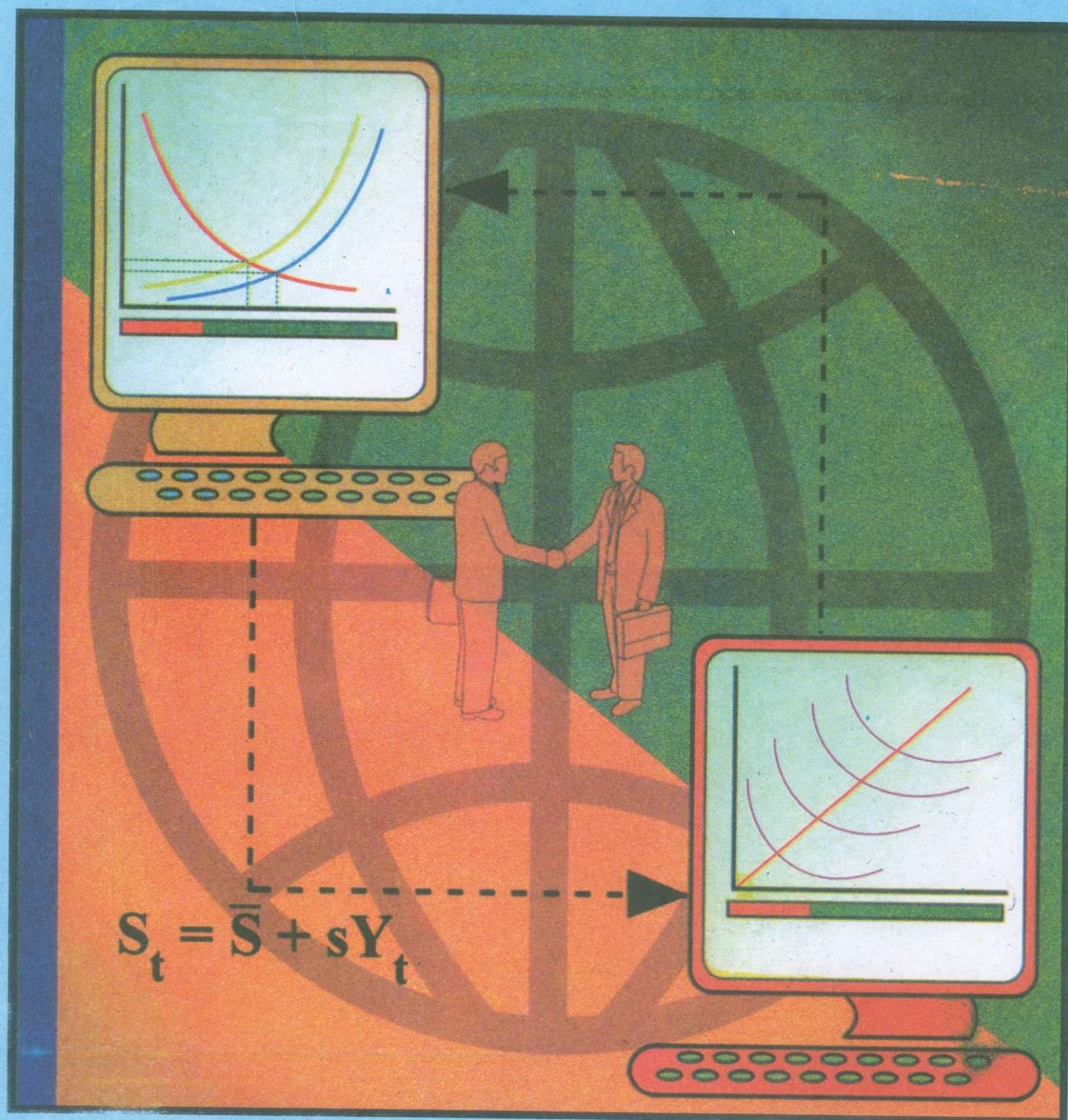




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MEC - 101
MICROECONOMIC
ANALYSIS



General Equilibrium

4

“शिक्षा मानव को बन्धनों से मुक्त करती है और आज के युग में तो यह लोकतंत्रा की भावना का आधार भी है। जन्म तथा अन्य कारणों से उत्पन्न जाति एवं वर्गगत विषमताओं को दूर करते हुए मनुष्य को इन सबसे ऊपर उठाती है।”

- इन्दिरा गाँधी

“Education is a liberating force, and in our age it is also a democratising force, cutting across the barriers of caste and class, smoothing out inequalities imposed by birth and other circumstances.”

- Indira Gandhi

Block

4

GENERAL EQUILIBRIUM**UNIT 11****General Equilibrium: Pure Exchange Model** **5****UNIT 12****General Equilibrium with Production** **27**

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BLOCK 4 GENERAL EQUILIBRIUM

Introduction

In this block we discuss general equilibrium analysis, which considers functioning of all markets simultaneously. Such a framework takes into account the feedback effects of other markets on the market being studied. See that this is in contrast to the method of partial equilibrium analysis, which examines the determination of price-quality combination in a market assuming that related markets are unaffected. The conditions of equilibrium are studied in a pure exchange economy in **Unit 11**. We present the discussion of Walrasian formulation of general equilibrium and spell out its solution using excess demand functions. It is highlighted that efficient allocation of resources is ensured in the economy in equilibrium under certain conditions. The next unit (**Unit 12**) extends the preceding discussion by including production into the model. We have shown that a free market in equilibrium will simultaneously satisfy technical, allocative and product-mix efficiencies. Moreover, two important theorems, in the presence of trade, are discussed in the general equilibrium framework before concluding the theme.

REPORT ON THE PROGRESS OF THE WORK

1890

The work of the year has been devoted to the study of the history of the Church in the United States, and to the preparation of a history of the Church in the United States, from the first settlement of the Pilgrims to the present time. The work has been divided into two parts, the first part dealing with the history of the Church in the United States, and the second part dealing with the history of the Church in the United States, from the first settlement of the Pilgrims to the present time. The work has been divided into two parts, the first part dealing with the history of the Church in the United States, and the second part dealing with the history of the Church in the United States, from the first settlement of the Pilgrims to the present time.

UNIT 11 GENERAL EQUILIBRIUM: PURE EXCHANGE MODEL

Structure

- 11.0 Objectives
- 11.1 Introduction
- 11.2 A Pure Exchange Economy
 - 11.2.1 Description of the Economy
- 11.3 Walrasian Equilibrium
 - 11.3.1 Excess Demand Functions
 - 11.3.2 Existence of Equilibrium Prices
- 11.4 Brouwer's Fixed Point Theorem
- 11.5 Mechanism for Attaining Walrasian Equilibrium
 - 11.5.1 Stability and Uniqueness
- 11.6 Competitive Equilibrium and Pareto Efficiency
 - 11.6.1 Edgeworth Box
 - 11.6.2 Pareto Optimal Allocations
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- 11.11 Exercises

11.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the usefulness of general equilibrium for an economy;
- appreciate the Walrasian formulation and solution of general equilibrium; and
- evaluate the efficiency of competitive equilibrium and welfare implications.

11.1 INTRODUCTION

Addressing the consumer and producer's objectives in the preceding analysis, we have considered only so-called partial equilibrium. Note that the attribute "partial" refers to looking at an equilibrium result in one market for a particular good only. It is just like a scenario you have worked out where the increased demand for agricultural products due to an increased income only and nothing has happened to other activities in the agricultural sector. That is to say, the impact of changed demand on markets such as inputs and employment has not come into effect. See that these other markets will also experience the change, which has been overlooked by us.

A model that includes the interdependencies of all the markets in the economy can account for the fact that if the equilibrium price in one market changes, the equilibrium prices and hence quantities in other markets are also affected. To understand such dimensions, we need a model that can accommodate the interactions of all market simultaneously and determine the properties of

equilibria in all the markets. We have to develop a general equilibrium model, in contrast to the partial equilibrium models used thus far.

11.2 A PURE EXCHANGE ECONOMY

11.2.1 Description of the Economy

Let us consider a pure exchange model where no production takes place. Consumers have initial bundles of goods, initial endowments. They exchange with each other these goods according to their preferences. For example, you have quantities of apples and I have oranges. We enter into an exchange, your apple and my oranges. Note that the exchange to take, you must be willing to consume my oranges and I am willing to consume your apples.

We can think of n consumers and k commodities in the economy. Each consumer has initial endowments and preferences. Whereas endowment refers to the commodity held by a consumer, her preferences are represented by a utility function $u_i = u_i(x_i^1, x_i^2, x_i^j, \dots, x_i^k)$ where $x_i = (x_i^1, x_i^2, x_i^j, \dots, x_i^k)$ is the i^{th} individual's consumption bundle.

We introduce a price system P such that $P = (P_1, P_2, \dots, P_k)$. Note that the economy you are presented with, does required payment terms of money as people trade one good for another (exchange in barter system). But the price we intend to use is for the exchange rates. For example, the price of one unit of good X is one unit good Y . Such a price, therefore, can be called relative price of good X . If the price of Potato is Rs. 5 per kg and the price of apple is Rs. 2.5 per kg, then the relative price of Potato in terms of apple is 2 (i.e., each unit of Potato is worth 2 units of apple). Similarly, the relative price of apples in terms of Potato is 0.5 (i.e., one unit of apple is worth half unit of Potato). Remember that we will use relative price in the following analysis. Imagine that the consumer i purchases x_i^j units of good j at price P_j . Then $P_j X_i$ gives the amount of expenditure incurred by her and to that extent her income stands reduced. On the other hand, when she delivers goods of equal quantity, the income $P_j x_i^j$ is added to her income.

To arrive at the equilibrium of the model, let us start with consumer's utility maximisation. See that the i^{th} consumer maximises

$U_i(X_i)$ subject to her budget constraint $PX_i = PW_i$. Remember that solution to this problem yields the demand functions $X_i = X_i(P, PW_i)$, $i = 1, 2, \dots, n$ and demand for each commodity depends on *all* prices and the initial endowment.

11.3 WALRASIAN EQUILIBRIUM

A solution to consumer's utility maximisation problem when we take prices as given yields the demand function for a consumer. In the equilibrium the aggregate demand cannot exceed the endowment. The *Walrasian Equilibrium* refers to a pair of price and consumption bundle (P^*, X^*) such that

$$X_i^* = X_i(P^*, P^*W_i) \text{ and}$$

$$\sum_{i=1}^n X_i^* \leq \sum_{i=1}^n W_i$$

If we assume that all markets are perfectly competitive and consumers participating there are price takers, then Walrasian equilibrium is called a *competitive equilibrium*.

11.3.1 Excess Demand Functions

Define the excess demand function

$$ED_i(P) = D_i(P) - S_i. \text{ That is,}$$

$$ED(P) = \sum_{i=1}^n X_i(P, P.W_i) - \sum_{i=1}^n W_i \text{ where}$$

X_i 's are demand functions for the i^{th} individual.

Making use of this notation, the equilibrium conditions can be written as

$$ED_i(P^*) = D_i(P^*) - S_i = 0.$$

This condition states that at the equilibrium prices, excess demand is to be equal to zero in all markets. Two interesting results immediately follow:

- 1) If there is equilibrium in the market for $n-1$ then in Walrasian equilibrium, the remaining market will also be in equilibrium.
- 2) One can only solve for the relative prices in the model. Attempts to solve for absolute prices require adoption of normalisation. This would involve making additional assumption of one of the prices is equal to 1. Another form of commonly employed normalisation is to assume that

$$P'_i = 1 \text{ with } P'_i = \frac{P_i}{\sum_{i=1}^n P_i}.$$

Several interesting features emerge from such a formulation:

- 1) The aggregate excess demand functions (and demand functions) is *homogenous of degree zero* in all prices. That is to say, if all prices were to double, the quantity demanded of every good would remain unchanged.
- 2) Demand functions are continuous. If prices were to change by only a small amount, quantities demanded would change by only small amount.
- 3) n excess demand functions are not independent of one another and the equations are related by the formula,

$$\sum_{i=1}^n P_i \cdot ED_i(P) = 0$$

This formulation is called *Walras' Law*. It states that the total value of excess demand is zero at any set of prices. There can be neither excess demand for all goods together nor excess supply. To prove this, take

$$P \cdot ED(P) = P \sum_i X_i(P, P.W_i) - \sum_i P W_i$$

$$= P \sum_i X_i(P, P.W_i) - P \sum_i W_i$$

$$= \sum_i P X_i - \sum_i P W_i$$

$$= 0 \text{ by budget constraint of } i^{\text{th}} \text{ individual.}$$

- 4) If for some price system P , we get all prices to be strictly positive and $(k-1)$ markets clear, then the k^{th} market also clears.

By Walras' law we have

$$[P_1 ED_1(P) + P_2 ED_2(P) + \dots + P_{R-1} ED_{R-1}(P)] + P_K ED_R(P) = 0.$$

If the first $k-1$ markets clear, then $ED_1(P) = ED_2(P) = \dots = ED_{R-1}(P) = 0$.

Hence, $P_K ED_K(P) = 0$. If $P_K > 0$, then $ED_R(P) = 0$, i.e., the k^{th} market also clears.

- 5) If a commodity is in excess supply in Walrasian equilibrium, then its price must be zero in equilibrium. That is, if (P^*, X^*) is a WE and $ED_g(P^*) < 0$, then $P_g^* = 0$.

Proof: Since (P^*, X^*) is a WE,

$$ED(P^*) \leq 0 \text{ and } P^* \geq 0, \text{ where } 0 \text{ denotes the vector } (0, 0, 0, \dots).$$

Consequently, for $P_j^* ED_j(P^*) < 0$ for all j .

However, by Walras law, $P^* ED(P^*) = 0$. If $ED_g(P^*) < 0$ and $P_g^* > 0$, then $P_g^* ED_g(P^*) < 0$.

We will have $P^* ED(P^*) < 0$ as all other terms $P_j^* ED_j(P^*) < 0$. This Contradicts the Walras law.

Example 1

Suppose there are two people (A and B), two goods (F and S), with initial endowment $(\omega_A^F, \omega_A^S), (\omega_B^F, \omega_B^S)$. A feasible allocation $X = (F_A, S_A), (F_B, S_B)$.

If prices are P_F and P_S , write the budget equations in terms of relative price.

The budgets are:

$$\text{A: Agent 1} \quad P_F F_A + P_S S_A = P_F \omega_A^F + P_S \omega_A^S$$

$$\text{B: Agent 2} \quad P_F F_B + P_S S_B = P_F \omega_B^F + P_S \omega_B^S$$

Since there is no money, we need to use relative prices. Let us express the prices of goods in terms of the second commodity S. Dividing both the budgets by P_S , we get

$$\text{Agent 1} \quad \frac{P_F}{P_S} F_A + S_A = \frac{P_F}{P_S} \omega_A^F + \omega_A^S$$

$$\text{Agent 2} \quad \frac{P_F}{P_S} F_B + S_B = \frac{P_F}{P_S} \omega_B^F + \omega_B^S$$

The price of F in terms of S is $p = \frac{P_F}{P_S}$ and the price of S in terms of S is 1.

Using $p = \frac{P_F}{P_S}$, we write

$$\text{Agent 1} \quad p F_A + S_A = p \omega_A^F + \omega_A^S$$

$$\text{Agent 2} \quad p F_B + S_B = p \omega_B^F + \omega_B^S$$

Example 2

In addition to information given in Example 1, you are told that the initial endowment is $(\omega_A^F, \omega_A^S) = (6, 2); (\omega_B^F, \omega_B^S) = (2, 4)$ and the preferences are represented by the following utility functions:

$$u_A(F_A, S_A) = (F_A \cdot S_A)$$

$$u_B(F_B, S_B) = (F_B \cdot S_B).$$

Find the general equilibrium in this economy.

