



Indira Gandhi
National Open University
School of Social Sciences

MEC-101
Microeconomic
Analysis

Block

6

ECONOMICS OF UNCERTAINTY

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BLOCK 6 ECONOMICS OF UNCERTAINTY

Introduction

In this block we analyse individual's decision-making behaviour in the uncertain and risky situations. So, the choice problems in deciding to (i) purchase lottery investing in stock markets; (ii) strike a deal with one's competitors and (iii) sign a contract in markets characterised by asymmetries information, is examined keeping in view that outcome of such event are uncertain. **Unit 17** presents the choice problem in risky situations when the probability of various outcomes is fairly well understood. Expected utility theory as developed by von Neumann-Morgenstern and its application with respect to insurance and gambling in the presence of risk averse, risk loving and risk neutral individuals is the main focus of examination. The succeeding unit (**Unit 18**) extends the analysis to insurance markets and it contains discussion on their functioning. The last theme of the block deals with information problems prevailing the markets. Thus, in **Unit 19**, an attempt is made to examine the ways individuals utilise the private information they held to their advantage and how the other party to a transaction devise mechanism to extract best response.



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UNIT 17 CHOICE IN UNCERTAIN SITUATIONS

Structure

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17.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the behaviour of individuals in risky decision-making situations;
- appreciate the tools adopted in arriving at decisions under risk and uncertainty; and
- examine the utility in the process of consumption associated with insurance.

17.1 INTRODUCTION

We have seen the behaviour of economic agents under conditions of certainty, i.e., the situation in which consequences of any choice made were fully known beforehand. However, once we bring in choices to be made under uncertainty, the theoretical prescriptions by far remain an uncovered domain. As our income levels fluctuate, price we pay change or health conditions worsened, we need to model choices accounting for such plausible events.

Also, see that most of decisions we take are forward looking: planning a holiday trip, to marry or to buy insurance are examples. Decision on these are usually taken on the basis of our beliefs about what is the optimal plan for present and future. Thus, these choices are made in the context of uncertainty. Consequently, there is a risk that the assumptions made in our plans may not

materialise. Anticipating such eventualities we resort to contingencies and probabilities. That is, if we want a realistic model of choice, it would be necessary to include in our models the effects of uncertainties.

17.2 BEHAVIOUR UNDER UNCERTAINTY: SOME OBSERVATIONS

In the presence of uncertain situations, people tend to show some behavioural traits. Look at the following observations:

- 1) People often do not want to play actuarially fair games.
- 2) People won't necessarily play actuarially favourable games.
- 3) People would not pay large amounts of money to play gambles with huge upside potential.

Example

Suppose Rita and Sita agree to flip a coin one time. If a head comes up, Rita will pay Sita Re. 1; in the event of the appearance of a tail Sita will pay Rita Re. 1. Try to understand this game taking Rita's view points. She expects two prizes available in the game:

- X_1 head prize $\rightarrow$ her payoff = -1, (as she loses Re. 1)
- X_2 tail prize $\rightarrow$ her payoff = +1 (as she wins Re. 1).

Expectations of the other player also run in the same line except for the reversed values of X_1 and X_2 .

Thus, the expected value of this game is

$$\frac{1}{2} X_1 + \frac{1}{2} X_2 = \frac{1}{2}(\text{Re.1}) + \frac{1}{2}(-\text{Re.1}) = 0$$

Remember that even when this game is played a number of times, the expected gains of the participants will not change.

Change this game slightly to assign $X_1 = \text{Rs.10}$ and $X_2 = \text{Re.1}$. Its expected value is

$$\begin{aligned} \frac{1}{2} X_1 + \frac{1}{2} X_2 &= \frac{1}{2}(\text{Rs.10}) + \frac{1}{2}(-\text{Re.1}) = 0 \\ &= \text{Rs. } (5 - 0.50) \\ &= \text{Rs. 4.50} \end{aligned}$$

When this game is played a large number of times, Sita will be a distinct gainer. Perhaps, Sita could pay Rita a small amount as an entry fee to participate in the game. In both its versions, the above game is called an *actuarially fair game*. It is observed that in many situations, people refuse to play actuarially fair games.

One way of appreciating above idea is to note your reaction if you are invited to participate in the above game with a bait of Rs. 10,000 instead of Re.1. You will, in all likelihood, decline to accept the offer. People often avoid playing games with bigger risk even if these are fair ones. For a formal treatment of the theme, it will be useful to touch upon the St. Petersburg Paradox.

17.2.1 St. Petersburg Paradox

A coin is flipped until a head appears on the n^{th} flip, at which the player is paid Rs. 2^n . If X_i represents the prize awarded when the first head turns up on the i^{th} trial, then

$$X_1 = \text{Rs.2}, X_2 = \text{Rs.4}, X_3 = \text{Rs. 6}, \dots\dots X_n = 2^n.$$

The probability of getting a head for the first time on the i^{th} trial is $\frac{1}{2}^i$. See that it is the probability of getting $(i - 1)$ tails and then a head. Hence, the probabilities of prizes, π_i are

$$\pi_1 = \frac{1}{2}, \pi_2 = \frac{1}{4}, \pi_3 = \frac{1}{8}, \dots, \pi_n = \frac{1}{2^n}.$$

The expected value of the game is

$$\sum_{i=1}^{\infty} \pi_i X_i = \sum_{i=1}^{\infty} 2^i \frac{1}{2^i} = 1 + 1 + 1 + \dots + 1 + \dots = \infty.$$

Since the expected value of the game is ∞ , how much would you be willing to pay to play this game? Perhaps, not more than a few rupees. Thus, the game in this sense not worth its large expected value (no taker to enter it).

As a solution to this paradox, it was argued that individuals attached negative 'utility value' to positive expected 'monetary value'. Because utility may rise less rapidly than the monetary value of the prizes, the utility value of the game will fall short of its monetary value. Thus, uncertain prospects are worth less in utility terms than certain ones, even when expected tangible payoffs are the same.

17.3 LOTTERIES

One way of characterizing such problems is to use the concept of a lottery. A lottery represents a pair of objects. First, X_i is a list of possible consequences of a decision and second is a list $P = (P_1, P_2, \dots, P_n)$ of probabilities with which we think of the occurrences of each consequence. Note that the number of probabilities is equal to the number of consequences in X . Each of the consequences could be a lottery. Thus, if $X = \{X_1, X_2, \dots, X_n\}$ be the space of constituted of all possible states of events and $P = \{p_1, p_2, \dots, p_n\}$ is a probability distribution, then we call the pair $L = \{X; P = X_1, X_2, \dots, X_n; p_1, p_2, \dots, p_n\}$ a lottery.

17.3.1 Simple Lottery

Simple lottery L is a list $L = (p_1, p_2, \dots, p_n)$ with $p_n \geq 0$ for all n and $\sum_n p_n = 1$ where p_n is the probability of outcome n occurring.

Example

You purchased a lottery ticket. The set of consequences consists of two elements: you either win or lose a prize. You also know that each of these consequences occurs with probability $\frac{1}{2}$.

17.3.2 Compound Lottery

A lottery over lotteries is called a compound lottery.

Given k simple lotteries $L_k = (p_1^k, \dots, p_N^k)$, $k = 1, \dots, K$ and probability $\alpha_k \geq 0$ with $\sum_k \alpha_k = 1$, the compound lottery $(L_1, \dots, L_k; \alpha_1, \dots, \alpha_k)$ is the risky alternative that yields simple lottery L_k with probability α_k for $k = 1, \dots, k$.

Example

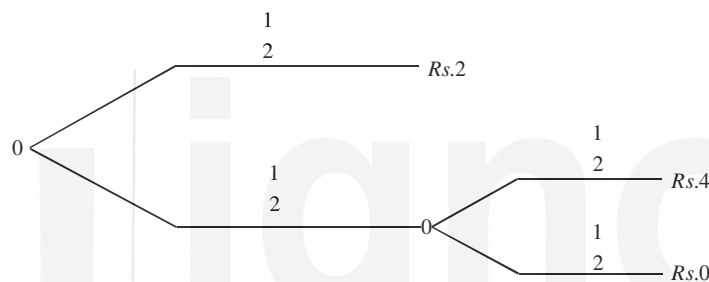
You flip a coin. If the coin comes up head in the first trial you get Rs.2. If tail appears, the coin is flipped again. If it comes up head in the second flip, you

get Rs.4, if tail comes up, the game stops and you get nothing. This is a compound lottery. There are two lotteries here:

In the first, there was a single consequence (you receive Rs.2) and you get the prize with probability 1. Come to the second lottery where each consequence occurs with equal probability. Thus, compound lottery involves $\frac{1}{2}$ chance that you will play the first lottery and get Rs.2 for sure. In another $\frac{1}{2}$ probability, you will play the second lottery and get either Rs.4 or 0.

17.3.3 Reduced Lottery

Every compound lottery can be presented as a reduced lottery which is same as that of the simple lottery where the consequences, as depicted in the above example, you receive either Rs.2, Rs.4 or 0 with probability $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. To appreciate the logic, you can present this example in terms of a decision tree.



From the decision tree, you can get the reduced form probabilities. The probabilities of the second lottery with prizes Rs.4 and 0 are obtained multiplying probabilities associated with each node, viz., $\frac{1}{2} \times \frac{1}{2}$ and

$$\frac{1}{2} \times \frac{1}{2}.$$

To see why compound lottery can be presented as a simple lottery, you may consider a situation where an individual is not sure, which of the lotteries, $L = \{p_1, p_2, \dots, p_N\}$ or $L' = \{p'_1, p'_2, \dots, p'_N\}$, she is facing. So she assigns probability α to it being L and $(1 - \alpha)$ to it being L' . Now, consider the resulting compound lottery

$$L'' = \{L, L'; \alpha, 1 - \alpha\}$$

Consequently, you find that

$$L'' = \{\alpha p_1 + (1 - \alpha) p'_1, \alpha p_2 + (1 - \alpha) p'_2, \dots, \alpha p_n + (1 - \alpha) p'_n\}$$

$= \alpha L + (1 - \alpha) L'$, which is a simple lottery. Thus, a compound lottery is an average of simple lotteries.

So, for any compound lottery $(L_1, \dots, L_k; \alpha_1, \dots, \alpha_k)$, we can calculate a corresponding reduced lottery as a simple lottery $L = (p_1, \dots, p_N)$ that generates the same ultimate distribution over outcomes. Taking probability of outcome n in the reduced lottery is

$$p_n = \alpha_1 p_n^1 + \alpha_2 p_n^2 + \dots + \alpha_k p_n^k \text{ for } n = 1.$$

That is, you simply add up the probabilities p_n^k of each outcome n in all lotteries, k , multiplying each p_n^k by the probability α_k of each lottery k .

17.3.4 Preferences Over Lotteries

While we need to choose among lotteries, there is no obvious way to do this. In the process of making a choice, however, it looks reasonable to assume, you will be able to express a preference over any pair of lotteries.

The basic premise of modelling preference is that you care only about the reduced lottery over final outcomes. Hence, you will be effectively indifferent to the compound lotteries underlying those reduced lotteries.

Let there be a set of lotteries $\mathcal{L}$ from which a choice is to be made. We assume that consumer has a preference relation $\succsim$ on $\mathcal{L}$. That is, if we select two lotteries, L and L' from space $\mathcal{L}$ and write $L \succsim L'$ implying thereby L is at least as good as lottery L' . If such preferences have the properties of completeness, reflexivity and transitivity, the individual would be able to rank her preferences. Recall that these concepts have been discussed in consumer's preference theory with certainty. But if you like, we will assume that $\succsim$ are such that

- for all x and y in X either $x \succsim y$ or $y \succsim x$ or both (completeness)
- $1.x+0.y \sim x$ (reflexive)
- for all x, y and z in X ,
if $x \succsim y$ and $y \succsim z$, then $x \succsim z$ (transitivity)

In the above, completeness indicates that the individual can compare any two lotteries, while transitivity means, if one lottery is at least as good as another, which in turn, is at least as good as the third, then this individual will rank the first lottery as being at least as good as the third.

Along with the rationality of preferences $\succsim$, let us bring in an additional regularity assumption ("continuity") to rule out certain discontinuous behaviour. Broadly, the continuity axiom says that small changes in probabilities do not change the nature of the ordering of two lotteries.

17.4 EXPECTED UTILITY THEORY

When the preference relations and continuity assumptions are satisfied, we can conclude that there exists a utility function

$U: \mathcal{L}(X) \rightarrow R$, which represents these preferences. That is,

$$L \succsim L' \Leftrightarrow U(L) \geq U(L')$$

For making this formulation workable, we need to take expectation of utilities. Before doing that, it will be useful to examine lotteries without uncertainty (or, sure winning lotteries). Let lotteries

$$L_1 = \{1, 0, 0, \dots, 0\}, L_2 = \{0, 1, 0, \dots, 0\} \dots L_N = \{0, 0, 0, \dots, 1\}$$

give 'no uncertainty' representation. Then it is not unreasonable to view utility of such a lottery as simply utility of having the corresponding state for sure. So we can write $u(x_k) = U(L_k)$. But it is generally not true that utility of a lottery is equal to the expected utility of the states given the probability distribution of the lottery.

17.4.1 Independence Axiom

The preference relation $\succsim$ on the space of simple lotteries $\mathcal{L}$ satisfies the independence axiom if for all

$$L, L', L'' \in \mathcal{L} \text{ and } \alpha \in (0, 1), \text{ we have}$$

$$L \succsim L' \text{ if and only if}$$

$$[\alpha L + (1-\alpha)L''] \succsim [\alpha L' + (1-\alpha)L'']$$

We interpret the above preference relation in words as follows:

When we mix each of two lotteries with a third one, then the preference ordering of the two resulting mixtures does not depend on, or independent of, the particular third lottery used. To see more specifically, suppose you choose L while comparing L and L' . Next suppose that, whether you choose L or L' with some probability $(1-\alpha)$, you will be facing a different lottery L'' .

Independence axiom says, this additional uncertainty *which is the same whatever your choice is*, should not matter for your choice.

Therefore, this axiom also says, the order (or frame) of lotteries is unimportant. That is,

$$[px + (1-p)y] \sim [(1-p)y + px]$$

Example

Consider a two-stage lottery as follows:

Stage I: Flip a coin to get head or tail.

Stage II: If it is head, flip again

Head yields Re.1 and tail yields Rs.0.75. If it is tail, roll a die with payoffs, Rs.0.10, Rs.0.17,... Rs.0.60, corresponding to outcomes 1 – 6. Now consider a single stage lottery, where you spin a pointer on a wheel with 8 areas: 2 areas of 90° representing Re.1 and Rs.0.75 and 6 areas of 30° each representing Rs.0.10, Rs.0.17,... Rs.0.60 each.

See that the single stage lottery has the same payouts at the same odds as the 2-stage lottery. Thus, as the compound lottery axiom says, the consumer is indifferent between these two.

Check Your Progress 1

- 1) You are offered this gamble: we will flip a coin. If it is head, I will give you Rs.10 million. If it is tail, you owe me Rs.9 million. Find its monetary value and point out if you would like to participate in the gamble.

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- 2) What is a lottery? Differentiate between simple and compound lotteries.

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- 3) What is continuity axiom?

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- 4) What is the meaning of independence axiom?

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- 5) Write down the expected utility theory.

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- 6) Two players have the opportunity to participate in a gamble with two possible outcomes as

Outcome	Probabilities
Rs.10	0.3
Rs.30	0.7

The players' utility functions for the money outcomes, are as follows:

Player 1: $U_1(M) = \sqrt{M+6}$ Player 2: $U_2(M) = (M+5)^2$.

Determine the difference in the amounts that you must offer to these two players.

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17.5 vNM EXPECTED UTILITY THEORY

von Neumann and Morgenstern showed that under the assumptions of rationality, continuity and independence, it is possible to choose a particular-utility function u , which represents the preferences and has an expected utility form:

$$U(L) = \sum_{i=1}^N p_i u(x_i) = E(u(x) | L)$$

Take note of the notation we are using in the discussion. The lower case u , defined over states, is called **Bernoulli utility function** while the upper case U , defined over lotteries, is known as von Neumann-Morgenstern (vNM) utility function. We will overlook the difference of usage here and call the utility formulation as vNM expected utility function.

The utility function u over states uniquely defines the preferences of an individual over the larger domain of the lotteries. It carries a lot more information than an arbitrary utility function over X . A monotonic transformation may, actually, distort some of this information. However, if the two utility functions are linear monotonic transformations of one another, and we can write

$w(x) = a u(x) + b$ where $a > 0$, $b =$ any real number,

then they represent the same preferences over uncertainty.

When we discuss the utility of a lottery in vNM expected utility form, we are essentially considering the expected utilities u_n of the N outcomes. That is to say, a utility function that has expected utility property must have the feature that the utility of a lottery is simply the (probability) weighted average of the utility of each outcome, viz.,

$$U\left(\sum_{k=1}^k \alpha_k L_k\right) = \sum_{k=1}^k \alpha_k U(L_k) \text{ for any } k \text{ lotteries}$$

$$L_k \in \mathcal{L}, k=1, \dots, k \text{ and probabilities } (\alpha_1, \dots, \alpha_k) \geq 0 \quad \alpha_k = 1.$$

17.5.1 Proof of Expected Utility Property

Suppose that the rational preference relation $\succsim$ on the space of lotteries $\mathcal{L}$ satisfies the continuity and independence axioms. Then $\succsim$ admits a utility representation of the expected utility form. That is, we can assign a number u_n to each outcome $n = 1, \dots, N$ in such a manner that for any two lotteries $L = (p_1, \dots, p_N)$ and $L' = (p'_1, \dots, p'_N)$, we have $L \succsim L'$ if and only if

$$\sum_{n=1}^N u_n p_n \geq \sum_{n=1}^N u_n p'_n$$

Proof: (Taken mostly from Autor, 1704) Assume that there are best and worst lotteries in $\mathcal{L}$, $\bar{L}$ and $\underline{L}$.

- 1) If $L \succ L'$ and $\alpha \in (0,1)$, then $L \succ \alpha L + (1-\alpha)L' \succ L'$ (due to the independence axiom)
- 2) Let $\alpha, \beta \in [0,1]$. Then $\beta \bar{L} + (1-\beta)\underline{L} \succ \alpha \bar{L} + (1-\alpha)\underline{L}$ if and only if $\beta > \alpha$.
- 3) For any $L \in \mathcal{L}$, there is a unique α_L such that $\alpha_L \bar{L} + (1-\alpha_L)\underline{L} \sim L$. (due to continuity axiom)
- 4) The function $U : \mathcal{L} \rightarrow \mathbb{R}$ that assigns $U(L) = \alpha_L$ for all $L \in \mathcal{L}$ represents the preference relation $\succsim$. From (3) above for any two lotteries $L, L' \in \mathcal{L}$, we have

$$L \succsim L' \text{ if and only if } [\alpha_L \bar{L} + (1-\alpha_L)\underline{L}] \succsim [\alpha_{L'} \bar{L} + (1-\alpha_{L'})\underline{L}].$$

Thus, $L \succsim L'$ if and only if $\alpha_L \geq \alpha_{L'}$.

- 5) The utility function $U(\cdot)$ that assigns $U(L) = \alpha_L$ for all $L \in \mathcal{L}$ is linear and therefore has the expected utility form.

We want to show that for any $L, L' \in \mathcal{L}$, and $\beta \in [0,1]$, we have $U(\beta L + (1-\beta)L') = \beta U(L) + (1-\beta)U(L')$.

By (3) above, we have

$$L \sim U(L)\bar{L} + (1-U(L))\underline{L} = \alpha_L \bar{L} + (1-\alpha_L)\underline{L}$$

$$L' \sim U(L')\bar{L} + (1-U(L'))\underline{L} = \alpha_{L'} \bar{L} + (1-\alpha_{L'})\underline{L}.$$

By the Independence Axiom

$$\beta L + (1-\beta)L' \sim \beta [U(L)\bar{L} + (1-U(L))\underline{L}] + (1-\beta) [U(L')\bar{L} + (1-U(L'))\underline{L}]$$

Rearranging terms, we have

$$\begin{aligned} \beta L + (1 - \beta)L' &\sim \beta U(L) + (1 - \beta)U(L') \quad \bar{L} \sim \beta(1 - U(L)) + (1 - \beta)(1 - U(L')) \quad \underline{L} \\ &= \beta U(L) + (1 - \beta)U(L') \quad \bar{L} \sim 1 - \beta U(L) - (1 - \beta)U(L') \quad \underline{L}. \end{aligned}$$

By (4) above, this expression can be written as

$$\begin{aligned} &\beta \alpha_L + (1 - \beta) \alpha_{L'} \quad \bar{L} \sim 1 - \beta \alpha_L - (1 - \beta) \alpha_{L'} \quad \underline{L} \\ &= \beta(\alpha_L \bar{L} + (1 - \alpha_L) \underline{L}) + (1 - \beta)(\alpha_{L'} \bar{L} + (1 - \alpha_{L'}) \underline{L}) \\ &= \beta U(L) + (1 - \beta)U(L'). \end{aligned}$$

So, we have established that a utility function satisfying the continuity and the Independence Axiom, has the expected utility property:

$$U(\beta L + (1 - \beta)L') = \beta U(L) + (1 - \beta)U(L').$$

In brief, we can say that, a person who has vNM expected utility preferences over lotteries will act as if she is maximising expected utility – a weighted average of utilities of each state, weighted by their probabilities.

