U})}{\partial p_i}$, i, j = 1,2

From the Shephard's Lemma: $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_i}=h_i((p_1,p_2,\bar{U}), i=1,2$.

So we have

(a) $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_1}=h_1((p_1,p_2,\bar{U})$ and (b) $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_2}=h_2((p_1,p_2,\bar{U})$

Differentiating both sides of (a) w.r.t. p_2: $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2 p_1}=\frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2}$

Differentiating both sides of (a) w.r.t. p_1: $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1 p_2}=\frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1}$

By Young's theorem the second order cross partial derivatives are equal, that is,

$$\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1 p_2}=\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2 p_1}\Rightarrow \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2}=\frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1}$$

2.5.5 Decomposition of Price

Own price effect captures the change in quantity demand *(ceteris paribus)* due to the change in its own price. Thus own price effect is $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1}$ for goods x and $(\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2})$ for goods y. Movement along the ordinary demand curve captures the own price effect. Price effect can be decomposed into Substitution Effect and Income Effect.

Own Substitution Effect is the effect of the change in own price of x (i.e., p_1) on the quantity demand of x, for a given level of utility. Movement along the compensated demand curve captures the own substitution effect.

Own substitution effect $\Longrightarrow \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} = \frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} \Big| U = \bar{U}$, is always negative irrespective of the nature of commodity $(i.e., \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} < 0)$.

Income Effect: Income Effect for a good (say,x) represents a change in quantity demanded due to the change in income (M), for the given prices. So income effect is captured by $\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}$. The sign can be positive, negative or zero according to the different characterization of the commodity. So if the income effect $\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} > 0$, the commodity is a normal good, if it is $\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} < 0$, the commodity is an inferior good and if it is $\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} = 0$, the commodity is perfectly income elastic.

Income effect for a price change works through the change in real income which changes the purchasing power of a consumer and therefore her demand. For a given money income as the price of one good changes (say, p_1), the income effect for a price change is $\left\{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} x(p_1, p_2, \bar{M})\right\}$. Note that income effect and income effect for a price change have opposite sign and different magnitude.

Consider good x being *normal good*. Suppose there is a fall in price of x from (p_1) to p_1' (so $p_1 < p_1'$), prices of other commodity and income (p_2, M) remaining unchanged. The change in demand of x due to the change in price is the whole own price effect. The fall in price of x (for a given income of the consumer) implies an increase in real income, as with the same nominal income consumer can buys more x or y or both. In order to capture the substitution effect, (according to the Hicksian principle), let us introduce a compensated change in income in such a way (in opposite direction to the price change) which enable the consumer to attain the initial utility (I_1) at the new lower price p_1'. Let AB (see Fig. 2.14a and b) be the initial budget line (having the slope $= \frac{p_1}{P_2}$) and the tangency point with the indifference curve I_1 defines the initial equilibrium at ε. Now the change in price will lead the budget line to rotate outward from AB to AC. The new equilibrium defined at the tangency point with the higher indifference curve I_1' and the budget line AC is ε'. Movement from ε to ε' captures the *price effect.*

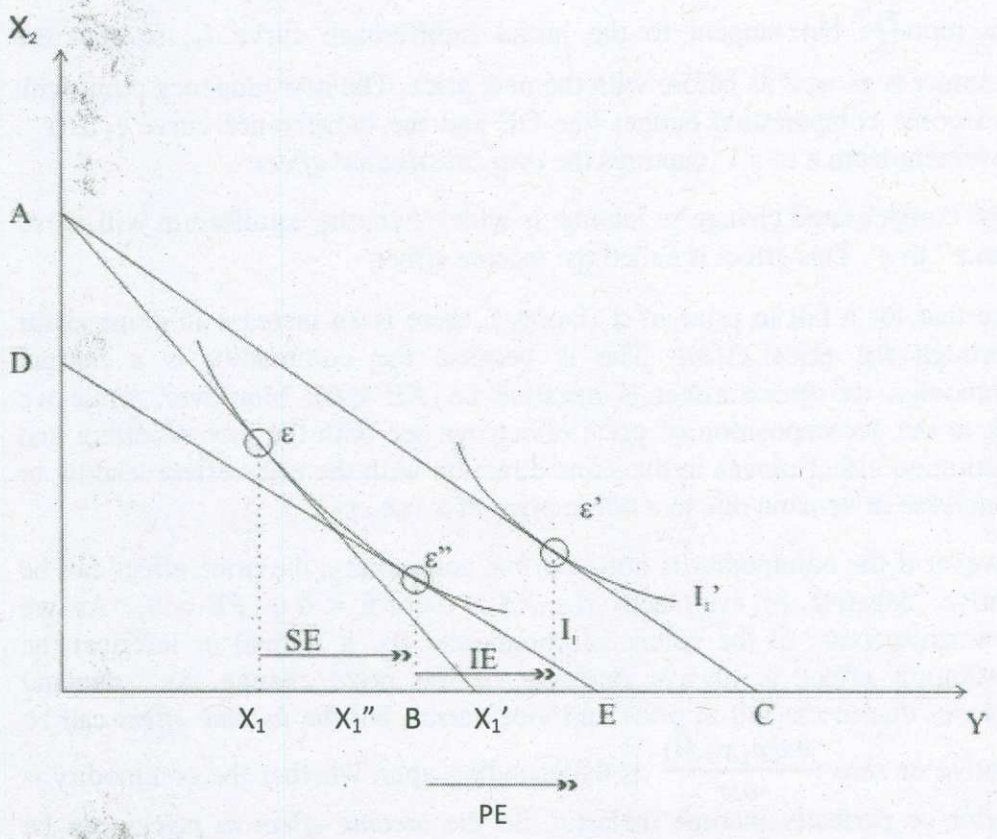

Fig. 2.14a: Decomposition of Price Effect (Normal Commodity)

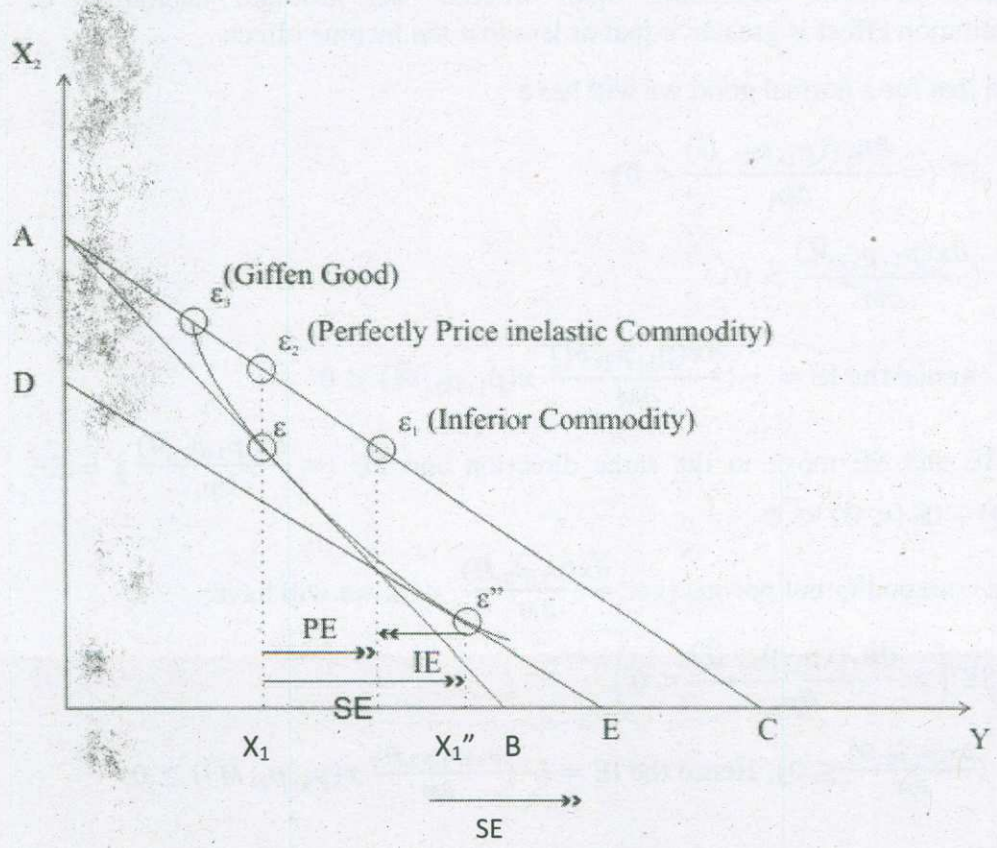

Fig. 2.14b: Decomposition of Price Effect (Inferior Commodity)

2.5.5 Decomposition of Price

Own price effect captures the change in quantity demand *(ceteris paribus)* due to the change in its own price. Thus own price effect is $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1}$ for goods x and $(\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2})$ for goods y. Movement along the ordinary demand curve captures the own price effect. Price effect can be decomposed into Substitution Effect and Income Effect.

Own Substitution Effect is the effect of the change in own price of x (i.e., p_1)

Let DE be the income compensated budget line, which will capture the initial new ratio $\frac{p_1'}{P_2}$ but tangent to the initial indifference curve I_1, so that the consumer is as well as before with the new price. The new tangency point with the income compensated budget line DE and the indifference curve I_1 is ε''. Movement from ε to ε'' , captures the *own substitution effect.*

If the compensated change in income is withdrawn, the equilibrium will move from ε'' to ε'. This effect is called the *income effect.*

Note that for a fall in price of x (i.e., p_1), there is an increase in demand for x through the price effect. The is because the commodity is a normal commodity, the (price effect is negative i.e., $PE < 0$). Moreover, while we look at the decomposition of price effect, we see both the income effect and substitution effect moves in the same direction with the price effect leading to an increase in demand due to a fall in price of x (i.e., p_1).

However if the commodity is not a normal commodity, the price effect can be positive, negative or even zero (i.e.,$PE > 0$ or $PE < 0$ or, $PE = 0$). As we know irrespective of the nature of commodity (be it normal or inferior) the substitution effect is always negative to the price change (i.e., demand increases due to the fall in price and vice versa), but the *income effect* can be negative or zero $(\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} \leq 0)$depending upon whether the commodity is inferior or perfectly income inelastic. So the *income effect in prices* can be positive or zero i.e.,$\{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\ x(p_1,p_2,\bar{M}) \geq 0\}$. In such cases the substitution effect (SE) and income effect (IE) move in opposite direction and price effect is the net of these two. Thus, the price effect can be negative, positive or zero, depending upon whether the absolute magnitude of substitution effect is greater, equal or less that the income effect.