Solution

For solving the problem, follow the steps:

- 1) find the demand for each consumer;
- 2) equate demand = supply in one of the markets to find the equilibrium prices;
- 3) plug the price in each consumer's demand, to find the equilibrium allocation

Step 1: Check that the demand curves are a result of Cobb-Douglas preferences. Note that exponents of the utility functions are equal. So, the consumer spends half of her income in each good. Thus the demand is

$$\text{Consumer 1: } F_A(P) = \frac{\frac{1}{2}(p \cdot 6 + 2)}{p}, S_A(P) = \frac{\frac{1}{2}(p \cdot 6 + 2)}{1}$$

$$\text{Consumer 2: } F_B(P) = \frac{\frac{1}{2}(p \cdot 2 + 4)}{p}, S_B(P) = \frac{\frac{1}{2}(p \cdot 2 + 4)}{1}$$

Step 2: demand = supply in the market for S. So,

$$\frac{1}{2}(p \cdot 6 + 2) + \frac{1}{2}(p \cdot 2 + 4) = 2 + 4$$

$$\text{or, } p = \frac{3}{4}.$$

Note that you have derived the equilibrium price ratio.

Step 3: Plug the prices in each consumer's demand.

$$\text{Consumer 1: } F_A(P) = \frac{\frac{1}{2} \cdot \frac{3}{4} \cdot 6 + 2}{\frac{3}{4}} = 4\frac{1}{3}; S_A(P) = \frac{\frac{1}{2} \cdot \frac{3}{4} \cdot 6 + 2}{1} = 3\frac{1}{4}$$

$$\text{Consumer 2: } F_B(P) = \frac{\frac{1}{2} \cdot \frac{3}{4} \cdot 2 + 4}{\frac{3}{4}} = 3\frac{2}{3}; S_B(P) = \frac{\frac{1}{2} \cdot \frac{3}{4} \cdot 2 + 4}{1} = 2\frac{3}{4}$$

You have checked that the equilibrium allocation is feasible. Here, $4\frac{1}{3} + 3\frac{2}{3} = 8$ and $3\frac{1}{4} + 2\frac{3}{4} = 6$, which would satisfy the requirement.

11.3.2 Existence of Equilibrium Prices

Existence of a set of equilibrium (relative) prices is tried to be seen by looking into the conditions that ensure the following:

- Ascertain that there exists at least one set of prices for which,
 - a) individuals maximise their utilities subject to their budget constraints; and
 - b) the demands are met by the existing stocks of the commodity in the hands of the consumers.

The idea then is to start with some initial arbitrary set of prices at which the economy is not in equilibrium. Some commodities may be in excess supply while others in excess demand. Some other market may be clearing at that price vector. Holding the other $(n-1)$ prices constant, find the equilibrium price in the market for good 1. Term this as provisional equilibrium price P_1' . Keeping P_1' and the other $(n-2)$ prices constant, solve for the equilibrium price in the market good 2. Call this price P_2' . By changing P_2 from its initial position to P_2' , you have disturbed the initial equilibrium price of market 1. Since the system of equations is Simultaneous, it is bound to happen. Using the provisional prices, P_1' and P_2' , solve per a provisional P_3' and the proof proceeds until a complete set of provisional relative prices has been calculated.

The phase of second interaction starts by holding $P_2' \dots P_n'$ constant and calculating the new equilibrium price (P_1'') for the first good. Proceeding the same manner, we get an entire new set of provisional relative prices ($P_1'', P_2'', \dots, P_n''$). The iteration goes on until a reasonable approximation to a set of equilibrium prices is achieve.

11.4 BROUWER'S FIXED-POINT THEOREM

The cumbersome process of arriving at the equilibrium price led to addition of other methods. Brouwer's fixed-point theorem is one of the tools used. It states:

Any continuous mapping $[F(x)]$ of a closed, bounded, convex set into itself has at least one fixed point (x^*) such that

$$F(x^*) = x^*$$

To understand the statement, suppose that $f(x)$ is a continuous function defined in the interval $[0, 1]$ and that it takes on values on the interval $[0, 1]$. Such a function then obeys the conditions of Brouwer's theorem that there exists some x^* such that $f(x^*) = x^*$. Look at Figure 11.1 to understand the idea. It can be seen from the figure that any function, which is continuous, has to cross the 45° line somewhere. The point of crossing is a fixed point, since f maps this point (x^*) into itself.

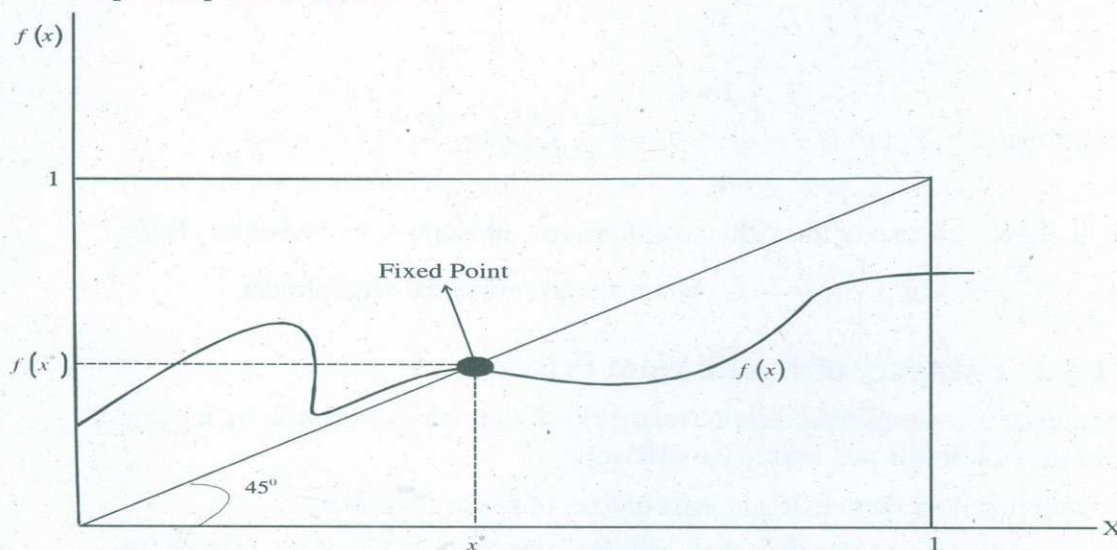


Fig. 11.1: Brouwer's Fixed-Point Theorem

The fixed-point theorem of Brouwer is developed by considering mapping defined on certain types of sets. These set are required to be closed, bounded and convex. While applying the theorem to the exchange model, we have outlined above, you have to choose a suitable way to normalising prices. If we consider the form of normalisation.

$$P'_i = \frac{P_i}{P_j}$$

and remember that at least one of the prices is non-zero, such transformed prices have the characteristics that $\frac{P'_i}{P'_j} = \frac{P_i}{P_j}$ and $\sum_{i=1}^n P'_i = 1$. Now the task is

to construct a continuous function that transforms one set of prices into another. The function is defined such that equilibrium is achieved by increasing the prices of goods that have excess demand while reducing those with excess supply.

Define mapping $F(P)$ such that

$$F^i(P) = P_i + Ed_i(P).$$

In order to ensure that new prices will be nonnegative, the mapping of F is either positive or zero. To include this condition, often we write, $F^i(P) = \text{Max}[P_i + Ed_i(P), 0]$. Moreover, the normalisation must satisfy the condition that $\sum_{i=1}^n F^i(P) = 1$.

Application of Brouwer's Theorem

With the above-mentioned normalisation process, there exists a point (P^*) that mapped into itself and

$$P^* = \text{Max}[P_i^* + Ed_i(P^*), 0] \text{ for all } i.$$

The key requirement for the existence of a Walrasian equilibrium is continuity of the aggregate excess demand function. This is the case if consumer preferences are convex.

11.5 MECHANISM FOR ATTAINING WALRASIAN EQUILIBRIUM

We have shown above that under certain conditions, an equilibrium price, P^* , can be found, for which demand is equal to supply. However, the underlying process through which the initial price P moves to P^* is seen through various mechanisms.

1) **Tatonnement Process:** A fictitious auctioneer is put in charge with calling out a price vector and agents respond by providing information about their demands and supplies. Only when the auctioneer calls that price vector for which the quantity demanded is identical to that which is supplied, trading is permitted to take place.

2) **Provisional Contract:** There could be recontracting in which buyers and sellers would enter into provisional contract before actual exchange goods takes place. Unless the agreed price vector is equilibrium one, provisional contracts remain unimplemented.

3) **Central Planning:** A central authority is established to whom the consumers report their excess demands at all prices. You have to remember

the planning processes of socialist economies to appreciate the notion of arriving at the equilibrium price. This was highlighted by Oskar Lange and Abba P. Lerner when they said that a socialist system can operate as efficiently as a capitalist one.

Check Your Progress 1

- 1) Why do you need to develop a general equilibrium model in contrast to partial equilibrium models?

.....

- 2) What is Walras Law?

.....

- 3) How are equilibrium prices set in a Walrasian equilibrium?

.....

11.5.1 Stability and Uniqueness

Stability of the Walrasian equilibrium can be evaluated by specifying that the prices will adjust according to the conditions of excess demand. If at some price, demand > supply, price would rise. Conversely, if demand < supply, price would fall. Thus, if we consider change in price over time and write the excess demand equation as

$$\frac{dp}{dt} = k [D(P) - S(P)] = k \cdot ED(P) \quad \text{for } k > 0,$$

price adjustment can be seen through a solution of the equation. Recall Taylor series given in MEC-103, and write the above equation as

$$\frac{dp}{dt} \cong k \cdot ED'(P^*) \cdot (P - P^*) \quad \text{as the Taylor approximation.}$$

You get a first-order differential equation. The general solution to it will be

$$P(t) = (P_0 - P^*) e^{kED'(P^*)t} + P^*,$$

where P_0 = initial price at time, $t = 0$.

Stability of the above system is ensured as

$$P(t) \rightarrow P^* \quad \text{when } t \rightarrow \infty.$$

For this to happen, $ED'(P^*) < 0$. In other words, an increase in price must reduce excess demand and a fall in price must increase excess demand.

We have seen that under certain conditions at least one Walrasian equilibrium exists. However, there could be a large number of equilibria, if equilibrium prices (P_s^*) are unstable. Such a possibility raises the question of uniqueness of Walrasian equilibrium. Without existence of a single equilibrium, it becomes difficult to predict the movement of an economy from a set of initial conditions when we change some of the parameters. For Walrasian equilibrium is to be unique, we need to incorporate additional assumptions. One assumption that can be made is that all goods are gross substitutes at all prices.

11.6 COMPETITIVE EQUILIBRIUM AND PARETO EFFICIENCY

In selecting an economy with pure exchange, we ignore the production at this point of our analysis. Such a framework helps us concentrate on allocation of goods that are *already produced*. Our concern now is to find a criterion of allocation agreed to be 'good' enough by the society. The framework of social welfare in which we looked for Pareto efficiency in Block 5 is invoked to see that competitive equilibrium ensures that.

11.6.1 Edgeworth Box

As we have already seen a useful graphical tool for describing the feasible allocations for the economy with 2 agents (A, B) and 2 goods (F, S) is the Edgeworth box.

Suppose the initial endowment is

$$\omega_A = (\omega_A^F, \omega_A^S)$$

$$\omega_B = (\omega_B^F, \omega_B^S)$$

Their consumption is given by,

$$x_A = (x_A^F, x_A^S)$$

$$x_B = (x_B^F, x_B^S)$$

If there were no trade, the consumption is

$$x_A = \omega_A$$

$$x_B = \omega_B \text{ and}$$

with trade, it is true that

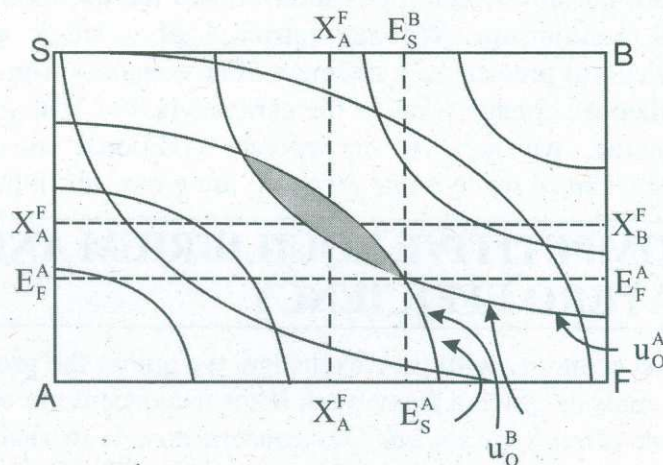
$$x_A^F + x_B^F = \omega_A^F + \omega_B^F$$

$$x_A^S + x_B^S = \omega_A^S + \omega_B^S.$$

The Edgeworth box combines the x-y axes of the two consumers such that when we allocate more to consumer A, there is less available for consumer B. Figure 11.2 shows the Edgeworth box for this economy. All the allocations inside the box are feasible. However, the preferences do not depend on the set of feasible allocation.

Let us start with position E in the figure, which depicts the positions of A and B, with their initial endowments. If trade is allowed where will they end up? It is not clear because either or both could be made better without making either worse off. However, we can say that they need to be *somewhere* in the lens

shaped region between u_A^0 and u_B^0 . Why? Because all these points Pareto dominate E, where Pareto dominate implies the following:



B's Indifference curve A's indifference curve

Fig. 11.2: Edgeworth Box

Consider two allocations x and x' . The allocation x' is said to Pareto dominate x if everyone prefers x' to x . In the figure one or both parties could be made better off without making the other worse off. Thus, there are potential gains from the trade.

Next, we look at the indifference curves of both consumers. We have assumed that the preferences are convex and increasing in both goods.

Note that it is possible to make both consumers better off by moving them to allocation inside the lens area formed by the indifference curves. That is, if agents consume their initial endowments, then this allocation will not be efficient. On the other hand, if there is a possibility to make both consumers better off, there is no reason for not doing that.

In the figure, equilibrium is determined at the point where the two IC's are tangent. Given the prices indicated by the slope of the price line, both agents maximise utility with respect to their budget set. Thus, at equilibrium, the MRS of the two agents are equal:

$$MRS_A = MRS_B = \frac{p_F}{p_S}.$$

An important result that you have to remember is, in the equilibrium only the relative price matters.

Check Your Progress 2

- 1) Suppose you are inside Edgeworth box that gives you two-people, two-commodity exchange scenario. Starting from E, the initial endowment, where will both parties end up if they are allowed to trade.

.....

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.....

- 2) In the Edgeworth box at stated above, are all points in the lens shaped region Pareto efficient?

.....

.....

3) What needs to be true at a Pareto efficient allocation?

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4) What is contract curve in an Edgeworth box you see in connection with exchange of two-goods?

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.....