To use this model, we need a utility function that bundles into an ordinal utility ranking. Note that such functions are defined up to an affine (i.e., positive linear) transformation. This means they are required to have more structure (i.e., are more restrictive) than standard consumer utility functions, which are only defined up to a monotone transformation.

17.6 EXPECTED UTILITY THEORY AND RISK AVERSION

Let us extend the discussion of expected utility preferences to link it with risk aversion. Note that ordinarily people dislike risk. Therefore, we can ask, how much a consumer would be willing to pay to avoid a *fair bet*. A fair bet is a lottery whose expected payoff is equal to zero. With such payoff, you either gain or lose money by playing them. That means, such lotteries expose you to risk, and we are trying to measure the aversion to such types of risk.

We conceive of concavity or convexity form of a vNM utility function where properties that are not generally preserved by arbitrary monotonic transformation. Then it will be possible to see that they carry a lot of information about individual attitude to risk. You can consider Jensen's inequality for that purpose which tells us that for any concave u ,

$$E(u(x)|L) = \sum_{i=1}^N P_i u(x_i) \leq u\left(\sum_{i=1}^N P_i x_i\right) = u(E_L(x)|L)$$

That is, an individual, whose preferences are represented by a concave utility function prefers to have the amount of money equal to the expected value of the lottery to having the lottery itself. Such an individual is known as *risk-averse* individual.

The above inequality is reversed for convex u and the individual will be called *risk-loving*. Finally, when u is linear, (i.e., $u(x)=x$), individual will consider only about the expected value of the lottery and will be indifferent to risk. In such a situation, she will be known as *risk-neutral* individual.

Consider the following three utility functions (due to Autor, 1704) characterising three different expected utility maximiser:

Case I: $u_1(w)=w$

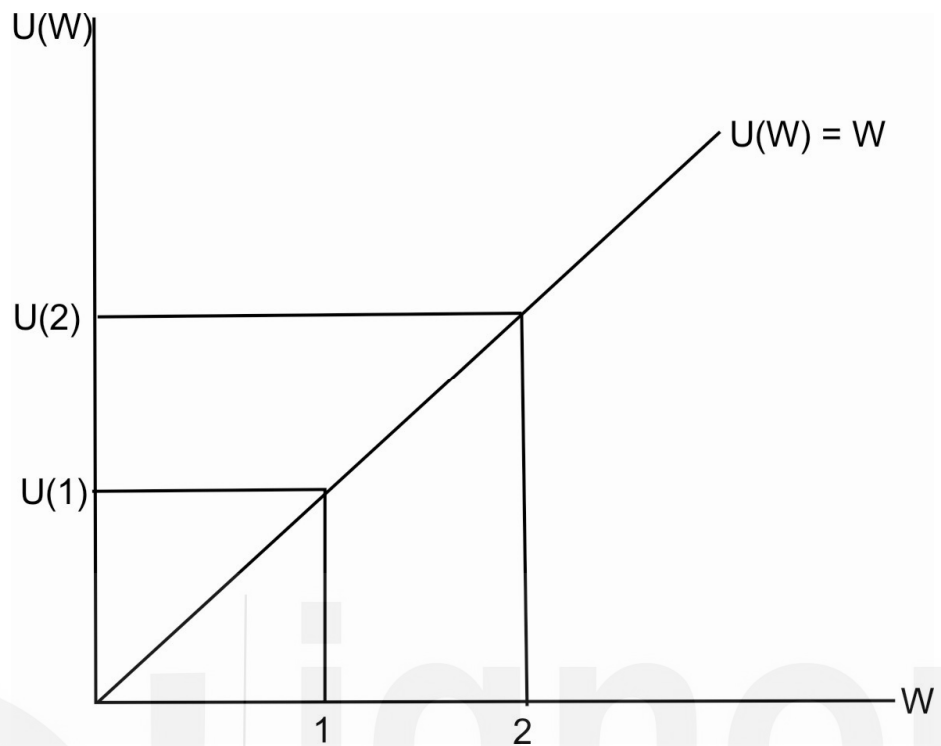


Fig. 17.1: Risk Neutral Consumer's Utility Function

Case II: $u_2(w)=w^2$

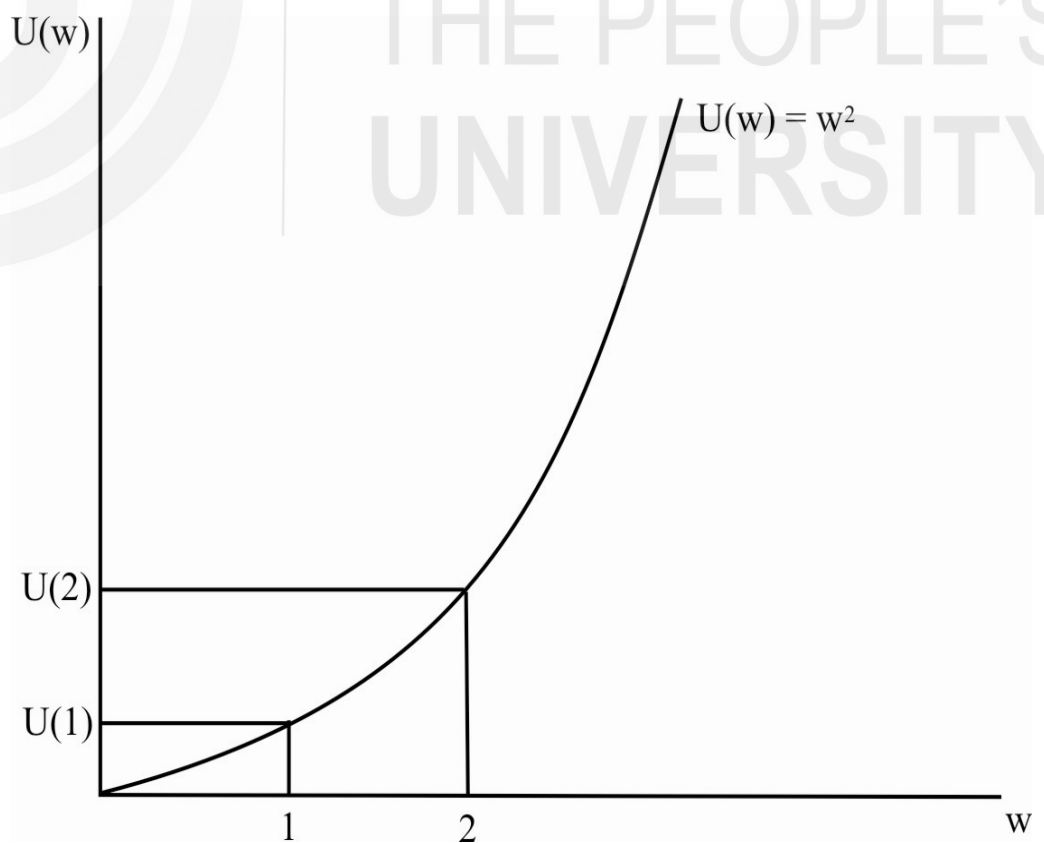


Fig. 17.2: Risk Loving Consumer's Utility Function

Case III: $u_3(2)=w^{1/2}$

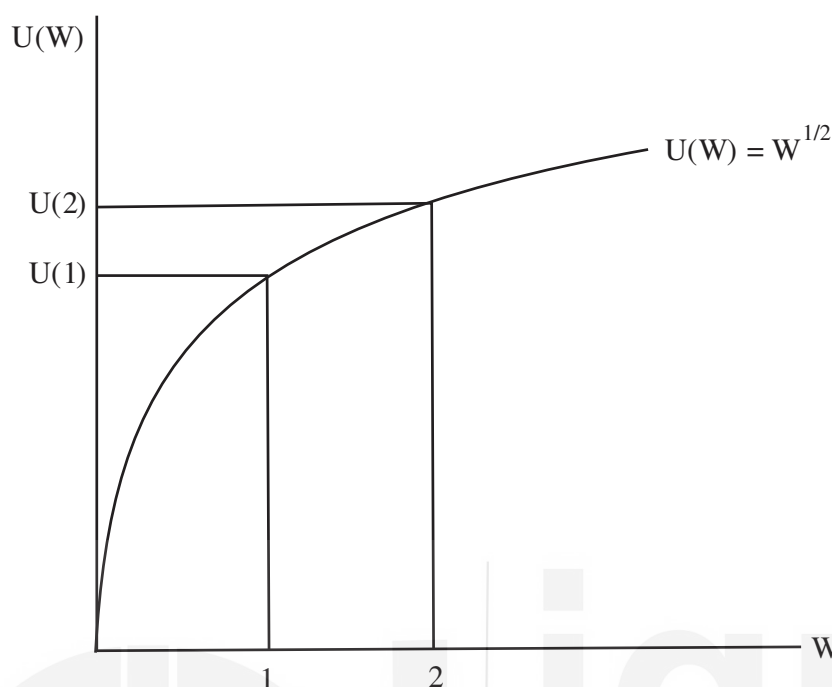


Fig. 17.3: Risk Averse Consumer's Utility Function

A consumer faces a lottery with 50/50 odds of either receiving two rupees or nothing, so that the expected monetary value of this lottery is Rs. 1. We will show that these three consumers differ in risk preference.

First, notice that $u_1(1) = u_2(1) = u_3(1) = 1$. That is, they all value one rupee with certainty equally. Consider the **Certainty Equivalent** for a lottery L that is a 50/50 gamble over Rs.2 versus 0. The certainty equivalent is the amount of cash that the consumer is willing to accept with certainty in lieu of facing lottery L.

The expected utility value as depicted in the above figures are:

- 1) $u_1(L) = .5 \cdot u_1(0) + .5 \cdot u_1(2) = 0 + .5 \cdot 2 = 1$
- 2) $u_2(L) = .5 \cdot u_1(0) + .5 \cdot u_1(2) = 0 + .5 \cdot 2^2 = 2$
- 3) $u_3(L) = .5 \cdot u_1(0) + .5 \cdot u_1(2) = 0 + .5 \cdot 2^5 = .71$

The certainty equivalent of lottery L for these three utility functions can be worked out as,

- 1) $CE_1(L) = U_1^{-1}(1) = \text{Re. } 1.00$
- 2) $CE_2(L) = U_2^{-1}(2) = 2^{.5} = \text{Rs. } 1.41$
- 3) $CE_3(L) = U_3^{-1}(0.71) = 0.71^2 = \text{Rs. } 0.51$

Thus, depending on the utility function, a person would pay Re.1, Re.1.41, or Rs.0.51 to participate in this lottery.

Although the expected monetary value $E(V)$ of the lottery is Rs.1.00, the three utility functions value it differently:

- 1) The person with U_1 is risk neutral: $CE = \text{Re. } 1.00 = E(\text{Value})$ Risk neutral
- 2) The person with U_2 is risk loving: $CE = \text{Rs. } 1.41 > E(\text{Value})$ Risk loving
- 3) The person with U_3 is risk averse: $CE = \text{Rs. } 0.50 < E(\text{Value})$ Risk averse.

The risk preference is decided by the concavity or convexity of the utility function. So we can write expected utility of wealth:

$$E(U(w)) = \sum_{i=1}^N p_i U(w_i) \text{ and}$$

utility of expected wealth:

$$U(E(w)) = U\left(\sum_{i=1}^N p_i w_i\right).$$

Accordingly, the Jensen's inequality yields:

- i) $E(U(w)) = U(E(w))$ Risk neutral
- ii) $E(U(w)) > U(E(w))$ Risk loving
- iii) $E(U(w)) < U(E(w))$ Risk averse

and an important result of expected utility theory can be obtained for a risk averse agent which states, given non-zero risk, the expected utility of wealth is less than the utility of expected wealth.

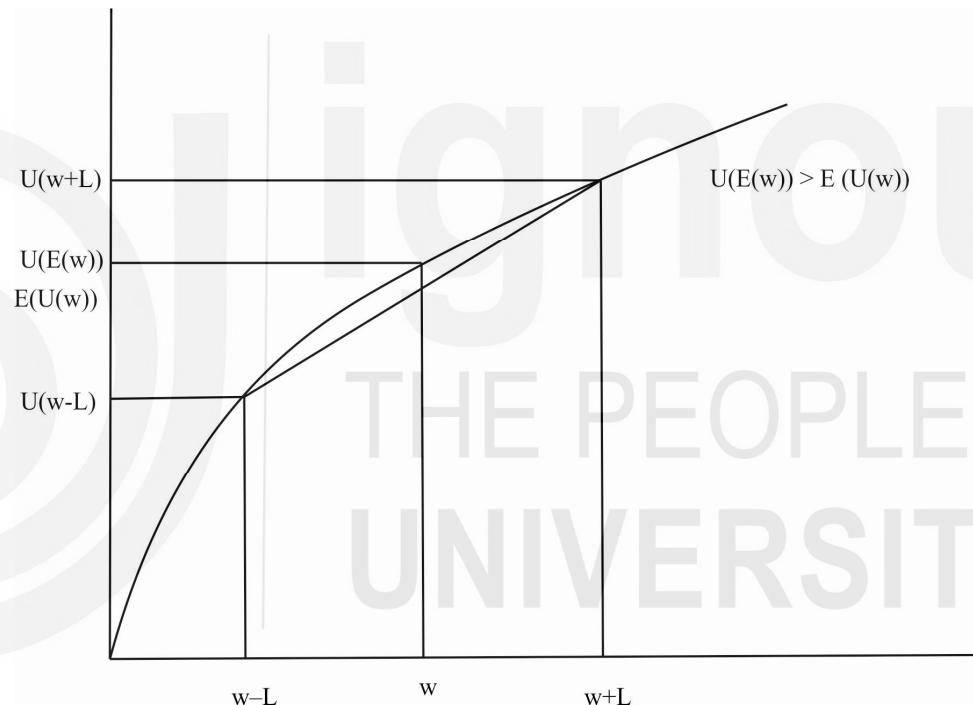


Fig. 17.4: Expected Utility of Wealth for Risk-averse Individual

The reason is, if wealth has diminishing marginal utility, which is true if $U(w) = w^{1/2}$, losses in terms of the cost of utility are more than equivalent monetary gains. Consequently, a risk averse agent is better off to receive a given amount of wealth with certainty than the same amount of expected wealth.

17.6.1 Arrow-Pratt Coefficient of Risk Aversion

The measure of absolute risk aversion is given by

$$r_A(x; u) = \frac{u''(x)}{u'(x)} \text{ and}$$

the following three statements are equivalent:

- i) $c(L; u) \leq c(L; w)$ for every lottery L
- ii) $u(x) = \varphi(w(x))$ for some increasing concave φ
- iii) $r_A(x; u) \geq r_A(x; w)$ for every state x in X

With the above conditions, it will be possible for you to conclude that individual with vNM within function u is more risk averse than the one with vNM utility function w .

You should keep in mind that this is only an incomplete ranking and it is possible that you can't compare two risk-averse individuals in this way. However, within an important subclass of utility functions, known as constant absolute risk-aversion (CARA) utility functions, the ranking is complete. These are the utility functions of the form

$$u(x) = -ae^{-rx} + b$$

for these, the coefficient of absolute risk-aversion

$$r_A(x; u) = r$$

There is a problem with CARA utility functions, however. Absolute risk-averse individuals are concerned only with *absolute* size of the risk they are exposed to. Whether a possible loss of Rs. 1000 represents .5%, 5% or 50% of her net worth, an individual with CARA preferences will view it in the same manner. Therefore, it may be useful to consider the Arrow-Pratt coefficient of relative risk-aversion, which can be written as

$$r_R(x; u) = -x \frac{u''(x)}{u'(x)}.$$

This formulation focuses on the size of the gamble as a proportion of one's wealth and constant relative risk-aversion implies decreasing absolute risk-aversion.

You will be dealing, mostly, with the constant relative risk-aversion (CRRA) utility functions of the form,

$$u(x) = \frac{x^{1-\sigma} - 1}{1-\sigma}, 0 < \sigma < 1$$

$$u(x) = \ln(x), \sigma = 1$$

$$\text{where } r_R(x; u) = \sigma$$

Check Your Progress 2

- 1) (Due to Sen, 1999) The vNM utility function of an individual is $u = m^{1/2}$. If her initial wealth is 36. Will she accept a gamble in which she wins 13 with a probability of $2/3$ and lose 11 with probability of $\frac{1}{3}$?

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- 2) Do you think that a risk-averse individual gamble or will a risk lover purchase insurance? Explain your answer.

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- 3) You are given three utility functions, viz.,

$$u_1(w) = w,$$

$$u_2(w) = w^2 \text{ and}$$

$$u_3(w) = w^{1/2}.$$

Explain how do they differ with respect to risk preferences.

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- 4) How do you measure risk aversion?

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- 5) Which of the following statements about utility theory are true?

- I) A risk averse person prefers a certain gain of E to a gamble with expected value of E.
 - II) If A_1 is preferred to A_2 , and A_2 is preferred to A_3 , then a probability p exists such that $U(A_2) = pU(A_1) + (1 - p)U(A_3)$.
 - III) For a risk averse person, the marginal utility of money is a decreasing function of the amount of money.
- a) I and II only
 - b) I and III only
 - c) II and III only
 - d) I, II and III
 - e) None of a,b,c,d, are correct.

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17.7 RISK AVERSION AND INSURANCE

We have drawn mostly from Autor, 1704 for developing this section. Consider insurance that is actuarially fair, meaning that the premium is equal to expected claims: Premium = $(p.A)$ where p is the expected probability of a claim, and A is the amount of the claim in event of an accident.

We start with a risk-averse person's decision to buy insurance by taking the initial endowment wealth w_0 , and L is the amount of the loss from an accident

$$\Pr(1 - p) : U(.) = U(w_0),$$

$$\Pr(p) : U(.) = U(w_0 - L)$$

If insured, the endowment is (incorporation the premium pA , the claim paid A if a claim is made, and the loss L):

$$\Pr(1 - p) : U(.) = U(w_0 - pA),$$

$$\Pr(p) : U(.) = U(w_0 - pA + A - L)$$

Expected utility if uninsured is:

$$E(U|I = 0) = (1 - p)U(w_0) + pU(w_0 - L).$$

Expected utility if insured is:

$$E(U|I = 1) = (1 - p)U(w_0 - pA) + pU(w_0 - L + A - pA) \dots (i)$$

We need to find out the amount of insurance that should be bought.

See that if the insurance can be bought they could buy upto their total wealth: $w_0 - pL$. To solve for the optimal policy that the agent should purchase, differentiate (i) with respect to A :

$$\frac{\partial U}{\partial A} = -p(1 - p)U'(w_0 - pA) + p(1 - p)U'(w_0 - L + A - pA) = 0$$

$$U'(w_0 - pA) = U'(w_0 - L + A - pA)$$

$A = L$, which implies that wealth is $w_0 - L$ in both states of the world (insurance claim or no claim)

Thus, a risk averse person will optimally buy full insurance if the insurance is actuarially fair. You can verify that for this kind of a consumer the expected utility rises with the purchase of insurance although expected wealth is unchanged.

Next, let us solve for how much the consumer would be willing to pay for a given insurance policy. Since insurance increases the consumer's welfare, she would be willing to pay some positive price in excess of the actuarially fair premium to defray risk. Thus, the agent is trying to equate the marginal utility of wealth across states. Because, the utility of average wealth is greater than the average utility of wealth for a risk averse agent.

The agent wants to distribute wealth evenly across states of the world, rather than concentrate wealth in one state. She will attempt to maintain wealth at the same level in all states of the world, assuming that she can costlessly transfer wealth between states of the world (which is what actuarially fair insurance allows the agent to do).

The above formulation of insurance problem is exactly analogous to convex indifference curves over consumption bundles. It can be seen that diminishing marginal rate of substitution across goods, which comes from diminishing marginal utility of consumption, causes consumer's to diversify across goods rather than specialise in single good; and diminishing marginal utility of wealth causes consumers to wish to diversify wealth across possible states of the world rather than concentrate it in one state.

The insurance problem would change if the consumer were risk loving. Such an individual would like to be at a corner solution where all risk is transferred to the least probable state the world, again holding constant expected wealth.

Example

Imagine the agent faced probability p of some event occurring that induces loss L . Moreover, assume that the policy pays $A = \frac{w_0}{p}$ in the event of a loss and costs pA . The wealth in the states of loss or no loss can be represented by

$$W(\text{No Loss}) = w_0 - p \frac{w_0}{p} = 0,$$

$$W(\text{Loss}) = w_0 - L - p \left(\frac{w_0}{p} \right) + \frac{w_0}{p} = \frac{w_0}{p} - L.$$

$$E(U) = (1-p)U(0) + pU \frac{w_0}{p} - L.$$

For a risk-loving agent, putting all of their eggs into the least likely basket maximises expected utility.

17.7.1 Operation of Insurance: State Contingent Commodities

We have seen above that risk preference generates demand for insurance. Let us now extend that discussion by taking insurance as a ‘state contingent commodity’. That is, a good, which you buy now but only consume if a specific state of the world arises. For example, when you buy insurance you are buying a claim on Re.1.00. Such insurance is purchased before the state of the world is known. You can only make the claim for the payout if the relevant state arises.

While analysing consumer behaviour, we’ve drawn indifference maps across goods X,Y. Now we will draw indifference map across states of the world: *good, bad*.

Consumers can use their endowment (equivalent to budget set) to shift wealth across states of the world via insurance, just like budget set can be used to shift consumption across goods, X, Y.