Note that for a normal good we will have

i) $SE\ (= \frac{\partial h_1((p_1,p_2,\ \bar{U})}{\partial p_1} < 0)$

ii) $(\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} > 0)$

hence the IE $= -(\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\ x(p_1,p_2,\bar{M}) < 0$

So IE and SE move in the same direction and PE $(= \frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1})$ = SE (< 0) + IE $\{< 0\} < 0$

For a commodity not normal (i.e., $\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} \leq 0$), we will have:

i) SE $\left(= \frac{\partial h_1((p_1,p_2,\ \bar{U})}{\partial p_1} < 0\right)$.

ii) $(\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} \leq 0)$. Hence the IE $= \{-(\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\ x(p_1,p_2,\bar{M})\} \geq 0$

Consequently, IE and SE move in opposite directions and hence the PE depends on the net effect of these two. There are three cases:

1.a) Suppose IE > 0. If $|SE| > |IE|$, it follows that PE < 0 and the law of demand still holds even when the commodity is an inferior one.

1.b) Suppose IE = 0. If $|SE| > |IE|$, it follows that PE < 0 and the law of demand still holds even though the commodity is an perfectly income inelastic.

2) Suppose IE > 0. If $|SE| = |IE|$, it follows that PE =0 and SE and IE are equal in magnitude (in absolute value) and the effects outweigh each other. So the net effect on PE = 0. It is a case of perfectly price inelastic commodity (represented by vertical demand curve).

3) Suppose IE >0. If $|SE| < |IE|$, it follows that PE >0. This is the case of Giffen commodity, represented by positively sloped demand curve, which exhibits violation of the law of demand.

2.5.6 Ordinary Demand Functions and Compensated Demand Functions

i) Ordinary demand functions are derived from the UMP and capture the entire price effect, which can be decomposed into substitution effect and income effect. On the other hand, the compensated demand functions are derived from the EMP and capture only the substitution effect.

ii) Since the substitution effect is always negative irrespective of the nature of the commodity, the compensated demand function are always negatively sloped, i.e., $(\frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} < 0,)$.

But the ordinary demand functions can be negatively sloped, vertical or positively sloped (viz., $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} < 0, \frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} = 0$ or, $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} > 0$) depending upon the nature of the commodity.

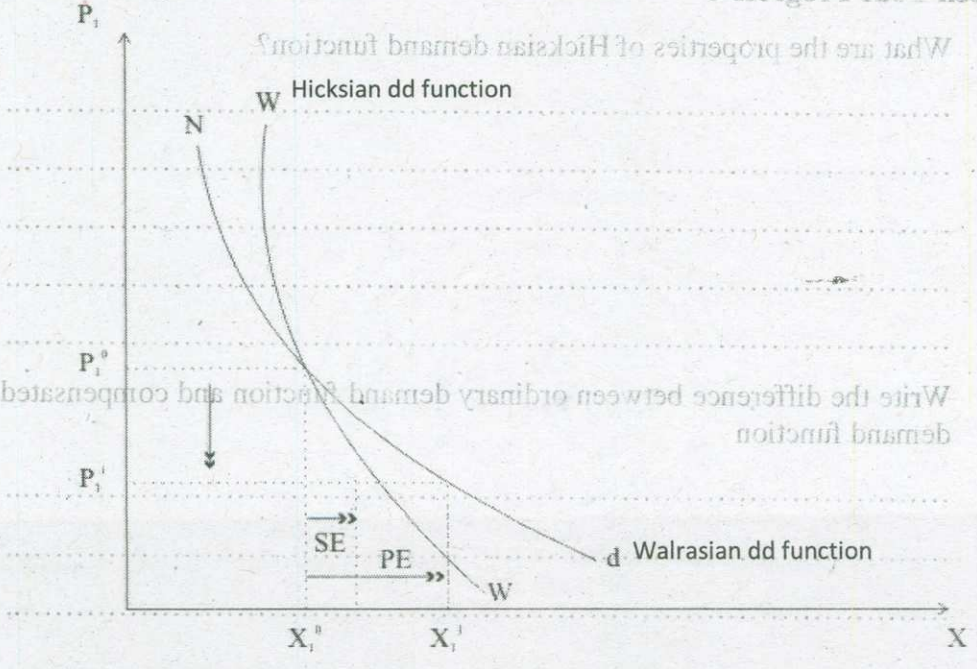

Fig. 2.15a: Ordinary and Compensated Demand Function (Normal Good)

iii) For a normal good, both the ordinary demand functions and the compensated demand functions are negatively sloped but as the former captures the entire price effect it is flatter than the later.

iv) For an inferior good, while the compensated demand functions are negatively sloped, the ordinary demand functions can negatively sloped, vertical or positively sloped depending upon whether $|SE| > |IE|$, $|SE| = |IE|$ or $|SE| < |IE|$.

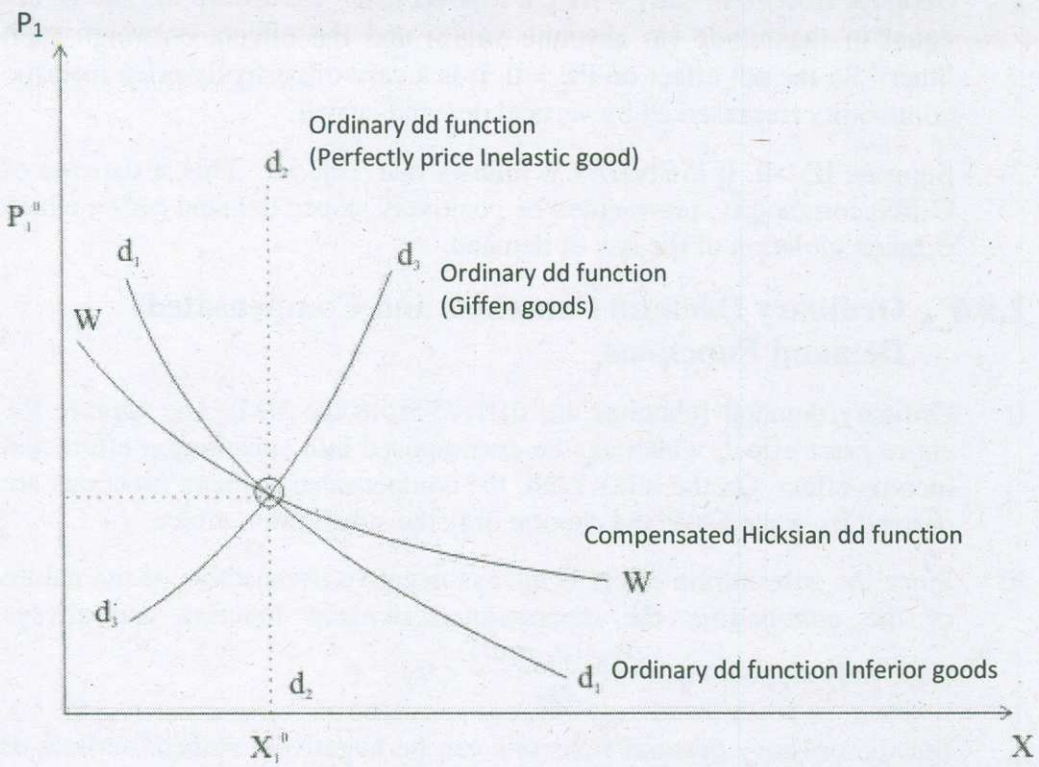

Fig. 2.15b: Ordinary and Compensated dd function (Inferior Commodity)

Check Your Progress 4

1) What are the properties of Hicksian demand function?

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2) Write the difference between ordinary demand function and compensated demand function

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3) What are the properties of expenditure function?

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2.6 BASIC DUALITY RELATIONS

In the last two sections we have discussed two optimisation problem:

A. UMP: Maximisation of utility $U(x,y)$ subject to the budget set (constraint) $p_1x + p_2y \leq \bar{M}$ and non-negativity constraints $x > 0$ and $y > 0$. The solutions of the UMP are

i) $x^* = x^*(p_1, p_2, \bar{M})$

ii) $y^* = y^*(p_1, p_2, \bar{M})$

These two are called **Marshallian or Walrasian or ordinary demand functions**. We also derived the **Indirect Utility function** $v(p_1, p_2, \bar{M})$ by plugging the equilibrium values of x^*, y^* in the direct utility function $U(x,y)$.

B. EMP: Minimisation of the expenditure, $\mathcal{E} = p_1x + p_2y$ subject to the utility constraint $U(x,y) \geq \bar{U}$ and non-negativity constraints > 0 and $y > 0$. The solutions of the EMP are

i) $x^* = h_1^*(p_1, p_2, \bar{U})$,

ii) $y^* = h_2^*(p_1, p_2, \bar{U})$

These two are called the **Hicksian or the compensated demand functions.** We also derived the **expenditure function** $\boldsymbol{e(p_1, p_2, \bar{U})}$, by plugging the equilibrium values of $x^* and\ y^*$ in the expenditure equation $\mathcal{E} = p_1x + p_2y$.

2.6.1 Duality Propositions

The link between the two optimisation problems (UPM and EMP) are provided by the **duality relations.** Following are the two duality propositions that state about the linkage.

Proposition 1: Suppose (i) $U(x,y)$ is continuous, (ii) preference relation satisfies local non satiation and (iii) both the problems of optimisation (UMP and EMP) have solutions. Let us define $U(x^*, y^*) = \bar{U}$. Then if (x^*, y^*) solves the UMP, (x^*, y^*) will also solve for EMP.

Proposition 2: Suppose (i) $U(x,y)$ is continuous, (ii) preference relation satisfies local non-satiation and (iii) both the problems of optimisation (UMP and EMP) have solutions. Let us define $\bar{M} = [p_1x^* + p_2y^*]$. Then if (x^*, y^*) solves the EMP, (x^*, y^*) will also solve for UMP.

2.6.2 Basic Duality Relations

Following are the four duality relations:

i) $(p_1, p_2, \bar{M}) \equiv h_1((p_1, p_2, v(p_1, p_2, \bar{M})), y(p_1, p_2, \bar{M}) \equiv h_2((p_1, p_2, v(p_1, p_2, \bar{M}))$.... (2.30)

ii) $h_1((p_1, p_2, \bar{U}) \equiv x(p_1, p_2, e(p_1, p_2, \bar{U})),\ h_2((p_1, p_2, \bar{U}) \equiv y(p_1, p_2, e(p_1, p_2, \bar{U}))$ ....(2.31)

iii) $v(p_1, p_2, e(p_1, p_2, \bar{U})) \equiv \bar{U}$ ..... (2.32)

iv) $e(p_1, p_2, v(p_1, p_2, \bar{M})) \equiv \bar{M}$ ..... (2.33)

From above it can be said that the same bundle which solves the UMP with given prices and income $(p_1, p_2, \bar{M})$ where $\bar{M} = e(p_1, p_2, \bar{U})$, solves the EMP with given prices and target utility level $v(p_1, p_2, \bar{M}) = \bar{U}$.

2.6.3 Slutsky Equation

We have seen that, generally, if the price of a commodity goes down, we buy more of it. This downward movement of the demand is due to two effects:

- Income effect: because it's less expensive, we have more purchasing power because it is a smaller drain on our personal finances.
- Substitution effect: because it offers more utility per unit of money, other alternatives become less attractive.