11.6.2 Pareto Optimal Allocations

Definition

An allocation x is Pareto optimal if it is feasible and there is no other feasible allocation that all consumers weakly prefer and at least one strictly prefers to x . Symbolically, a feasible allocation

$$x = (F_A S_A), (F_B S_B)$$

is Pareto optimal if there is no other feasible allocation $\tilde{x} = (\tilde{F}_A \tilde{S}_A)(\tilde{F}_B \tilde{S}_B)$ such that

$$(\tilde{F}_i, \tilde{S}_i) \succeq_i (F_i, S_i) \text{ for all } i \in (A, B)$$

$$\text{and } (\tilde{F}_i, \tilde{S}_i) \succ_i (F_i, S_i) \text{ for some } i \in (A, B)$$

Thus, it is impossible to make one agent better off without making other worse off. In the figure the indifference curves of A, B are tangent and at this point all gains trade are exhausted. This not always true however. If the preferences are not convex, then it is not true. Moreover, even in the case of strictly convex preferences, the set of Pareto optimal allocation may contain other points, besides the options of tangencies of indifference curves.

Essentially, therefore, we look for the set of tangency points of the indifference curves of A and B. The set forms the *contact curve* (CC) and the allocations that lie along this curve are Pareto efficient. Moreover, the set of choices of the agents agree to trade will lie somewhere on CC that passes through the lens defined by the points interior to u_A^0 and u_B^0 .

Example 1: (Due to Bar, M)

Find the Pareto optimal allocations of the following problem.

Solution

You are given the description of a pure exchange economy with two agents (1 and 2); two goods (x,y) and initial endowment $(\omega_x \omega_y)$. Solve for

$$\max_{x^1, y^1} u^1(x^1, y^1)$$

$$\text{subject to } u^2(x^2, y^2) = \bar{u}$$

$$x^1 + x^2 = \omega_x^1 + \omega_x^2$$

$$y^1 + y^2 = \omega_y^1 + \omega_y^2$$

We want to maximise the utility of agent 1 when the utility of agent 2 is given. If we translate the problem to Pareto optimal formulation it is equivalent to saying that it is impossible to make agent 1 better off without making agent 2 worse off. The last two constraints of the problem give the feasibility constraints. To solve the problem substitute the feasibility constraints into the first constraint which produces the modified version of the problem as

$$\max_{x^1, y^1} u^1(x^1, y^1)$$

s.t.

$$u^2(\omega_x^1 + \omega_x^2 - x^1, \omega_y^1 + \omega_y^2 - y^1) = \bar{u}$$

Denote the total amounts of X and Y in the economy by $\omega_x = \omega_x^1 + \omega_x^2$, $\omega_y = \omega_y^1 + \omega_y^2$. The problem now is

$$\max_{x^1, y^1} u^1(x^1, y^1)$$

s.t.

$$u^2(\omega_x - x^1, \omega_y - y^1) = \bar{u}$$

Set the **Lagrangian** such that

$L = u^1(x^1, y^1) - \lambda [u^2(\omega_x - x^1, \omega_y - y^1) - \bar{u}]$. Then the first order conditions (F.O.N.C.) would be given by

- 1) $L_{x^1} = u_{x^1}^1(x^1, y^1) + \lambda u_{x^1}^2(\omega_x - x^1, \omega_y - y^1) = 0$ (minus sign follows from the chain rule).
- 2) $L_{y^1} = u_{y^1}^1(x^1, y^1) + \lambda u_{y^1}^2(\omega_x - x^1, \omega_y - y^1) = 0$, where

$u_x^i(x^i, y^i)$, $u_y^i(x^i, y^i)$ denote the partial derivatives of u^i with respect to x and y .

$$(1):(2) \quad \frac{u_{x^1}^1(x^1, y^1)}{u_{y^1}^1(x^1, y^1)} = \frac{u_{x^1}^2(\omega_x - x^1, \omega_y - y^1)}{u_{y^1}^2(\omega_x - x^1, \omega_y - y^1)}$$

The left hand side is the slope of agent 1 indifference curves (MRS^1) and the right hand side is the slope of agent 2 indifference curves (MRS^2). Thus, if we have "well behaved" preferences, we can find the set of Pareto optimal (PO) allocations using.

$$MRS^1 = MRS^2$$

Example 2: (Due to Bar, M)

Suppose that the initial endowment is $(\omega_x^1, \omega_y^1) = (6, 2)$; $(\omega_x^2, \omega_y^2) = (2, 4)$. The preferences are represented by the following utility functions:

$$u^1(x^1, y^1) = x^1 \cdot y^1$$

$$u^2(x^2, y^2) = x^2 \cdot y^2$$

Notice that the upper indexes are not powers. They correspond to the number of the agent, agent 1 and agent 2.

Find the set of all P.O. allocations.

Solution: Following the above example, we have $MRS^1 = MRS^2$ such that

$$\frac{u_x^1(x^1, y^1)}{u_y^1(x^1, y^1)} = \frac{u_x^2(\omega_x - x^1, \omega_y - y^1)}{u_y^2(\omega_x - x^1, \omega_y - y^1)}$$

$$\frac{y^1}{x^1} = \frac{y^2}{x^2}$$

$$\frac{y^1}{x^1} = \frac{6 - y^1}{8 - x^1}$$

$$8y^1 - x^1 y^1 = 6x^1 - x^1 y^1$$

$$\text{Thus, } y^1 = \frac{3}{4}x^1 \text{ (the set of Pareto optimal allocations).}$$

From Figure 11.3, we can see the set of Pareto optimal allocations. All points of tangency between the agents' indifference curves give it. You can also see that giving all the goods in the economy to one person is efficient in the Pareto sense. However, most people will say that it is "unfair". Our analysis does not say that all that efficiency is all that matters. However, if we have two alternatives to achieve the same social goal, and one alternative is not efficient while the other one is, most people will advice in favor of the efficient alternative. Why? Because, it is possible to make some people better off, without making the rest worse off.

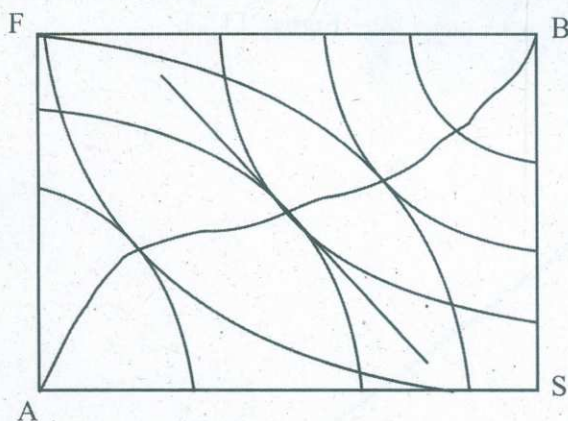


Fig. 11.3: Contract Curve in Edgeworth Box

11.6.3 Process of Reaching Equilibrium Through Trade

You have already seen the task assigned to an auctioneer for deciding the equilibrium prices. We apply the same logic to our example of food (F) and shelter (S) for reading the contract curve. See that in the position of initial endowment, all the goods are consumed. In this sense, the market clears. However, such an allocation is not Pareto efficient. Therefore, the auctioneer announces some price after which both the parties trade. What they preferred at such prices might be Pareto efficient but not necessarily market clearing. So, there must be re-auction at new set of prices.

To understand the price adjustment mechanism of the auctioneer, which ultimately produces an equilibrium, you may recall the option of charging the relative price. According to the position of excess demand, price, in our

example, $\frac{P_F}{P_S}$, will have to be varied. When the auctioneer get the price ratio correct, the market clears and there is no excess demand or supply for any good.

Example: (Due to Edgar Preugschat Lecture Notes)

We continue with an economy with 2 agents, 2 goods presented in the Edgeworth box. Present consumer i's problem as

$$\max u(F_i, S_i)$$

subject to

$$P_F F_i + P_S S_i = P_F \omega_i^F + P_S \omega_i^S.$$

We can rewrite the budget constraint as a function of $S_i(F_i)$, i.e., S_i as a function of F_i :

$$\text{Thus, } P_F F_i + P_S S_i = P_F \omega_i^F + P_S \omega_i^S.$$

Solve for S_i :

$$S_i = \frac{P_F \omega_i^F + P_S \omega_i^S - P_F F_i}{P_S} = \underbrace{\frac{P_F \omega_i^F + P_S \omega_i^S}{P_S}}_{\text{intercept}} - \underbrace{\frac{P_F}{P_S}}_{\text{slope}} F_i.$$

It will useful to recall the utility maximisation problem seen in case of individual consumer. It was seen that the slope of the budget line was determined by the negative of the price ratio. If you bring in the contrast now, the increment depends on both, the prices and the endowments of the goods.

So, to find the budget line, we need to find the endowment point in the two-goods diagram. Then, given the slope by the price ratio, draw the budget line through the endowment point (see Figure 11.4).

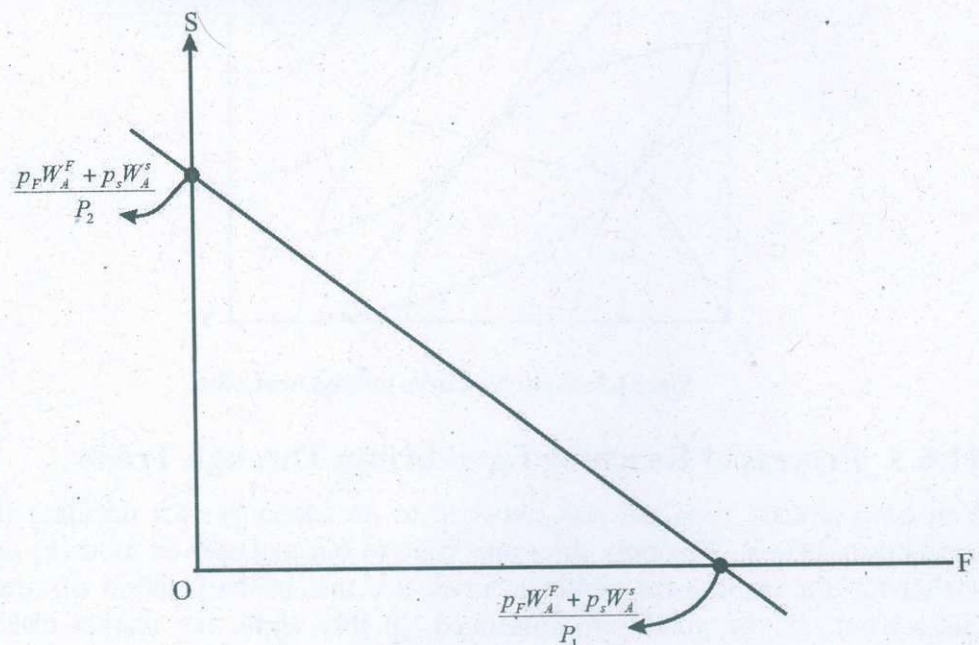


Fig. 11.4: Budget Line of Consumer

Note that you can consume your endowment independent price. So,

$$\omega_i^F = F_i$$

$$\text{or, } \omega_i^S = S_i.$$

Now we bring together the budget line of two consumers in the Edgeworth box. In Figure 11.5 the two 2-goods diagrams are put-together. The size of the box is determined by the endowments. The axes for F and S are of length equals to the sum of the endowments, i.e., the length of the F side is $\omega_A^F + \omega_B^F$ and the length of the S side is $\omega_A^S + \omega_B^S$.

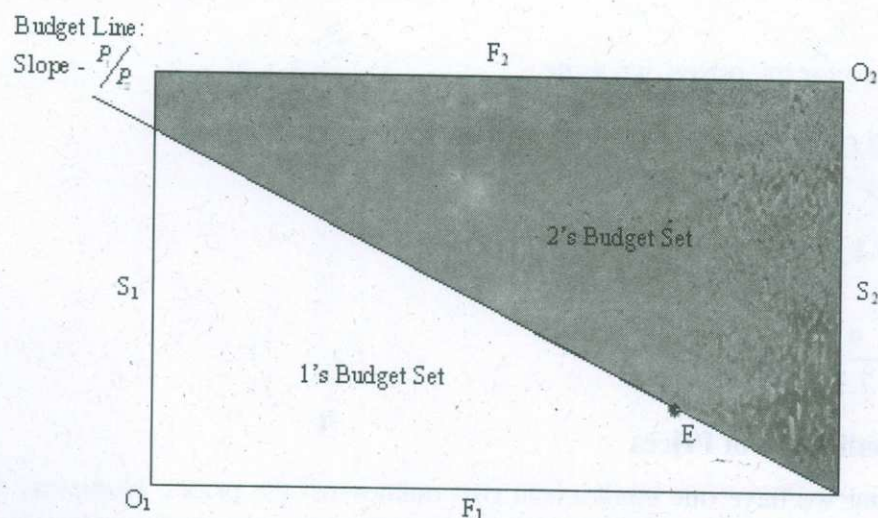


Fig. 11.5: Edgeworth Box with Agents Budget Set

Example:

There are two people (1 and 2) in the economy. They consume two goods x_1^i and x_2^i with their initial endowments ω_1^i and ω_2^i . If both have identical Cobb-Douglas preferences, compute the equilibria of the following:

$$\max (x_1^i)^{0.6} \cdot (x_2^i)^{0.4}$$

subject to

$$P_1 x_1^i + P_2 x_2^i = P_1 \omega_1^i + P_2 \omega_2^i.$$

You are given the information that the endowments for the agents (1 and 2) are

$$\text{Agent 1: } \omega_1^1 = 7; \omega_2^1 = 5$$

$$\text{Agent 2: } \omega_1^2 = 3; \omega_2^2 = 7.$$

First we need to compute the demands:

$$x_1^i(p_1, p_2) = 0.6 \frac{p_1 \omega_1^i + p_2 \omega_2^i}{p_1} \quad i=1,2 \text{ (i denotes the agent)}$$

$$x_2^i(p_1, p_2) = 0.4 \frac{p_1 \omega_1^i + p_2 \omega_2^i}{p_2}$$

Setting demand = supply in equilibrium, we get for (good1):

$$\sum_{i=1}^N \hat{x}_1^i = \sum_{i=1}^N \omega_1^i \text{ for } i=1,2$$

In our example:

$$x_1^1 + x_1^2 = \omega_1^1 + \omega_1^2. \text{ Thus,}$$

$$0.6 \frac{p_1 \omega_1^1 + p_2 \omega_2^1}{p_1} + 0.6 \frac{p_1 \omega_1^2 + p_2 \omega_2^2}{p_1} = \omega_1^1 + \omega_1^2$$

When the values for the endowments are plugged, you get,

$$0.6 \frac{p_1 7 + p_2 5}{p_1} + 0.6 \frac{p_1 3 + p_2 7}{p_1} = 7 + 3 = 10$$

To solve for the prices, we write,

$$4.2 + 3.0 \frac{p_2}{p_1} + 1.8 + 4.2 \frac{p_2}{p_1} = 10$$

$$(3.0 + 4.2) \frac{p_2}{p_1} = 10 - 4.2 - 1.8 = 4$$

$$\frac{p_2}{p_1} = \frac{4}{7.2} = \frac{5}{9} \approx 0.5556$$

Normalisation of Prices

See that we have one equation in two unknowns, the prices. However, since we have a pure exchange economy without money, only relative prices matter, i.e., only the price ratio is important. That means, we can “normalise” on of the prices to 1:

$$P_1 = 1$$

$$P_2 = 5/9$$

Remark: This result also comes from the homogeneity of degree 0 of the demand functions. Remember; by definition of homogeneity of degree 0 in prices we have: $d_x(p_x, p_y) = d_x(\alpha p_x, \alpha p_y)$ for some $\alpha > 0$. Set $\alpha = 1/p_x$,

$$\text{then we get } d_x(p_x, p_y) = d_x\left(1, \frac{p_y}{p_x}\right).$$

Now we can compute the individual demands for good 1:

$$x_1^1 = 0.6 \frac{1 \cdot 7 + 5/9 \cdot 5}{1} \approx 5.87$$

$$x_1^2 = 0.6 \frac{1 \cdot 3 + 5/9 \cdot 7}{1} \approx 4.13$$

Let us check, whether demand actually equals supply:

$$x_1^1 + x_1^2 = \omega_1^1 + \omega_1^2$$

$$5.87 + 4.13 = 10 = 7 + 3$$

Second Market

We solve for the other market, i.e., the market for good 2.