Example

Two states of world, good and bad. Wealth in these two states and probability of occurrence of the states are given as, $w_g = 117$

$$w_b = 40$$

$$\Pr(g) = P = 0.75 \text{ and } \Pr(b) = (1 - P) = 0.25.$$

Then, $E(w) = 0.75(117) + .25(40) = 100$ and $E(u(w)) < u(E(w))$ if agent is risk averse.

Let us look at Figure 17.5 to assess the consumer’s optimal decision.

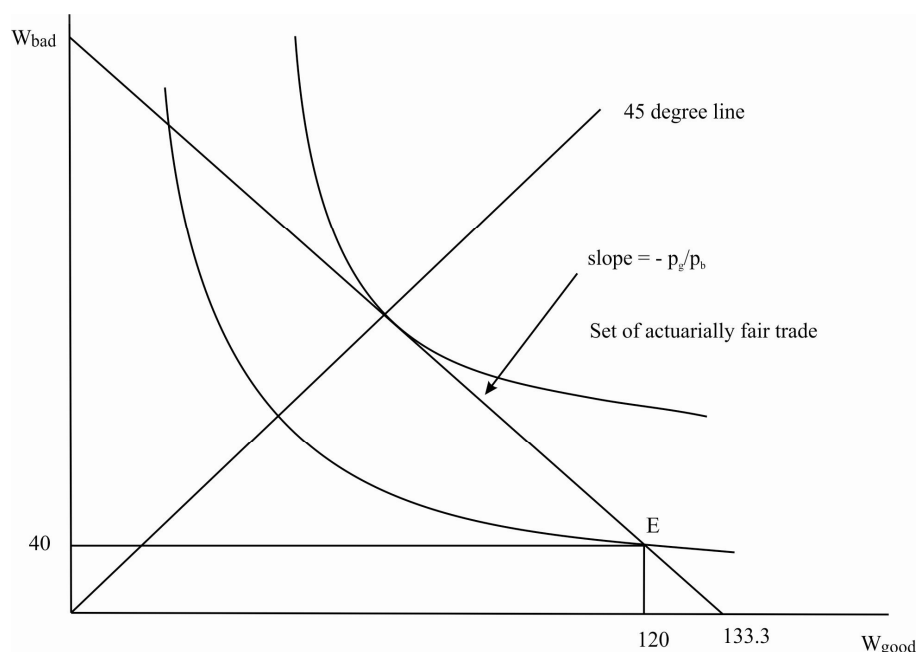


Fig. 17.5: Consumer's Optimal Decision in Insurance

Let's say that this agent can buy actuarially fair insurance.

If you want Re.1.00 in *Good state*, this will sell of Rs.0.75 prior to the state being revealed.

If you want Re.1.00 in *Bad state*, this will sell for Rs.0.25 prior to the state being revealed.

As you can observe the price set is such that these are the expected probabilities of making the claim. So, a risk neutral agent (say LIC of India) could sell you insurance against bad states at a price of Rs.0.25 and insurance against good states (assuming you wanted to buy it) at a price of Rs.0.75.

The price ratio is therefore

$$\frac{X_g}{X_b} = \frac{P}{(1-P)} = 3.$$

The set of fair trades among these states can be viewed as a budget set and the slope is $-\frac{P}{(1-P)}$. Now we bring in indifference curves. Recall that the utility

of this lottery (the endowment) is:

$$u(L) = Pu(w_g) + (1-p)u(w_b).$$

Along an indifference curve

$$\begin{aligned} dU &= 0 = Pu'(w_g)\partial w_g + (1-P)u'(w_b)\partial w_b, \\ \frac{\partial w_b}{\partial w_g} &= \frac{Pu'(w_g)}{(1-P)u'(w_b)} < 0. \end{aligned}$$

Provided that $u()$ concave, these indifference curves are bowed towards the origin in probability space. We can then prove that indifference curves are convex to origin by taking second derivatives. But intuition is straightforward. Flat indifference curves would indicate risk neutrality – because for risk neutral agents, expected utility is linear in expected wealth.

Convex indifference curves means that you must be compensated to bear risk. Thus, if I gave you Rs.133.33 in good state and 0 in bad state, you are strictly worse off than getting Rs.100 in each state, even though your expected wealth is

$$E(w) = 0.75 \cdot 133.33 + 0.25 \cdot 0 = 100.$$

So, I would need to give you more than Rs.133.33 in the good state to compensate for this risk.

It is easy to see that there are potential utility improvements from reducing risk. In the figure, $u_0 \rightarrow u_1$ is the gain from shedding risk. Also notice from Figure 17.5 that along the 45° line, $w_g = w_b$. But if $w_g = w_b$, this implies that

$$\frac{\partial w_b}{\partial w_g} = \frac{Pu'(w_g)}{(1-P)u'(w_b)} = \frac{P}{(1-P)}.$$

Hence, the indifference curve will be tangent to the budget set at exactly the point where wealth is equated across states. This is a very strong restriction that is imposed by the expected utility property: The slope of the indifference curves in expected utility space must be tangent to the odds ratio.

Example

Suppose you have a friend who has Rs.10,000 as her initial wealth and a car of worth Rs.2,100. There is a probability $p = 0.1$ that her car may be stolen.

She has a utility function, $u(x) = x^{1/2}$ that values her total wealth, i.e., the initial money and the value of the car. What are the possible realisations of her wealth, W ?

Solution: We have

$$W = \begin{array}{l} 12,000, \text{ with } pr = 0.9 \\ 10,000, \text{ with } pr = 0.1 \end{array}$$

Her expected wealth is

$$\begin{aligned} \bar{W} &= E[W] \\ &= 12,100 \times 0.9 + 10,000 \times 0.1 \\ &= 11,890 \end{aligned}$$

Since her utility function is

$$u(x) = x^{1/2},$$

her expected utility is

$$\begin{aligned} E[u[W]] &= 0.9 \times 12,100^{1/2} + 0.1 \times 10,000^{1/2} \\ &= 0.9 \times 110 + 0.1 \times 100 \\ &= 109 \end{aligned}$$

We can see from her utility function that she is risk-averse. This can be said because the utility of having $\bar{W}$ for sure is

$$\begin{aligned} u(\bar{W}) &= 11,890^{1/2} \\ &= 109.04 \end{aligned}$$

So, $u(\bar{W}) > E[u(W)]$.

Suppose she does not choose to have $\bar{W}$ for sure but instead intends to buy insurance. Let the insurance company offers to pay her $c = 2,100$ if her car is stolen. For which it demands a premium p_m that is made before the uncertainty is revealed.

We need to solve the amount she is willing to pay for such insurance. Solve and find the value of p_m that leaves her indifferent between having insurance or not having insurance.

The expected utility under insurance is

$$\begin{aligned} E[u(\text{insurance})] &= 0.9 \cdot u(12,100 - p_m) + 0.1 \cdot u(\overbrace{2,100}^c - p_m) \\ &= (12,100 - p_m)^{1/2} \end{aligned}$$

The utility under insurance with the utility of no insurance is equated, so that we have

$$\begin{aligned} (12,100 - p_m)^{1/2} &= 109 \\ 12,100 - p_m &= 109^2 \\ 12,100 - 11,881 &= p_m \\ p_m &= 219 \end{aligned}$$

the insurance company would be willing offer such a contract if values expected profits only. In that case,

$$\begin{aligned} E[\pi] &= 219 - 0.9 \times 0 + 0.1 \times 2100 \\ &= 219 - 210 \\ &= 9 > 0 \end{aligned}$$

So, the insurance company is making positive expected profits and should agree with the contract.

Check Your Progress 3

- 1) The probability of a house catching fire is $\frac{1}{10,000}$. An estimate of damage caused in case of fire is Rs.1 million. What is the expected loss due to the fire? If an insurance company asks for a premium of Rs.150, would you pay it? Explain your answer.

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- 2) (Due to Maddala and Miller, 1989) Your utility function is

$$u(y) = 100y - 10y^2$$

you are asked to choose between two prospects:

- a) $y = 30$ and $y = 50$ each with probability $\frac{1}{2}$.
b) $Y = 40$ with probability 1 (certainty)

Which one will you choose? What is the certainty equivalent income for choice a? Define the cost of risk and risk premium for prospect a.

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17.8 LET US SUM UP

In this unit we have discussed the expected utility theory to help take decision in uncertain and risky situations. It is seen that a person who has vNM expected utility preferences over lotteries will act as if she is maximising the utility as a weighted average of each state, weights being their probabilities. In this formulation, the utility function is made operational by ordinal utility ranking of individual preferences. Consequently, the utility functions are defined up to a positive linear transformation. Agents could be risk averse, risk loving or risk neutral depending on the type of utility functions being concave, convex or linear. An important insight gained from the expected utility theory is concerning a risk averse agent who has the expected utility of wealth less than the utility of expected wealth. Based on the expected utility theory, we explain why risk-averse individuals will buy insurance and risk-loving ones will pay to gamble.

17.9 KEY WORDS

Agent: A person who makes economic decisions for other economic actor.
Certainty Equivalent: Amount of income received with certainty that yields the same utility as the expected utility associated with a given distribution of uncertain incomes.

Expected Utility Theory: Individuals behave as if their objective were to maximise expected utility.

von Neumann-Morgenstern Utility: A ranking of outcomes in uncertain situations such that individuals choose among these outcomes on the basis of their expected utility values.

Risk Averse: Representation of the behaviour of economic agent who is unwilling to undertake a fair gamble.

Risk Loving: Representation of the behaviour of economic agent who undertakes a fair game.

Risk Neutral: Representation of the behaviour of economic agent who is indifferent between accepting or rejecting fair gamble.

Risk Premium: Fraction of expected income that an individual would be willing to give up in exchange for certainty.

17.10 SOME USEFUL BOOKS

Autor, H. David (1704), *Uncertainty, Expected Utility Theory and the Market for Risk*, Lecture note, available on Internet site, oc.mit.edu

Maddala, G. S. and Ellen Miller (1989), *Microeconomics: Theory and Applications*, McGraw-Hill, USA.

Nicholson, W. (1992), *Microeconomic Theory*, 5th edition, the Dryden press, Harcourt Brace Jovanovich, Orlando.

Sen, Anindya (1999), *Microeconomics: Theory and Applications*, Oxford University Press, New Delhi.

Varian, Hall (3rd Edition), *Microeconomic Analysis*, W. W. Norton and Company, Inc., New York.

17.11 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) No. People would not pay large amounts of money to play gambles with large upside potential.
- 2) Do yourself.
- 3) Do yourself.
- 4) Do yourself.
- 5) Do yourself.
- 6) Rs.2

Check Your Progress 2

- 1) Expected utility is $= (2/3)\sqrt{36+13} + \frac{1}{3}\sqrt{(36-11)} = \frac{19}{3} > \sqrt{36}$. Gamble should be accepted.
- 2) Do yourself.
- 3) In terms of certainty equivalent.
- 4) Do yourself.
- 5) d

- 1) Get the expected loss, Rs. 100 and answer
- 2) Do yourself.

17.12 EXERCISES

- 1) The loss of utility for carrying an umbrella is $\frac{1}{2}$. If it rains and you don't have any umbrella your utility drops by 3 units, while it drops by 1 unit if you have an umbrella. If the probability of rain is $\frac{1}{2}$, would you carry an umbrella?

Ans. Expected utility of carrying an umbrella is higher than not to carry it and so you would carry an umbrella.

- 2) You are given a decision process with three possible outcomes, as shown below:

k	O(k)
1	a loss of Rs.500
2	a loss of Rs.35
3	a profit of Rs.1700

You formulate a reference lottery as a gamble with payoff O(3) if you win a O(1) if you lose, with win probability q. Your utility value for money is

$$U(M) = \begin{cases} \sqrt{2M + 2400} & \text{if } M \geq 50 \\ M & \text{if } M < 50. \end{cases}$$

For what value of q are you indifferent between outcome O(2) and the reference lottery?

Ans. 0.8

- 3) Each of decision makers X, Y and Z has the opportunity to participate in a game with payoff uniformly distributed on (0, 10000). Assume that X, Y, Z value assets of amount $W \geq 0$ according to the following utility functions.

Decision Maker	Utility Function
X	$u(W) = \sqrt{W}$
Y	$u(W) = W / 100$
Z	$u(W) = (W / 1000)^2$

What decision makers would not be willing to pay more than Rs.5000 to participate in the game?

Ans. X and Y only.

- 4) You have assets of Rs.10,000 and are facing a loss of Rs.3600, with probability .002. You are indifferent between paying Rs.G for insurance protection and assuming the risk of loss personally. You value total assets of amount $w \geq 0$ according to the utility function $U(w) = \sqrt{w}$. Determine G. (**Ans.** Rs.8).

UNIT 18 INSURANCE CHOICE AND RISK

Structure

- 18.0 Objectives
- 18.1 Introduction
- 18.2 Reduction of Risk
 - 18.2.1 Risk Pooling
 - 18.2.2 Risk Spreading
 - 18.2.3 Risk Transfer
- 18.3 Problems in Insurance Markets
 - 18.3.1 Moral Hazard
 - 18.3.2 Adverse Selection
- 18.4 Modelling Insurance Market with Adverse Selection
 - 18.4.1 Case of Homogenous Risk Pool
 - 18.4.2 Case of Heterogenous Risk and Private Information
 - 18.4.3 Pooling Equilibrium
 - 18.4.3.1 Failure of the Pooling Equilibrium
 - 18.4.4 Separating Equilibrium
 - 18.4.4.1 Failure of the Separating Equilibrium
- 18.5 Let Us Sum Up
- 18.6 Key Words
- 18.7 Some Useful Books
- 18.8 Answer or Hints to Check Your Progress
- 18.9 Exercises

18.0 OBJECTIVES

After going through this unit, you will be able to:

- understand the functioning of insurance market;
- appreciate risk defraying options for insurance;
- evaluate the problems posed to insurance market by adverse selection and moral hazard; and
- derive the equilibrium condition in the presence of adverse selection.

18.1 INTRODUCTION

In the preceding unit, we have seen that people, in general, are risk averse and would be willing to buy insurance. Viewed from such a perspective, insurance is an exchange in which you make a payment in order to get rid of a gamble – that is, to avoid or reduce a risk. However, if everyone is risk averse, a seller of insurance seems a part of the group. You cannot then explain the existence of an insurance market without saying that insurance company is risk lover. To come to a definite conclusion, it will be necessary for us to examine the operation of insurance market.

18.2 REDUCTION OF RISK

There are three distinct mechanisms by which insurance market can operate: risk pooling, risk spreading and risk transfer.

18.2.1 Risk Pooling

Risk pooling is mechanism to defray risk. Its operation depends on the *Law of Large Numbers*.

Law of large numbers: In repeated independent trials with probability p of success in each trial, the chance that the percentage of success differs from the probability p by more than a fixed positive amount $e \geq 0$ converges to zero as number of trials n goes to infinity for every positive e . If you recall from the probability theory covered in MEC-003, the toss of a fair coin n times will yield an expected value of $\frac{1}{2}$ for getting a head as $n \rightarrow \infty$. Along with it, the variance (see Appendix at the end) will tend to decline. Therefore, while we cannot predict the number of heads when a coin is tossed once, we can do so with, say, 10,000 tosses. Thus, the lesson is, by *pooling* many independent risks, insurance companies can treat uncertain outcomes as almost certain.

Example (See David Autor, 2004)

Let us say that there is a 0.04% chance that my house will be burgled. If it happens, I lose Rs.250,000. Prior to facing this risk, if I calculate the expected loss it will be Rs.1,000 (i.e., $.004 \times 250,000$). If I am risk averse, it is costlier in expected utility terms for me to bearing this risk. Now, suppose that there are 100,000 owners who are likely to face a loss of similar amount, Rs. 250,000, in case of burglary and probability of occurring such an event remains the same as that in my case. If all these owners put Rs.1,00 into a pool, the total amount of collection will be Rs.100,00,000. In expectation, 400 ($100,000/.004$) of us will have our homes burgled. The pool therefore will have to pay Rs.100 million as compensation. See that such an amount is just equal to the collection model for the pool. The net result is, every one who participated in this pool was better off to be relieved of the risk.

Note that there will be some variation around 400, the expected household being burgled. But the law of large number ensures that such a variance tends to zero as the pool becomes large and the risks are independent. Thus, from our example, if we calculate the variance from a binomial distribution, we get

$$V(\text{Fraction lost}) = \frac{P_L(1 - P_L)}{n}$$

where P_L = probability of loss

$$n = 10,000.$$

$$= \frac{(.004)(1 - 0.004)}{100,000} = 3.984 \times 10^{-8}$$

$$\text{So } SD(\text{Loss}) = \sqrt{3.984 \times 10^{-8}} = 0.0002$$

Since the binomial distribution is approximately normally distributed when n is large, the above result can be seen in terms of a confidence interval of

$$P[\text{Loss} \in (0.004 \pm 1.96 * 0.0002)] = .95.$$

That is, there is a 95% chance that homes likely to burgled fall in the interval of 361 to 439. The compensation of loss per policyholder therefore will be paid in the range of Rs. 924.50 to Rs .1,075.50 which is true in 95% cases.

18.2.2 Risk Spreading

The mechanism of risk pooling we have discussed above works only if the risks are independent between two groups of people. You will find many

events, especially the natural catastrophes, such as flood, earthquake and nuclear war where risk are not independent. Thus, when a catastrophic event is likely to affect many people simultaneously, risk-pooling mechanism will not work and we have to look for other means to insure.

A basic idea to work with is spreading risk such that it covers some of the unaffected people in the scheme of insurance. As the utility function of risk averse people is convex, taking away small portion of money from each has a lower social cost compared to asking for a lot from a few people. Usually, private insurance companies don't enter in such markets. But often government intervenes by transferring money among parties. For example, on this type of approach, you should remember the compensation being collected for all kinds of disaster relief.

Risk spreading does not defray the total amount of risk faced by the society. Nevertheless, it improves social welfare as the aggregate loss for the society is lessened than that of the individual loss.

Example

A society has 100 people each with vNM utility function $u(w) = \ln(w)$ and a wealth of Rs.500. Suppose that one of them experiences a loss of Rs. 200. Her utility loss is

$$L = u(300) - u(500) = -0.511.$$

Consider this loss when distributed over the entire population. Then we have,

$$L = 100[\ln(498) - \ln(500)] = -0.401.$$

The result is, aggregate loss smaller than the individual loss.

Risk spreading, however, does not result in Pareto improvement as some members of the society are compensated by taking from some others.

18.2.3 Risk Transfer

Consider a logarithmic utility function which exhibits declining absolute risk aversion. So more wealthier you are, the lower is your cost of bearing a fixed monetary amount of risk. When utility cost of risk is declining in wealth, less wealthy people could pay more wealthy people to bear the risk. Consequently, both parties would be better off.

Example

The utility function is represented as $u(w) = \ln(w)$. Let an individual face a 50 percent chance of losing Rs.100. The willingness of this person to pay for eliminating this risk depends on her initial wealth.

If we assume the initial wealth is Rs.200, the expected utility can be obtained as

$$u(L) = 0.5 \ln 200 + .5 \ln 100 = 4.952$$

The certainty equivalent of this lottery is $\exp(4.952) = 141.5$. Therefore, the individual would be willing to pay Rs.8.50 to defray this risk.

Let there be an individual with initial wealth of Rs.1,000. We assume that the utility function is same with that of the above case. Then the expected utility is

$$u(L) = .5 \ln 1000 + .5 \ln 900 = 6.855$$

The certainty equivalent of this lottery is $\exp(6.855) = 948.6$. Hence, this agent would be willing to pay only Rs.1.4 to defray the risk. See that the wealthy member could fully insure the poor member at a cost of Rs.1.4 while

the poor member would be willing to pay Rs.8.5 for this insurance. Therefore, they may agree for a price between Rs.1.4 and Rs. 8.5. Such an outcome will be Pareto improvement.

18.3 PROBLEMS IN INSURANCE MARKETS

We have seen the operation of insurance market through the mechanism of defraying risk. More importantly, we also observed that through such market mechanism, except for risk spreading, Pareto improvement can be affected as some people can be made better off without making any one worse off. Thus, efficient insurance markets have the potential of improving social welfare.

Notwithstanding such theoretical insights, there are instances where you come across uninsured people. Every the basic markets such as health insurance and life insurance exhibit such features.

Insurance, which exists in many markets, is incomplete. For example, suppose an individual is contemplating insurance that would pay off in the event she contacted an incurable illness. Although she might choose a policy that covered all anticipated medical expenses, complete cover of income loss might not be chosen since the illness may foreclose some consumption possibilities. So, insurance may be offered with deductions or caps. At times, the coverage may be denied altogether. Thus, we need to explain the persistence of incomplete insurance markets.

Factors Behind Incomplete Insurance Market

There are constraints like non-availability of credit to buy an insurance and people cannot afford for a policy of their own. So, even if you know that you have a disease, which demands good deal of expenditure, it may be too late for you to buy insurance; and you bear the risk. Similarly, there are non-diversifiable risk, which cannot be insured. You may think of nuclear war, where all of us will face identical risk simultaneously and there is no way to get back the loss through insurance. These apart, let us look for two important structural constraints, which is why it makes sense to have less-than-full insurance, viz., **Moral Hazard and Adverse Selection**. We will return to these concepts for a greater discussion in the next unit and for the present acquaint ourselves with their implications in the insurance market.

18.3.1 Moral Hazard

Moral hazard is used to describe the fact that people tend to engage in riskier behaviour when they are insured against losses resulting from their behaviour. For example, drivers may take more risks on the road when they know the insurance company will pay for damages. Moral hazard is a problem if such a fact causes someone not to be able to insure against risk that would be insurable if it didn't exist.