What *Eugen Slutsky* managed to do was to find an equation that decomposes this effect that is based on *Hicksian and Marshallian demand curves*.

From the duality relation: $h_1((p_1, p_2, \bar{U}) \equiv x(p_1, p_2, e(p_1, p_2, \bar{U}))$

Differentiating both sides by p_1: $\frac{\partial h_1((p_1, p_2, \bar{U})}{\partial p_1} =$

$$\frac{\partial x(p_1, p_2, e(p_1, p_2, \bar{U}))}{\partial p_1} + \frac{\partial x(p_1, p_2, e(p_1, p_2, \bar{U}))}{\partial e(.)} \frac{\partial e(p_1, p_2, \bar{U})}{\partial p_1} \frac{\partial h_1((p_1, p_2, \bar{U})}{\partial p_1}$$

$$= \frac{\partial x(p_1, p_2, e(p_1, p_2, \bar{U}))}{\partial p_1} + \frac{\partial x(p_1, p_2, e(p_1, p_2, \bar{U}))}{\partial e(.)} h_1((p_1, p_2, \bar{U})$$

{By Shephard's Lemma we have $\frac{\partial e(p_1, p_2, \bar{U})}{\partial p_1}$

$$= h_1((p_1, p_2, \bar{U}) \frac{\partial h_1((p_1, p_2, \bar{U})}{\partial p_1}$$

$$= \frac{\partial x(p_1, p_2, \bar{M})}{\partial p_1} + \frac{\partial x(p_1, p_2, \bar{M})}{\partial M} x(p_1, p_2, \bar{M})$$

{By duality, $h_1((p_1, p_2, \bar{U}) = x(p_1, p_2, e(p_1, p_2, \bar{U})) = x(p_1, p_2, \bar{M})\}$

$$\frac{\partial x(p_1, p_2, \bar{M})}{\partial p_1} = \frac{\partial h_1((p_1, p_2, \bar{U})}{\partial p_1} + \{-\frac{\partial x(p_1, p_2, \bar{M})}{\partial M} x(p_1, p_2, \bar{M})\}$$

Own Price Effect $\left(\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1}\right)$ = **Substitution Effect** $\left(\frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1}\right)$+ **Income Effect**$\{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\, x(p_1,p_2,\bar{M})\}$

In a two commodity framework there are four sets of **Slutsky equations** (showing the decomposition of price effect into income and substitution effect).

Own Price Effect:

i) $$\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} + \{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\, x(p_1,p_2,\bar{M})\} \quad \ldots\ldots\ldots (2.34)$$

ii) $$\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_2} + \{-\frac{\partial y(p_1,p_2,\bar{M})}{\partial M}\, y(p_1,p_2,\bar{M})\} \quad \ldots\ldots\ldots (2.35)$$

Cross Price Effect:

iii) $$\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_2} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2} + \{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M}\, y(p_1,p_2,\bar{M})\} \quad \ldots\ldots\ldots (2.36)$$

iv) $$\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_1} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1} + \{-\frac{\partial y(p_1,p_2,\bar{M})}{\partial M}\, x(p_1,p_2,\bar{M})\} \quad \ldots\ldots\ldots (2.37)$$

Slutsky Matrix (or Substitution Matrix)

$$\boldsymbol{S} = \begin{bmatrix} \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} & \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2} \\ \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1} & \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_2} \end{bmatrix} \quad \ldots\ldots\ldots (2.38)$$

$$\mathbf{S} = \begin{bmatrix} \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1^2} & \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2 p_1} \\ \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1 p_2} & \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2^2} \end{bmatrix} \quad \ldots\ldots\ldots (2.39)$$

Since we know by Shephard's Lemma, $\frac{\partial e(p_1,p_2,\bar{U})}{\partial p_i} = h_i((p_1,p_2,\bar{U}), i = 1,2$

$\Rightarrow \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1^2} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1}$ and $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2^2} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_2}$

Similarly, $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2 p_1} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2}$ *and* $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1 p_2} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1}$.

By Young's theorem the second order cross partial derivatives are equal. So $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_1 p_2} = \frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_2 p_1}$

Slutsky matrix is negative definite or negative semi definite symmetric matrix, the leading principal minors are of alternative in sign starting from the negative sign.

Since the expenditure function $e(p_1,p_2,\bar{U})$ is concave function in (p_1,p_2), $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_i^2} < 0$, i = 1, the first principal minor $S_1 < 0$ and the second principal minor $S_2 = S \geq 0$.

Check Your Progress 5

1) What are the four basic duality relations through which equilibrium consumer choice are solved?

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2) In a two commodity framework there are four sets of Slutsky equations (showing the decomposition of price effect into income and substitution effect). What are these?

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3) Why Slutsky matrix is negative definite or negative semi definite symmetric matrix?

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2.7 LET US SUM UP

The **consumption bundle** is the set of goods and services that the consumer would like to consume. Axioms of the theory of consumer choice help making the assumption that a consumer can make a choice between any two bundles—the assumption of **complete preferences**. The consumer prefers one to another or is indifferent between the two. We refer to this as complete preferences. Consumers are able to make comparisons and choices. We assume that consumers' preferences are **transitive preferences.** If a consumer prefers Bundle A to Bundle B and prefers Bundle B to Bundle C, then the consumer prefers A to C if preferences are transitive. We assume that there is **nonsatiation** for at least one good.

Consumers try to make the choices that maximise their satisfaction given their limited resources. A consumer's budget constraint shows the possible combinations of different goods she can buy given her income and the prices of the goods. The slope of the budget constraint equals the relative price of the goods.

An increase in income shifts the budget constraint outward. A change in the price of one of the goods pivots the budget constraint.

A consumer's indifference curves represent her preferences. An indifference curve shows all the bundles that give the consumer a certain level of happiness. The consumer prefers points on higher indifference curves to points on lower ones.

The slope of an indifference curve at any point is the marginal rate of substitution – the rate at which the consumer is willing to trade one good for the other. The consumer optimises by choosing the point on her budget constraint that lies on the highest indifference curve. At this point, the marginal rate of substitution equals the relative price of the two goods. When the price of a go consumer's choices can be broken down into two consumer's choices can be broken down into two effects.

The substitution effect is the change that arises because a price change encourages greater consumption of the good that has become relatively cheaper. It is represented by a movement along an indifference curve. When the substitution effect is removed the income effect is the residual effect in the change in consumption that arises because of a lower price makes the consumer better off in terms of increased affordability. It is represented by a movement from a lower indifference curve to a higher one.

2.8 KEY WORDS

Marshallian Demand Function (uncompensated demand function) : Specifies what the consumer would buy in each price and income or wealth situation, assuming it perfectly solves the utility maximisation problem.

Engel Aggregation : Share-weighted income elasticities must always sum to one.

Hicksian Demand Function (compensated demand function) : Corresponds to the **demand** of a consumer over a bundle of goods that minimises their expenditure while delivering a fixed level of utility.

Bounded Set : A set of numbers with both an upper bound and a lower bound.

Closed Set : A set that contains all of its accumulation points, as the set of points on and within a circle

Concave Function : Every line segment joining two points on its graph does not lie above the graph at any point

Convex Function : Every line segment joining two points on its graph does not lie below the graph at any point.

Check Your Progress 5

1) What are the four basic duality relations through which equilibrium consumer choice are solved?

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Continuous Function : A function for which sufficiently small changes in the input result in arbitrarily small changes in the output.

Eulers' Theorem : For a function F(L,K) which is homogeneous of degree n, $(\partial F/\partial L)L + (\partial F/\partial K)K = nF(L,K)$.

Giffen Good : A product that people consume more of as the price rises and vice versa

Homogenous Function : A function which satisfies the condition $f(tx, ty) = t^n(x, y)$, for some integer n.

Inferior Commodity : A type of good whose demand declines when income rises.

Locally Non-Satiated : For any bundle of goods there is always another bundle of goods arbitrarily close that is preferred to it.

Monotonic Function : A function which is either entirely nonincreasing or nondecreasing.

Normal Good : A good that experiences an increase in demand as the real income of an individual or economy increases.

2.9 SOME USEFUL BOOKS

Jehle, by Geoffrey A. and Philips J. Reny (2011): *Advanced Microeconomic Theory*, Pearson Education, India.

Kreps, David M. (1998): *A Course in Microeconomic Theory*, Prentice Hall, India, New Delhi

Mas-Colell, Michael D. Whinston and Jerry R. Green (2007): *Microeconomic Theory*, Oxford University Press, Indian Edition.

Synder, Christopher and Walter Nicholson (2008): *Fundamentals of Microeconomics*, Cengage Learning, Indian Edition.

Varian, Hal R. (2006): *Intermediate Micro Economics: A Modern Approach*, 7th edition, East West Press Pvt. Ltd, Indian Edition.

Varian, Hal R. (2010): *Microeconomic Analysis*, 3rd Edition, WW. Norton and Company. Indian Edition.

2.10 ANSWERS OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) Completeness, transitivity, reflexive, continuity and monotonicity

2) Consumption set, feasible set and preference relation

3) For the *uniqueness* of a solution from the utility maximisation exercise we need the property of **strict quasi-concavity** of the utility function. For, if the utility function $U(x)$ quasi-concave then the indifference curves are convex to the origin and if $U(x)$ is strictly quasi-concave, then (x^*) is unique.

4) Preference function remains unchanged.

Check Your Progress 2

1) Yes. The indirect utility function is decreasing in p. Another way to show it - the fall in all prices means that the previously optimal bundle, x^0 prices means that the previously optimal bundle, x thus it must be $v(p^1, y) = v(p^0, y)$.

2) Yes.

3) No. Engel aggregation, $\sum_{i=1}^{n} s_i \eta_i = 1$ and income shares are non-negative.

4) Yes.

5) Yes. The indirect utility function is decreasing in p. Another way to show it - the fall in all prices means that the previously optimal bundle, x^0 prices means that the previously optimal bundle, x thus it must be that $v(p^1, y) = v(p^0, y)$.

Check Your Progress 3

1) A property of an equation that exists if independent variables are increased by a constant value, then the dependent variable is increased by the value raised to the power of 0. In other words, for any changes in the independent variables, the dependent variable does not change.

2) $x_1^*=p_2/p_1$; $x_2^*=(m-p_2)/p_2$; Yes because the demand is downward sloping.

3) Purchase more of x and less of y.

4) $x_1(p_1, p_2, m) = \frac{m}{p_1+2p_1^{\frac{1}{2}}p_2^{\frac{1}{2}}}$; $x_2(p_1, p_2, m) = \frac{m}{2p_2+p_1^{\frac{1}{2}}p_2^{\frac{1}{2}}}$.