By Walras law, if one market clears the second also clears (if the total number of market is 2); thus using the prices computed above, we get the equilibrium demands for the market of good 2. Thus, from

$$P_1 = 1$$

$$P_2 = 5/9, \text{ we have}$$

$$\begin{aligned}
 & 0.4 \frac{1 \cdot 7 + 5/9 \cdot 5}{5/9} + 0.4 \frac{1 \cdot 3 + 5/9 \cdot 7}{5/9} \\
 &= 0.4 \cdot \frac{9}{5} \cdot (7 + 5/9 \cdot 5) + 0.4 \cdot \frac{9}{5} \cdot (3 + 5/9 \cdot 7) \\
 &= 0.4 \cdot \frac{9}{5} \cdot 7 + 0.4 \cdot 5 + 0.4 \cdot \frac{9}{5} \cdot 3 + 0.4 \cdot 7 \\
 &= \underbrace{5.04 + 2.0}_{\text{Agent 2's Demand (good 2)}} + \underbrace{2.16 + 2.8}_{\text{Agent 2's Demand (good 2)}} = 7.04 + 4.96 = 12 \\
 &= \underbrace{\omega_2^1}_{\text{Endowment (supply) of Agent 1 (good 2)}} + \underbrace{\omega_2^2}_{\text{Endowment (supply) of Agent 2 (good 2)}}
 \end{aligned}$$

Tangency Condition

In equilibrium (see Figure 11.6), we have that

$$MRS^{\text{Agent 1}} = MRS^{\text{Agent 2}} = \frac{P_{x_1}}{P_{x_2}}.$$

Let us verify this:

The MU's are:

For agent 1:

$$\begin{aligned}
 MU_{\text{good 1}}^{\text{Agent 1}} &= 0.6 (x_1^1)^{-0.4} (x_2^1)^{0.4} \\
 MU_{\text{good 2}}^{\text{Agent 1}} &= 0.4 (x_1^1)^{0.6} (x_2^1)^{-0.6}
 \end{aligned}$$

For agent 2:

$$\begin{aligned}
 MU_{\text{good 1}}^{\text{Agent 2}} &= 0.6 (x_1^2)^{-0.4} (x_2^2)^{0.4} \\
 MU_{\text{good 2}}^{\text{Agent 2}} &= 0.4 (x_1^2)^{0.6} (x_2^2)^{-0.6}
 \end{aligned}$$

Dividing the MUs to get MRS and evaluating at the equilibrium quantities gives:

$$MRS^{\text{Agent 1}} = \frac{0.6 (x_1^1)^{-0.4} (x_2^1)^{0.4}}{0.4 (x_1^1)^{0.6} (x_2^1)^{-0.6}} = 1.5 (x_1^1)^{-1} (x_2^1)^1 = 1.5 \frac{7.04}{5.87} \approx 9/5$$

$$MRS^{\text{Agent 2}} = \frac{0.6 (x_1^2)^{-0.4} (x_2^2)^{0.4}}{0.4 (x_1^2)^{0.6} (x_2^2)^{-0.6}} = 1.5 (x_1^2)^{-1} (x_2^2)^1 = 1.5 \frac{4.96}{4.13} \approx 9/5$$

which is the same as the price ratio:

$$\frac{P_{x_1}}{P_{x_2}} = \frac{1}{5/9} = 9/5$$

Let us summarise the general equilibrium of this economy:

The equilibrium of the above economy is:

a vector of prices $p = (1, 5/9)$

and an allocation $\{(\hat{x}_1^1, \hat{x}_2^1)(\hat{x}_1^2, \hat{x}_2^2)\} = \{(5.87, 7.04), (4.13, 4.96)\}$.

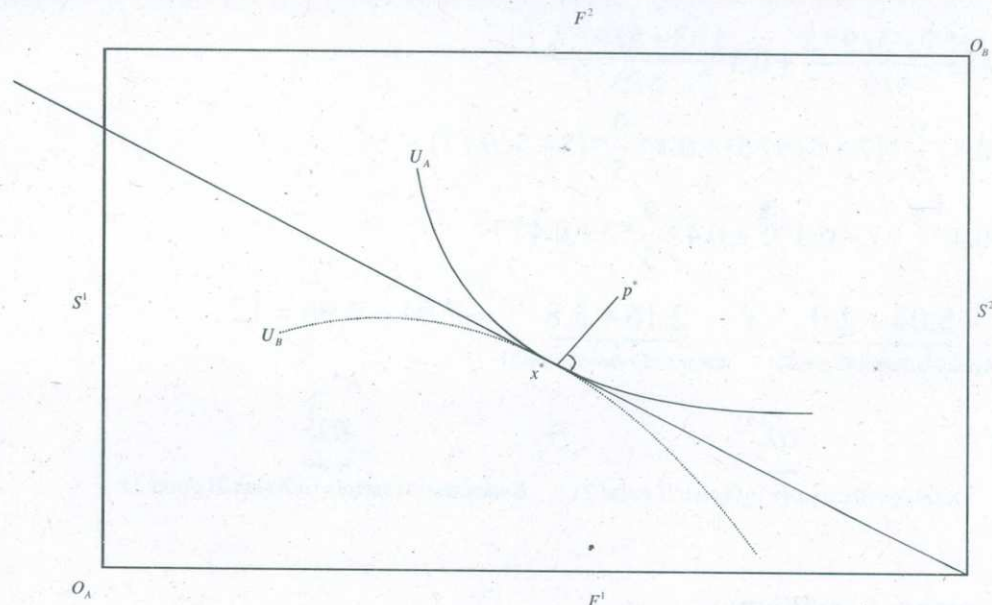


Fig. 11.6: Equilibrium of Two-goods Economy

11.6.4 Pareto Efficiency of the Market Allocation

Pareto Efficiency

Technical Definition: An allocation of goods among individuals:

$$X = \left\{ (x_1^1, x_2^1, \dots, x_L^1), (x_1^2, x_2^2, \dots, x_L^2), \dots, (x_1^N, x_2^N, \dots, x_L^N) \right\}$$

is called efficient, if

- 1) the allocation X is feasible, i.e., the allocation is available such that

$$\sum_{i=1}^N x_l^i = \sum_{i=1}^N \omega_l^i \text{ for all goods } l=1, \dots, L \text{ and if}$$

- 2) there is no other allocation

$$Y = \left\{ (y_1^1, y_2^1, \dots, y_L^1), (y_1^2, y_2^2, \dots, y_L^2), \dots, (y_1^N, y_2^N, \dots, y_L^N) \right\}$$

- a) that is feasible, i.e., $\sum_{i=1}^N y_l^i = \sum_{i=1}^N \omega_l^i$
- b) such that Y is (weakly) preferred by all consumers, i.e., for all consumers $I = 1, 2, \dots, N$,

$$U^i(y_1^i, y_2^i, \dots, y_L^i) \geq U^i(x_1^i, x_2^i, \dots, x_L^i) \text{ and}$$

- c) Y is strictly preferred for at least one consumer j :

$$U^j(y_1^j, y_2^j, \dots, y_L^j) > U^j(x_1^j, x_2^j, \dots, x_L^j)$$

Pareto Efficiency and the Edgeworth box

In the Edgeworth box, if we go back to our last example, the Pareto efficient allocations are described by all allocations $X = \left\{ (x_1^1, x_2^1), (x_1^2, x_2^2) \right\}$ such that

- 1) the allocation X is feasible, i.e., the allocation is available,

$$\begin{aligned} X_1^1 + X_1^2 &= \omega_1^1 + \omega_1^2 = 7 + 3 = 10 \\ X_2^1 + X_2^2 &= \omega_2^1 + \omega_2^2 = 5 + 7 = 12 \end{aligned} \quad \text{and}$$

- 2) there is no other allocation Y , where

$$Y = \{(y_1^1, y_2^1), (y_1^2, y_2^2)\}$$

- a) that is feasible, i.e.,

$$y_1^1 + y_1^2 = \omega_1^1 + \omega_1^2 = 7 + 3 = 10$$

$$y_2^1 + y_2^2 = \omega_2^1 + \omega_2^2 = 5 + 7 = 12$$

such that

- b) Y is weakly preferred by all consumers, i.e.,

for all consumers $i = 1, 2$

$$(y_1^i)^{0.6} \cdot (y_2^i)^{0.4} \geq (x_1^i)^{0.6} \cdot (x_2^i)^{0.4}$$

and

- c) Y is strictly preferred for one particular j of them (either Agent 1 or Agent 2):

$$(y_1^j)^{0.6} \cdot (y_2^j)^{0.4} > (x_1^j)^{0.6} \cdot (x_2^j)^{0.4}$$

Given these pieces of information, we try to address an important question, how a decentralised economy, where all agents decide independently about their demands, can be called good for the society, although no one cares about what other people do, i.e., everyone acts selfish. To find an answer to this question, we will narrow the expression for "good for the society" down to the notion of Pareto efficiency. Then we can state a very interesting relationship between a general competitive equilibrium and Pareto efficiency.

The two Fundamental Welfare Theorems of Economics

First Fundamental Welfare Theorem

Every competitive equilibrium is Pareto efficient (given that preferences are not (locally) satiated).

That is to say a free market in equilibrium is Pareto efficient, provided the following conditions true (as we have seen in Block 5):

- 1) No externalities
- 2) Perfect competition
- 3) No transaction costs
- 4) Full information

Second Fundamental Welfare Theorem

Under some (fairly weak) assumptions, every Pareto efficient allocation can be decentralised as a competitive equilibrium (i.e., a price vector can be found such that if the endowments are set equal to the (initial) Pareto efficient allocation, all agents will demand exactly these endowments (which is the Pareto allocation.) given the prices.

Check Your Progress 3

- 1) Does the first Welfare Theorem guarantee that the market allocation will be fair or equitable?

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- 2) Is there a trade-off between enlarging and dividing the pie?

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- 3) How do we know that allocative efficiency will be satisfied?

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11.7 LET US SUM UP

In this unit, we have discussed the usefulness of general equilibrium analysis over that of partial equilibrium and looked into the formulation and solution of general equilibrium problems in a pure exchange model. We highlighted the idea of general equilibrium as perceived by Walras and touched up his solution procedure with the help of excess demand functions. A major result brought out through the analysis is the solution obtained through relative prices. Existence of at least one equilibrium was proved and Brouwer's fixed-point theorem was explained. The process of attaining Walrasian equilibrium through fictitious auctioneer has been dealt with. The stability of the equilibrium was seen with the help of price adjustment according to the conditions of excess demand. The uniqueness of the equilibrium, on the other hand, was seen by bring in substitute goods. In the last part of the discussion, we have examined the efficiency of the competitive equilibrium and elaborated on Pareto efficiency conditions. It has been pointed out that a free market equilibrium is Pareto efficient provides preferences of the consumers are convex and continuous.

11.8 KEY WORDS

Contract Curve: The set of all the efficient allocations of goods among those individuals in an exchange economy. Each of these allocations has the property that no one individual can be made better off without making someone else worse off.

Convexity Assumptions: Assumptions about the shapes of individuals' utility functions and firms' production functions. Based on the presumption that the relative marginal effectiveness of a particular good or input diminishes as the quantity of that good or input increases. Important because they ensure that the application of first-order conditions will indeed yield a true maximum.

Offer Curve: A curve showing those trades an individual would willingly make away from a particular initial endowment at alternative price ratios.

Pareto Optimality: An allocation of resources in which no one individual can be made better off without making someone else worse off.

11.9 SOME USEFUL BOOKS

Bar Michael (2003), *General Equilibrium – Pure Exchange Economy*, Lecture Notes, available in Internet.

Nicholson, Walter (1992), *Microeconomic Theory: Basic Principles and Extensions*, The Dryden Press, Harecourt Bral Jovanovich, Orlando.

Sen, Anindya (1999), *Microeconomics, Theory and Applications*, Oxford University Press.

11.10 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) To capture the interactions of all markets simultaneously.
- 2) Total excess demand of the economy must be equal to zero.
- 3) By an auctioneer or central planner.

Check Your Progress 2

- 1) It is not fully clear because either or both could be made better off without making either worse off. But they have to be somewhere inside the lens of curves.
- 2) No. All of the points in the lens region are Pareto superior, but on a subject are Pareto efficient.
- 3) The indifference curves (say A and B) are tangent.
- 4) All Pareto efficient allocations lie on this curve.

Check Your Progress 3

- 1) No. Giving everything to A in the initial endowment would be Pareto efficient as would be giving everything to B.
- 2) Yes. The Second Welfare Theorem proves that.
- 3) Because all agents face the same prices (relative price).

11.11 EXERCISES

- 1) Suppose than an economy has just two consumers, A and B and two commodities (1 and 2). The endowments of the two agents are respectively $\omega_A(\omega_A^1, \omega_A^2)$ and $\omega_B(\omega_B^1, \omega_B^2)$. The utility functions are Cobb-Douglas: $u_A = (x_A^1)^\alpha (x_A^2)^{1-\alpha}$, and $u_B = (x_B^1)^\beta (x_B^2)^{1-\beta}$. Assuming $0 < \alpha, \beta < 1$, show that consumption demands are functions only of relative prices, if you know that the price of the goods (x_1 and x_2) are p_1 and p_2 and the agents agree to adhere to a feasible allocation.
- 2) Discuss the idea that competitive market equilibrium leads to Pareto efficient allocation. Under what conditions you should agree with such a viewpoint.
- 3) How do you establish the efficient exchange in an Edgeworth box. Discuss the process through which gains from trade are exhausted in the equilibrium in the box.
- 4) Consider the following exchange economy. There are two gods and two consumers. Two goods are called food and shelter and two consumer are called 1 and 2. Consumer 1 has utility function $U_1(f, s) = .4 \ln(f) + .6 \ln(s)$ (where f is the amount of food 1 consumes, and s is the amount of shelter). Consumer 2 has utility function $U_2(f, s) = .5 \ln(f) + .5 \ln(s)$.

Consumer 1 is endowed with 10 units each of shelter and food. Consumer 2 is endowed with 10 units of shelter and 5 units of food.

- a) What is the Walrasian equilibrium of this economy? (If there are more than one equilibria, give them all.)
 - b) Suppose a social dictator wished to implement an allocation that makes $U_1(f,s) + U_2(f,s)$ as large as possible at the equilibrium. Give all the possible reallocations of the endowment that give the dictator's optimal endowment as a Walrasian equilibrium.
- 5) To illustrate the role of the assumptions used in proving results, give examples of the following (illustrate using an Edgeworth box).
- i) An exchange economy with no competitive equilibrium relative to particular initial endowments.
 - ii) An exchange economy with an infinite number of competitive equilibria relative to particular initial endowments.
 - iii) A Pareto optimum, which cannot be sustained as a competitive equilibrium.
 - iv) A competitive equilibrium, which is not Pareto optimal.
- 6) Consider an economy with 15 consumers and 2 goods. Consumer 3 has a Cobb-Douglas utility function $U^3 = \ln X + \ln Y$. At a certain Pareto efficient allocation, consumer 3 has an allocation (10,5). What are the competitive prices that support this Pareto optimal allocation? (Varian 17.9).