Example

Suppose you have a driver with a car worth Rs.20,000. It will meet an accident with probability $p = 0.05$. But if the driver is fully insured, he will drive more recklessly with $p = 0.1$. Uninsured, the driver faces a gamble with a 95% chance of Rs.20,000 and 5% chance of no loss. The expected value of this gamble is Rs.19,000. Because the driver is risk averse, his certainty equivalent for this gamble is Rs.18,500. Therefore, the maximum premium he'll pay is Rs.1500. But from the insurance company's perspective, the chance of an accident is 15% indicating the expected loss (minimum insurance premium) is $[0.1 (\text{Rs.20,000})] = \text{Rs.2000}$. Notice that this premium is more than the driver is willing to pay. As a result, no full insurance policy gets sold.

The best possible outcome would be if the driver were fully insured but continued to drive carefully, just as he were not insured. But once he is insured, he'll drive recklessly just because he's already got the policy. If he could care for the consequences of his behaviour, he would (prior to buying insurance) agree to a self-imposed commitment to driving carefully in future.

One solution to deal with such a problem is to charge higher premiums for people with records of claims before for accidents. Such a provision will compel the parties to act carefully even when insured.

To have less-than-full insurance, in the form of deductibles or co-payments is another solution. Suppose that a potential loss of Rs.10,000 is enough to induce the driver above to drive carefully. Then the insurance company could offer a policy to insure a loss upto Rs.10,000, with a minimum premium of $[0.05(\text{Rs.}10,000)] = \text{Rs. } 500$. Because it's better than no insurance at all, the driver will buy the policy. However, this is a second best solution, given that full insurance with careful behaviour is the best.

18.3.2 Adverse Selection

Adverse selection is a problem that arises when an insurer cannot differentiate between two groups of people with different risks (say, high and low risks) and has to charge the same premium from both. The problem arises because an average premium charged from the two different risk groups, which might result to be too high for the lower-risk group. Due to such an approach, some members of the lower-risk group go uninsured.

Example

Suppose an insurance company in a competitive market cannot ascertain the difference between smokers and non-smokers. Smokers have a 20% probability of getting lung cancer, whereas non-smokers have only 10%. Assume that smokers constitute one-half of the population. As a result, from the viewpoint of insurance company, a person randomly chosen from the population as a whole has a 15% chance of getting cancer.

Suppose that cost of treating lung cancer is Rs.20,000. The average fair premium for the whole population is $[0.15(\text{Rs.}20,000)] = \text{Rs.}3,000$. The fair premium for the smokers alone would be $[0.2(\text{Rs.}20,000)] = \text{Rs.}4,000$, and the fair premium for the non-smokers alone would be $[0.1(\text{Rs.}20,000)] = \text{Rs.}2,000$. Thus, you know that risk-averse people will accept any fair insurance policy, whereas for the smokers, the average fair premium is better than fair. So they have an incentive to buy insurance at the average fair premium.

Next, look for the decision of non-smokers. The average fair premium is higher than the fair premium for this group. If they are risk averse, they'll be willing to pay more than the fair premium of Rs.2,000 and anyone with a certainty equivalent lower than Rs.17,000 (i.e., $\text{Rs.}20,000 - \text{Rs.}3,000$) will buy the insurance. Now, consider a case with a certainty equivalent higher than Rs.17,000. She would find herself worse off with the insurance than without it. As a result, a non-smokers will choose not to get insured.

The consequences of such an outcome lead to inefficiency. If non-smokers don't buy insurance, and the insurance companies know this is true, then any person buying a policy is more likely to be a smoker than a non-smoker. If, say, only half of the non-smokers buy, then 2 out of 3 policy buyers will be smokers. That means the insurance company will have to charge more than the Rs.3,000 average fair premium to break even. But if the premium rises, then even more non-smokers will decide not to buy. That means the population of insurance buyers will be even more skewed towards smokers, leading to yet another premium hike. In the extreme, it could turn out that only smokers get insured.

Averse selection does not always produce the extreme result of only the riskiest people getting insured. The more general result is just that policy buyers, as a group, tend to be a riskier group than the population as a whole. Also some of the lower-risk people in the population are uninsured. Generation of inefficient outcome increases as not only people go uninsured, but also insurance companies attract high-risk people due to their inability to distinguish between high and low-risk types.

To deal with this problem, the most common method employed is to distinguish between different risk groups. Many health insurance companies require a test to find out customers who are smokers, and they charge higher premiums to them.

Check Your Progress 1

- 1) Discuss the mechanism of defraying risk in insurance markets.

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- 2) In the insurance market, what is the problem of moral hazard.

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- 3) Discuss the problem of adverse selection in insurance market.

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- 4) Suppose you are a risk averse person, and your risk attitude can be represented by the following concave vNM utility function: $u(w) = \sqrt{w}$, where w denotes wealth. Suppose your initial wealth is Rs.100, and hence your initial utility is $\sqrt{100} = 10$.

Imagine you are offered the following gamble: with probability $\frac{1}{2}$ you'll win Rs.156, and with probability $\frac{1}{2}$ you'll lose Rs.100. Would you take that gamble?

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18.4 MODELLING INSURANCE MARKET WITH ADVERSE SELECTION

We will discuss the important features of Rothschild-Stiglitz theorem, which basically highlights the following result:

When information is private, giving rise to the problem of adverse selection, market outcomes cannot produce Pareto efficiency. Particularly, when there is imperfection in the market such that one of the parties to a transaction is better informed than the other, inefficiencies would crop up. You will notice equilibrium of such a market that violates the first welfare theorem.

Let us see how the model works. For this purpose, we will follow the presentation as well as notations of *Autor, 2004*. Suppose a risk-averse individual faces two states of the world in which her initial wealth is characterised by no-accident and accident.

If there is no accident, wealth is w and with accident, wealth is $w - d$ (where $d > 0$ stands indicates damage).

So, each person's wealth endowment is given as

$$W_i = \begin{pmatrix} \Pr(p_i) & w \\ \Pr(1 - p_i) & w - d \end{pmatrix}.$$

Note that there is no insurance coverage for w .

Imagine that a person is insured. Then her endowment is changed such that

$$W_i = \begin{pmatrix} \Pr(p_i) & w - \alpha_1 \\ \Pr(1 - p_i) & w - \alpha_2 \end{pmatrix}$$

where the vector $\alpha = (\alpha_1, \alpha_2)$ describes the insurance contract. You can think that the insurance premium α_1 is paid in both the accident and no-accident states. Hence, α_2 is the net payout of the policy in event of accident.

If you denote the probability of an accident as p , then an individual purchases insurance when the expected utility of being insured exceeds the expected utility of being uninsured, i.e.,

$$(1 - p) \cdot u(w - \alpha_1) + p \cdot u(w - d + \alpha_2) > (1 - p) \cdot u(w) + p \cdot u(w - d).$$

On the other hand, the insurance company will sell a policy if expected profits are non-negative, i.e.,

$$\alpha_1 - p(\alpha_1 + \alpha_2) \geq 0.$$

Let us assume a competitive market, so that this equation holds with equality. Therefore, in equilibrium:

$$\alpha_1 - p(\alpha_1 + \alpha_2) = 0.$$

To derive the equilibrium conditions of the model, we follow the formulation Rothschild-Stiglitz who propose the following conditions:

- 1) No insurance contract makes negative profits (break-even condition).
- 2) No contract outside of the set offered exists that, if offered, would make a non-negative profit. For, if there were a potential contract that could be offered and would be more profitable than the contracts offered in equilibrium, then the current contracts cannot be an equilibrium.

18.4.1 Case of Homogenous Risk Pool

Let us start with the simplest case to see how it works.

Assume that all potential insured have the same probability of loss, $p : p_i = p$ for all i and as we have already assumed that all losses are equal to d .

See Figure 18.1, which presents the state preference diagram taken from risk aversion and insurance discussed earlier. Note that from the initial endowment

$E = (w, w - d)$, the fair odds line extends with slope $\frac{-(1-p)}{p}$, reflecting the

odds ratio between the accident and no-accident states. As we have shown a risk averse agent will optimally purchase full insurance. Following from the von Neumann-Morgenstern expected utility property, the highest indifference

curve tangent to the fair odds line has slope $\frac{-(1-p)}{p}$ at its point of tangency

with the fair odds line, which is where it intersects the 45° line. At this point, wealth is equalised across states. So, the tangency condition yields

$$\frac{(1-p)u'(w-\alpha_1)}{pu'(w-d+\alpha_2)} = \frac{(1-p)}{p} \Rightarrow w-\alpha = w-d+\alpha_2.$$

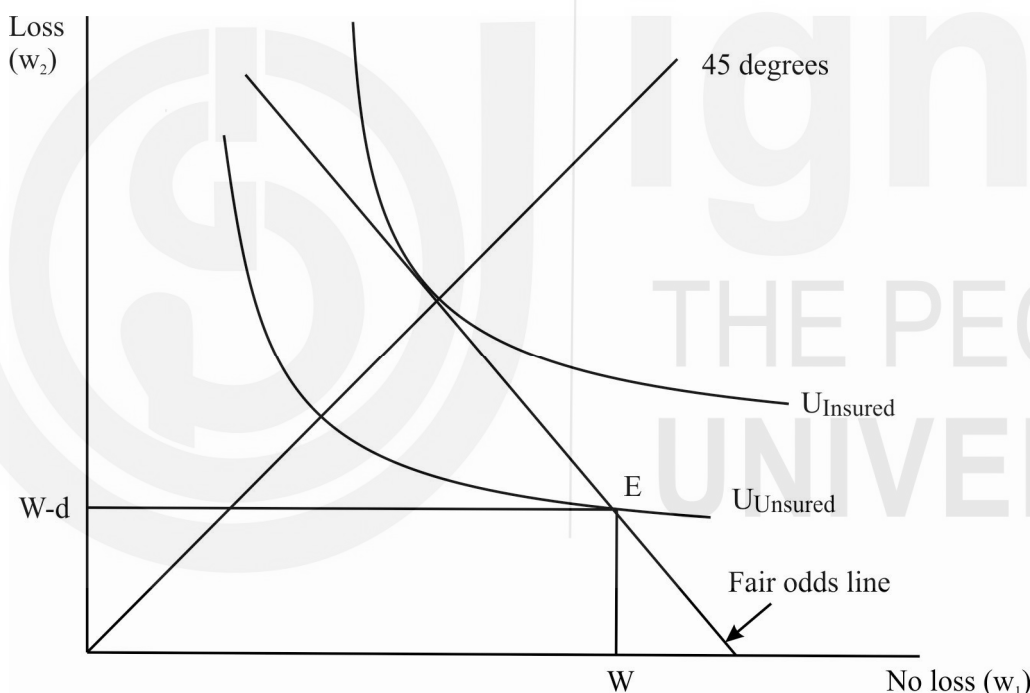


Fig. 18.1: Equilibrium with Risk Aversion and Insurance

In this simple case, insurance companies will be willing to sell the policy $\alpha = (pd, (1-p)d)$ since they break even. It is also important to note that when the above tangency condition is satisfied, there is no alternative profit-making policies that could potentially be offered.

18.4.2 Case of Heterogenous Risk and Private Information

We extend the model to the case with:

- 1) **Heterogeneity:** The loss probability p varies across individuals. Specifically, assume two types of insurance buyers:

H : Probability of loss p_h ,

L : Probability of loss p_l ,

with $p_h > p_l$.

These buyers are otherwise identical in w and the amount of loss d in the event of an accident (and their utility functions $u(\cdot)$). Only their odds of loss differ.

- 2) **Private information:** Assume that individuals know their risk type p_i but this information is not known to insurance companies. (Note: private information without heterogeneity is not meaningful; if everyone is identical, there is no private information.)

Given asymmetric information, H and L, there are two possible classes of equilibria in the model:

- 1) 'Pooling equilibrium' — All risk types buy the same policy.
- 2) 'Separating equilibrium' — Each risk type (H, L) buys a different policy.

Note that under pooling equilibria, the market is not able to distinguish among the types of insurance buyers whereas under separating equilibria, it does. The different groups "separate out" by taking different actions.

We'll discuss these possibilities.

18.4.3 Pooling Equilibrium

In a pooling equilibrium, both high and low risk types buy the same policy. That is, the insurer could offer a policy whose premium is based on the average probability of loss.

The equilibrium construct requires that this policy lie on the aggregate fair odds line. This implies that the policy earns neither negative nor positive profits. To arrive at the equilibrium, let us define λ as the proportion of the high-risk population such that the expected share of the population experiencing a loss is given as

$$\bar{p} \equiv \lambda p_h + (1 - \lambda) p_l.$$

On the other hand, the expected share experiencing no loss is given as

$$1 - \bar{p} = 1 - (\lambda p_h + (1 - \lambda) p_l).$$

If we look at the aggregate fair odds line's slope from Figure 18.2, it is given by

$$-\frac{1 - \bar{p}}{\bar{p}}.$$

Notice that in the 'pooling' policy, A must lie on the aggregate fair odds line. For, if it is above, that would be unprofitable. Therefore, we would not have it in equilibrium. If it is below, there would be positive profits. That means, we cannot have it in equilibrium.

You may note that the figure is drawn with $|MRS_{w_1, w_2}^H| < |MRS_{w_1, w_2}^L|$. It is important for the analysis. To appreciate the underlying reason, define

$$w_1 = w + \alpha,$$

$$w_2 = w - d + \alpha_2.$$

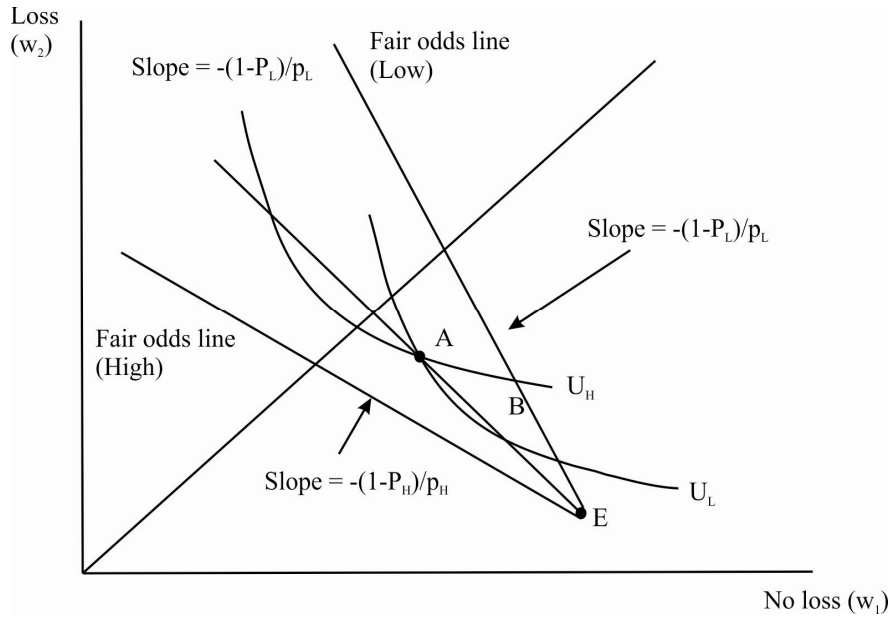


Fig. 18.2: Pooling Equilibrium in Insurance

From the vNM property, we know the following:

$$MRS_{w_1, w_2}^H = -\frac{dw_2}{dw_1} = \frac{u'(w - \alpha_1)(1 - p_h)}{u'(w - d + \alpha_2)p_h},$$

$$MRS_{w_1, w_2}^L = -\frac{dw_2}{dw_1} = \frac{u'(w - \alpha_1)(1 - p_L)}{u'(w - d + \alpha_2)p_L}.$$

Since both types, High and Low, agents are otherwise identical, we can write, $u_h(w) = u_l(w)$. This implies that

$$\frac{MRS_H}{MRS_L} = \frac{1 - p_h}{p_h} \cdot \frac{p_l}{1 - p_l} = \frac{1 - p_h}{1 - p_l} \cdot \frac{p_l}{p_h} < 1.$$

Thus, the slope of the indifference curve for type H is less steep than for type L. We know that the probability of loss is lower for type L. Therefore, type L must get strictly more income than H in the loss state to compensate for income taken from the no loss state. From this observation you can say that L types have steeper indifference curves for transfers of income between loss and no-loss states.

As seen above, the pooling equilibrium involves a cross-subsidy from L to H types, i.e., L types pay more than their expected cost and H types pay less than their expected cost. While both H, L types pay the same premium, H makes more claims which is met through cross subsidisation. Herein lies the problem for sustenance of the equilibrium which we see below.

18.4.3.1 Failure of the Pooling Equilibrium

Although in Figure 18.2, A satisfies one of the equilibrium conditions, i.e., the breaks even condition by virtue of its being on the aggregate fair odds line, it fails in case of another. See that no potentially competing contract can make a non-negative profit. For example, if another insurance company offers a policy like point B in the figure, types H will not be the gainers. As B lies strictly below U_H , so H types are happier with the current policy.

However, consider L types. They strictly prefer this policy as B is above U_L . Since B lies above the fair odds line for the pooling policy it offers a better deal to this group. However, it fails to provide much insurance as it lies closer to E than that of A. This is attractive to L types because they would rather have a little more money and a little less insurance since they are cross-

subsidising the H types. It is clear that for the opposite reasons, H types would prefer the old policy. So, when policy B is offered, all L types change to B, and the H types stick with A. Because B lies below the fair odds line for L types it is profitable if it attracts L types. But A cannot be offered without the L types participating as it requires the cross-subsidy.

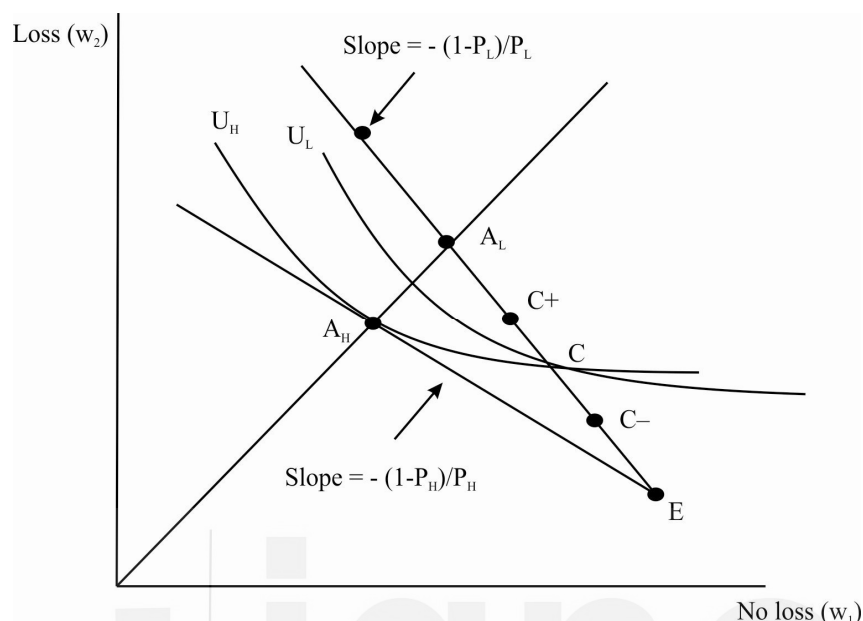


Fig. 18.3: Separating Equilibrium

Consequently, the pooling equilibrium cannot exist. It is always undermined by a 'separating' policy that skims off the L types from the pool. See that the insurer enters into a process whereby it tries to select the most favourable individuals with expected losses below the premium charged.

Free entry leads to 'cream skimming' of low-risk from the pool and makes it difficult or impossible for individuals with high expected losses to purchase private insurance. This causes pooling policy to lose money because only high risk remains and pooling policy disappears.

In above result we see how it makes sense to separate the profitable group from that of the less profitable one. If an insurance company loses money on one group but makes it back on another, there is a strong incentive to separate the profitable from the unprofitable group and charge them different prices or just drop the unprofitable group, thereby undermining the cross-subsidy.

20.4.4 Separating Equilibrium

We have seen above that the pooling equilibrium is not feasible. Therefore, let us consider a 'separating equilibrium' instead. For that purpose, it will be useful to examine Figure 18.3. In the figure we have Just as in case of the pooling equilibrium two fair odds line corresponding to the two different risk groups. Points A_L and A_H give full-insurance points for the two risk groups. Group L has higher wealth because its odds of experiencing a loss are lower. In the figure, C on the fair odds line is for the L group. The indifference curve from the full-insurance point for the H group crosses the fair odds line for the group L at this point. Moreover, the indifference curve U_L that intersects point C is steeper than the corresponding curve for the group H. Thus, we have

$$\left| MRS_{w_1, w_2}^H \right| < \left| MRS_{w_1, w_2}^L \right|,$$

as has been shown earlier.

On the basis of the Figure 18.3, we can make the following observations concerning the two groups, H and L:

- i) C is the best policy the insurance company could offer to the L types that would not at the same time attract H types.
- ii) If the company offers the policy C^+ on the figure to L types, they would strictly prefer it. However, the problem with it is that H types would also prefer it. Such a policy therefore would produce a pooling equilibrium. In addition, C^+ involves cross-subsidy if H types take it because U_H is the indifference curve for the fully-insured type H. If there is a point that H types prefer to full-insurance, it can only mean that the policy is subsidised.
- iii) If the company offers a policy C^- , H types would not select it. With such an offer, L types would strictly prefer the original policy, C. Consequently, any policy like C^- will be dominated by C.