5) You can get your answer by substituting the Marshallian demands that you found into the definition of indirect utility which is
$v(p, m) = u(x_1(p, m), x_2(p, m))$ and
$v((p_1, p_2, m)) = \frac{m}{4p_2+p_1+4p_1^{\frac{1}{2}}p_2^{\frac{1}{2}}}$.

Check Your Progress 4

1) Properties of the expenditure function $E(p_x, p_y, u) = E(u, p)$ where $p = (p_x, p_y)$.

   1) Non decreasing in p. That is, if $p' \geq p$ then $E(u, p') \geq E(u, p)$.

   2) Homogenous of degree one in p. That is, $E(u, \lambda p) = \lambda E(u, p)$ for $\lambda > 0$.

   3) Concave in p. That is, $E(u, \lambda p + (1-\lambda)) \geq \lambda E(u, p) + (1-\lambda)E(u, p)$ for $0 \leq \lambda \leq 1$

4) Continuous in p. That is, $E(u,p)$ is continuous as a function of p for ≥ 0 .

The expenditure function has the same properties as the cost function.

2) Ordinary demand functions are derived from the utility maximisation problem and capture the entire price effect, which can be decomposed into substitution effect and income effect. Whereas the compensated demand functions are derived from the expenditure minimisation problem and capture only the substitution effect.

3) i) $e(p_1, p_2, \bar{U})$ is continuous function in $p_1, p_2, and\ \bar{U}$.

Since, $e(p_1, p_2, \bar{U}) = \{p_1 h_1{}^*(p_1, p_2, \bar{U}) + p_2 h_2{}^*(p_1, p_2, \bar{U})\}$, and $h_i{}^*(p_1, p_2, \bar{U})$ is continuous function in $(p_1, p_2, \bar{U})$, so it follows that $e(p_1, p_2, \bar{U})$ is continuous function in $p_1, p_2, and\ \bar{U}$.

ii) $e(p_1, p_2, \bar{U})$is strictly increasing in U and decreasing in price vector $p = p_1, p_2$.

(A) $e(p_1, p_2, \bar{U})$is strictly increasing in U

Check Your Progress 5

1) Four duality relations given in sub-section 2.6.2 are:

i) $(p_1, p_2, \bar{M}) \equiv h_1((p_1, p_2,\ v(p_1, p_2, \bar{M})), y(p_1, p_2, \bar{M}) \equiv h_2((p_1, p_2, v(p_1, p_2, \bar{M}))$

ii) $h_1((p_1, p_2, \bar{U}) \equiv x(p_1, p_2, e(p_1, p_2, \bar{U}))$, $h_2((p_1, p_2, \bar{U}) \equiv y(p_1, p_2, e(p_1, p_2, \bar{U}))$

iii) $v(p_1, p_2, e(p_1, p_2, \bar{U})) \equiv \bar{U}$

iv) $e(p_1, p_2, v(p_1, p_2, \bar{M})) \equiv \bar{M}$

From above it can be said that the same bundle which solves the utility maximisation problem (UMP) with given prices and income $(p_1, p_2, \bar{M})$ where $\bar{M} = e(p_1, p_2, \bar{U})$, solves the expenditure minimisation problem (EMP) with given prices and target utility level $v(p_1, p_2, \bar{M}) = \bar{U}$.

2) Own Price Effect:

i) $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_1.} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_1} + \{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} x(p_1, p_2, \bar{M})\}$

ii) $\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_2} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_2} + \{-\frac{\partial y(p_1,p_2,\bar{M})}{\partial M} y(p_1, p_2, \bar{M})\}$

Cross Price Effect:

iii) $\frac{\partial x(p_1,p_2,\bar{M})}{\partial p_2} = \frac{\partial h_1((p_1,p_2,\bar{U})}{\partial p_2} + \left\{-\frac{\partial x(p_1,p_2,\bar{M})}{\partial M} y(p_1, p_2, \bar{M})\right\}$

iv) $\frac{\partial y(p_1,p_2,\bar{M})}{\partial p_1} = \frac{\partial h_2((p_1,p_2,\bar{U})}{\partial p_1} + \{-\frac{\partial y(p_1,p_2,\bar{M})}{\partial M} x(p_1, p_2, \bar{M})\}$

3) Since the expenditure function $e(p_1, p_2, \bar{U})$ is concave functon in (p_1, p_2), $\frac{\partial^2 e((p_1,p_2,\bar{U})}{\partial p_i^2} < 0$, i = 1, the first principal minor $S_1 < 0$ and the second principal minor $S_2 = S \geq 0$. Such a feature generates the negative definite matrix.

2.11 EXERCISES AND ANSWER/HINTS

1) Consider the expenditure function of a consumer who consumes only two goods, $e(p,u) = \frac{(2p_1+p_2)}{2}$ where u is a feasible utility level and $p_1, p_2 > 0$ are the goods' prices.

a) Write down the expenditure minimisation problem (for this part you can use the general notation)

b) Derive this consumer's Hicksian demand functions.

c) Derive this consumer's indirect utility function $v(p,y)$ for any $y \geq 0$.

d) Derive this consumer's Marshallian demand functions $x_1^* \; and \; x_2^*$.

e) Suppose $y = 6$ and the good's prices (p_1, p_2) change from $(1,1)$ to $(2,1)$. Compute the income and substitution effects of the price change on the quantities demanded of each good.

Ans.: a) $min_x \; p_1x_1 + p_2x_2 \; s.t. u(x_1,x_2) \geq u$.

b) use Shephard's lemma: $\hat{x}_1(p,u) = \frac{\partial e(p,u)}{\partial p_1} = u \; and \; \hat{x}_2(p,u) = \frac{u}{2}$.

c) use duality holding p fixed, $v(y \mid p) = e^{-1}(y \mid p)$ and thus $v(p,y) = \frac{2y}{(2p_1+p_2)}$

d) use Roy's identity, $x_1^*(p,y) = -\frac{\frac{\partial v}{\partial p_1}}{\frac{\partial v}{\partial y}} = -\frac{4y(2p_1+p_2)^{-2}}{2(2p_1+p_2)^{-1}} = \frac{2y}{(2p_1+p_2)}$

and similarly, $x_2^*(p,y) = \frac{y}{(2p_1+p_2)}$

e) plug in the numbers using the Slutsky equation at $u = v(p^0, y)$.

1. Consider the expenditure function of a consumer who consumes only two goods, $e(p,u) = \frac{(2p_1+p_2)}{2}$ where u is a feasible utility level and $p_1, p_2 > 0$ are the goods' prices.

f) Write down the expenditure minimisation problem (for this part you can use the general notation)

g) Derive this consumer's Hicksian demand functions.

h) Derive this consumer's indirect utility function $v(p,y)$ for any $y \geq 0$.

i) Derive this consumer's Marshallian demand functions $x_1^* \; and \; x_2^*$.

j) Suppose $y = 6$ and the good's prices (p_1, p_2) change from $(1,1)$ to $(2,1)$. Compute the income and substitution effects of the price change on the quantities demanded of each good.

Ans.: a) $min_x\, p_1x_1 + p_2x_2\, s.t. u(x_1, x_2) \geq u$.

b) Use Shephard's lemma: $\hat{x}_1(p,u) = \frac{\partial e(p,u)}{\partial p_1} = u \; and \; \hat{x}_2(p,u) = \frac{u}{2}$.

c) Use duality holding p fixed, $v(y \mid p) = e^{-1}(y \mid p)$ and thus $v(p,y) = \frac{2y}{(2p_1+p_2)}$

d) Use Roy's identity, $x_1^*(p,y) = -\frac{\frac{\partial v}{\partial p_1}}{\frac{\partial v}{\partial y}} = -\frac{4y(2p_1+p_2)^{-2}}{2(2p_1+p_2)^{-1}} = \frac{2y}{(2p_1+p_2)}$ and similarly, $x_2^*(p,y) = \frac{y}{(2p_1+p_2)}$.

e) Plug in the numbers using the Slutsky equation at $= v(p^0, y)$.

2) A consumer has the utility function over goods x and y, $u(x,y) = 12x^2y^4$.

Let the price of good x be given by px, let the price of good y be given by py, and let income be given by M.

a) What is the slope of the consumer's indifference curve at the consumption bundle, (1,1) ?

b) Derive the consumer's generalized demand function for goods x and y.

c) If we have $p_x = 2$, $p_y = 2$, and $M = 24$, compute the utility maximising consumption bundle.

Ans: a) The marginal rate of substitution is the absolute value of the slope of the indifference curve. The MRS can be calculated as

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{24xy^4}{48x^2y^3} = \frac{y}{2x}$$

Evaluated at the point (1,1) by plugging x = 1 and y = 1 into the above formula, the MRS is 1/2. Therefore, the slope of the indifference curve is −1/2.

(b) Set up the Lagrangean expression, $L = 12x^2y^4 + \lambda[M - p_xx - p_yy]$. By differentiating L with respect to x, y, and λ, and setting the derivatives equal to zero, the resulting first order conditions are:

$$24xy^4 - \lambda p_x = 0, 48x^2y^3 - \lambda p_y = 0, and\; M - p_xx - p_yy = 0.$$

Solving the first two equations for λ and equating the expressions, we have $\lambda = \frac{24xy^4}{p_x} = \frac{48x^2y^3}{p_y}$. We can simplify and solve for y, yielding $y = \frac{2xp_x}{p_y}$. Substituting this expression into the budget equation, we have

$$0 = M - p_xx - \frac{2xp_x}{p_y}p_y = \text{M} - 3p_x\text{x}.$$

Solving for x, we have the generalized demand function for x, $x = \frac{M}{3p_x}$. Plugging (2) into (1), we get the generalized demand for y, $y = \frac{2(\frac{M}{3p_x})p_x}{p_y} = \frac{2M}{3p_y}$.

c) Simply substitute $p_x = 2, p_y = 2,$ and $M = 24$ into the generalized demand functions from part (b), to get $x = 4$ and $y = 8$.

2) Assume a person has a utility function U = XY, and money income of Rs.10,000, facing an initial price of X of Rs.10 and price of Y of Rs.15. If the price of X increases to Rs.15, answer the following questions:

a) What was the initial utility maximising quantity of X and Y?

b) What is the new utility maximising quantity of X and Y following the increase in the price of X?

c) What is the Hicks compensating variation in income that would leave this person equally well off following the price increase? What is the Slutsky compensating variation in income?

d) Calculate the pure substitution effect and the real income effect on X of this increase in the price of X. Distinguish between the calculation of these effects using the Hicksian analysis vs. the Slutsky analysis.

Ans.:

a) There are two related approaches. Both approaches require the simple derivation of the first order condition for maximising utility subject to a budget constraint: MU(x) /MU(y) = P(x)/(P(y) i.e, Y / X = 10 / 15 = .67, or Y = .67 X and X = 1.5 Y.