UNIT 12 GENERAL EQUILIBRIUM WITH PRODUCTION

Structure

- 12.0 Objectives
- 12.1 Introduction
- 12.2 Set Up of the Problem
- 12.3 Edgeworth Box for Production
 - 12.3.1 Production Possibility Frontier (PPF)
 - 12.3.1.1 Rate of Product Transformation
 - 12.3.1.2 Shape of Production Possibility Frontier
 - 12.3.1.3 Opportunity Cost and Supply
 - 12.3.2 PPF for Food and Shelter: An Example
- 12.4 Consumption Optimisation
- 12.5 Product-mix Efficiency and the Optimum
- 12.6 Explanation on Determination of Equilibrium Prices
- 12.7 General Equilibrium Price Setting and Efficiency
 - 12.7.1 Productive (Technical) Efficiency
 - 12.7.2 Allocative Efficiency
 - 12.7.3 Product Mix Efficiency
- 12.8 Link between Factor and Goods Markets
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12.0 OBJECTIVES

After going through this unit, you will be able to:

- formulate general equilibrium problems with production;
- determine simultaneous clearance of factor and product markets in the presence consumer choices; and
- evaluate important general equilibrium related theorems with international trade.

12.1 INTRODUCTION

In the preceding unit, we have discussed the general equilibrium in a pure exchange economy. Under such a set up, goods have already been produced, found, inherited, or *endowed*, and the only issue is how they should be distributed and consumed. The analysis undertaken has produced efficient

allocation of resources among consumers. Connections between markets and efficiency in an exchange economy are established giving rise to two most fundamental results of general equilibrium analysis, viz., first and second welfare theorems. Both offer a mechanism for determining optimal *production*, consumption and exchange.

The present unit extends the above analysis by adding production to the model. As a result, we will allow the consumers not only to be trading goods as in case of pure exchange but also as producers they will be making goods. Needless to say that they will use factors of production to produce final consumption goods. Thus, we will come closer to more real life scenario of functioning of the economy. Of course such an attempt will add some complexity to the analysis. The net result, however, is the usefulness of talking about international trade and comparative advantage in an integrated framework. The general equilibrium determining the patterns of production will be derived as an outcome of the model.

12.2 SETUP OF THE PROBLEM

Recall the case of pure exchange model delineated earlier. There are two consumers A and B and two goods, say, food (F) and shelter (S). While attempting to include production in the model, we consider two factors of production, capital (K) and labor (L). These factors are the endowments used to transform into both or either of the two F and S for final consumption.

Let's also assume that food is labour intensive and shelter is capital intensive. That means for any given price ratio P_f/P_s , the food industry will demand relatively more labor per unit output and the shelter industry will demand relatively more capital.

From the perspective of the economy, such a problem translates into finding the most appropriate of the factors of production in order to produce goods F and S and to distribute these goods amongst consumers A and B . The design will be such that there is no other feasible configuration which would allow an increase in any of the individual's welfare without decreasing the other individual's welfare.

In order to appreciate formulation above, we look for a process of the market allocating resources such that the following three efficiencies are achieved:

- 1) Capital and labour factors are used to produce each good efficiently: Productive Efficiency
- 2) Each consumer consumes of each good (food, shelter) efficiently: Allocative Efficiency
- 3) Total resources are allocated to each sector efficiently: Product mix Efficiency

To find solution for these efficiencies, we need to solve the above mentioned three problems, viz., productive, allocative and product mix efficiencies simultaneously so as to arrive at a general equilibrium with production. Such a solution is different from the equilibrium derived in the case of pure exchange discussed in the preceding unit where the attainment of only one, allocative efficiency, was solved for.

12.3 EDGEWORTH BOX FOR PRODUCTION

Just like we have done in case of pure exchange, the Edgeworth box is drawn to examine allocation of production between goods. The axes of the box will

reflect the allocations of K and L to the production of F and S . Steps followed in drawing the box are as follows:

It may be useful keep in mind that the optimisation problem in production relies on the *maximisation of total output production*. Given that there are limited amounts of capital and labor, it is postulated as,

$$\begin{aligned} \text{Max } & F(K_F, L_F) + S(K_S, L_S) \\ \text{s. t. } & K_F + K_S = \bar{K}; L_F + L_S = \bar{L} \end{aligned}$$

We start by looking at the production of goods F and S as two different optimisation problems. The firm has to decide quantities of capital and labour to allocate for the production of good F and S . The left and right sides of the diagram below show the *isoquants* corresponding to each production process.

It may be noted that the further the indifference curve is from the origin, the higher the level of output.

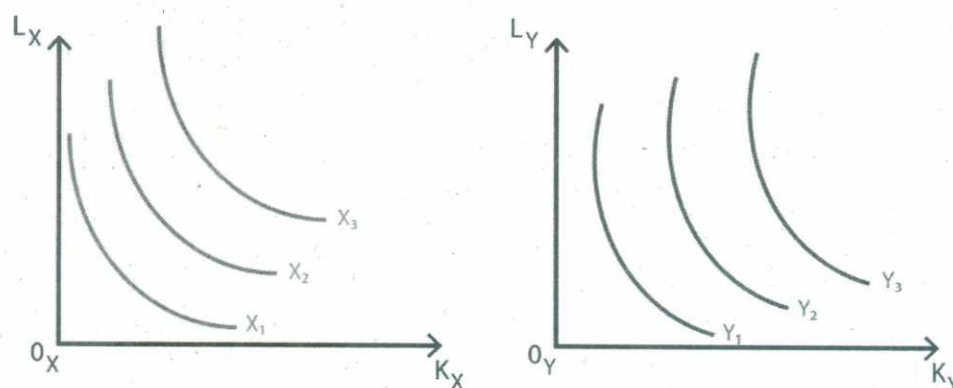


Fig. 12.1: Isoquants for Commodities Food and Shelter

By plotting these two diagrams together we obtain the *Edgeworth box* for production (See Figure 12.2). In the figure, the Southwest corner represents the origin for food while the Northeast corner represents the origin for shelter. It then shows the technically efficient ways to allocate fixed amounts of K and L between production of these two outputs. The figure makes it easier to compare quantities of capital and labour used while comparing quantities of goods F and S being produced. The line joining O_x and O_y gives the locus of their efficient points. Along this line the *RTS* of L for K in producing the good F is equal to *RTS* of L for K in production of S .

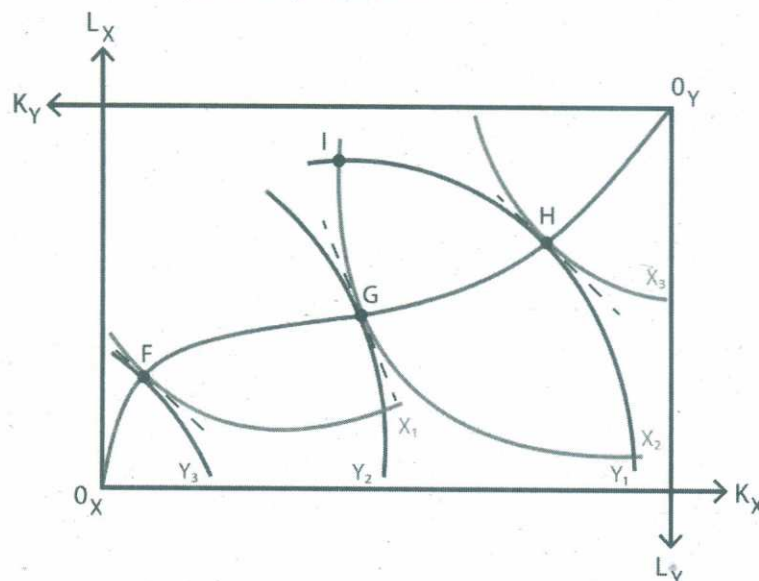


Fig. 12.2: Edgeworth Box for Technical Efficiency

Note that

- i) production isoquants are like indifference curves;
- ii) contract curve traces out the set of Pareto efficient allocations, i.e., where the production isoquants for the two goods are tangent. Production rather than consumption is plotted here as K and L are transformed into food and shelter; and
- iii) points on this contract curve equate the Marginal Rate of Technical Substitution (MRTS) among goods. In case of pure exchange seen in the previous unit, the ratio of marginal utilities across consumers has been equated to plot the graph.

So, along a production isoquant:

$$S = S(K, L),$$

$$dS|_{S=\bar{S}} = \frac{\partial S}{\partial K} dK + \frac{\partial S}{\partial L} dL = 0,$$

$$-\frac{dK}{dL} = \frac{\partial S/\partial L}{\partial S/\partial K} = MRTS_S.$$

And

$$F = F(K, L),$$

$$dF|_{F=\bar{F}} = \frac{\partial F}{\partial K} dK + \frac{\partial F}{\partial L} dL = 0,$$

$$-\frac{dK}{dL} = \frac{\partial F/\partial L}{\partial F/\partial K} = MRTS_F.$$

Hence, along the Contract Curve, we have $MRTS_F = MRTS_S$. That is to say, when the marginal rate of technical substitution is equal for both goods, all available inputs are being used, which translates into a purely efficient production process. The points of tangency are called the Efficient Production Set, and the rate of technical substitution is equated among factors at this point.

Let us consider a case where $\frac{\partial F/\partial L}{\partial F/\partial K} > \frac{\partial S/\partial L}{\partial S/\partial K}$ instead of being equal.

That is, the marginal product of labour in the Food sector was higher than in the Shelter sector. Then more labour should be allocated to Food and more capital to Shelter until these marginal products are equated. It will be possible to raise output in either (or, both) sectors without lowering output in the other simply by reallocating factors between them. In other words, there are 'gains from trade' among these industries.

12.3.1 Production Possibility Frontier (PPF)

PPF is a graph representing production tradeoffs of an economy given fixed resources. It shows the maximum possible production level of one commodity when the production level of the other is kept fixed, given the existing state of technology. This curve helps in determining what quantity of a non-essential good an economy can afford to produce without jeopardizing the required production of essential goods. We make use of the Edgeworth box to get the PPF.

It is to be noted that in the Edgeworth box above, points on the contact curve represent all Pareto efficient distributions, such as F , G or H where the

isoquants are tangent to one another. I is not Pareto efficient, since going from I to either G or H would result in an increase in the production of one of the goods without giving up the production of the other. From this curve we can derive the production possibility frontier (PPF), which shows the quantities of goods F and S being produced.

The efficiency locus in Figure 12.3 shows the maximum output of S that can be produced for any pre-assigned output of F . We have used this information to construct a production possibility frontier. In the figure the $OxOy$ locus has been taken from Figure 12.2. It is transferred onto a graph with F and S outputs on the axes X and Y . At O_X , for example, no resources are devoted to F production; consequently, S output is as large as is possible with the existing resources. Similarly, at O_Y , the output of F is as large as possible. The other points on the production possibility frontier (say, F , G and H) are derived from the efficiency locus in an identical way.

12.3.1.1 Rate of Product Transformation

The slope of the production possibility frontier shows how F output can be substituted for S output when total resources are held constant. For example, for points near O_X on the production possibility frontier, the slope is a small negative number—say, $(-1/4)$; this implies that, by reducing S output by 1 unit, F output could be increased by 4. Near O_Y , on the other hand, the slope is a large negative number (say, -5), implying that S output must be reduced by 5 units in order to permit the production of one more F . The slope of the production possibility frontier, then, clearly shows the possibilities that exist for trading S for F in production. The negative of this slope is called the rate of product transformation (RPT) or marginal rate of product transformation (MRPT).

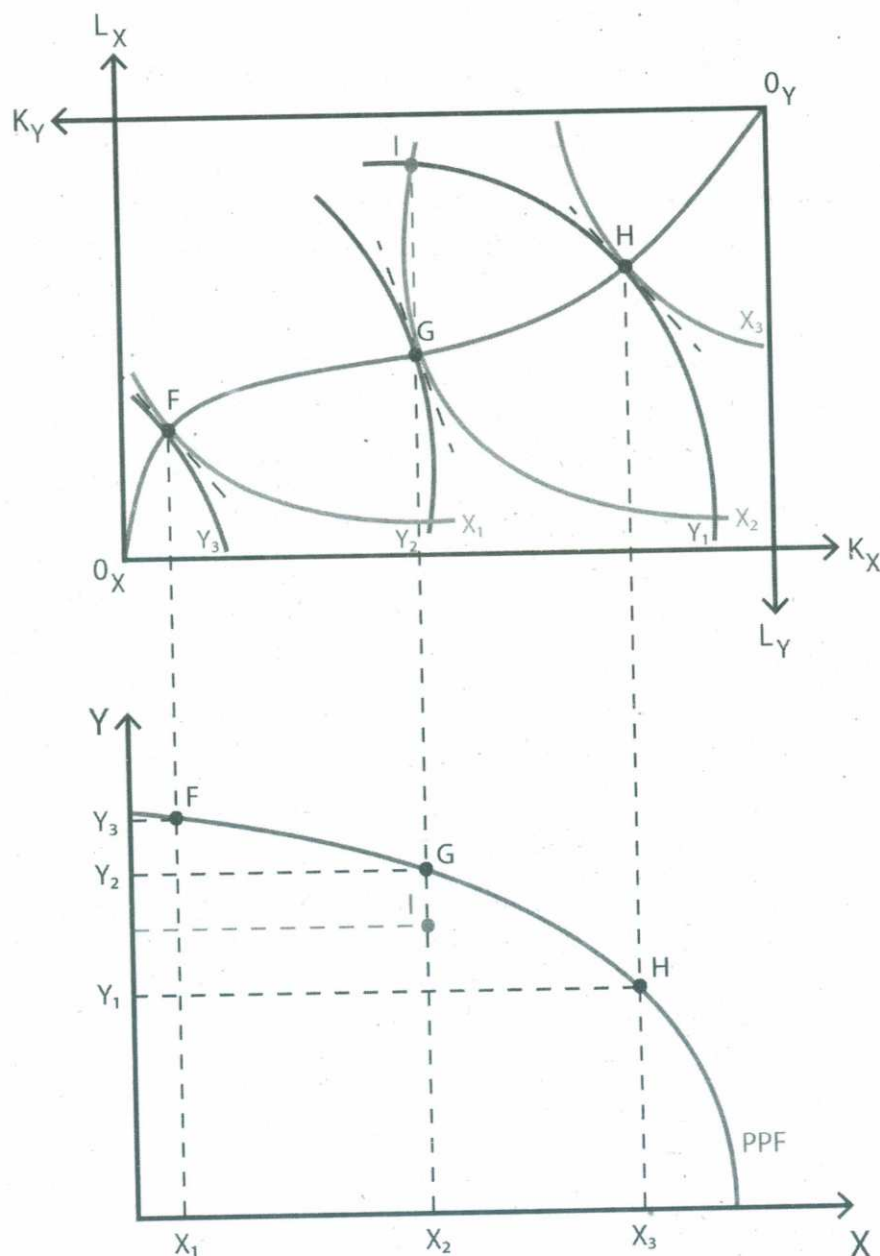
Mathematically, RPT (of F for S) = $-(\text{slope of production possibility frontier}) = -dy/dx$ (along O_X, O_Y). The RPT records how F can be technically traded for S while continuing to keep the available productive inputs efficiently employed.