So, C in the figure defines the ‘separating constraint’ for types H, L. Any other policy that is more attractive to H types would result in pooling. In equilibrium, we get policies A_H and C where type H chooses A_H and type L chooses C. Both policies break even since each lies on the fair odds line for the insured group. In the equilibrium H risk types are fully insured and L risk types are only partly insured.

It is important to note that equilibrium arrived in the market is influenced by the preferences of H risk buyers. The insurance providers maximise the welfare of L risk buyers subject to the constraint that they don’t attract H risk buyers. Despite the constraints imposed by the H risk types on L risk types, they are not better off. You have a solution where one group loses without gains to the other.

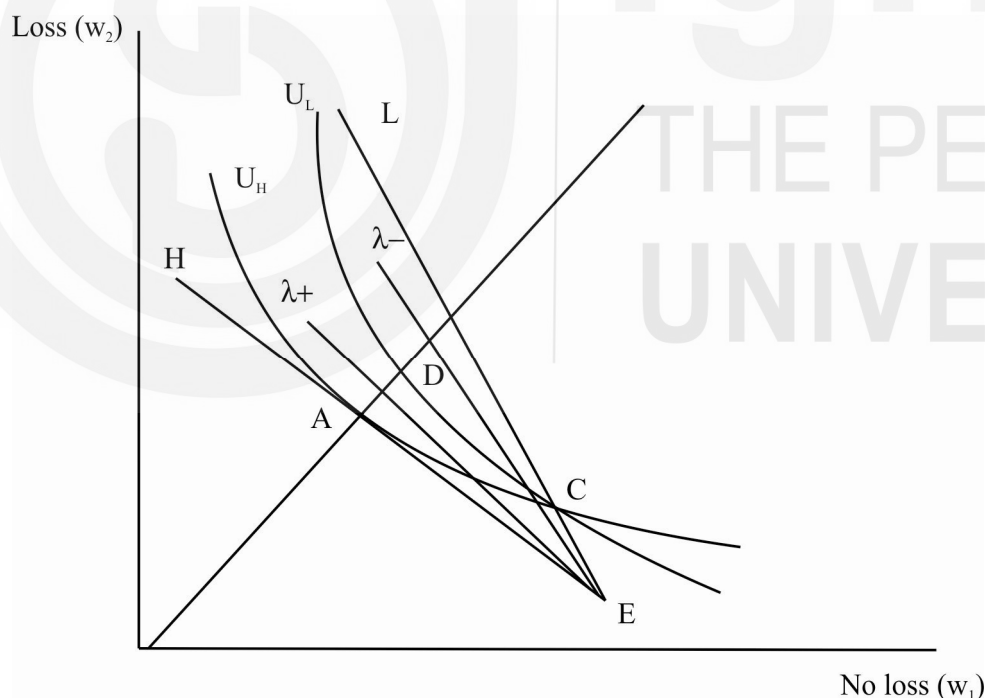


Fig. 18.4: Failure of Separating Equilibrium

18.4.4.1 Failure of the Separating Equilibrium

We are now in a position to examine the policies that constitute a separating equilibrium above. See that in Figure 18.4, both types, H and L, would buy the policy D because of its location above each of their indifference curves when purchasing policies A_L and C. However, D is a pooling policy. As we have already seen above, it cannot exist in equilibrium. In the following therefore we will examine the circumstances when D can be offered by an insurance firm.

It is easy to see that the insurance company would offer D when it is more profitable than that of the pair (A_L, C) , which yields zero profits. The profitability of D in turn is determined by the share of high-risk claimants in the population. Let us define such a share as λ . From the figure, it can be seen that the slope of the fair odds line for pooling policies depends on λ . So,

- i) when the value of λ is larger, pooling odds line lies closer to the H fair odds line;
- ii) when the value of λ is smaller, the pooling odds line lies closer to the L fair odds line.

Construct two different cases λ^+ (greater share of high risk population) and λ^- (greater share of low risk population) such that these give the pooling odds lines. When you consider λ^+ (i.e., population is mostly high risk type), a pooling policy D will lead to breaking up of separating equilibrium. Since D lies above the fair odds line, it becomes unprofitable and hence cannot be offered. By contrast, consider λ^- (i.e., population is mostly low risk type). The pooling policy D is profitable here since it lies below the fair odds line. But it breaks the separating equilibrium.

Check Your Progress 2

- 1) Define the concepts of separating equilibria.

- 2) Show that there are no separating equilibria in which both types of consumers purchase insurance.

- 3) Suppose you have a friend who is exactly like you: his vNM utility function is also $u(w) = \sqrt{w}$, and his initial wealth is also Rs.100. You propose a split the gamble evenly with him, i.e., if you win, each of you would win Rs.78, and if you lose, each of you would have a loss Rs.50. Discuss on the benefit of risk pooling of the problem.

18.5 LET US SUM UP

In this unit we have discussed the operation of insurance market through the mechanism of defraying risk, identified the major problems posed to it due to adverse selection and moral hazard. The equilibrium processes attained in the presence of adverse selection are also examined.

The risk reduction mechanism adopted by the insurance markets depend on the law of large numbers, which helps evaluate uncertain outcomes with almost certainty. Consequently, risk pooling risk, spreading and risk transfer tools are adopted. The risk pooling and risk transfer mechanisms result in Pareto improvement in the society.

The fact that insurance market exhibits the persistence of either no insurance or less than full insurance is tried to be explained. While credit constraints could be a major factor for people to not buying insurance, there are structural problems like adverse selection of moral hazard, which result in less than full insurance as a viable option. Insurers may have less information about potential risks than the insurance purchasers resulting in adverse selection.

Insurers unable to monitor for the behaviour of insured leads to moral hazard problems. Such problems give rise to self-selection in insurance markets. When the resulting equilibria exist, which are seen by considering pooling and separating equilibrium, often found to be Pareto inefficient.

18.6 KEY WORDS

Adverse Selection: Depict a situation when buyers and seller do not of have equal excess to information and transaction takes place. Trade executed is biased to favour the agent with better information.

Cream-Skimming or Creaming Off: A rational transactor trying to select the most favourable cases out of a set of options. For example, insurance companies try to cover the "best" or lowest risks from a population, and avoid the high risks.

Moral Hazard: Effect of insurance encouraging risk an insurance party takes; an unshaved action emerges that affects the probability expected before hand and triggers payment from the insurer.

Pooling Equilibrium (in insurance): High and low risk consumers of insurance are grouped together with a common premium and cover. Thus, when high-risk individuals and low risk individuals choose the same contract the equilibrium is pooling.

Risk Pooling: The practice of bringing several risks together for insurance purposes in order to balance the consequences of the realisation of each individual risk.

Risk Transfer: The shifting of risk through insurance.

Risk-spreading: Diversification or spreading of one's wealth among a large number of different assets to ensure that if one area goes sour, one may still be doing well in some other.

Self Selection: Consequence of a contract that induces only one group to participate

Separating Equilibrium (in insurance): When high-risk individuals and low risk individuals choose different contracts, the equilibrium is separating.

18.7 SOME USEFUL BOOKS

Autor, H. David (2004), *Adverse Selection, Risk Aversion and Insurance Markets*, Lecture note, available on Internet site www.core.org.in.

Maddala, G. S. and Ellen Miller (1989), *Microeconomics: Theory and Applications*, McGraw-Hill, USA.

Nicholson, W.(1992), *Microeconomic Theory*, 5th edition, the Dryden Press, Harcourt Brace Jovanovich, Orlando.

Sen, Anindya (1999), *Microeconomics: Theory and Applications*, Oxford University Press, New Delhi.

Varian, Hall (3rd Edition), *Microeconomic Analysis*, W. w. Norton and Company, Inc., New York.

18.8 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) See Section 18.2
- 2) See Sub-Section 18.3.1
- 3) See Sub- Section 18.3.2
- 4) If one naively looks at the expected monetary value, he'll find that on average you'll win $(156 - 100)/2 = 28$, and hence expects you to take that gamble. However, since you're risk averse, the gamble is worth less than its EMV to you. If fact, if you take the gamble, your expected utility will be $[u(100 + 156) + u(100 - 100)]/2 = (\sqrt{256} + \sqrt{0})/2 = 16/2 = 8$, which is lower than your initial utility, and hence you would not take the gamble. If fact, if the gamble is forced upon you, it'll be as if Rs.36 is robbed from you. In the latter case, your utility will also be $u(100 - 36) = u(64) = \sqrt{64} = 8$.

Check Your Progress 2

- 1) See Section 18.4 and answer
- 2) See Section 18.4 and answer
- 3) By reducing the size of the gamble to a half, your resulting utility will be $[u(100 - 50) + u(100 + 78)]/2 = (u(50) + u(178))/2 = (\sqrt{50} + \sqrt{178})/2 = (7.071 + 13.341)/2 = 10.206$, which is higher than your initial utility. The same is true for your friend, so it is not that you're better off at the expense of your friend. Note that half a gamble looks like a gift of Rs.4.162 to each of you.

18.9 EXERCISES

- 1) The market for insurance is competitive: insurers are risk neutral and make zero expected profits. What unit price will be charged to the agents?
- 2) Julie owns a house worth Rs.100,000. She cares only about her wealth, which consists entirely of the house. In any given year, there is a 20% chance that the house will burn down. If it does, its scrap value will be Rs. 30,000. Julie's utility function is $U = x^{1/2}$
 - a) Draw Julie's utility function.
 - b) Is Julie risk-averse or risk preferring?
 - c) What is the expected monetary value of Julie's gamble?
 - d) How much would Julie be willing to pay to insure her house against being destroyed by fire?

- e) Say, Harish is the president of an insurance company. He is risk-neutral and has a utility function of the following type: $U = x$. Between what two prices could a beneficial insurance contract be made by Julie and Harish.
- 3) Suppose that Julie and Harish both have a good paying job, that pays Rs.120,000 a year to each of them. Due to the situation of the economy, they both have a risk of getting laid off of 5%. Harish has utility function over money given by $U = x^{1/2}$, and Julie has a utility function over money given by $U = x^{1/4}$.
- Without risk pooling, what is the mean and the variance of the “job gamble” that each of them faces?
 - What is the expected utility of the “job gamble” for each of them without risk pooling?
- Suppose Julie and Harish pool their risk by agreeing to split their income no matter what happens.
- With risk pooling, what is the mean and the variance of the new “job gamble” that each of them faces?
 - What is the expected utility for each of them with risk pooling?
- 4) (*Due to Mukherjee D.*) An insurance market has a large number of customers that belong to two risk categories: high and low, with probabilities of an accident being p_h and p_l respectively, where $1 > p_h > p_l > 0$. A given fraction $\lambda \in (0,1)$ of the customers are the high-risk types. There is a single consumption good. All customers have the same income Y in the absence of an accident, and suffer a damage of $d \in (0,Y)$ in the event of an accident. They share a common von Neumann-Morgenstern utility function u , which is strictly increasing, strictly concave and differentiable. There are a large number of risk-neutral insurance firms competing to offer insurance policies to these customers. An insurance policy is described by a premium m paid by the customer to the insurance company when there is no accident, and a net payout r from the insurance company to the customer in the event of an accident.
- Write down the expected utility of an insurance customer of either type, and the expected profit of the insurance company, as a function of the insurance policy (r, m) .
 - Draw indifference curves of either type of customer in (r, m) space, and describe their properties (slope, curvature etc.).
 - Draw the zero expected-profit locus for either type of customer in (r, m) space.
 - Draw the line of contracts that insure either type of customer perfectly in (r, m) space.
- 5) (*Due to Nicholson, W.*) Suppose there is a 50-50 chance that a risk-averse individual with a current wealth of Rs.20,000 will contract a debilitating disease and suffer a loss of Rs.10,000.
- Calculate the cost of actuarially fair insurance in this situation and use utility-of-wealth graph to show that the individual will prefer fair insurance against this loss to accepting the gamble uninsured.

b) Suppose two types of insurance policies were available:

- 1) A fair policy covering the complete loss.
- 2) A fair policy covering only half of any loss incurred.

Calculate the cost of the second type of policy and show that the individual will generally regard it as inferior to the first.

- 6) (*Due to Nicholson, W.*) Ms. Fogg is planning an around-the-world trip on which plans to spend Rs.10,000. The utility from the trip is function of how much she actually spends on it (y) given by

$$U(y) = \ln Y.$$

- a) If there is a 25 percent probability that Ms. Fogg will lose Rs.1000 of her cash on the trip, what is the trip's expected utility?
- b) Suppose that Ms. Fogg can buy insurance against losing the Rs.1000 (say, by purchasing traveler's cheques) at an "actuarially fair" premium of Rs.250. Show that her expected utility is higher if she purchases this insurance than if she faces the chance of losing the Rs.1000 without insurance.
- c) What is the maximum amount that Ms. Fogg would be willing to pay to insure her Rs.1000?

Mean Variance Analysis Risk Assets (Based on Nicholson, W.)

Usually individuals prefer more assets to less. However, with risk aversion, risk is seen as a variation in return. So we can measure risk through variance and represent preferences such that individuals make choice by looking at mean and variance. There is one section of risk analysis, which indicates that individuals' choice among assets often is decided by the mean and variance of the return. We consider a random variable X with outcomes $X_1, X_2, \dots, X_n$ occurring with probabilities $p_1, p_2, \dots, p_n$, so that the expected value (mean) is given by

$$E(X) = \sum_{i=1}^n p_i X_i = \mu_X$$

$$Var(X) = \sigma_x^2 = E[(X - \mu_x)^2] = \sum_{i=1}^n p_i (X_i - \mu_x)^2$$

Let the utility-of-wealth function is represented by a quadratic function

$$U(X) = a + bW + cW^2 \text{ where } b > 0, c < 0.$$

Moreover, for all relevant values of W , $b + 2cW > 0$. If W is a random variable, then $E[U(W)]$ is a quadratic function of the mean of W (μ_w) and its variance (σ_w^2). Thus,

$$\text{expected utility} = a + b\mu_w + c(\mu_w^2 + \sigma_w^2).$$

The expected utility function has a positive slope for the indifference curves between μ_w and σ_w^2 so that the individuals accept a higher σ_w^2 only when a higher μ_w is assured. That is,

$$\frac{d\sigma_w^2}{d\mu_w} = \frac{-b - 2c\mu_w}{C} > 0.$$

Along any indifference curve,

$$\frac{d^2\sigma_w^2}{d^2\mu_w} > 0$$

indicating the $\mu_w - \sigma_w^2$ indifference curves have a diminishing MRS.

Take an individual with a quadratic utility-of-wealth function who plans investment in two independent assets, X and Y with $\mu_x = \mu_y$; $\sigma_x^2 = \sigma_y^2$.

See that if

$$Z = \frac{X + Y}{2}, \text{ then}$$

$$\mu_z = \mu_x = \mu_y \text{ whereas}$$

$$\sigma_z^2 = \frac{\sigma_x^2}{4} + \frac{\sigma_y^2}{4} = \frac{\sigma_x^2}{2}.$$

What we derive is that the individual is better off with a mixed investment in the sense that holding both assets X and Y result in a reduced variance.

Portfolio Separation

Let us assume that there are exactly two different securities that an investor can buy. These are, 1) risk-free cash and 2) a risky asset X. Further, assume that a fraction α ($0 \leq \alpha \leq 1$) of wealth (W) is invested in X. So investment is given by αX . The portion of W not invested is $(1 - \alpha)W$. Let this portion be invested in risk-free cash. Thus, the total net wealth (NW) is given by $NW = \alpha X + (1 - \alpha)W$. If NW is a random variable, then

$$\text{mean: } \mu_{NW} = \alpha\mu_X + (1 - \alpha)W \text{ and}$$

$$\sigma_{NW}^2 = \alpha^2 \sigma_X^2 \text{ (as W is a constant)}$$

We can solve for utility maximising quadratic value of α of the utility function

$U(W) = a + bW + cW^2$, taking a, b, c, μ_X and σ_X^2 as parameters.

If α^* is the utility maximising value of α , then $\frac{\partial \alpha^*}{\partial \sigma_X^2} < 0$ and if $(\mu_X - W)$ is constant, then,

$$\frac{\partial \alpha^*}{\partial W} < 0.$$

That is, since the variance increases, person with a quadratic utility function invests less in risky asset.

UNIT 19 ECONOMICS OF INFORMATION

Structure

- 19.0 Objectives
- 19.1 Introduction
- 19.2 The Principal-Agent Framework
- 19.3 Moral Hazard Problem
 - 19.3.1 Hidden Action Modelling
- 19.4 Adverse Selection in Markets
 - 19.4.1 Hidden Information Modelling
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19.0 OBJECTIVES

After going through this unit, you will be able to:

- understand some of the important issues that arise in the market when information is imperfect;
- evaluate the problems of decision making with asymmetric information; and
- see that applications of Principal-Agent model with problems of hidden action, hidden information and efficient wage determination.

19.1 INTRODUCTION

Decision making in the presence of uncertainty encounters problem due to peculiarities of information. We have seen in the preceding unit that inefficiencies creep into the insurance contracts on account of adverse selection. As we saw, just because one party held better information on the risk involved (the insured knew her health status better than the insurer), the market outcome was a distorted one. In another instance (in case of driver's insurance) lack of perfect monitoring on the part of insurer resulted in insured choosing to act differently after buying the insurance, which led to influence the utility of the insurer. Information asymmetries of these types are pervasive in many other economic relations. It is not difficult for you to find,

- customers know more about their tastes than the firm;
- firms know more about their costs than the government; and
- employees know more about their capabilities than the employer.

The net result of all these instances is that individuals who hold such private information are expected to manipulate it to their advantage. Thus, holding the information acts as if they have monopoly power over its price. In order to take that into consideration, we need to know some of the important concepts information and analytical tools available for drawing inferences.

The following discussion is introductory in nature. It attempts to bring in some basic concepts of information economics and provides a preliminary

analytical framework while seeking solution to problems. In the process you will be exposed to designing of basic contract between two parties taking into account the constraints imposed by the prevailing institutional settings. For advanced level discussion, which we will not cover presently, you will resort to the insights of game theory with asymmetric information (see Blocks 7&8) for solving many of such problems.

Remember that studies of behaviour under asymmetric information involve strategic interaction among agents. A worker, for example, might have better idea on her capabilities than the employer. However, by carefully observing her behaviour, the employer might be able to infer the worker's capabilities. Consequently, he might design incentive structure to get best work efforts. On the other hand, good workers might want to be known as efficient or otherwise, according to the payment package being offered.

Another aspect of solution to present discussion, you may take note of, is the designing of incentives. Recognition of private information being in the possession of some agents, necessitates additional cost to be incurred for arriving at optimal solution among the parties.

In the following we start with *Principal-Agent framework* to highlight the problems encountered due to asymmetric information and extend the discussion to cover applications such as *hidden action*, *hidden information* and *efficiency wage* model.

19.2 THE PRINCIPAL-AGENT FRAMEWORK

We start with the situations of asymmetric information in which one agent knows something that another does not prior to contracting. One person, the principal, wants to delegate a task to another, the agent, to undertake some action, which is costly to her (the agent). See that principal's welfare depends on what the agent does.

What we observe in the principal-agent problems is

- there is a contract between the principal and the agent;
- information gap exists between the principal and the agent; and
- such a gap has implications for the decision of the contract they sign.

Note that a contract is an agreement between the two parties to act in some specified way. The agent is willing to undertake the task as long as her net utility from performing it is at least as large as she can earn from her next best opportunity. The agent, if hired, has to decide whether to work hard or not. Hard work involves incurring disutility for the agent. So all other things equal, she would prefer not to work hard. As a result, the value of the work would be lower and the principal will get very little from the deal. In order to ensure that the contract results in gainful trade, the principal must make provisions, may be by providing incentives (like bonus), so that the agent puts in her best efforts.

Implicit in signing of a contract is the fact that it is enforceable. Without such a provision no one will enter a contract. Courts are entrusted with the task of enforcing it. To avoid its no-enforceability or even to minimise the chance of court cases, which are expensive and time consuming, attempts are made to design contract in such a way that each party chooses to adhere to its terms and conditions. In other words, contracts are designed to be self-enforcing.

Once we limit ourselves to a self-enforcing contract, the principal-agent framework reduces to one of principal offering the agent a contract, which is either accepted or rejected by the agent.

Let us present the principal-agent problem in terms of a manager-worker example: the manager wants the worker to put in as much effort as possible for producing maximum output. However, the worker rationally wants to make a choice that would maximise her own utility given the effort and incentive payment scheme. We use the notation of Varian (1992) while introducing the problem.

Let x be the output received by the principal. The agent has a set of feasible actions A from which she chooses actions a and b . If we assume that there is no uncertainty, output will be completely determined by the actions of the agent. We write output-effort relationship as $x = x(a)$, cost of action a as $c(a)$ and incentive payment from the principal to the agent as $s(x)$. In this formulation, the utility function of the principal is given by output minus the incentive payment, i.e., $x - s(x)$. The utility function of the agent, on the other hand, is decided by the difference between incentive payment and the cost of the action, $[s(x) - c(a)]$ giving her utility function: $u[s(x) - c(a)]$.

The maximisation problem of the principal is given by

$$\max_x [x - s(x)]$$

Subject to the constraint $s(x) - c(a)$, imposed by the agent's optimising behaviour.

There are two types of constraints when the agent is involved. First, the agent may have another opportunity available to her, which gives her some reservation level of utility. In order that the agent is willing to participate in the contract, the principal must be willing to give her at least this reservation level. This is called the *participation constraint* or individual *rationality constraint*. Secondly, given the incentive schedule chosen by the principal, the agent will pick the best action for herself. This type of constraint is called *incentive compatibility constraint*. The second constraint implies that irrespective of the terms of the contract, the agent will maximise her own utility.