Y = 333.33, and X = 1.5Y = 500

b) Y = no change; X = 333.33, a reduction of 167.67.

c) Original utility was U = XY = (333.33)(500) = 166,665. Hicksian compensating variation of income (HCV) = Rs.12,247.50 - Rs.10,000 = Rs.2,247.50.

Original M was Rs.10,000, so the Slutsky compensating variation (SCV): is much easier to calculate: At the new prices the money income required to consume the original X,Y bundle of X = 500, Y = 333.33 is simply: M = Rs.15 (500) + Rs.15 (333.33) = Rs.12,499.95.

d) If no compensating variation is actually paid, the full reduction in the consumption of X is from 500 to 333.33 or 167.67. How much of this 167.67 reduction is due to a pure substitution effect and how much is due to a real income effect.

Hicksian analysis: the movement along that original indifference curve representing the pure substitution effect is 500 – 408.25 = 91.75. Then, the remaining change in X of 408.25 – 333.33 = 74.92 is the real income effect (the result of now taking that Rs.2,247.50 away from the person, so there is a parallel shift to the left to the lower indifference curve at X = 333.33 and Y = 333.33).

Slutsky derivation of substitution and income effects: "hypothetical" increase in M of Rs.2,499.95 to M = Rs.12,499.95. The pure substitution effect related to X is 500 – 416.67 = 83.33 and the real income effect is then the "residual" of 416.67 – 333.33 = 83.34.

UNIT 3 THEORY OF DEMAND: SOME RECENT DEVELOPMENTS

Structure

3.0 Objectives

3.1 Introduction

3.2 Recent Developments in Demand Analysis: Linear Expenditure Systems

3.3 Theory of Consumer Surplus

3.4 Theory of Inter-Temporal Consumption

3.5 Elementary Theory of Price Formation: Demand-Supply Analysis

3.6 Cobweb Model

3.7 Lagged Adjustment in Interrelated Markets

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3.10 Some Useful Books

3.11 Answers or Hints to Check Your Progress Exercises

3.0 OBJECTIVES

In this unit, we will discuss some of the recent development in demand analysis. First, we will look at an important implication of utility maximisation exercise viz., linear expenditure system. Then we move on to another important theory in consumer behaviour called consumer surplus, where we introduce three different types of definition with their graphical interpretation. In the next section, we introduce a more advance theory of consumer behaviour where consumers present decision depend on her future concerns. The price determination in the market is covered next. Then we move on to explaining a dynamic model called Cobweb model, which will explain the dynamic stability property of the equilibrium of Demand-Supply analysis. Finally, we will discuss a model related to lagged adjustment in interrelated markets.

This unit will enable you to:

- determine the optimum choice of a consumer under linear expenditure system;
- evaluate consumer surplus in different markets;
- decide the optimum choice under two period analysis of consumer behavior;
- determine price under Demand-Supply analysis;
- find the nature of equilibrium under Demand-Supply analysis; and
- assess the equilibrium under lagged adjustment in interrelated markets.

3.1 INTRODUCTION

The basic theory of consumer behaviour discussed in the previous unit can be extended in many directions, and can be applied to cover optimal behaviour for

a variety of specific types of utility functions. Some of these extensions and specific applications are discussed here. In the market, prices of all goods are given to the consumers. They can't influence the price by changing their own decisions. Some times prices are also given to the individual firm i.e., in some cases, firms also are not able to charge prices that they want and have to settle with the price prevailing in the market. There ware considerable interest therefore among the economist to explain the price formation in different types of markets. The Demand-Supply analysis is the most important among them. Once the equilibrium is achieved the second most important question came to mind is the question of stability of that equilibrium. There are many approaches to determine the stability property of equilibrium. Among them Cobweb model is simplest and quite elegant in nature.

3.2 RECENT DEVELOPMENT IN DEMAND ANALYSIS: LINEAR EXPENDITURE SYSTEMS

For many years economic theorists analysed the optimal behaviour of consumers while econometricians estimated consumer demand and expenditure relations, with little communication between the two. Theorists would provide examples that were of little aid for empirical work, and econometricians would estimates relations that had little connection with the theory of utility maximisation. Fortunately, as days passed on, the gap between theory and empirical evidence has lessened, and a number of theoretically strong examples that allow empirical estimation have been developed. In this section we present one of such examples.

Consider the utility function

$$U = \alpha_1 \ln(q_1 - \gamma_1) + \alpha_2 \ln(q_2 - \gamma_2)$$

with the domain $q_1 > \gamma_1$ and $q_2 > \gamma_2$. The γ's may be interpreted as minimum subsistence quantities and are positive. The α's are also positive. Applying the positive monotonic transformation $U' = U/(\alpha_1+\alpha_2)$ we get,

$$U' = \beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2)$$

The coefficients β_1 and β_2 ($\beta_1 + \beta_2 = 1$) are called "share" parameters.

The consumer's objective is to maximise her utility subject to budget constraint. So, she will try to solve the problem given below.

Maximise $\beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2)$

Subject to $q_1 > 0$

$q_2 > 0$

$y \geq p_1 q_1 + p_2 q_2$

We set Lagrange function of the above maximisation exercise as

$$Z = \beta_1 \ln(q_1 - \gamma_1) + \beta_2 \ln(q_2 - \gamma_2) + \lambda(y - p_1 q_1 - p_2 q_2)$$

and set its first partial derivatives equal to zero (we assume interior solution of this maximisation problem):

$$\frac{\partial Z}{\partial q_1} = \frac{\beta_1}{q_1 - \gamma_1} - \lambda p_1 = 0$$

MFC-101

$$\frac{\partial Z}{\partial q_2} = \frac{\beta_2}{q_2 - \gamma_2} - \lambda p_2 = 0$$

$$\frac{\partial Z}{\partial \lambda} = y - p_1 q_1 - p_2 q_2 = 0$$

It can be easily verified that the second order condition for the maximisation is satisfied. By evaluating the above three equation one can also find out that the marginal utility of income is decreasing.

Solving the above equations for optimal quantities gives the demand functions,

$$q_1 = \gamma_1 + \frac{\beta_1}{p_1}(y - p_1\gamma_1 - p_2\gamma_2)$$

and $$q_2 = \gamma_2 + \frac{\beta_2}{p_2}(y - p_1\gamma_1 - p_2\gamma_2)$$

Multiplying the first equation of the above two demand functions by p_1 and the second by p_2 we get the expenditure functions

$$p_1 q_1 = p_1\gamma_1 + \beta_1(y - p_1\gamma_1 - p_2\gamma_2)$$

and $$p_2 q_2 = p_2\gamma_2 + \beta_2(y - p_1\gamma_1 - p_2\gamma_2)$$

which are linear in income and prices, and thus suitable for linear regression analysis.

Check Your Progress 1

1) Consider the utility function $U = q_1^{\delta_1} . q_2^{\delta_2}$. Find out the linear expenditure function.

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3.3 THEORY OF CONSUMER SURPLUS

In this section, we discuss the basic concept of consumer surplus and its derivation. A consumer normally pays less for a commodity than the maximum amount that she would be willing to pay rather than forego its consumption. Consumer surplus therefore in crude sense is the difference between what consumer willing to pay and what she actually pays. Several measures of such consumer's surplus have been proposed. We will discus three of them. Attention is limited to a consideration of the good under investigation and a composite commodity called "money", with consumption quantities of q and M respectively. Let the distance OA in Figure 3.3.1 represents the consumer's income. She achieves a tangency solution at point D on indifference curve I_2. If she were unable to consume Q, she would be at A on the lower indifference

curve I_1. She would have to be given an income increment of AB dollars to restore her to indifference curve I_2. This increment, called compensating income variation, is denoted by c, and provides a measure of consumer's surplus.