12.3.1.2 Shape of Production Possibility Frontier

The production possibility frontier illustrated in Figure 12.3 exhibits an increasing RPT. For output levels near O_X , relatively little S must be sacrificed to obtain one more F ($-\frac{dS}{dF}$ is small). Near O_Y , on the other hand, additional F may be obtained only by substantial reductions in S output ($-\frac{dS}{dF}$ is large). In this section we will show why this concave shape might be expected to characterise most production situations.

In that analysis we need to recognise that RPT is equal to the ratio of the marginal cost of F (MC_F) to the marginal cost of y (MC_S). Intuitively, such feature Figure 12.3: Derivation of Production Possibility Frontier from Distribution of Resources for Production is to be expected only. The reason is as follows: Suppose, for example, that F and S are produced only with labor. If it takes two labour hours to produce one unit of more F , we can say that MC_F is equal to 2. Similarly, if it takes only one labour hour to produce an extra unit of S , then MC_S is equal to 1. Thus two units of S must be forgone to provide enough labour so that F may be increased by one unit. Hence, the RPT is 2 and is equal to the ratio of the marginal costs of the two goods. More

formally, suppose that the costs (say, in terms of the "disutility" experienced by factor suppliers) of any output combination are denoted by $C(F, S)$.



Along the production possibility frontier, $C(F, S)$ will be constant because the inputs are in fixed supply. So we can write the total differential of the cost function as $dC = (\partial C / \partial F) \cdot dF + (\partial C / \partial S) \cdot dS = 0$ for changes in F and S along the production possibility frontier. Rearranging the above equation yields

$$RPT = -dS/dF \text{ along } O_x, O_y = -\frac{\partial C / \partial S}{\partial C / \partial F} = \frac{MC_F}{MC_S}$$

which establishes that RPT is a measure of the relative marginal costs of the two goods.

The RPT is also expected to rise for clockwise movements along the production possibility frontier. We can show this by the rising ratio of MC_F to MC_S as output F expands and S output contracts.

There are several other possible reasons which determine the shape of PPF. They are:

- 1) Diminishing returns in each sector: Efficiency falls as scale increases.
- 2) Specialised inputs: Some capital and labour is more suited to food or shelter production.
- 3) Differing factor intensities of products: If Shelter is more capital intensive than Food, then the marginal productivity of the Shelter sector will decline as all the labor diverted for its production. Similarly, Food will experience decline in marginal productivity as we allocate all of the capital. So, even if there are no diminishing returns in each sector, it will operate if we force a sector to use a comparatively less technically productive mix of inputs.

12.3.1.3 Opportunity Cost and Supply

In the production possibility curve there are many possible efficient combinations of the two goods and the production more of one good necessitates cutting back on the production of the other good. Thus PPF shows the opportunity cost as the cost of producing more unit of F can be readily measured by the requirement of reduction in S . The cost of one more unit of F is therefore best measured as the RPT (of F for S) at the prevailing point on the production possibility frontier. As such a cost increases with the production of more F , it represents the formulation of supply in a general equilibrium context.

Note the features of the PPF:

- 1) The points along the Efficient Production Set can also be drawn in final good (F, S) space rather than factor (K, L) space.
- 2) The PPF represents the alternative combinations of two final goods that can be produced with fixed quantities of inputs.
- 3) The frontier of this set corresponds to the technically efficient, feasible combinations of outputs.
- 4) The points inside the set are feasible but are not technically efficient as defined above.
- 5) Points outside of this set might be technically efficient and are certainly more desirable than the points on the frontier but are not feasible.
- 6) Note that the slope of the PPF is not the $MRTS$ between K and L , that is, the slope of the production isoquants.
- 7) Instead, the slope of the PPF is the $MRTS$ between F and S , which we call the $MRPT$, Marginal Rate of Product Transformation. It is the rate of technical transformation from one good to the other (at the margin).

12.3.2 PPF for Food and Shelter: An Example

For simplicity, assume that Food and Shelter use only labor in this example:

$$F = L_F^{0.5},$$

$$0.5S = L_S^{0.5}$$

Budget constraint: $L_F + L_S = 100$, which implies $F^2 + 4S^2 = 100$.

To obtain the PPF, totally differentiate:

$$2FdF + 8SdS = 0$$

$$-\frac{dF}{dS} = \frac{F}{4S}$$

Note that the absolute value of this expression increases as F rises — the marginal product of labour in the S sector rises as more labour is allocated to the F sector, and vice versa.

In this case, the curvature of the PPF comes from diminishing returns in each sector ((b/c) the exponent of each production function is less than unity).

So note that the following points are all technically efficient allocations on the PPF:

$$F = 10, S = 0$$

$$F=0, S=5$$

$$F=5, S=4.3$$

Check Your Progress 1

- 1) Using the Edgeworth box diagram, answer the following:
 - a) What does a vertical movement inside the box signify? And a horizontal movement?

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 - b) Why is a competitive equilibrium on the contract curve?

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- 2) Does the production possibilities frontier show for each level of output of one good, the maximum amount of the other good that can be produced.

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- 3) What determines the opportunity cost along a PPF?

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- 4) An economy has a constant cost PPF. What do you know about the slope?

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It is difficult to rank the consumption of bundles of goods without the knowledge of distribution of the products, especially if a bundle consists of different amounts of each good. The difficulty arises as some of the bundles may have more products of a good but less of another. The optimisation problem in terms of our example therefore will be to *maximise the utility* of individuals *A* and *B* subject to a limited total amount of goods *F* and *S*. We can state the maximisation problem as,

$$\text{Max } U_A(F_A, S_A) + U_B(F_B, S_B)$$

$$\text{s. t. } F_A + F_B = \bar{F}; S_A + S_B = \bar{S}$$

Since we have the production of two goods *F* and *S*, their optimal distribution is to be worked out between two individuals, *A* and *B*. We follow the same steps as is done above in case of production of *F* and *S*. Here, we'll plot *indifference curves* corresponding to the amounts of goods *F* and *S* consumed by *A* and *B*. As may be seen from the Figure-2 below amounts of two goods consumed by *A* is given on the left and the amounts consumed by *B* on the right.

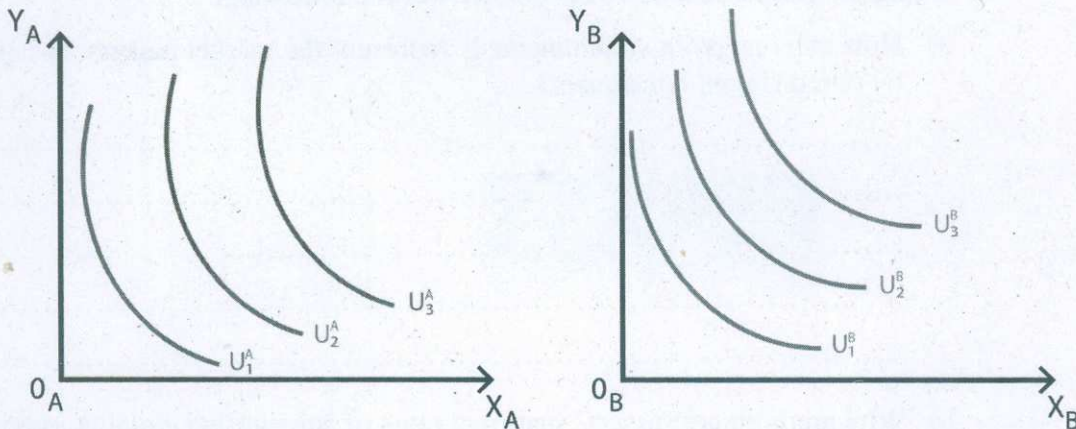


Fig. 12.4: Distribution Utility Levels of Consumption of Goods

Just like the procedure adopted in case of production above, we use the Edgeworth box to graph the different distributions that can be given between two individuals, *A* and *B*, and two goods, *F* and *S*. In the box, the further the indifference curve is from the origin, the higher the level of utility enjoyed by the consumer.

Although all the points in the graph (see Figure 12.5) are feasible, not all are efficient. Why? Due to given the utilities and preferences of consumers. The indifference curves join all the points that give consumers the same level of utility. By connecting all points of tangency between the indifference curves of both individuals, the contract curve is constructed. Such a curve represents all the Pareto efficient allocations. The tangency between indifference curves is the point where both consumers have an equal *marginal rate of substitution* for goods *F* and *S*, and are therefore not willing to trade between them, as it would result in a lower utility.

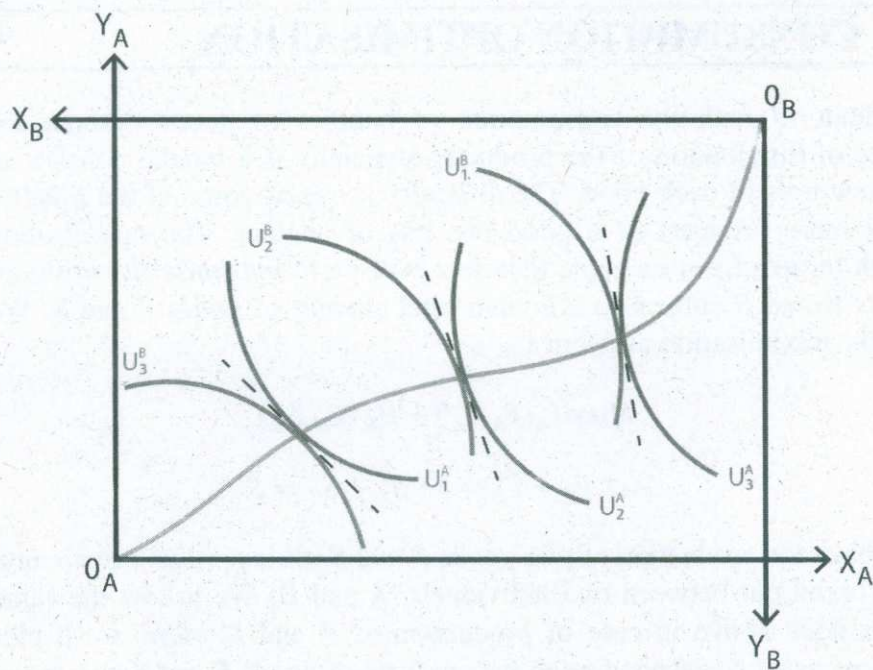


Fig. 12.5: Edgeworth Box for Allocative Efficiency

Check Your Progress 2

1) Using the Edgeworth box diagram, answer the following:

- a) How can one point simultaneously represent the market baskets owned by two different consumers?

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- b) Why are both consumers' marginal rates of substitution equal at every point on the contract curve in the Edgeworth box?

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2) Explain why goods will not be distributed efficiently among consumers if the MRPT is not equal to the consumers' marginal rate of substitution.

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12.5 PRODUCT-MIX EFFICIENCY AND THE OPTIMUM

The discussion above considers different parts of the economy by putting the production and consumption activities separately. When combined together we can examine the function of the economy as a whole. Accordingly the optimisation problem needs to be reset considering both consumption and production. Basically, we solve the following optimisation problem in which the level of production must be efficient:

$$\begin{aligned} \text{Max } & U_A(F_A, S_A) + U_B(F_B, S_B) \\ \text{s.t. } & F_A + F_B = \bar{F}; S_A + S_B = \bar{S} \\ & \bar{F} + \bar{S} = \text{PPF} \end{aligned}$$

It may be seen that the optimisation problem is already seen in case consumption above. So in the solution we have the same marginal rate of substitution equalisation between F and S . Additionally, these two must have equality in the slope of the PPF , i.e., the *marginal rate of transformation* (MRT). So we have

$$MRT_{FS} = MRS_{FS}^A = MRS_{FS}^B$$

However, the solution derived above is not unique. There are multiple points where the above condition holds. Only when output price is decided in accordance with input prices mentioned before, we will get a unique solution. As may be seen from the diagram below, if output prices were to be P_F and P_S , the equilibrium would be at point E . However, if output prices were instead P'_F and P'_S , the equilibrium would be point E' .

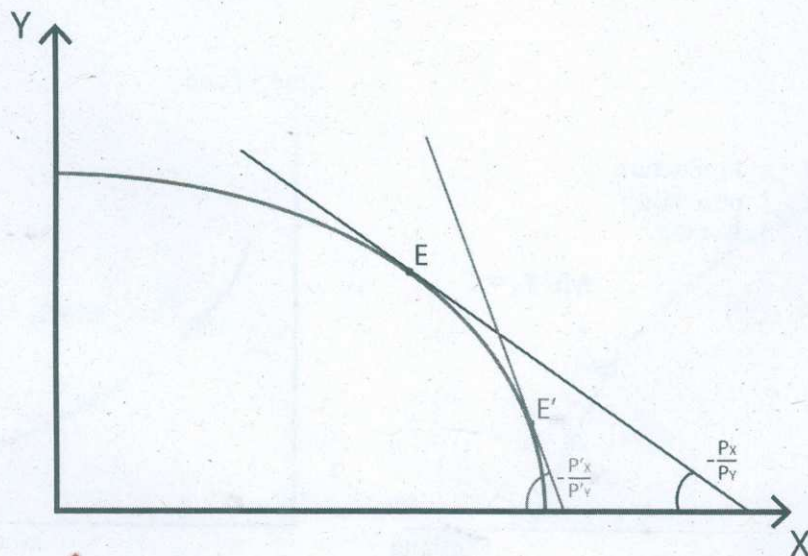


Fig. 12.6: Equilibrium Price and PPF

Consider the overall equilibrium that emerges in the diagram below when prices are set at P_F and P_S , and the equilibrium point is E , consumers A and B will consume both goods F and S in different amounts. Note that these quantities are given by the equilibrium in consumption, point E on the contract curve. Equilibrium in the production process is given by point E on the production possibility frontier. It is a general equilibrium because the marginal rate of substitution is equal to the marginal rate of transformation; that is, slopes of the indifference curves are equal to the slopes of the production possibility frontier.