Full Information Model

Let us start with the simplest example in which the principal has full information about the agent's costs and actions, and devises an incentive scheme for the agent.

In order to appreciate the problem, we take the output as a function of ' a ', the action taken by the agent. The principal is interested to extract best effort from the agent. Let the best effort is b which can be thought of as a contrast to a , the alternative action. Now, the problem of optimal incentive scheme $s(\cdot)$ can be written as

$$\max_{b, s(\cdot)} x(b) - s(x(b))$$

such that $s(x(b)) - c(b) \geq \bar{u} \dots \dots \dots (2.1)$

$$\text{and } s(x(b)) - c(b) \geq s(x(a)) - c(a) \text{ for all } a \text{ in } A \dots \dots \dots (2.2)$$

In the above problem, (2.1) is the participation constraints whereas (2.2) is the incentive compatibility constraint (the agent finds it optimal to choose b). The underlying mechanism of the above problem lies in that the principal chooses agent's action b , albeit indirectly, while designing the incentive payment function. The constraint, which the principal faces, is to make sure that the agent would like to take the action, which the principal wants her to take.

To understand the solution of the above maximisation problem, we can break it into two parts. First, ignore the incentive compatibility constraints and consider the objective function along with the participation constraint. Recall that we have assumed full information for the principal so that we solve the case when effort is being observed. This means, we can drop the agent's compatibility constraint, since we can condition pay directly on the action and do not have to induce her to take the right action using pay as function of output. We see that for any x , the principal wants $s(x)$ to be as small as possible. That is, the principal will pay the agent only enough to meet the participation constraint:

$$s(x(b)) - c(b) \geq \bar{u}.$$

Then the principal's problem is to choose b to maximise $(x(b)) - c(b) \geq \bar{u}$.

The first order condition is $x'(b) = c'(b)$ and the solution is, say, b^* .

Second, the principal has to pick a function $s(\cdot)$ such that it is in the agent's interest to choose b^* , given $s(\cdot)$. That is, s must be such that $s(x^*) - c(b^*) \geq s(x(a)) - c(a)$ for all a in A .

There are various ways of devising an appropriate incentive scheme. We consider two in the following:

Target Output Scheme

$$\text{Let } s(x^*) = \begin{cases} u + c(b^*) & \text{if } b = b^* \\ -\infty & \text{otherwise.} \end{cases}$$

A target output of x^* is set and the agent is paid her reservation price if she reaches the target and otherwise receives punishment.

Linear Incentive Payment Scheme

Set $s(x(a)) = x(a) - F$. The agent pays a lump sum fee F to the principal and then receives the entire output produced.

Note two things about these solutions to the full-information principal-agent problem:

- 1) The incentive compatibility constraint is not really 'binding'. Once the optimal level of output is chosen, the agent may just be told that production will be stopped unless she chooses the effort level principal wants.
- 2) Since the incentive compatibility is never binding, we are in effect implementing a first-best outcome. This means that the outcome is Pareto efficient. Remember, Pareto efficient allocations are solutions to problem where you maximise one person's utility, subject to the others attaining a given level of utility.

Let us now look at the difficulties associated with the above incentive schemes.

- i) The postulated input-output relationship may not be perfectly determinate. Thus, the low output produced may not be due to lack of effort but could be due to some noise (bad luck). In view of such a possibility, the agent, if paid only on the basis of output production would refuse to participate. In order to satisfy the participation constraint, the principal must arrange for payment to the agent that equals her reservation level of utility. Thus, the scheme of payment, which must be positive for different levels of output, may have to be evolved.

This problem is known as a hidden action incentive problem and arises due to the fact that the action of the agent is not perfectly observable by the principal.

- ii) The incentive scheme is designed with the presumption that the principal knows the type of agent, her utility function. However, there may be many different types of agents with different utility functions or cost functions. Consequently, the principal cannot observe the objective function of the agent perfectly. This type of incentive design problem is known as the hidden information problem.

We will take up these two types of incentive problems below.

Check Your Progress 1

- 1) Explain why asymmetric information between buyers and sellers can lead to a market failure when a market is otherwise perfectly competitive.

.....

- 2) Explain the difference between adverse selection and moral hazard in insurance markets. Can one exist without the other?

.....

- 3) Why might a seller find it advantageous to signal the quality of her product? How are guarantees and warranties a form of market signalling?

.....

19.3 MORAL HAZARD PROBLEM

As apparent from the above discussion, we speak of moral hazard when the agent takes an action that affects her utility and the utility of the principal. Remember the driver's insurance example presented in the preceding unit. After being insured, a driver may drive recklessly as damage in the event of an accident will be borne by the insurance company. The net result is change in the utility of both the parties to insurance. Mark the information imperfection in such a behaviour. The principal does not observe the agent's action but only the outcome, which is an imperfect signal of this action. The action chosen by the agent is not Pareto optimal.

The principal will try to influence the action of the agent by conditioning agent's utility on the only variable, which is observable, viz., the outcome. You can add a number of examples describing this kind of behaviour apart from the one cited in case of driver's insurance.

Examples

- Insurance companies cannot observe the self-protection efforts of the insured;
- Patients cannot ascertain the effort put in by a doctor to find out the causes of a disease.
- A landowner cannot observe the efforts of a tenant while cultivating the land.
- In a firm the manager cannot observe the efforts of the workers.
- The shareholders are uncertain about the manager's efforts in profit maximisation.

If we try for a first-best (i.e., Pareto optimal) solution of moral hazard problem and follow the principal-agent framework, we may assume that the principal can observe agent's action. In that formulation, the principal orders the agent to take the optimal action and will choose the payment, which achieves the optimal risk sharing.

To appreciate the above option, let us consider the employer-employee example. We will follow the presentation module, sequence and notation of *Meunier, Valerie 2005* for this part of the discussion.

19.3.1 Hidden Action Modelling (see, *Meunier, Valerie 2005*)

The employer (the principal) hires the employee (the agent) to execute a project (say, developing a software on financial security). Let us assume that there are two possible outcomes of the project: it succeeds or fails. When the project succeeds, it yields an outcome x_s in money terms. On the other hand, it may also fail with the associated outcome of x_f and $x_s > x_f \geq 0$. The principal is risk neutral, as he faces many independent risks and thus can diversify the associated risks to his relationship with the employees. Through such an arrangement, the principal would be interested only on the expected revenue.

The agent, in our example, can choose among two levels of efforts e : she can work ($e = 1$) or bunk ($e = 0$). We can normalise the cost of effort for the agent to e without introducing any distortion to our analysis.

Let us start with a contract agreement in which the employer offers the worker a wage w once the project is finished. We assume the worker's utility to be

$u(w) = v(w) - e$, where $v' > 0$ and $v'' < 0$, i.e., utility is increasing and concave in wealth. Therefore, the agent is risk-averse.

Symmetric Information: First Best Option

Start with the assumption that the principal is able to verify the agent's action. He can therefore "force" the agent to choose $e = 1$. The contract that can be offered by the principal is $(e = 1, w_s, w_f)$ consisting of a level of effort and a wage contingent on the project's outcome. The principal's expected utility (payoff) increases with the level of effort, since

$$p_s(1)(x_s - w_s) + (1 - p_s(1))(x_f - w_f) > p_s(0)(x_s - w_s) + (1 - p_s(0))(x_f - w_f).$$

The agent will accept the contract if it yields an expected payoff at least as high as what she could get elsewhere, i.e.,

$$p_s(1)v(w_s) - 1 + (1 - p_s(1))v(w_f) \geq \underline{u},$$

where $\underline{u}$ denotes the agent's reservation utility. If the wages w_s and w_f are not high enough, the agent will decline the offer and get $\underline{u}$ somewhere else.

The principal's problem is to maximise his expected utility, subject to the agent's participation constraint (i.e., subject to the agent accepting the contract), that is,

$$\max_{w_s, w_f} p_s(1)(x_s - w_s) + (1 - p_s(1))(x_f - w_f)$$

$$\text{subject to } p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 \geq \underline{u}.$$

Now, we write the Lagrangian of the problem as

$$L = p_s(1)(x_s - w_s) + (1 - p_s(1))(x_f - w_f) + \lambda [p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 - \underline{u}]$$

The FOC of this optimisation problem are

$$\frac{\partial L}{\partial w_s} = 0,$$

$$\text{or, } -p_s(1) + \lambda p_s(1)v'(w_s) = 0 \Leftrightarrow v'(w_s) = \frac{1}{\lambda} \dots\dots\dots (3.1)$$

$$\frac{\partial L}{\partial w_f} = 0,$$

$$\text{or, } -(1 - p_s(1)) + \lambda(1 - p_s(1))v'(w_f) = 0 \dots\dots\dots (3.2)$$

$$\text{or, } v'(w_s) = \frac{1}{\lambda}$$

$$\lambda [p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 - \underline{u}] = 0 \dots\dots\dots (3.3)$$

From the conditions (3.1) and (3.2) we write $v'(w_f) = \frac{1}{\lambda}$. Since we have assumed $v'' < 0$, this equality can only be verified if $w_s = w_f$. When the agent is risk-averse and the principal is risk-neutral, he offers total insurance to his employee by offering a wage that is independent from the outcome of the project.

Thus, we get the first-best solution that a risk-averse worker if employed by a risk-neutral principal would get a wage independent of any contingency.

If in the equilibrium, $w_s = w_f = w^*$, the agent participation constraint is

$$v(w^*) - 1 \geq \underline{u}.$$

This constraint must bind. Otherwise the principal could lower the wage offered w^* by a small amount, without affecting the agent's participation, but thereby increasing his own revenue.

Thus, the first-best solution is

$$e = 1, w_s = w_f = w^*,$$

$$\text{such that } v(w^*) = \underline{u} + 1$$

$$\text{or equivalently, } w^* = v^{-1}(\underline{u} + 1).$$

Asymmetric Information

If we introduce the more realistic case where the principal cannot verify the agent's level of effort, we get the following result:

The principal offers a contract with a constant wage w^* , the agent will decide to bunk: because she minimises her cost. The principal cannot verify this. The principal anticipates such a behaviour, and chooses the wage accordingly. What he is trying to do is to offer a wage that maximises his expected profit, subject to $e = 0$.

The principal can design a contract of the form (w_s, w_f) , such that it pays for the agent when she actually works, i.e.,

$$p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 \geq p_s(0)v(w_s) + (1 - p_s(0))v(w_f)$$

or, $p_s(1) - p_s(0) \geq \frac{v(w_s) - v(w_f)}{v(w_s) - v(w_f)}$

or, $p_s(1) - p_s(0) \geq 1$ (3.4)

If the agent accepts the contract, we get

$$p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 \geq \underline{u}. \quad (3.5)$$

Thus, in order to get $(w_s - w_f)$ increasing as $(p_s(1) - p_s(0))$ decreases we must have from (3.4) above $w_s > w_f$. If there is success, the wage has to increase as its probability decreases towards $p_s(0)$. It becomes more difficult to distinguish someone who works from someone who bunks.

It is easy to recall that condition (3.4) is incentive compatibility constraint (IC) we have seen earlier. The principal must give the agent an incentive to actually do the work. Such a design is compatible with the agent's preferences, such that when the agent maximises her objective function, she chooses the right level of effort.

The other condition (3.5) is participation constraint, or individual rationality constraint (PC or IR). It must be binding because otherwise, it would mean that the principal would decrease both w_s and w_f by a small amount ε , which would not affect the IC constraint and would increase his expected payoff.

We can write the principal's optimisation problem as

$$\max_{w_s, w_f} p_s(1)(x_s - w_s) + 1 - p_s(1)(x_f - w_f)$$

subject to IC: $p_s(1) - p_s(0) \geq \frac{v(w_s) - v(w_f)}{v(w_s) - v(w_f)}$ and

IR: $p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 \geq \underline{u}.$

Therefore, the Lagrangian is

$$L = p_s(1)(x_s - w_s) + 1 - p_s(1)(x_f - w_f) - \lambda(p_s(1) - p_s(0) - \frac{v(w_s) - v(w_f)}{v(w_s) - v(w_f)}) - \mu(p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 - \underline{u}).$$

The Kuhn-Tucker conditions, which you can see in MEC-003, are

$$\frac{\partial L}{\partial w_s} = 0$$

or, $-p_s(1) + \lambda(p_s(1) - p_s(0))v'(w_s) + \mu p_s(1)v'(w_s) = 0$ (3.6)

$$\frac{\partial L}{\partial w_f} = 0$$

$$\text{or, } -(1 - p_s(1)) - \lambda(p_s(1) - p_s(0))v'(w_f) + \mu(1 - p_s(1))v'(w_f) = 0 \quad (3.7)$$

$$\text{or, } \lambda \begin{pmatrix} p_s(1) & p_s(0) & v(w_s) & v(w_f) & 1 \end{pmatrix} = 0 \quad (3.8)$$

$$\mu(p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 - \underline{u}) = 0 \quad (3.9)$$

Let us write conditions (3.6) and (3.7) as

$$\frac{1}{v'(w_s)} = \mu + \lambda \left(1 - \frac{p_s(0)}{p_s(1)} \right) \quad (3.10)$$

$$\frac{1}{v'(w_f)} = \mu + \lambda \left(1 - \frac{1 - p_s(0)}{1 - p_s(1)} \right). \quad (3.11)$$

Now, we have to shown that both constraints, IC and IR are binding.

Suppose IC is not binding such that $\lambda = 0$; conditions (3.10) and (3.11) would then yield $\frac{1}{v'(w_s)} = \frac{1}{v'(w_f)} = \mu$ and we get $w_s = w_f$. But then, $v(w_s) = v(w_f)$

and IC will not be satisfied. Therefore, we must have $\lambda > 0$, and $[p_s(1) - p_s(0)][v(w_s) - v(w_f)] = 1$.

Next, suppose IR is not binding such that $\mu = 0$. Condition (3.7) will not be satisfied as the LHS would always be negative. Therefore, you must have $\mu > 0$ and $p_s(1)v(w_s) + (1 - p_s(1))v(w_f) - 1 = \underline{u}$.

With the constraint binding and $\mu > 0$, the payment in general to the agent will vary with the outcome. Thus, we have a situation where the principal desires the action which imposes high costs on the agent so the payment to the agent will depend on the behaviour of the fraction $\frac{p_x(0)}{p_x(1)}$.

This ratio is called the likelihood ratio. It indicates the result x signals that the effort was high (i.e., $e = 1$). When the ratio is smaller, $p_x(1)$ is higher with respect to $p_x(0)$ and the signal that $e = 1$ is stronger. Use of the likelihood ratio for the construction of the optimal incentive scheme suggests that we can use regularity conditions found in statistics to solve the behaviourable problem of optimal scheme. Let us return to our example of employer-employee above.

We have assumed that $w_s > w_f$. It holds if $\frac{p_s(0)}{p_s(1)} < \frac{p_f(0)}{p_f(1)}$. This implies, the

ratio $\frac{p_x(0)}{p_x(1)}$ decreases when the outcome x increases, just a statistical property, which we call monotonous likelihood property.

Using the fact that IC and IR bind in the equilibrium, we can have

$$\text{IC: } v(w_s) - v(w_f) = \frac{1}{p_s(1) - p_s(0)}$$

$$\text{IR: } p_s(1)v(w_s) - v(w_f) + v(w_f) = \underline{u} + 1.$$

Solving for $v(w_s)$ by substituting IC and IR, we get $v(w_s) > v(w_f)$, $(w_s) = \underline{u} + \frac{1 - p_s(0)}{p_s(1) - (0)}$. Since we know that the agent is not fully insured, and her wage will decrease in case of failure. Note that this provides her an incentive to work.

In the above, we have tried to get the second-best situation where the principal cannot observe the action taken by the agent, but the outcome only. If the employee (agent) receives a constant wage, she will choose the effort which minimises her cost. Such an action, in general, will not be optimal. Solving the moral hazard problem, therefore, essentially implies that the employer (principal) offers the employee a contract that trade off risk sharing and incentives.

The optimal incentive scheme can take on very complicated forms. The factors, which influence the optimal incentive could be profit created by additional effort, precise assessment of desired activities agent's risk tolerance and response to incentives.

19.4 ADVERSE SELECTION IN MARKETS

Market for Lemons

Look at a model of market where two parties (principal and agent) have different information about the qualities of the goods being sold. Suppose there are 100 people in this market who want to sell their used cars and another 100 who want to buy used cars. Everyone knows that 50 of the cars are 'plums' (good quality cars) and 50 are 'lemons' (bad quality cars). The owner of the car selling knows its quality whereas the prospective buyers do not have such private information.

The owner of a bad car (lemon) is willing to sell it for Rs.50,000 while the owner of a good car (plum) wants to get at least Rs.100,000. The buyers of car are willing to pay Rs.60,000 for lemons and Rs.110,000 for plums.

If the quality of a car is easily verifiable, then there will be no problems in this market. Lemons will be sold for some amount between Rs.50,000 and Rs.60,000, while plums will be sold for a price between Rs.100,000 and Rs.110,000. But what happens to the market if the buyers cannot observe the quality of the car?

In such a situation, buyers will have to guess how much a car is worth. Let us make some simplifying assumption about the form that this guess takes: a buyer is willing to pay the expected value of the car. On the basis of the prices described above, the buyer's expected value is worked out to be

$\frac{1}{2}60,000 + \frac{1}{2}110,000 = \text{Rs.}85,000$. That is, the buyer would be willing to pay this amount for a car. See that this price is below that of the owner a plum. So this group of owner will not sell their cars. In that event, the only cars offered for sale in the market will be the lemons.

But if the buyers are certain that only lemons will be sold, they will not be prepared to pay Rs.85,000. The equilibrium price in this market, therefore, will have to be somewhere between Rs.50,000 and Rs.60,000. With such a price range, only the owners of lemons will offer their cars for sale and no plum will come to the market. It is important to see the market behaviour through this example: buyers of plums are willing to pay more than the price offered by their owners, but no transactions of plums taking place. We have a market failure instance.

Search for the source of market failure will indicate that there is an externality between the sellers of good cars and bad cars. The seller of a bad car is affecting the purchasers' perceptions of the quality of the average car on the market. Such a possibility lowers the price that they are willing to pay for the average car. Consequently, the people who are trying to sell good cars are

thrown out of the market. The signals emerging from the market is that low-quality items are offered for sale.

After George Akerlof pointed out the problem in 1970, it has been used extensively to analyse other markets. See that you are classifying information by *what there is lack of information about*. So, it will be possible for you put adverse selection in the category of hidden information. That is, one side of the market – either buyers or sellers – has better information than the other side. If we say, the agent has some private knowledge about her cost or valuation, which the principal does not know, then we have the case of adverse selection or hidden knowledge. First, the principal offers the agent a contract, with the asymmetric information. Then, the agent takes an observable action and the outcome is realised. Here, the principal's problem stems not from the future unobservability of the agent's action, but the realisation that he might be dealing with different *types* of people.

Insurance Market and Adverse Selection

Let us return to the example of the insurance markets discussed in Unit 21. We repeat those results here by spelling out the problem explicitly, in principal-agent framework. You know that the insurance company has less information about the risks regarding illness of the insured than the insured individuals themselves.

Suppose that there are two groups of individuals, the high-risk group (H) and the low-risk group (L). Take their probabilities of illness or accident as P_H and P_L with $P_H > P_L$. Both groups buy insurance and the company cannot distinguish them. The weighted average probability of illness for the whole group given by

$$\bar{P} = P_H \cdot \frac{H}{H+L} + P_L \cdot \frac{L}{H+L}$$

In this formulation $P_H > \bar{P} > P_L$. If the cost of illness is C, then the insurance premium I for a full coverage will be

$$I \geq C\bar{P}$$

If individuals know their own risks, since $\bar{P} > P_L$, the ones with low risk may not be willing to buy insurance whereas the high risk individuals would be willing to accept the offer. When low-risk agents drop out, the insurance company has to raise the premium, and only the high-risk individuals will buy the insurance. The low-risk individuals will go without any insurance.

To by pass the problem, the insurance company can get the low-risk people to reveal themselves by offering coinsurance or some deductibility scheme. This process is known as “self-selection.” Such a move will help the insurance company to infer the risk characteristics of individuals. However, the information problem is not completely solved. The information conveyed by the individual's choice of a particular contract depends on the set of contracts available to the individual.

With two groups of individuals with different probabilities of disaster (illness), there cannot, theoretically speaking, be a single insurance policy. If at all, there have to be two insurance policies, and it can be shown that the high-risk individual obtains complete insurance and the low-risk individual obtains only partial insurance.

We can incorporate other examples of adverse section as well. Try to consider the following:

- **Labour Contract:** A firm may offer only low wage without knowing the potential workers, which will give rise to recruiting least productive workers. These are the types of workers who would be willing to accept the offer made.
- **Financial Markets:** A bank may fail to observe the risk-return characteristics of a project. Consequently, it will extend credit facilities to bad projects while rationing credit to good projects.

For more instances of adverse selection you may also note the following contracts:

- **Landlord–Tenant Case:** The landlord delegates the cultivation of his land to a tenant who will be the only one to observe the exact local weather condition (i.e., the private information).
- **Legal Attorney–Client Case:** A client delegates her defence to an attorney who only has the private knowledge of the difficulty of the case.
- **Investor – Broker Deals:** An investor would delegate the management of her portfolio to a broker, who only knows the market conditions.