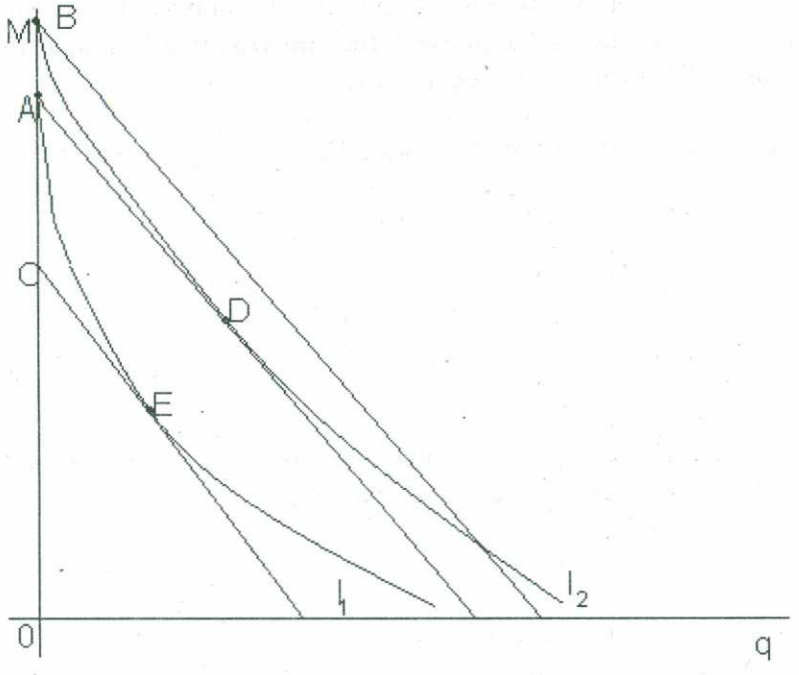


Fig. 3.1: Consumer Surplus

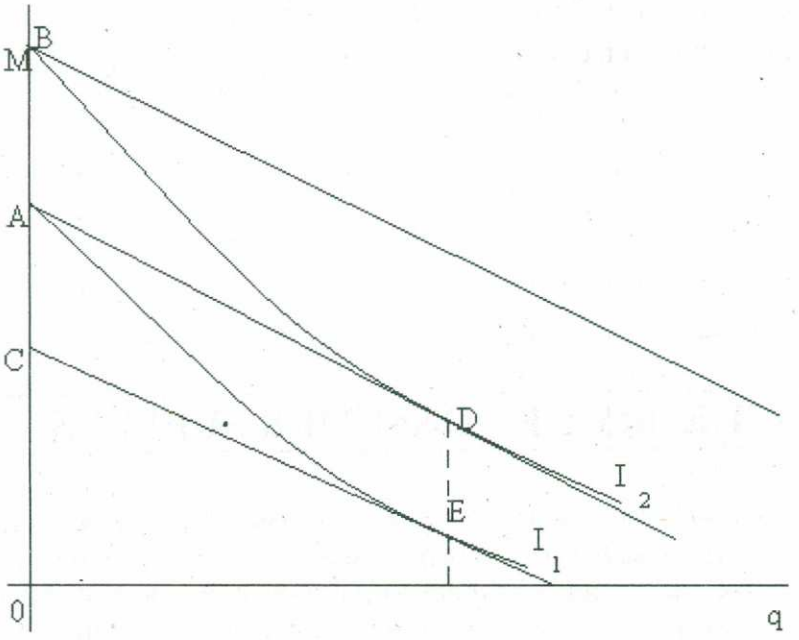


Fig. 3.2: Consumer Surplus

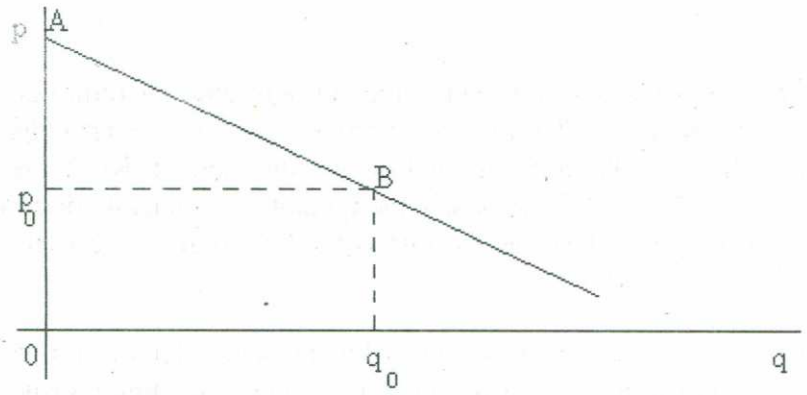


Fig. 3.3: Consumer Surplus

At the given prices, the consumer would be willing to forgo AC dollars of income rather than lose her opportunity to consume good Q. With income OC, her consumption is at E, which is on the same indifference curve as A. The amount corresponding to AC is called equivalent income variation and is denoted by e. It provides an alternative measure of consumer's surplus. A third measure is provided by the demand curve in Figure 3.3 for the price-quantity combination p_0q_0. It equals the area ABp_0, which is the difference between the area lying under the demand curve $OABp_0$ and the consumer's expenditure Op_0Bq_0, and is denoted by s.

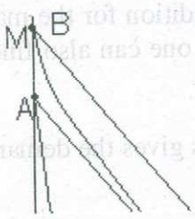
It can be shown that $c \geq s \geq e$. The strict inequalities hold for the case pictured in Figure 3.1 as a consequence of the income effect. If the consumer were to pay more to consume the good, her demand would decline because of her lower effective income, and the area under the demand curve would exceed the amount that she would pay rather than forego consumption of the good. Figure 3.2 depicts a case in which the income effect is zero throughout. A perpendicular such as the line through D and E connects points with the same marginal rate of substitution. The indifference curves are "parallel" with a constant vertical distance between a pair of indifference curves. In this case AB=AC and the three measures of consumer's surplus are the same.

3.4 THEORY OF INTER-TEMPORAL CONSUMPTION

In the previous two units, we have been concerned with choices among contemporaneous commodities. An important class of choices made by consumers, however, relates to consumption over time, that is, how one allocates income earned in different time periods to consumption. It seems that when income is earned in an uneven pattern, individuals attempt to "smooth out" their consumption through borrowing and lending. In this way, people's consumption varies less than their income.

We began this discussion by considering consumption in just two-time period. Denote the present as period 1 and the future (next year) as period 2, and consumption in period 1 and 2 as x_1 and x_2. Suppose a person earns x_1^0 in the present (this year) and x_2^0 in the future (next year). Suppose also that this individual can borrow and lend in the "capital market" at rate of interest r. What this means is any income y not spent this year can be loaned to others, in return for which the consumer receives some greater amount $y + r\,y = y(1 + r)$ next year. Alternatively, the consumer can increase present consumption by some amount y and repay $y\,(1 + r)$ next year. The opportunity cost of

curve I_1. She would have to be given an income increment of AB dollars to restore her to indifference curve I_2. This increment, called compensating income variation, is denoted by c, and provides a measure of consumer's surplus.



consuming income y this year is thus forgoing consumption of y (1 + r) next year.

The price of present consumption is thus (1 + r) units of future consumption; alternatively, the price of future consumption is (1 / (1 + r)) units of present consumption. We commonly say that the present value of Rs. Y one year from now is Rs. y / (1 + r); this is merely the quantity, y, times its price in terms of present consumption. The interest rate is the "premium for earlier availability of goods".

Wealth, W, in the present, is defined as the present value of current and future income. The consumer's budget constraint is that she cannot spend more than her wealth, i.e.,

$$x_1 + \frac{x_2}{1+r} = x_1^0 + \frac{x_2^0}{1+r} = W \quad \text{---------- (a)}$$

The consumer maximises $U(x_1, x_2)$ subject to equation (a).

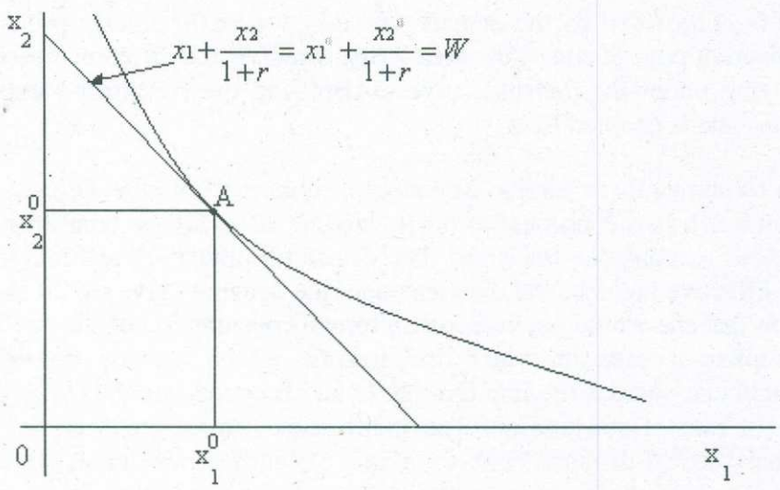


Fig. 3.4: Maximisation of utility subject to wealth constraint

Though we are using "income" and "consumption" interchangeably as arguments in the utility function, it is well to remember, as pointed out by economist I. Fisher, that "income" really consists of consuming something. "Saving" (or dissaving) is just a way of rearranging consumption over time. Income is realized when it is consumed.

The model is depicted in Figure 3.4. The budget line has slope $\frac{dx_1}{dx_2} = -(1+r)$, the price of x_1 in terms of x_2, and passes through the endowment point A, (x_1^0, x_2^0). An increase in the interest rate represents an increase in the price of the present consumption, and has the effect of rotating the wealth constraint clockwise through A.

The Lagrange function for this problem is

$$L = U(x_1, x_2) + \lambda[(x_1^0 - x_1) + \frac{(x_2^0 - x_2)}{(1+r)}] \quad \text{---------- (b)}$$

assuming strict interior solution, producing the first order conditions

$$\frac{\partial L}{\partial x_1} = \frac{U(x_1, x_2)}{\partial x_1} - \lambda = 0 \quad \text{---------- (b.1)}$$

$$\frac{\partial L}{\partial x_2} = \frac{U(x_1, x_2)}{\partial x_2} - \frac{\lambda}{(i+r)} = 0 \quad \text{---------- (b.2)}$$

and the constraint

$$\frac{\partial L}{\partial \lambda} = x_1 + \frac{x_2}{1+r} = x_1^0 + \frac{x_2^0}{1+r} = 0 \quad \text{---------- (b.3)}$$

Combining equations (b.1) and (b.2) yields

$$\frac{\dfrac{U(x_1, x_2)}{\partial x_1}}{\dfrac{U(x_1, x_2)}{\partial x_2}} = (1+r) \quad \text{---------- (c)}$$

Equation (c) says that the consumer's marginal value of present consumption, U_1/U_2 (where $U_1 = \frac{U(x_1, x_2)}{\partial x_1}$ and $U_2 = \frac{U(x_1, x_2)}{\partial x_2}$), equals the opportunity cost of present consumption, in terms of future consumption forgone. It will simplify the algebra if we let $p = \frac{1}{(1+r)}$, the price of future consumption. Assuming the sufficient second-order conditions hold, the first-order conditions can be solved for the Marshallian demand functions

$$x_1 = x_1^M (p, x_1^0, x_2^0)$$

and, $$x_2 = x_2^M (p, x_1^0, x_2^0)$$

We can gain greater insight into the model by deriving the Slutsky equation, separating out the substitution effect and the wealth (income) effect.

The Hicksian demands can be derived minimising the endowment in either period so as to achieve some arbitrary indifference level U^0. We can therefore state the model as

Minimise

$$x_1^0 = x_1 + p(x_2 - x_2^0)$$

subject to

$$U(x_1, x_2) = U^0$$

The Lagrange function for this problem is then

$$L = x_1 + p(x_2 - x_2^0) + \lambda(U^0 - U(x_1, x_2))$$

Assuming the first and sufficient second-order conditions hold, the implied first-order equations can be solved for the Hicksian demands

$$x_1 = x_1^U(p, U^0)$$

and $$x_2 = x_2^U(p, U^0)$$

Substituting these demands into the objective function produces a minimum "expenditure" type of function

$$x_1^*(p, U^0) = x_1^U + p(x_2^U - x_2^0)$$

The fundamental identity linking between the Marshallian and Hicksian demands is therefore

$$x_i^U(p, U^0) \equiv x_i^M(p, x_1^*(p, U^0), x_2^0) \text{ for i = 1, 2,}$$

producing the famous Slutsky equation

$$\frac{dx_i^M}{dp} \equiv \frac{\partial x_i^U}{\partial p} + (x_2^0 - x_2^U)\left(\frac{\partial x_i^M}{\partial x_i^0}\right)$$

If the interest rate increases, the price of future consumption, p, decreases. This produces a pure substitution effect towards less present and greater future consumption: $\frac{\partial x_2^U}{\partial p} < 0$. However, a change in the interest rate produces an attendant wealth effect. An increase in the endowment of present income is the same as an increase in wealth from any source, since income can be traded back and forth across time periods. Assume that consumption in both time periods enters the utility function as normal goods, so that $\frac{\partial x_i^M}{\partial x_1} > 0$. The income or, more properly, the wealth term on the right hand side of the Slutsky equation, indicates that if, for example, the consumer is a net borrower in period 1, so that $(x_2^0 - x_2^U) < 0$, the substitution effect will be reinforced by the wealth effect. In this case, an increase in the interest rate, in addition to making present consumption relatively more expensive, also lowers the consumer's wealth, producing an additional reduction in present consumption. If the individual is a net lender in period 1, the wealth and substitution effects oppose one another: an increase in the interest rate raises present wealth and leads to greater present consumption.

Check Your Progress 2

1) Suppose utility function of the consumer is $U(x_1, x_2) = x_1^{0.5} x_2^{0.5}$. Consumer lives two periods. In period 1 her income in Rs 1000 i.e., $x_1^0 = 1000$ and in period two she has no income i.e., $x_2^0 = 0$. The market rate of interest is 50%. Find out the optimum consumption in each period.