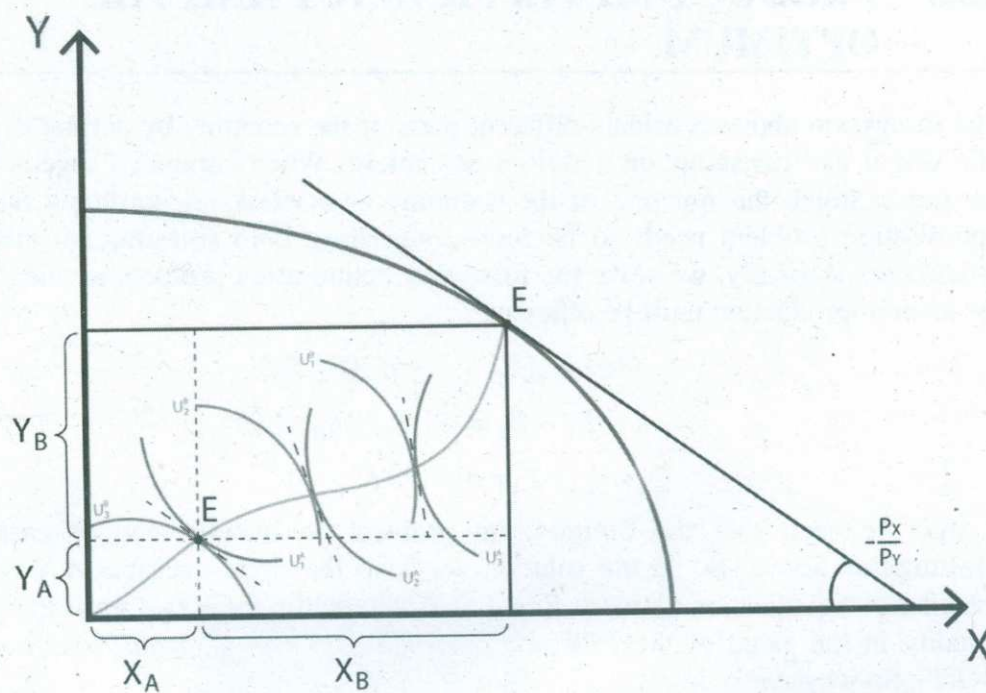
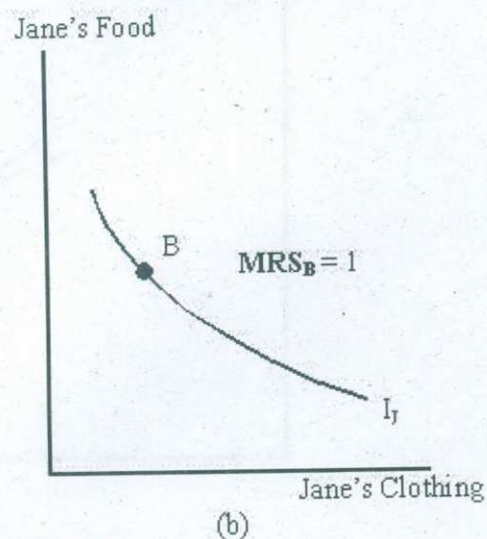
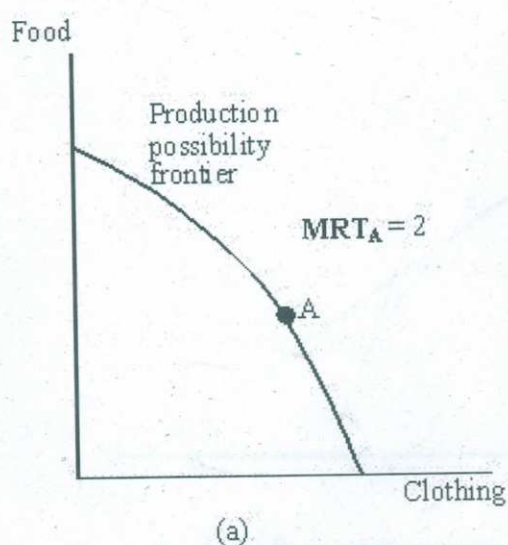


Fig. 12.7: General Equilibrium with Production and Consumption

Check Your Progress 3

- At the product mix A in panel a of the diagram below, the Marginal Rate of Transformation is 2. If Jane's Marginal Rate of Substitution at bundle B in panel b is 1 then what combination of food and clothing production will make everyone better off?



- 2) When would you say that Perfectly Competitive Markets Satisfy the Conditions for Pareto Efficiency?

- 3) When can you have efficiency in production and in consumption, and yet there is still room for a Pareto improvement?

12.6 EXPLANATION ON DETERMINATION OF EQUILIBRIUM PRICES

We have seen above how equilibrium goods prices are determined with the help of *PPF*. The *PPF* shows all of the efficient combinations of supply of goods and consumer preferences help provide demand for goods. So, if we assume an aggregate utility function, which can be represented using a set of 'community indifference curves', these curves would reflect the aggregation of individual preferences reflecting societal willingness to trade off among goods.

The equilibrium price ratio P_F/P_S equates demand and supply for both goods. At this point, the community indifference curve is tangent to the *PPF*. We may also note that the existence of the equilibrium depends upon:

- 1) No transaction costs
- 2) Full information
- 3) No externalities
- 4) Preferences satisfy utility axioms
- 5) No market power (firms and consumers are small relative to the market)

When these conditions are satisfied,

- 1) each consumer takes price as given and maximises utility subject to her budget constraint;
- 2) each producer takes prices as given and maximises profits given these prices;
- 3) the "Law of One Price" prevails — Each good sells at a single price regardless of who is buying or selling it. So the identity of sellers and buyers is immaterial;
- 4) the market clears (as seen by the tangency of the *PPF* and the community indifference curve).

12.7 GENERAL EQUILIBRIUM PRICE SETTING AND EFFICIENCY

Price setting under general equilibrium satisfies three property of efficiency viz.,

- 1) Productive ('technical') Efficiency
- 2) Allocative Efficiency
- 3) Product mix Efficiency

We explain below the process of their satisfaction.

12.7.1 Productive (Technical) Efficiency

Productive efficiency requires that we cannot produce more food without producing less shelter. The marginal condition that expresses this idea is

$$\frac{\partial F/\partial L}{\partial F/\partial K} = \frac{\partial S/\partial L}{\partial S/\partial K}$$

$$MRTS_F = MRTS_S$$

Law of one price guarantees that this condition will hold in equilibrium since both sectors face same factor prices for *K* and *L* and we have:

$$\frac{\partial F/\partial L}{\partial F/\partial K} = \frac{\partial S/\partial L}{\partial S/\partial K} = \frac{P_L}{P_K}$$

Each sector equates the ratio of marginal productivities of factors to the price ratio and so these ratios are equated across sectors. Therefore, productive efficiency is satisfied in equilibrium.

12.7.2 Allocative Efficiency

On the consumer side, the analogous requirement to efficient production is efficient allocation of goods, i.e., Efficient Consumption ↔ Allocative efficiency:

$$MRS_A = MRS_B,$$

$$\left(\frac{\partial u/\partial F}{\partial u/\partial S} \right)_A = \left(\frac{\partial u/\partial F}{\partial u/\partial S} \right)_B$$

Such condition hold along with price ratio, in this case the relative prices of final goods (F and S) rather than the prices of factors K and L , is $MRS_A = MRS_B = \frac{P_F}{P_S}$.

As we have seen in the Edgeworth box e condition is satisfied. So, the market equilibrium also satisfies allocative efficiency.

Note that mapping between efficient consumption and efficient production:

Problem	Scarce inputs	Outputs	Efficiency conditions
Efficient consumption	F, S	$u_A = u_A(F_A, F_B)$ $u_B = u_B(F_A, F_B)$	$MRS_A^{FS} = MRS_B^{FS}$ $\left(\frac{u^F}{u^S}\right)_A = \left(\frac{u^F}{u^S}\right)_B$
Efficient production	K, L	$S = S(K_S, L_S)$ $F = F(K_F, L_F)$	$MRTS_A^{FS} = MRTS_B^{FS}$ $\frac{\partial F/\partial L}{\partial F/\partial K} = \frac{\partial S/\partial L}{\partial S/\partial K}$

12.7.3 Product Mix Efficiency

Suppose that we have produced F and S efficiently using K and L and consumers A and B have consumed these goods efficiently. But still something would be missing. What is it?

We could have produced a suboptimal mix of goods — too much food and too little shelter or vice versa. The market condition that guarantees that this does not occur is the tangency between the community indifference curve and the PPF.

$$\text{So along the PPF: } MRPT_{FS} = -\frac{dF}{dS}.$$

For a given price ratio: $MRPT_{FS} = -\frac{dF}{dS} = \frac{P_F}{P_S}$. That means for a given price ratio, the marginal rate of product transformation must be equal to the price ratio.

Why? If we assume $P_F/P_S > 1$, so it costs more than 1 unit of shelter to buy one unit of food, then it needs to be the case that at the technical level that to produce more than one unit of food, it would require more than 1 unit of shelter (or, more accurately, the K and L required to produce that shelter). Had this not been the case, we would be making more food and less shelter.

Similarly, along the community indifference curve, it will also be the case that

$$MRS_{FS} = \frac{\partial U/\partial F}{\partial U/\partial S} = \frac{P_F}{P_S}$$

So, given the equilibrium price vectors, we'll have $MRS_{FS} = MRPT_{FS} = \frac{P_F}{P_S}$.

12.8 LINK BETWEEN FACTOR AND GOODS MARKETS

We've now established the equilibrium conditions for efficient consumption, efficient production and efficient mix.

But we are still missing one thing. We also need to show that there is an appropriate linkage between the factor and goods markets. Without this, how do we know that the price vector that clears the goods market is compatible with the price vector that clears the factor market?

This is actually straight-forward to show. What we require is,

$$\frac{\partial u / \partial F}{\partial u / \partial S} = \frac{\partial S / \partial K}{\partial F / \partial K} = \frac{\partial S / \partial L}{\partial F / \partial L}$$

This equation says that the marginal rate of substitution in consumption must be equated with the marginal rate of transformation. Notice the inversion here, however, the marginal utility of F is relatively high (relative to S), then we want the marginal product of K in production of F to be relatively low (relative to the marginal product of K in S). The reason is that if we get a lot of utility from consumption of F at the margin, we are willing to allocate relatively more K to its production.

A simple example should clarify:

- Suppose you can produce two goods, cake and soft drink by using sugar.
- Assume further that there is diminishing marginal returns to each, so that the more cake you make, the more sugar is used at the margin and similarly for soft drink.
- Assume finally that you greatly prefer eating cake to soft drink on average, but that you have diminishing marginal utility of consumption of each.
- It is therefore efficient for you to allocate sugar to cake (rather than soft drink) until there is very low marginal productivity of sugar in cake and relatively high marginal productivity in soft drink. In other words, since you value cake more at the margin, you are happy to use most of sugar for cake, even though the marginal physical productivity of sugar is low in this use.

To know that the marginal conditions above are satisfied we have to examine the tangency condition between the community indifference curve and the PPF.

We know that $MRS_{FS} = MRTP_{FS}$. Consider shifting one unit of K from production of F to production of S . Using the PPF, we have:

$$-\frac{dF}{dS} = \frac{\partial S / \partial K}{\partial F / \partial K} = MRPT_{FS},$$

$$-\frac{dF}{dS} = \frac{\partial S / \partial L}{\partial F / \partial L} = MRPT_{FS}.$$

But from the tangency condition between the community indifference curve and the PPF, we know that $MRS_{FS} = \frac{\partial u / \partial F}{\partial u / \partial S} = MRPT_{FS}$. This implies that $\frac{\partial u / \partial F}{\partial u / \partial K} = \frac{\partial F / \partial K}{\partial S / \partial K}$ and $\frac{\partial u / \partial F}{\partial u / \partial S} = \frac{\partial F / \partial L}{\partial S / \partial L}$.

12.9 LINK BETWEEN GOODS AND FACTOR PRICES

A question lingers in the mind is: what is the relation between the prices of goods and the prices of factors?

To find an answer, one can demonstrate that:

$$P_K = (\partial F / \partial K) P_F = (\partial S / \partial K) P_S,$$

$$P_L = (\partial F / \partial L) P_F = (\partial S / \partial L) P_S,$$

$$\frac{P_K}{P_L} = \frac{(\partial F / \partial K) P_F}{(\partial S / \partial L) P_S} = \frac{(\partial S / \partial K) P_S}{(\partial F / \partial L) P_F}$$

In equilibrium it will be true since we know that $\frac{P_F}{P_S} = \frac{\partial U / \partial F}{\partial U / \partial S}$.

That is, the price of capital and labour depends on their marginal contributions to production of F and S . In particular,

$$P_K = (\partial U / \partial F)(\partial F / \partial K) = P_F(\partial F / \partial K),$$

$$P_L = (\partial U / \partial F)(\partial F / \partial L) = P_F(\partial F / \partial L).$$

We can write the analogous equations for P_K and P_L in the production of S rather than F .

These equations indicate that P_K and P_L reveal the marginal contribution of K and L to consumer utility through their production of consumer goods F and S . So we can write

$$\frac{P_K}{P_L} = \frac{\partial U / \partial K}{\partial U / \partial L}.$$

That is to say, the prices of factors K and L mirror their 'indirect utility' through their transformation into goods F and S .

12.10 LET US SUM UP

In this unit we have shown that a free market in equilibrium will simultaneously satisfy all the three efficiency criteria:

- 1) Technical efficiency
- 2) Allocative efficiency
- 3) Product Mix efficiency

For a set of given variables, viz., factor endowments K and L , preferences u_A, u_B and technologies $F = F(K, L)$, $S = S(K, L)$, we obtain results like,

- 1) The market will *all by itself* give

$$F^*, S^*, (P_F/P_S)^*, (P_F/P_S)^*, (P_K/P_S)^*, (P_L/P_S)^*.$$

Also note that of the four possible prices, only three are needed to be the first three. The choice of P_S as the 'numeraire' is a normalisation; any of the four prices can serve this purpose;

- 2) Goods and factor markets will both clear with neither excess demand nor excess supply;
- 3) All gains from trade among consumers and reallocation of resources across goods will be exhausted;
- 4) In other words, the equilibrium will be Pareto efficient.

It is however, useful to keep in mind that these results are derived under the satisfaction of stringent conditions of

—Perfectly competitive markets

—Full information

—No transaction costs

—No externalities

Of course we are not realistic in thinking that market operates satisfying these conditions. Nevertheless it is remarkable to realise that a free market economy can potentially solve this set of fundamental efficiency problems without any centralised decision making. Therefore, it is often suggested that free market economies have considerable potential to generate many desirable outcomes. As a precaution results generated from general equilibrium model need to be seen by allowing relaxation above mentioned conditions into the model.

12.11 KEY WORDS

Allocative Efficiency(Consumption Efficiency): An occurrence when there is an optimal distribution of goods and services, taking into account consumer's preferences. At the output level where the price equals the Marginal Cost (MC) of production since the price that consumer's are willing to pay is equivalent to the marginal utility that they get. Therefore the optimal distribution is achieved when the marginal utility of the good equals the marginal cost.

Production Possibility Frontier: A locus of output of an economy when resources are optimally allocated.

Product Mix Efficiency: Equality between individuals' preferences and the production Possibilities, a condition necessary to ensure that the right goods are produced such that the marginal rate of substitution (MRS) for any two goods must equal the rate of product transformation (RPT) of the two goods.

Rate of Product Transformation (RPT): The rate at which one output can be traded for another in the productive process while holding the total quantities of inputs constant. The RPT is the absolute value of the slope of the production possibility frontier.

The Law of one Price: A good must sell for the same price in all location. It constitutes the basis of the theory of purchasing power parity.

Social Welfare Function: A mapping of individual utility functions to an aggregate utility function.

Technical Efficiency (productive efficiency): Productive efficiency is concerned with producing goods and services with the optimal combination of inputs to produce maximum output for the minimum cost. To be productively efficient means the economy must be producing on its production possibility frontier.

12.12 SOME USEFUL BOOKS

Kreps, D. (1990) *A Course in Microeconomic Theory*, Princeton, NJ: Princeton University.

Mas-Colell, A., M. Whinston, and J. Green (1992), *Microeconomic Theory*, New York, NY: Oxford University Press.

Nicholson, Walter (1992), *Microeconomic Theory: Basic Principles and Extensions*, The Dryden Press, Harecourt Bral Jovanovich, Orlando.

Varian, H. (1992) *Microeconomic Analysis*, 3rd ed. New York, NY: W.W. Norton.