Signalling and Screening

When we consider the consequences of asymmetric information we see the following problem: In the lemons model the problem was that sellers of high quality products did not know the quality when products of both high and low quality are marketed side by side. A solution to the problem suggested was that if sellers of a high-quality product could find some activity that was less costly for them than for sellers of a lower-quality product, then it might pay them to undertake this activity as a signal of higher quality. The buyers, too, would learn that the signal was associated with higher quality.

In the insurance example, we have discussed a form of signalling. An individual with low risk is more willing to co-insure than an individual with a high risk. Thus, the level of co-insurance was used as a potential signal of risk level. Another application considered was in education. Since education increases productivity it can act as a signal of high productivity if workers with high productivity can acquire it at a lower cost than workers with low productivity.

Consider two kinds of workers, H and L , with high productivity P_H and low productivity P_L . Assume that $P_H > P_L$. Then the average productivity of the workers can be written as

$$\bar{P} = P_H \cdot \frac{H}{H+L} + P_L \cdot \frac{L}{H+L}$$

where productivity is measured in terms of the value of goods produced. When employers cannot distinguish between workers, they may pay each worker a wage $W = \bar{P}$. However, by acquiring a *signal* such as education workers with higher productivity can distinguish themselves from the other category. If all high-productivity workers are educated whereas none of the low-productivity workers are, the employers will find out the high-productivity workers and pay them a wage $W_H = P_H$. On the other hand, the low-productivity workers are paid a wage $W_L = P_L$. Thus, there will be equilibrium in the market.

We now look for the condition under which it can take place. Suppose that the costs of education are C_H for the high-productivity workers and C_L for the

low-productivity workers. In that case, we must have $C_H < C_L$ and $C_L > P_H - P_L$ to ensure that it is not worthwhile for the low-productivity worker to invest in education. Moreover let us take $C_L < P_H - P_L$, so that it is worthwhile for the high productivity worker to invest in education. Thus, when education is used as a signal, those with education get a wage $W_H = P_H$ and those with no education get a wage $W_L = P_L$. Finally, we must have the condition that

$$C_H < (P_H - P_L) < C_L.$$

We have in a simplified way described above an equilibrium, which is called a separating equilibrium. In that the two- group of workers separate themselves by getting or not getting education. However, there are other equilibria also. Suppose that $C_L > P_H - P_L$ in which the low-productivity workers do not find it worthwhile to invest in education. Since every worker gets a wage $\bar{P}$ if no one gets educated and there is no separation, if $C_L > P_H - \bar{P}$ then it would not be worthwhile for the high-productivity workers to get educated either. Consequently, there will be equilibrium with no one getting educated. Therefore, multiple equilibria are possible with asymmetric information.

Recall that the problem of adverse selection arises from the fact that the principal has less information than the agent about the risk involved. We have seen two examples of this. Akerlof's model of the market for lemons is discussed to highlight the problems created. When buyers are uncertain about the quality of used cars and offer to pay only a price for average quality, the better quality cars disappear from the market. This is due to the fact that the sellers of these cars do not get a fair price. In the second example, an insurance company is uncertain about the risk quality of the applicants and offers an insurance premium appropriate for average risk. As a result, individuals with lower risk drop out as they consider the premium very high. Thus, the individuals with high risk get insurance.

19.4.1 Hidden Information Modelling (see Tian, G., 2005)

The discussion of this sub-section is based on the lecture notes of Tian, G., (2005) prepared as a part of microeconomic theory. We have taken liberty of adopting the sequence and notation used in the note of the Sub-section 12 and borrowed freely the work. The information gap between the principal and the agent has some fundamental implications for the design of the contract they sign. For an efficient use of economic resources, some information rent must be given up to the private agent holding information. Thus, we look for second-best solutions in the contract. In such a situation the principal trades off his desire to reach allocative efficiency by giving information rent to the agent to induce information revelation.

The main objective of this section is to find the optimal rent extraction efficiency trade-off faced by the principal. We seek to find the mechanism of designing his contractual offer to the agent under the set of incentive feasible constraints: incentive and participation constraints. In general, incentive constraints are binding at the optimum, showing that adverse selection clearly impedes the efficiency of trade.

Consider a consumer or a firm (the principal) who wants to delegate to an agent the production of q units of a good. The value for the principal of these q units is $S(q)$ where $S' > 0$, $S'' < 0$ and $S(0) = 0$.

The production cost of the agent is unobservable to the principal, but it is known that there is a fixed cost F and the marginal cost belongs to the set $\Phi = \{\underline{\theta}, \bar{\theta}\}$. The agent can be either efficient ($\underline{\theta}$) or inefficient ($\bar{\theta}$) with respective probabilities v and $1 - v$. That is, she has the cost function

$$C(q, \underline{\theta}) = \underline{\theta}q + F \quad \text{with probability } v \quad (4.1)$$

or

$$C(q, \bar{\theta}) = \bar{\theta}q + F \quad \text{with probability } 1 - v \quad (4.2)$$

Denoted by $\Delta\theta = \bar{\theta} - \underline{\theta} > 0$ the spread of uncertainty on the agent's marginal cost. This information structure is exogenously given to the players.

Contracting Variables: Outcomes

The contracting variables are the quantity produced q and the transfer t received by the agent. Let A be the set of feasible allocations that is given by

$$A = \{(q, t) : q \in \Re + t \in \Re\} \quad (4.3)$$

These variables are both observable and verifiable by a third party such as a benevolent court of law.

The Complete Information Optimal Contract

First-Best Production Levels

To start with a simpler framework, let us first suppose that there is no asymmetry of information between the principal and the agent. By equating the principal's marginal value and the agent's marginal cost, we get the efficient solution levels. So, first-order conditions are

$$S'(q^*) = \underline{\theta} \quad (4.4)$$

and

$$S'(\bar{q}^*) = \bar{\theta} \quad (4.5)$$

With complete information, efficient production levels q^* and $\bar{q}^*$ should be both carried out if their social values, respectively $\underline{W}^* = S(q^*) - \underline{\theta}q^* - F$ and $\bar{W}^* = S(\bar{q}^*) - \bar{\theta}\bar{q}^* - F$, are non-negative.

Since

$$S(q^*) - \underline{\theta}q^* \geq S(\bar{q}^*) - \underline{\theta}\bar{q}^* \geq S(\bar{q}^*) - \bar{\theta}\bar{q}^*$$

by definition of $\underline{\theta}$ and $\bar{\theta} > \underline{\theta}$, the social value of production when the agent is efficient, $\underline{W}^*$, is greater than when she is inefficient, namely, $\bar{W}^*$.

The trade is carried out as long as production is socially valuable for the least efficient type,

$$\bar{W}^* = S(\bar{q}^*) - \bar{\theta}\bar{q}^* - F \geq 0. \quad (4.6)$$

As we enter into trade the fixed cost F does not have any role other than justifying the existence of a single agent. You can set it to zero from now. In addition, since the principal's marginal value of output is decreasing, the optimal production of an efficient agent is greater than that of an inefficient agent, i.e., $q^* > \bar{q}^*$.

In order that the agent enters into a contract, the principal must offer her a utility level that is at least as high as that the agent obtains outside the relationship.

These are called the agent's participation constraints. If we normalise to zero the agent's outside opportunity utility level (sometimes called her quo utility level), the participation constraints are

$$\underline{t} - \underline{\theta}q \geq 0, \quad (4.7)$$

$$\bar{t} - \bar{\theta}q \geq 0 \quad (4.8)$$

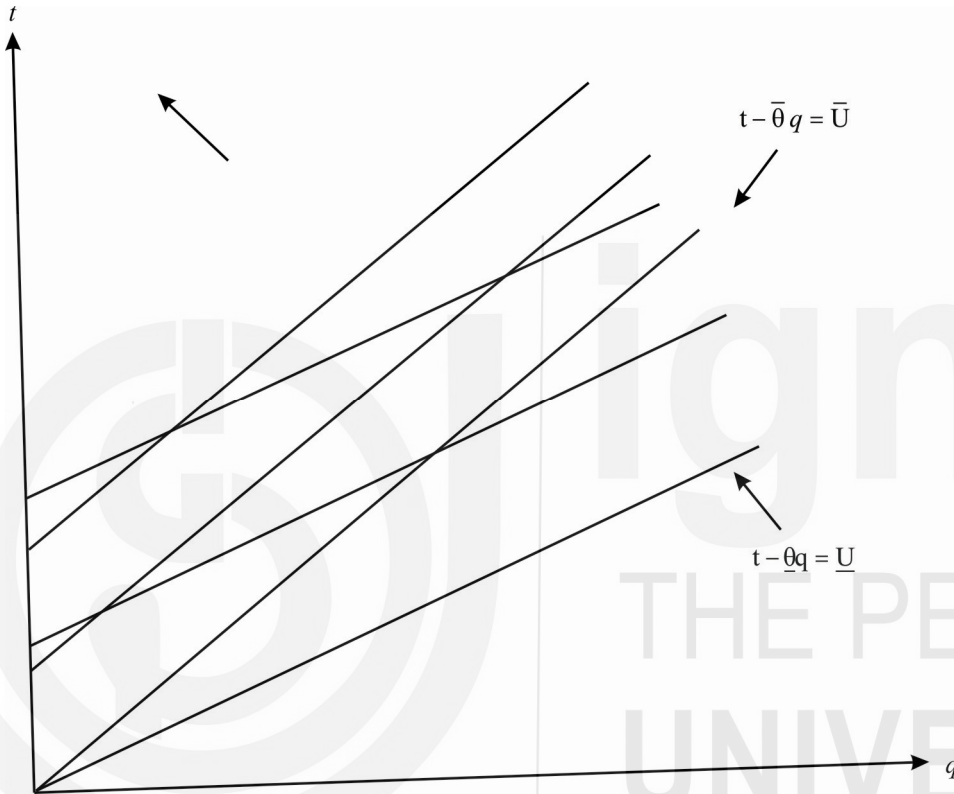


Fig. 19.1: Indifference Curves of Efficient and Inefficient Agent-types

In the assumption of complete information, the principal makes the following take-it-or-leave-it offers to the agent: If $\theta = \bar{\theta}$ (resp. $\underline{\theta}$), the principal offers the transfer $\bar{t}^*$ (resp. $\underline{t}^*$) for the production level $\bar{q}^*$ (resp. $\underline{q}^*$) with $\bar{t}^* = \bar{\theta}\bar{q}^*$ (resp. $\underline{t}^* = \underline{\theta}\underline{q}^*$). The agent accepts the offer and makes zero profit. Therefore, the complete information optimal contracts are $(\underline{t}^*, \underline{q}^*)$ if $\theta = \underline{\theta}$ and $(\bar{t}^*, \bar{q}^*)$ if $\theta = \bar{\theta}$.

You can represent complete information optimal contract with the help of a graph. See Figure 19.2 for that purpose.

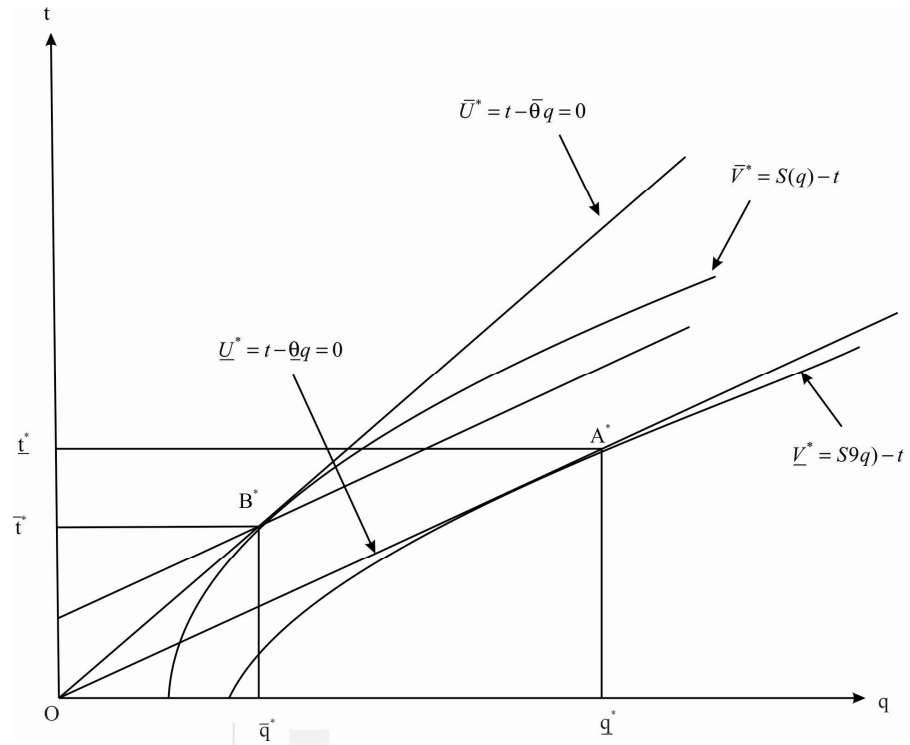


Fig. 19.2: First Best Contracts

It can be seen from Figure 19.2 that the indifference curves for efficient and inefficient agents cross only once as shown in the above figure. This property is called the single-crossing or Spence-Mirrlees property.

The complete information optimal contract (A^* , B^*) is given in Figure 19.2. It can be seen that since the iso-utility curves of the principal correspond to increasing levels of utility when one moves in the southeast direction, the principal reaches a higher profit when dealing with the agent of efficient type. We denote by $\bar{V}^*$ (resp. $\underline{V}^*$) the principal's level of utility when he faces the $\bar{\theta}$ –(resp. $\underline{\theta}$ –) type. When the principal has the complete information, he will design the contract, such that $\bar{V}^* = \bar{W}^*$ (resp. $\underline{V}^* = \underline{W}^*$).

Incentive Feasible Contracts

Incentive Compatibility and Participation

The problem with the above contract scheme is that it does not satisfy the incentive compatibility. Let us assume that the marginal cost θ is the agent's private information, and the principal offers the menu of contracts $\{(\underline{t}^*, \underline{q}^*); (\bar{t}^*, \bar{q}^*)\}$, agents with type $\underline{\theta}, \bar{\theta}$. But you can see from Figure 19.2 above that B^* is preferred to A^* by both types of agents. Hence offering the menu (A^* , B^*) fails to have the agents self-selecting properly within this menu. The efficient type agent will have an incentive to mimic the inefficient one and will select the contract B^* . The result is complete information optimal contracts can no longer be implemented under asymmetric information.

In order to be incentive compatible, i.e., for agent $\underline{\theta}$, $(\underline{t}, \underline{q})$ is weakly preferred to $(\bar{t}, \bar{q})$ and for agent $\bar{\theta}$, $(\bar{t}, \bar{q})$ is weakly preferred to $(\underline{t}, \underline{q})$, we must have

$$\underline{t} - \underline{\theta}\underline{q} \geq \bar{t} - \underline{\theta}\bar{q} \quad (4.9)$$

and

$$\bar{t} - \bar{\theta}\bar{q} \geq \underline{t} - \bar{\theta}\underline{q} \quad (4.10)$$

Furthermore, for a menu to be accepted, it must satisfy the following two participation constraints:

$$\underline{t} - \underline{\theta}\underline{q} \geq 0 \quad (4.11)$$

$$\bar{t} - \bar{\theta}\bar{q} \geq 0 \quad (4.12)$$

Thus, a menu of contracts is incentive feasible if it satisfies both incentive and participation constraints (4.9) through (4.12).

The inequalities (4.9) through (4.12) present the set of incentive feasible menus of contracts. In the presence of asymmetric information between the principal and the agent, these additional constraints have to be imposed on the resource allocation.

Special Cases

Bunching or Pooling Contracts: Depicts a situation in which $\underline{t} = \bar{t} = t^p$ and $\underline{q} = \bar{q} = q^p$. That is, the contract designed for each type of agent coincides and both types of agent accept the contract.

Shutdown of the Least Efficient Type: Depicts a case, when one of the contracts is the null contract (0,0) and only the non-zero contract (t_s ; q_s) is accepted by the efficient type. Then, (4.9) and (4.11) reduce to

$$t^s - \underline{\theta}q^s \geq 0. \quad (4.13)$$

The incentive constraint of the low efficient type reduces to

$$0 \geq t^s - \bar{\theta}q^s. \quad (4.14)$$

See that in both cases, the number of constraints have been reduced and the incentive and participation constraints have taken the same form.

By adding (4.9) and (4.10), we get

$$\underline{q} \geq \bar{q} \quad (4.15)$$

We will call condition (4.15) an implementability condition.

When (4.15) holds, we can have transfers $\bar{t}$ and $\underline{t}$ such that the incentive constraints (4.9) and (4.10) both hold. It will be sufficient to take those transfers such that

$$\bar{\theta}(\bar{q} - \underline{q}) \leq \bar{t} - \underline{t}(\bar{q} - \underline{q}). \quad (4.16)$$

Information Rents

Under complete information, the principal is able to maintain all types of agents at their zero status quo utility level. At the first-best, their respective utility levels $\underline{U}^*$ and $\bar{U}^*$ satisfy

$$\underline{U}^* = \underline{t}^* - \underline{\theta}\underline{q}^* = 0 \quad (4.17)$$

and

$$\bar{U}^* = \bar{t}^* - \bar{\theta}\bar{q}^* = 0 \quad (4.18)$$

Generally this will not be possible anymore under incomplete information, at least when the principal wants both types of agents to be active.

Take any menu $\{(\bar{t}, \bar{q}); (\underline{t}, \underline{q})\}$ of incentive feasible contracts and consider the utility level that a $\underline{\theta}$ -agent would get by mimicking a $\bar{\theta}$ -agent. The high-efficient agent would get

$$\bar{t} - \underline{\theta}\bar{q} = \bar{t} - \bar{\theta}\bar{q} + \Delta\theta\bar{q} = \bar{U} + \Delta\theta\bar{q} \quad (4.19)$$

Thus, as long as the principal insists on a positive output for the inefficient type, $\bar{q} > 0$, the principal must give up a positive rent to a $\underline{\theta}$ -agent. This information rent is generated by the informational advantage of the agent over the principal.

We use the notations $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$ and $\bar{U} = \bar{t} - \bar{\theta}\bar{q}$ to denote the respective information rent of each type.

The Optimisation Program of the Principal

The principal offers a menu of contracts without knowing type of agent. Then, the principal's optimisation problem is

$$\begin{aligned} \max_{\{(\bar{t}, \bar{q}); (\underline{t}, \underline{q})\}} & v(S(\underline{q}) - \underline{t}) + (1-v)(S(\bar{q}) - \bar{t}) \\ \text{subject to} & (4.9) \text{ to } (4.12). \end{aligned}$$

We will assess the distributive impact of asymmetric information through the information rent. Using the information rents $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$ and $\bar{U} = \bar{t} - \bar{\theta}\bar{q}$, we can replace transfers in the principal's objective function as functions of information rents and outputs so that the new optimisation variables are expressed as $\{(\underline{U}, \underline{q}), (\bar{U}, \bar{q})\}$.

With this change of variables, the principal's objective function can then be rewritten as

$$\underbrace{v(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-v)(S(\bar{q}) - \bar{\theta}\bar{q})}_{\text{expected allocative efficiency}} - \underbrace{(v\underline{U} + (1-v)\bar{U})}_{\text{expected information rent}} \quad (4.20)$$

In this formulation, the first term denotes expected allocative efficiency, and the second term denotes expected information rent. It implies that the principal accepts some distortions away from efficiency in order to decrease the agent's information rent. So we are looking for second best solutions.

The incentive constraints (4.9) and (4.10), when seen in terms of information rents and outputs, are

$$\underline{U} \geq \bar{U} + \Delta\theta\bar{q}, \quad (4.21)$$

$$\bar{U} \geq \underline{U} + \Delta\theta\underline{q}. \quad (4.19)$$

Similarly, the participation constraints (4.11) and (4.12) are changed to

$$\underline{U} \geq 0 \quad (4.23)$$

$$\bar{U} \geq 0. \quad (4.24)$$

Thus, the principal's optimisation problem is

$$\begin{aligned} \max_{\{(\bar{t}, \bar{q}); (\underline{t}, \underline{q})\}} & v(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-v)(S(\bar{q}) - \bar{\theta}\bar{q}) - (v\underline{U} + (1-v)\bar{U}) \\ \text{subject to} & (4.21) \text{ to } (4.24). \end{aligned}$$

Let us use superscript SB in these solutions to represent second-best options.

The Optimal Contract Under Asymmetric Information

In the optimum, the incentive compatibility and participation constraints must bind. To solve the problem, let us first consider contracts without shutdown, such that $\bar{q} > 0$ and try to eliminate the irrelevant conditions.

Note that the $\underline{\theta}$ -agent's participation constraint (4.23) is always strictly-satisfied. Indeed, (4.24) and (4.21) immediately imply (4.23). (4.19) also seems irrelevant because the difficulty comes from a $\underline{\theta}$ -agent willing to claim that she is inefficient rather than efficient. When these are taken into account, we are left with only two remaining constraints, viz., the $\underline{\theta}$ -agent's incentive compatible constraint (4.21) and the $\bar{\theta}$ -agent's participation constraint (4.24). Both constraints must be binding at the optimum of the principal's problem. Thus,

$$\underline{U} = \Delta\theta\bar{q} \quad (4.25)$$

$$\text{and} \quad \bar{U} = 0 \quad (4.26)$$

Substituting (4.25) and (4.26) into the principal's objective function, we obtain a reduced form of the problem (P') with outputs as the only choice variables:

$$\max_{\{\underline{q}, \bar{q}\}} v \left(S(\underline{q}) - \underline{\theta}\underline{q} \right) + (1-v) \left(S(\bar{q}) - \bar{\theta}\bar{q} \right) - (v\Delta\theta\bar{q}).$$

If we recall the solution obtained in case of full information setting, it can be seen that the asymmetric information has changed the principal's optimisation simply by the subtraction of the expected rent that has to be given up to the efficient type. The inefficient type gets no rent, but the efficient type $\underline{\theta}$ gets information rent that she could obtain by mimicking the inefficient type $\bar{\theta}$. Such a rent depends only on the level of production requested from this inefficient type.