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3.5 ELEMENTARY THEORY OF PRICE FORMATION: DEMAND-SUPPLY ANALYSIS

In this section, we discuss the elementary theory of price formation. Demand curve in the market is derived from the aggregate consumer demand and supply curve is derived from the aggregate firms supply. Since market demand curve for a good is the sum total of demand for that good of all individual consumers and since demand curve for a good for an individual consumer is derived from its utility maximisation, so along the demand curve consumer's optimizing behaviour is always fulfilled. That means each point on the demand curve represents that consumers are willing to purchase the corresponding demand quantity with corresponding price.

We consider perfect competition prevail in the market. In short, run perfectly competitive supply curve of a commodity in the market or industry is determined from supply curve of an individual firm, where supply curve of commodity of individual firm is derived from profit maximising objective of that firm. Hence, along the market supply curve-optimising behaviour of the firm is fulfilled. That means each point on the market supply curve represents that firms are willing to supply the corresponding supply quantity with corresponding price.

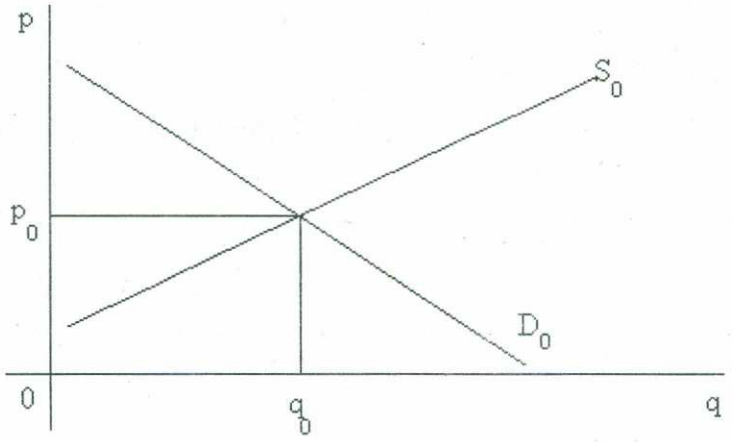

Fig. 3.5: Demand-Supply Equilibrium

Clearly, at the point of intersection between market demand and supply curve, exchange will take place between consumers and producers, as both of them simultaneously fulfilled their optimizing behaviour. Corresponding price and aggregate quantity are short run equilibrium price (say p_0) and aggregate quantity (say q_0) respectively, which is shown in Figure 3.5.

The process of adjustment of short run equilibrium of a competitive market takes place in the following way. Generally, by adjusting price of the commodity equilibrium in short run perfect competition is achieved as given below:

It is assumed that for any excess demand (or excess supply) prices will increase (or decrease). According to this behaviour of the market, price adjustment in disequilibrium will take place by a mechanism, which is known as auctioneer mechanism.

Suppose there is an invisible referee who controls the market price according to the above behavioural assumptions. Producers supply their quantity on the basis of existing market price. Suppose, the referee initially specifies a particular price on the basis of which producers and consumers specify their supply and demand respectively. Then suppose the referee observed that supply quantity is larger than the demand quantity i.e., we have excess supply of the commodity.

If producers fail to supply their entire supply quantity at the existing price, then according to the behavioural assumption, the referee specifies a lower price of that commodity. Producers will be discouraged and will supply lower quantity and consumers will be encouraged and will demand higher quantity. Thus, in both ways excess supply of the commodity decreases. Suppose it is observed that demand quantity is larger than the supply quantity in aggregate. We have excess demand for that commodity. The referee again specifies a larger price level. Hence, producers will increase their supply and consumers will decrease their demand. Thus, in both ways excess demand for that commodity goes down. This process will continue till the referee specifies a particular price at which corresponding demand and supply quantities are equal. That means supply quantity offered by the producer is demanded by the consumer at the corresponding price. So, both consumer and producer fulfilled their optimising behaviour simultaneously. The exchange of commodity will take place at this price and quantity. These price and quantity are the equilibrium price and quantity in the market respectively.

Check Your Progress 3

1) Describe the way of converging a disequilibrium point to equilibrium in a perfectly competitive market.

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3.6 COBWEB MODEL

Concept of dynamic stability: A market equilibrium is said to dynamically stable only when disequilibrium price and quantity move and over time reach to any equilibrium, otherwise it is dynamically unstable. Movement of price and quantity in disequilibrium over time depends on behaviour of the market. Hence, analysis of dynamic equilibrium and stability depends on behavioural assumption of the market. Dynamic analysis is of two types. Here we consider discrete time analysis. Below we discuss Cobweb model in details.

Cobweb model analyses dynamic equilibrium and stability of a competitive market with the following behavioural assumptions:

- We consider linear demand and supply functions for simplicity.
- Both demand and supply functions depend on time, where time is a discrete matter (in that sense it is a dynamic model).
- Supply quantity at any time, t, depends on previous period's price and

$S_t = c + d P_{t-1}$ ---------- (i)

But demand quantity at any time, t, depends on the price at that time

$D_t = a + b P_t$ ---------- (ii)

That means demand quantity is instantaneously determined at the existing price but there is a lag in supply quantity with respect to price since supply of output requires some amount of time.

- Behaviour of the market is such that as soon as supply quantity comes into the market, entire quantity is demanded at that period by adjusting price so that market is clear in each period. Thus, at any time, t, $S_t = D_t$ --------(iii)

Since demand and supply functions are linear, a, b, c and d are constant. On the basis of these we now analyze dynamic equilibrium and stability.

At equilibrium of the market $D_t = S_t$ holds where price doesn't change over time (i.e., $P_t = P_{t-1} = P_e$, where P_e is the equilibrium price). Hence, at equilibrium

$$a + b P_e = c + d P_e$$

or, $(a-c) = (d-b) P_e$

or, $$P_e = \frac{(a-c)}{(d-b)}$$

This is the equilibrium price, which is assumed to be positive such that initially equilibrium exists. Now we find the time path of price from behavioural assumptions.

$$D_t = S_t$$

or, $a + b P_t = c + d P_{t-1}$

or, $b P_t - d P_{t-1} = (c-a)$

or, $$P_t - \frac{d}{p} P_{t-1} = \frac{(c-a)}{b}$$ ---------- (iv)

The time path of price is a first order linear non-homogeneous difference equation. Its solution consists of a particular solution (P_p) and a complementary solution (P_c), viz.,

$$P_t = P_p + P_c$$ ---------- (v)

To find out P_p we put $P_t = P_{t-1} = \bar{P}$ (say) into the equation (iv), and have

$$\bar{P}(1 - \frac{d}{b}) = \frac{c-a}{b}$$

or, $$\bar{P}(\frac{b-d}{b}) = \frac{c-a}{b}$$

or, $$\bar{P} = \frac{a-c}{d-b} = P_e$$

Since P_e exists, P_p also exists and it is equilibrium price and independent of time.

To find out P_c, we put $P_t = x^t$ in the homogeneous part of the difference equation (equation – (iv)), and write

$$x^t - \frac{d}{b}x^{t-1} = 0$$

or, $$\frac{x^t}{x^{t-1}} - \frac{d}{b}\frac{x^{t-1}}{x^{t-1}} = 0$$

or, $$x = \frac{d}{b},$$

which is the characteristic root. Therefore, $P_c = m\,x^t$, where m is any unknown integral constant. From equation (v) we get

$$P_t = P_e + m\,x^t \text{ ---------- (vi)}$$

giving the general solution.

Suppose at initial time, $t = 0$, $P_t = P_0$ is known. Then from equation (vi), we have,

$$P_0 = P_e + m\,x^0$$

or, $$P_0 = P_e + m$$

or, $$m = P_0 - P_e,$$

So the definite solution of time path of price is

$$P_t = P_e + (P_0 - P_e)(\frac{d}{b})^t \text{ ---------- (vii)}$$

Here P_0, P_e, d and b all are known and for each level of time price can be determined from equation (vii).

If $P_t \to P_e$ as $t \to \infty$ (i.e., if actual price, P_t, moves towards equilibrium price and overtime reaches to P_e), then market equilibrium is dynamically stable (otherwise it is dynamically unstable). This requires $(\frac{d}{b})^t \to 0$ as $t \to \infty$ (since then only $P_t \to P_e$ as $t \to \infty$ where $P_0 - P_e$ is constant and does not change over time). Now $(\frac{d}{b})^t \to 0$ as $t \to \infty$ only when

$$|\frac{d}{b}| < 1$$

or, $$|\frac{1}{b}| < |\frac{1}{d}| \text{ ---------- (viii)}$$

This is the condition for dynamic stability (as in this case we have convergence to equilibrium from any disequilibrium over time). That means absolute slope of the demand curve $|\frac{dP_t}{dD_t}| = |\frac{1}{b}|$ should be lower than the absolute slope of the supply curve $|\frac{dP_t}{dS_t}| = |\frac{1}{d}|$, i.e., demand curve should be flatter than supply curve for dynamic stability.

When $|\frac{d}{b}|>|$ then we have divergence from equilibrium over time (from equation (vii)) we have dynamically unstable equilibrium, and at that situation $|\frac{1}{b}|>|\frac{1}{d}|$ i.e., demand curve is steeper than supply curve.

When $|\frac{d}{b}|=1$, we have neither convergence to equilibrium nor divergence from equilibrium (from equation (vii)). So it is also dynamically unstable.At that situation $|\frac{1}{b}|=|\frac{1}{d}|$ i.e., slope of the demand curve is equal to the slope of the supply curve in absolute sense.

If $(\frac{d}{b})>0$, then we have monotonic time path of price from equation (vii). If $(\frac{d}{b})<0$, then we have cyclical time path of price from equation (vii).

Case 1: Suppose demand curve is downward sloping and supply curve is upward sloping i.e., d>0 and b<0, therefore $(\frac{d}{b})<0$.

Case A: If $|\frac{1}{b}|<|\frac{1}{d}|$, then we have cyclical convergence to equilibrium. It's a dynamically stable equilibrium.

Case B: If $|\frac{1}{b}|=|\frac{1}{d}|$, then we have regular cycle around equilibrium. It's a dynamically unstable equilibrium.

Case C: If $|\frac{1}{b}|>|\frac{1}{d}|$, then we have cyclical divergence from equilibrium. It's a dynamically unstable equilibrium.

Case 2: Suppose demand curve and supply curve are upward sloping i.e., d>0 and b>0 hence $(\frac{d}{b})>0$.

Case D: If $|\frac{1}{b}|<|\frac{1}{d}|$ then we have monotonic convergence to equilibrium. It's a dynamically stable equilibrium.

Case E: If $|\frac{1}{b}|=|\frac{1}{d}|$ then demand and supply curve are coincide if their intercepts are also same then we have infinite number of equilibrium and there is no need for dynamic stability analysis or demand and supply curve are parallel to each other (when intercepts are not equal) then equilibrium does not exist. Hence, there is no need for stability analysis in this case.