12.13 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

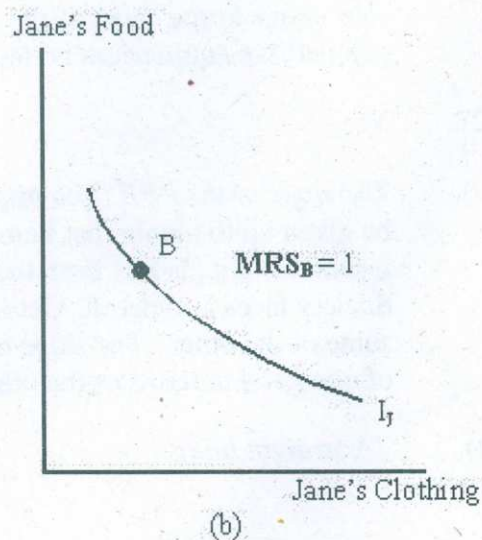
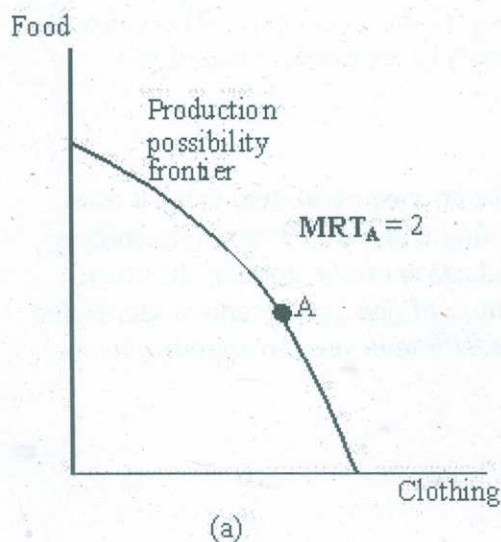
- 1.a) The horizontal and vertical dimensions of the Edgeworth box measure the total quantities of the two goods. Vertical and horizontal movements indicate that one party has given away some of one good without getting anything in return. If both goods are economic goods, a voluntary trade would not be shown by such movements.
- 1.b) A competitive equilibrium is one point on the production contract curve. It is the intersection of the production contract curve and a line passing through the initial allocation with a slope equal to the ratio of prices. For a competitive equilibrium to hold, each producer must use inputs so that the slopes of the isoquants are equal to one another, and also equal to the ratio of the prices of the two inputs. Therefore, the competitive equilibrium is (technically) efficient in production.
- 2) Yes
- 3) *The slope of the PPF.* The **opportunity cost** of an item is what must be given up to obtain that item. Moving along a *PPF* involves shifting resources (e.g., labor) from the production of one good to the other. Society faces a tradeoff: Getting more of one good requires sacrificing some of the other. The slope of the PPF tells you the opportunity cost of one good in terms of the other.
- 4) [A straight line]

Check Your Progress 2

- 1.a) The Edgeworth box allows us to represent the distribution of two goods between two individuals. The box is formed by inverting the indifference curves of one individual and superimposing those on the indifference curves of another individual. Each point in the box thus represents a different allocation of the two goods between the two individuals.
- 1.b) The contract curve is the set of points where the indifference curves of the two individuals are tangential. The MRS is equal to the (negative) slope of the indifference curves, and if two curves are tangent at a point, their slopes are also equal at that point. Thus, by definition, the MRS between the two goods is equal for the two individuals if we assume convex indifference curves.
- 2) If $MRPT \neq MRS$, we could reallocate inputs in producing outputs to leave consumers better off. If $MRPT \neq MRS$, the ratio of marginal cost will not be equal to the ratio of the prices. We could increase the output of one good, sell this output in the market, and use the proceeds to increase the production of the other good, leaving the consumer better off than in the initial position. Therefore, the initial allocation was not Pareto efficient. Only when $MRPT = MRS$ will consumers be left worse off with a reallocation of input between the production processes.
- 3) The utility possibilities frontier plots the utility levels of the two consumers for all points on the contract curve. One consumer's utility is plotted on the vertical axis and the other consumer's utility is shown on the horizontal axis. While points that lie between the origin and the utility possibilities frontier are feasible, they are not efficient because they are not on the frontier, and trades are possible that will make one or both individuals better off without making anyone worse off. Points outside the frontier are not feasible unless the individuals are given greater amounts of one or both goods.

Check Your Progress 3

- 1) At the product mix A in panel a of the diagram below, the Marginal Rate of Transformation is 2. If Jane's Marginal Rate of Substitution at bundle B in panel b is 1 then what combination of food and clothing production will make everyone better off



By producing two more units of Food and one less unit of clothing, it is possible to make everyone better off.

- 2) Consumers maximize utility taking prices as given. From utility max $MRS = \text{price ratio}$. If consumers are price takers, they all face the same prices and therefore all their marginal rates of substitution have to be equal. The competitive equilibrium satisfies consumption efficiency.

Firms maximize profits. From profit maximization of competitive firms (if the number of firms is sufficiently large) $P = MC$.

For any two goods, consumers set $MRS = p_X/p_Y$. From profit max a firm produces an amount of X where $p_X = MC_X$, and a firm produces an amount of Y where $p_Y = MC_Y$. This implies $p_X/p_Y = MC_X/MC_Y$, but $MC_X/MC_Y = MRT$ and therefore $MRS = MRT$. The competitive equilibrium satisfies product mix efficiency.

- 3) When we produce too much of one good and not enough of the other. That means product mix efficiency has to put together both sides, consumers and producers.

12.14 EXERCISES

- 1) Consider an economy with production of two goods, A and B, and two people, Sita and Baxi. There are a total of 4 units of good A and 8 units of good B. If the economy is Pareto efficient, what will be MRPT.

Ans. Baxi always requires $B=2A$ and Sita has an $MRS=B/A$. Exchange equilibrium thus required Sita's $MRS=B/A=2A/A=2$. Hence whatever formula accounts for the MRPT, the value of the MRPT=2.

- 2) individuals have a utility function $U=A*B$. The PPF for A and B is a straight line with a constant MRPT. With the resources in the economy, at most 20 units of B can be produced, and at most 10 units of A.
- What is the MRPT in this economy?
 - Find the formula for the MRS for each person.
 - How much A and how much B will be produced if the economy is Pareto efficient? Explain. (Hint, remember we need the $MRS = MRPT$.)
- 3.a) Using the two point formula for slope: $(x-y)/(x'-y') = (0-20)/(10-0) = -2$. The negative of this is the MRPT.
- $MRS = B/A$ for both individuals because of symmetric preferences.
 - We need $MRS = MRPT$, but we know the $MRPT=2$. Hence we need the $MRS = 2$ as well. Efficient production where $B/A = 2 \rightarrow B = 2A$. We also know that $B = 20 - 2A$ from the PPF. Hence $2A = 20 - 2A$ implying $4A = 20$ or, $A = 5$. Then $B = 10$. This is the only point on the PPF where $B = 2A$.
- 4) In the preceding question suppose Al can produce only A and Sue can produce only B.

- a) Draw the Edgeworth box of that illustrates efficient production and the initial allocation.
- b) Show the exact point at which they will end up. Explain why they end up there.

Let's assume, because it has to end up there, that the economy starts with the efficient amount of production that we found in question 5, so $B=10$ and $A=5$. Al produces 5 A and Sue produces 10 B. Since Al produces all of the A and Sue all of the B, the initial allocation is at the purple dot. We know both have an $MRS = 2$ and $B = 2A$ for both Sue and Al, which means they end up right in the middle along line, where each consumes 2.5 A and 5 B.

- 5) "Because all points on a contract curve are efficient, they are all equally desirable from a social point of view." Do you agree with this statement? Explain.

If society is only concerned with efficiency and not with equity, then all points on the contract curve are equally desirable. Since it is impossible to make comparisons of utility between individuals, economics focuses on efficiency. But most societies are also concerned with equity (i.e., whether the final allocation is fair, however that might be defined). In this case, all points on the contract curve are not equally desirable.

- 6) How is the production possibilities frontier related to the production contract curve?

We can graph the quantities of each good produced (each point in the Edgeworth box) on a graph, where the vertical axis represents the output of one good and the horizontal axis represents the output of the other good. The production contract curve is represented in this graph as the production possibilities frontier. It is analogous to the utility possibilities frontier for consumers. Points inside the frontier are feasible but inefficient; points outside the frontier are infeasible and attainable only when more inputs become available or technological change increases productivity. Points on the production possibilities frontier are the same as those on the production contract curve. The difference is that the production contract curve measures inputs on the axes and the production possibilities frontier measures outputs on the axes.

- 7) The marginal rate of transformation, $MRPT$, is equal to the absolute value of the slope of the production possibilities frontier, PPF , and measures how much of one output must be given up to produce one more unit of the other output. The total cost of all inputs is the same at each point on the PPF because the amount of each input is fixed. Therefore, when we move down along the frontier, the cost of producing one output is reduced by the same amount as the cost of producing the other output is increased. Suppose the $MRPT$ is 4, then we must give up 4 units of the output on the vertical axis to get one more unit of the output on the horizontal axis.

This means that the total cost of producing the 4 units is the same as the total cost of producing the one unit, so the marginal cost of the good on the horizontal axis is 4 times the marginal cost of the good on the vertical axis.

- 8) Explain why goods will not be distributed efficiently among consumers if the $MRPT$ is not equal to the consumers' marginal rate of substitution.

If the marginal rate of transformation, $MRPT$, is not equal to the marginal rate of substitution, MRS , we could reallocate inputs in producing output to leave the consumers and producers better off. If the MRS is greater than the $MRPT$, then consumers are willing to pay more for another unit of a good than it will cost the producers to produce it. Both consumers and producers can therefore be made better off by arranging a trade somewhere between what consumers are willing to pay and what the producers have to pay to produce the extra unit. Note also that when $MRT \neq MRS$, the ratio of marginal costs will not be equal to the ratio of prices. This means that one good is being sold at a price below marginal cost and one good is being sold at a price above marginal cost. We should increase the output of the good whose price is above marginal cost and reduce the output of the other good whose price is below marginal cost.

- 9) Why can free trade between two countries make consumers of both countries better off?

Free trade between two countries expands each country's effective production possibilities frontier and allows each country to consume at a point above its original production possibilities frontier. Since each country has a comparative advantage in the production of some good or service, trade allows each country to specialise in the area where it has this advantage. It trades these outputs for those more cheaply produced in another country. Therefore, specialisation benefits consumers in both countries.

- 10) If Country A has an absolute advantage in the production of two goods compared to Country B, then it is not in Country A's best interest to trade with Country B. True or false? Explain.

This statement is false. A country can have an absolute advantage in the production of all goods, but it still will have a comparative advantage only in the production of some goods. Suppose Country A requires 4 units of labor to produce good 1 and 8 units of labor to produce good 2, whereas Country B requires 8 units and 12 units respectively. Country A can produce both goods more cheaply, so it has an absolute advantage in the production of both goods. However, trade is based on comparative advantage, which depends on how much of one good must be given up for one more unit of the other good. Here, Country A must give up 2 units of good 1 for another unit of good 2 whereas Country B must give up only 1.5 units of good 1 for another unit of good 2. Country B therefore has a comparative advantage in producing good 2. Likewise, Country A has a comparative advantage in producing good 1 and should trade good 1 to Country B for good 2.

- 11) Jane has 3 liters of soft drinks and 9 sandwiches. Bob, on the other hand, has 8 liters of soft drinks and 4 sandwiches. With these endowments, Jane's marginal rate of substitution (MRS) of soft drinks for sandwiches is 4 and Bob's MRS is equal to 2. Draw an Edgeworth box diagram to show whether this allocation of resources is efficient. If it is, explain why. If it is not, what exchanges will make both parties better off?

Given that $MRS_{Bob} = MRS_{Jane}$, the current allocation of resources is inefficient. Jane and Bob could trade to make one of them better off without making the other worse off. Although we do not know the exact shape of Jane and Bob's indifference curves, we do know the slope of both indifference curves *at the current allocation*, because we know that $MRS_{Jane} = 4$ and $MRS_{Bob} = 2$. At the current allocation (point A in the diagram), Jane is willing to trade 4 sandwiches for 1 drink, or she will give up 1 drink in exchange for 4 sandwiches. Bob is willing to trade 2 sandwiches for 1 drink, or he will give up 1 drink in exchange for 2 sandwiches. Jane will give 4 sandwiches for 1 drink while Bob is willing to accept only 2 sandwiches in exchange for 1 drink.

Any exchange that leaves both parties inside the lens-shaped area between points A and B will make both better off. For example, if Jane gives Bob 3 sandwiches for 1 drink, they will be at point C. Jane is better off because she was willing to give up 4 sandwiches but only had to give up 3. Bob is better off because he was willing to accept 2 sandwiches and actually received 3. Jane ends up with 4 drinks and 6 sandwiches and Bob ends up with 7 drinks and 7 sandwiches, and both are better off than at point A.

- 12) In the context of our analysis of the Edgeworth production box, suppose that a new invention changes a constant-returns-to-scale food production process into one that exhibits sharply increasing returns. How does this change affect the production contract curve?

In an Edgeworth production box, the production contract curve is made up of the points of tangency between the isoquants of the two production processes. A change from a constant-returns-to-scale production process to a sharply-increasing-returns-to-scale production process does not necessarily imply a change in the shape of the isoquants. One can simply redefine the quantities associated with each isoquant such that proportional increases in inputs yield greater-than-proportional increases in outputs. Under this scenario, the marginal rate of technical substitution would not change. Thus, there would be no change in the production contract curve.

However, if the change to a sharply-increasing-returns-to-scale technology involved a change in the trade-off between the two inputs (a change in the shape of the isoquants), then the production contract curve would change.

For example, suppose the original production functions for food and the other good (call it shelter) were both $Q = LK$ and $MRTS = K/L$ with then the shape of the isoquants would not change if the new production function for food were $Q = K^2L^2$ with $MRTS = K/L$. In both cases the production contract curve would be linear. The shape of the production contract curve would become nonlinear, however, if the new production function for food were $Q = L^2K$ with $MRTS = 2(K/L)$ assuming the production function for shelter was unchanged.

- 13) Suppose there are 2 consumers, Ara and Bahar, and two goods good X and good Y. Ara's utility function is given by $U_A = XY$ and Bahar's utility function is given by $U_B = X^{1/4}Y^{3/4}$. Both goods are produced with labor and capital and good X's production function is $f_X =$

$(K_X L_X)^{1/2}$ while good Y's production function is $f_Y = (K_Y L_Y)^{1/2}$. Suppose there are 8 units of labor and 8 units of capital in this economy. Suppose Ara receives one unit of good X. How many units of good Y must Ara receive and how many units of good X and good Y must Bahar receive for the allocation to be Pareto efficient?

Ans. Production efficiency: We have 8 units of labor and eight units of capital. If for both goods the capital labor ratio is equal to one, then $K_X/L_X = 1 = MP_{LX}/MP_{KX}$ and $K_Y/L_Y = 1 = MP_{LY}/MP_{KY}$. So $MRTS_{X \text{ capital for labour}} = MRTS_{Y \text{ capital for labour}}$ $MRTS_X \text{ capital for labor}$. Moreover PPF: $Y = 8 - X$.

For product mix efficiency $MRS = MRT, Y_A/X_A = Y_B/X_B = 1$.

The allocation $K_X = L_X = 2.5$, $K_Y = L_Y = 5.5$, $X_A = 1$, $X_B = 1.5$, $Y_A = 1$, $Y_B = 4.5$ is Pareto efficient. We have shown that with this allocation of resources and consumption goods all three conditions of Pareto efficiency are satisfied.

NOTES

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