Case F: If $|\frac{1}{b}|>|\frac{1}{d}|$ then we have monotonic divergence from equilibrium. It's a dynamically unstable equilibrium.

Case 3: Suppose demand curve is downward sloping and supply curve is upward sloping i.e., d<0 and b<0, therefore $(\frac{d}{b}) > 0$.

Case G: If $|\frac{1}{b}| < |\frac{1}{d}|$ then we have monotonic convergence to equilibrium. It's a dynamically stable equilibrium.

Case H: If $|\frac{1}{b}| = |\frac{1}{d}|$ then demand and supply curve are coincide if their intercepts are also same then we have infinite number of equilibrium and there is no need for dynamic stability analysis or demand and supply curve are parallel to each other (when intercepts are not equal) then equilibrium does not exist. Hence, there is no need for stability analysis in this case.

Case I: If $|\frac{1}{b}| > |\frac{1}{d}|$ then we have monotonic divergence from equilibrium. It's a dynamically unstable equilibrium.

Check Your Progress 4

1) In Cobweb model under what condition equilibrium is dynamically stable when there is linear downward sloping demand and linear upward sloping supply curve.

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3.7 LAGGED ADJUSTMENT IN INTERRELATED MARKETS

Here we discuss the Walrashian dynamic stability analysis of a competitive commodity market. Let us consider the following assumptions:

- initially equilibrium exists short run competitive commodity market
- both demand and supply function are linear and depend on time where time is discrete.
- both demand and supply at any time, t, depend on price at that time and demand and supply functions are respectively given by

  $D_t = a + b P_t$ ---------- (i)

  $S_t = c + d P_t$ ---------- (ii)

- behaviour of the market is such that for positive (or negative) excess for commodity price rises (or falls) in next period. This can be captured by the following equation:

  $(P_t - P_{t-1}) = K E (P_{t-1})$ ---------- (iii)

  where $E (P_{t-1}) = (D_{t-1} - S_{t-1}) = a + b P_t - c - d P_{t-1} = (a-c) + (b-d) P_{t-1}$ --- (iv)

Note that (iv) is the excess demand at time (t-1) and K>0. For simplicity, K is taken as a constant and it represents the speed of adjustment of price.

[Note: At time (t-1) if E $(P_{t-1})>0$, then price should increase in the next period according to the behavioural assumption i.e., $P_t>P_{t-1}$ i.e., $(P_t - P_{t-1})>0$ which is captured by equation (iii) with the restriction that K>0]

At equilibrium $P_t = P_e$ at which $D_t = S_t$ holds. Therefore,

$$a + b P_e = c + d P_e$$

or, $(a-c) = (d-b) P_e$

or, $P_e = \frac{(a-c)}{(d-b)}$ [on the assumption that initial equilibrium exist where price must be positive].

Now we analyse whether the equilibrium price (P_e) is dynamically stable or not. For that we find out the time path of price from the behavioral assumption of the market viz.,

$$(P_t - P_{t-1}) = K E (P_{t-1}) = K [(a-c) + (b-d) P_{t-1}]$$

or, $(P_t - P_{t-1}) = K (a-c) + K(b-d) P_{t-1}$

or, $P_t - [1 + K (b-d)] P_{t-1} = K(a-c)$ –(v)

The time path of price is represented by the first order linear non-homogeneous difference equation since by assumption a, b, c, d and K are constants. Solution of the time path of price (P_t) consists of a particular solution (P_p) and a complementary solution (P_c). Thus,

$$P_t = P_p + P_c \text{ ---------- (vi)}$$

To find out P_p, we put, $P_t = P_{t-1} = P'$ (say), into the difference equation of price and get

$$P' [1 - \{1 + K (b-d)\}] = K (a-c)$$

or, $P' (d-b) = (a-c)$

or, $P' = P_e = \frac{(a-c)}{(d-b)} > 0$ since $P_e>0$.

This is the particular solution; note that it is nothing but inter-temporal equilibrium price, which is a constant and independent of time.

[**Note:** In fact, here the inter-temporal price is equal to static equilibrium price where equilibrium does not change over time].

To find out P_c, we put $P_t = x^t$, into the homogeneous part of the difference equation of price and get

$$x^t - x^{t-1}[1+K(b-d)] = 0$$

$$\frac{x^t}{x^{t-1}} - \frac{x^{t-1}}{x^{t-1}}[1+K(b-d)] = 0$$

$$x = [1+K(b-d)]$$

It is the characteristic root. Therefore, complementary solution is $P_c = m\ x^t$, where m is any unknown integral constant. So from equation (vi) we have

$$P_t = P_p + m\ x^t \text{ ---------- (vii)},$$

which is the general solution.

Suppose at t = 0 (i.e., at initial time) $P_t = P_0$ is given. Hence, from equation (vii) we get

$$P_0 = P_e + m\ x^0$$

or, $$m = P_0 - P_e$$

Therefore, definite solution of the time path of price is

$$P_t = P_p + (P_0 - P_e)\ [1 + K\ (b\text{-}d)]^t \text{ ---------- (viii)}$$

When $P_t \to P_e$ as $t \to \infty$, we have convergence to equilibrium from any disequilibrium price over time. So equilibrium is dynamically stable in Walrashian sense, which requires that

$$|1+K\ (b\text{-}d)| < 1$$

When [1 + K (b-d)] >0, we have a monotonic time path of price. Alternatively, we get a monotonic convergence to equilibrium over time when 0 < [1 + K (b-d)] < 1. Therefore,

$$K\ (b\text{-}d) < 0 \text{ or, } (b\text{-}d) < 0 \text{ or, } b<d,$$

which automatically holds when demand curve is downward sloping [i.e., b<0] and supply curve is upward sloping [i.e., d>0]. Equilibrium is dynamically unstable when demand curve is upward sloping and supply curve is downward sloping (as b>0 and d<0 so b<d does not hold)

When both demand and supply curves are upward sloping, the Walrashian dynamic stability requires b<d or, $(\frac{1}{b}) > (\frac{1}{d})$. That means $|\frac{1}{b}| > |\frac{1}{d}|$ [since d>0 and b>0 and price is measured in vertical axis]. So, the supply curve is flatter than the demand curve. Otherwise, it would be dynamically unstable.

When both demand and supply curves are downward sloping, the Walrashian dynamic stability requires b<d or $(\frac{1}{b}) > (\frac{1}{d})$. That means $|\frac{1}{b}| < |\frac{1}{d}|$ [since d<0 and b<0]. Hence, the demand curve should be flatter than the supply curve. Otherwise, it is dynamically unstable.

Check Your Progress 5

1) Derive the time path of a disequilibrium point under Walrashian dynamic stability analysis.

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3.8 LET US SUM UP

In this unit, we learn some advance topics in consumer behaviour theory. We started with the theory of linear expenditure. It gives a demand function from which one can derive the expenditure function by minimising the expenditure subject to a given level of utility. The special features of this theory is the curvature of the expenditure function, which turns out to be linear. Linear expenditure function is extremely useful particularly when some one want to do an empirical study on utility maximisation. The usefulness of the linear expenditure system emanates from the fact that ordinary least square estimation needs linear function to estimate. Next, we move on to the theory of consumer surplus. Consumer surplus is broadly defined as the difference between what consumer willing to pay for a particular commodity bundle and what she actually pays for that commodity bundle. Here we discussed three different ways of finding out consumer surplus and their relationship to each other. In the next section, we analysed the theory of inter-temporal consumption. This theory is more realistic than the previous theory of consumer behaviour because in it, the consumer lives in two periods and there is a notion of saving. The theory can be easily upgraded to multi-period decision making under certainty. Elementary theory of price formation under demand-supply analysis is the next theme that was discussed. The concept of demand and supply curve was introduced, where the demand curve was derived from utility maximising objective of the consumer and the supply curve from the profit maximising objective of the firm. Therefore, in the intersecting point between demand and supply curve both firm and consumer fulfill their objective. The market equilibrium point was thus presented. In the next section, we moved on to the dynamic stability analysis part and introduced the Cobweb model. A model for Walrashian dynamic stability analysis was taken up subsequently. In the Cobweb model, we derived the nature as well as the stability part of the equilibrium under different demand and supply curves. When demand curve was downward sloping and supply curve was upward sloping, the equilibrium was stated to be dynamically stable if absolute slope of the demand curve is less than the absolute slope of the supply curve (i.e., demand curve is relatively flatter than the supply curve). In the last section, we brought in a model of lagged adjustment in interrelated markets and discussed the Walrashian dynamic stability. In this model, we observed that under standard case (i.e., demand curve is downward sloping and supply curve is upward sloping) equilibrium was dynamically stable if slope of the demand curve was greater than the slope of the supply curve, which indeed happened for standard cases.

1.9 KEY WORDS

Linear Expenditure Systems : A general linear formulation of demand and algebraically imposed theoretical restrictions of additivity, homogeneity, and symmetry.

Consumer Surplus : Defined as the difference between the total amount that consumers are willing and able to pay for a good or service and the total amount that they actually do pay.

Inter-Temporal Consumption : Explain people's preferences in relation to consumption and saving over the course of their lives.

Cobweb Models	:	Explain irregular fluctuations in prices and quantities that may appear in some markets.
Interrelated Markets	:	Relationship that exists when the values of related variables move in the markets.
Walrashian Dynamic Stability	:	Equilibrium was dynamically stable if slope of the demand curve was greater than the slope of the supply curve.

3.10 SOME USEFUL BOOKS

Koutsoyiannis, A. (1979), *Modern Microeconomics*, Second edition, London: Macmillian.

Varian, Hal (1992), *Microeconomic Analysis*, W.W. Norton & Company, Inc., New York

3.11 ANSWERS OR HINTS TO CHECK YOUR PROGRESS EXERCISES

Check Your Progress 1

1) **Hint:** Set up the Lagrange function and solve the first order condition, assuming interior solution exists.

$L = q_1^{\delta_1}.q_2^{\delta_2} + \lambda(y-p_1q_1-p_2q_2)$

Check Your Progress 2

1) **Hint:** The Lagrange function of the problem is

$L = x_1^{0.5} x_2^{0.5} + \lambda[(1000-x_1)-(1/1.5)x_2]$.

The first-order conditions are

$0.5x_1^{-0.5}x_2^{0.5}-\lambda=0$ – (a)

$0.5x_1^{0.5}x_2^{-0.5}-(1/1.5)\lambda=0$ – (b)

$1000-x_1=(1/1.5)x_2$ – (c)

Dividing equation (a) by (b) we get

$x_2/x_1 = 1.5$ – (d)

Solving equation (c) and (d) we get the optimum consumption

$x_1^*=500$ and $x_2^*=750$

Check Your Progress 3

1) See Section 3.5

Check Your Progress 4

1) See Section 3.6 Case 1
2) See Section 3.6 Case 3
3) See Section 3.6 Case 2

Check Your Progress 5

1) See Section 3.7

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