The first order conditions are then given by

$$S'(\underline{q}^{SB}) = \underline{\theta} \quad \text{or} \quad \underline{q}^{SB} = \underline{q}^*. \quad (4.27)$$

$$\text{and} \quad (1-v) \left(S'(\bar{q}^{SB}) - \bar{\theta} \right) = v\Delta\theta. \quad (4.28)$$

The second of the above equations, i.e., (4.28) expresses the important trade-off between efficiency and rent extraction which arises under asymmetric information.

To validate our approach based on the sole consideration of the efficient type's incentive compatible constraint, it is necessary to check that the omitted incentive compatible constraint of an inefficient agent is satisfied. i.e., $0 \geq \Delta\theta\bar{q}^{SB} - \Delta\theta\underline{q}^{SB}$. This latter inequality follows from the second-best schedule of outputs since we have $\underline{q}^{SB} = \underline{q}^* > \bar{q}^* > \bar{q}^{SB}$ with a downward output distortion for the inefficient type, $\bar{q}^{SB} < \bar{q}^*$ with

$$S'(\bar{q}^{SB}) = \bar{\theta} + \frac{v}{1-v} \Delta\theta \quad (4.29)$$

We have said above that the complete information optimal contract (A^* ; B^*) was not incentive compatible. Let us now construct a diagram (see Figure 19.3) with an incentive compatible contract (B^* ; C).

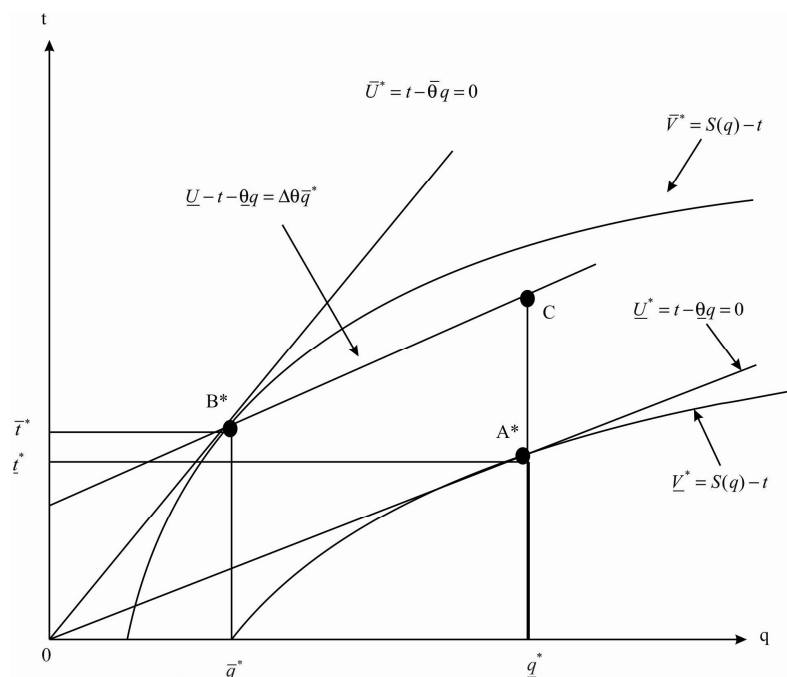


Fig. 19.3: Rent Extraction to Implement the First Best Outputs

See that it has the same production levels by giving a higher transfer to the agent producing q^* as shown in Figure 19.3.

It can be seen that the contract C is on the $\underline{\theta}$ -agent's indifference curve passing through B*. Hence, the $\underline{\theta}$ -agent is now indifferent between B* and C. (B* ; C) becomes an incentive compatible menu of contracts. The rent that is given up to the $\underline{\theta}$ -firm is now $\Delta\theta\bar{q}^*$. This contract is not optimal by the first order conditions (4.27) and (4.28). The optimal trade-off finally occurs at $(A^{SB}; B^{SB})$ as shown in Figure 19.4 below.

Shutdown Policy

If the first-order condition in (4.29) has no positive solution, $\bar{q}^{SB}$ should be set at zero. We are in the special case of a contract with shutdown. B^{SB} coincides with 0 and A^{SB}

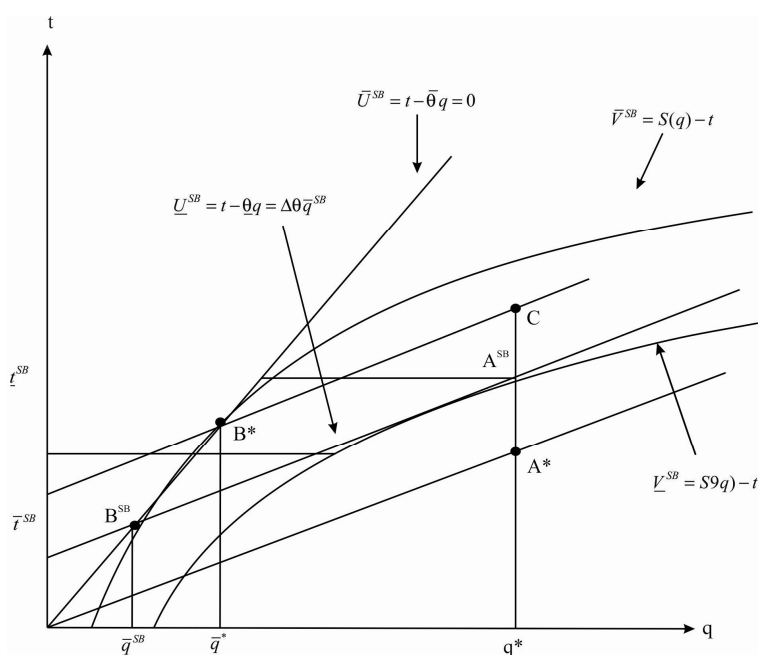


Fig. 19.4: Optimal Second-best Contract SSB and BSB.

with A^* in Figure 19.4. No rent is given up to the $\underline{\theta}$ -firm by the unique non-null contract $(\underline{t}^*, \underline{q}^*)$ offered and selected only by agent $\underline{\theta}$. The benefit of such a policy is that no rent is given up to the efficient type.

The shutdown policy is dependent on the status quo utility levels. Suppose that, for both types, the status quo utility level is $U_0 > 0$. Then, from the principal's objective function, we have

$$\frac{v}{1-v} \Delta \theta \bar{q}^{SB} + U_0 \geq S(\bar{q}^{SB}) - \bar{\theta} \bar{q}^{SB}. \quad (4.30)$$

Thus, for v large enough, shutdown occurs.

Check Your Progress 2

- 1) Explain why bonus and profit-sharing payment schemes are likely to resolve principal-agent problems, whereas a fixed wage payment will not.

.....

- 2) What is the importance of incentive compatibility and participation constraints in finding solution in a market with asymmetric information?

.....

- 3) In the context of asymmetric information, how will you find second best solutions?

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19.5 EFFICIENCY WAGE MODEL

Efficiency wage models are developed with the input of imperfect information where the participation constraint of employees are slack because of informational problems or others like limited liability constraints. The basic issue raised in these concerns with, why there is unemployment. A dominant view is unemployment arises because wages need to be above the market clearing level in order to give incentives to workers. In fact, it is the combination of unemployment and high wages that make more attractive for workers.

Such a configuration materialise because,

- i) unlike other forms of capital, human being can choose their levels of effort; and
- ii) it is costly for firms to determine how much effort workers are exerting.

Consider a situation where all workers come receive the market wage and there is no unemployment. In such an eventuality, the worse thing that can happen to a worker is that she will be fired and instantaneously rehired. There is therefore no penalty for exerting effort ('shirking'). To induce workers not to shirk, firm pays above-market wages. So job loss impose penalty. The problem is, if one firm pays above-market wages, then other will follow the suit. When such a situation prevails, the incentive not to shirk disappears. However, the unemployment results since wages are above the natural equilibrium level and unemployment creates its own penalty for shirking.

Let us see the how these models present the problems. Suppose a firm pays markets clearing wage w^* and the workers shirk. When they are caught for shirking and are fired (dismissed), they get hired elsewhere for the same wage w^* . Thus, the threat of being fired is not a cost to a worker and she has no incentive to be productive. To induce the worker for not shirking, a firm must offer higher wage. In such an arrangement, once a worker is fired and gets hired by another firm, she receives only the market clearing wage w^* . Thus, when the difference between shirking and non-shirking wages is substantial, it induces the workers to be productive. The wage at which no shirking occurs is the efficiency wage (w_e).

If shirking prevails in all the firms, every one of these is paying wages w_e , that is greater than w^* . Does such a feature therefore indicate that worker will shirk even with higher wages? You may think yes, because they will be hired at a higher wage by other firms if they get fired. However, you may be wrong. Why? See that all firms are offering wages higher than w^* . As wages are higher, the demand for labour is lower than that of the market clearing quantity. So, when a worker is fired even with an incentive-based wage, she may remain unemployed on some time interval.

Shirking in the labour market is shown in Figure 19.5. It can be seen that with no shirking, the demand for labour (D_L) intersects the supply of labour (S_L) to give market clearing wage W^* and full employment of labour L^* . With shirking, however, individual firms are willing to pay more than W^* , which we may call no shirking wages (NSC). The NSC curve shows the minimum wage to be given to worker in order not to shirk with NSC. The equilibrium wage will be determined at w_e and labour employment will be L_e . As the NSC curve depicts, the lowest wage that firms can pay to avoid shirking, will not exceed the labour supply curve. So there will be always some unemployment in equilibrium.

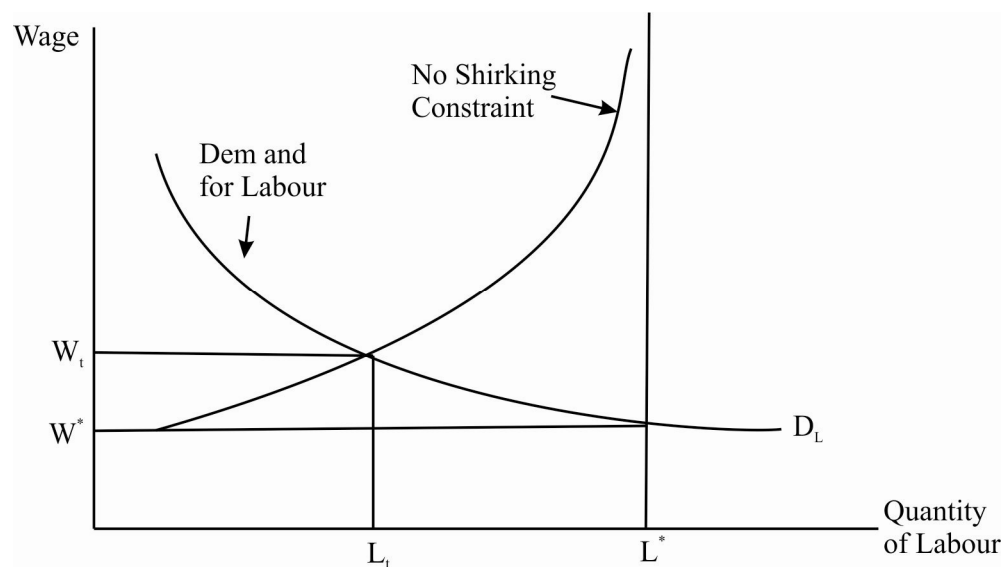


Fig. 19.5: Equilibrium in Shirking Model

Important features of the Shapiro-Stiglitz efficiency wage model is presented in the following:

Basic Setup

Let us assume that there are N identical, risk neutral workers. They face an instantaneous utility is a function of wages and effort 'normalised' as:

$$u(w, e) = w - e.$$

In the above equation e is either 0 or a constant $e > 0$ i.e., it is a binary variable and not continuous.

The unemployed workers receive wage $\bar{w} \geq 0$ and the effort level $e = 0$.

The worker leaves the job with separation rate per unit time b that is exogenous. Intertemporal interest/discount rate is taken as r .

The workers objective function is

$$\max_e E \int_0^{\infty} u(w(t), e(t)) \exp(-rt) dt$$

We have the workers choice at any point in time in which she decides to exert effort or not. The odds of a worker getting caught if shirking are q . If fired, the worker enters unemployment, waits for a new job. The job finding rate, which will be determined in the equilibrium is a

Asset Equations

We consider that the value of employment is the flow value of the wage minus the capital loss of unemployment scale of its probability.

To appreciate the valuation problem, remember that a typical worker may transfer from the current state of unemployment to possible future state of employment. The rate at which this transfer occurs may depend on the individual's actions, in two states:

One state being 'employed and working hard' and another being 'employed and taking it easy'. The model includes the following asset values:

V_u = Present discounted life time value of currently being unemployed

V^n = Value of currently being employed and working hard

V^s = Value of currently being employed and shirking.

When working hard, a worker puts up effort e and has productivity 1. When shirking, there is no effort and no productivity from this worker.

To find the asset value of employment, we take a short discrete time interval: $[0, t]$. Then $V_e = wt + (1 - rt) [btV_u + (1 - bt)V_e]$ (5.1)

Notice we are for now taking V_u as given. The wt term gives the flow of wages, and the second expression is the discounted value of future outcome, bearing in mind that $(1 - rt) \approx \exp(-rt)$.

Rewriting (5.1) above we get

$$\begin{aligned} V_e &= \frac{wt + (1 - rt)btV_u}{1 - (1 - rt)(1 - bt)} \\ &= \frac{wt + btV_u - rbt^2V_u}{rt + bt - rbt^2} \end{aligned}$$

Let us move to continuous time so that it allows $t \rightarrow 0$ and we write

$$V_e = \frac{w + bV_u}{r + b} - rV_e - w - b(V_u - V_e).$$

In the above equation, notice that value of employment is the flow value of the wage minus the capital loss of unemployment scaled by its probability.

We now want to consider this problem separately for a shirker versus non-shirker:

$$rV_e^s = w + (b + q)(V_u - V_e^s), \dots \dots \dots (5.1')$$

$$rV_e^n = w - e + b(V_u - V_e^n) \dots \dots \dots (5.1'')$$

In (5.1') above, rV_e^s is the return to the asset v^s assuming a common interest rate r . The first term on the right-hand side gives the current return to wage w for a shirking worker, who does not put up any effort. The second term is the expected value of the change in the asset, $(b + q)(V_u - V_e^s)$, where $(b + q)$ is the total rate of change that occurs for a shirker (remembering the fact that a shirker also suffers a probability q of being caught and then fired).

When a worker loses her job, $V_u - V_e^s$ yields the asset change. In general, this change is negative as utility loss is associated with losing ones job.

In case (5.1''), the first part of right side, i.e., $w - e$ indicates the lower current return. Why? Because the worker puts up effort e while working hard, lowering thereby the current utility. In the second term, i.e., $b(V_u - V_e^n)$, the capital loss is suffered with smaller probability as q is missing. Intuitively, this equation shows that if the worker is checked now, she keeps her job and continues receiving the return $w - e$.

Rearranging (5.1') and (5.1'') above we get

$$V_e^s = \frac{w + (b + q)V_u}{r + b + q} \text{ and } V_e^n = \frac{(w - e) + bV_u}{r + b}$$

No Shirking Condition (NSC)

The NSC condition is on the wage, which makes it attractive for a worker to work hard instead of shirking. This is the essential part of the solution to the model. Thus, it requires the workers to prefer to exert effort. So the NSC is simply $V_e^n > V_e^s$.

Note that whenever this condition is binding, the solution chosen by the firm will imply that $V^n = V^s$. This must be the case at equilibrium, since in case of a reverse condition, $V^n > V^s$, the non-shirking constraint has no influence on firm's wage decision. It will be shown below that this is contradictory because firm would always try to set the wage as low as it could. Let us now find a constraint on w such that

$$\hat{w} \equiv w \geq rV_u + \frac{e}{q}(r + b + q) \dots \dots \dots (5.3)$$

We get the wage level what is called the non-shirking wage $w(ns)$ by converting the above inequality to equality, i.e.,

$$w = rV_u + \frac{e}{q}(r + b + q) = w(ns).$$

The inequality (5.3) implies $q(V_e^s - V_u) \geq e$.

Note that $q \rightarrow 0$ implies no penalty. From the above equation therefore we get that if there were not penalty associated with unemployment, everyone will shirk.

Demand for Labour from Firm Side

We turn to demand for labour. Let us suppose that there are M identical firms, each with a production function $Q_i = f(L_i)$ with f increasing and strictly concave, i.e., $F'' < 0$. The aggregate production is $Q = F(L)$.

Assume each worker contributes one unit of labor unless she shirks. Firms compete by offering wage packages only. Assume that full employment is efficient, i.e., $F'(N) > e$. The monitoring technology q is exogenous. Therefore, the only way to punish shirkers is to fire them.

Equilibrium of the Model

In equilibrium, each firm takes wages and employment levels at other firms as given, and finds it optimal to offer the going wage. So, firms will not pay more than the NSC wage: $w^* = \hat{w}$ and we'll have $f'(L_i) = F'(L) = \hat{w}$. That is to say, when wages are higher, workers will value jobs better. There could be two reasons for this result, viz., (i) high wages increase utility and (ii) low employment due to high wages leads to long unemployment spells. Firms will continue to offer lower wages until NSC binds.

An immediate result of low wage is workers shirking. In such circumstances workers only moderately prefer work to unemployment. Moreover, they also know that high employment leads to shorter unemployment spells. Firms realise this and raise wages until NSC satisfied.

Endogenising Unemployment

We need to find an expression for V_u . For that you may proceed the similar way as in case of (1') and (1''). From the worker's problem, consider V_u , such that

$$rV_u = \bar{w} + a(V_e - V_u), \quad (5.4)$$

where a is a job acquisition rate i.e., the rate at which the capital gain $V_e^n - V_u$ is realised, and $\bar{w}$ is the current return from being unemployed, i.e., unemployment benefit.

See that V_e must be equal to V_e^n in equilibrium. For, it can never be efficient for firms to hire workers who shirk. If they shirk ever, they shirk always, so they are unproductive.

We can now solve equations (5.2) and (5.4) together we get,

$$rV_e = \frac{(w - e)(a + r) + \bar{w}b}{a + b + r}$$

and

$$rV_u = \frac{(w - e)a + \bar{w}(b + r)}{a + b + r}$$

In the above equation we do not have q as no one is shirking in equilibrium. By plugging back rV_u into the NSC (5.3) above, we get

$$\hat{w} \geq \bar{w} + e + \frac{e}{q}(a + b + r). \quad (5.5)$$

On the basis of this inequality, we get two additional implications viz.,

- i) higher the unemployment benefit, higher is the critical wage.
- ii) higher the flow out of unemployment (a), higher is the critical wage.

We are now in a position to solve the parameters of the model.

In the long-run equilibrium, we have the following steady state relationship between total labour force N , employment L and unemployment $U = N - L$. Thus, it must be the case that flows out of unemployment equal flows in.

Hence,

$$a(N - L) = bL \quad a = \frac{bL}{N - L}.$$

Since $N - L = U$ (unemployment), we have $aU = a(N - L)$. This gives the number of workers who find jobs at any given time. On the other hand, bL gives the number of workers who lose their jobs at any given time (with no shirking).

Substituting back into (5.5), we get

$$\hat{w} \geq \bar{w} + e + \frac{e}{q} \frac{bN}{N - L} + r.$$

Define now $u = \frac{N - L}{N} = \frac{U}{N}$ as the rate of unemployment, and $1 - u = \frac{L}{N}$ as the rate of employment. Thus, from above, $a(N - L) = bL$

$$\text{or, } a = \frac{bL}{N - L} = b \frac{L}{N} \frac{N}{(N - L)} = \frac{b(1 - u)}{u} = \frac{b}{u} - b. \text{ So,}$$

$$a + b = \frac{bL + b(N - L)}{N - L} = \frac{bN}{N - L}.$$

Using $a(N - L) = bL$ and $w = \bar{w} + e + \frac{e}{q}(a + b + r)$, we rewrite to get

$$\begin{aligned} \hat{w} &\geq \bar{w} + e + \frac{e}{q} \frac{b}{u} + r \\ &= w(ns) \end{aligned} \quad (5.6)$$

Equation (5.6) shows that the non-shirking wage requires some unemployment at equilibrium. Thus, (5.6) becomes a quasi-labor supply function. As the employment rate approaches 0, the critical wage tends toward infinity. In equilibrium $F'(L) = \hat{w} = w^*$, i.e., labor demand intersects quasi-labor supply.

Check Your Progress 3

- 1) What is an efficiency wage? Explain why it is profitable for the firm to pay an efficiency wage when workers have better information about their productivity than firms do.

- 2) Discuss the importance of no shirking condition (NSC) in Efficiency Wage Model.

19.6 LET US SUM UP

This unit presents the effect of uncertainty that access to information emanates from asymmetric information, i.e., when parties entering a trade have unequal access to information in the market. We have seen that presence of asymmetric information creates inefficiency precluding the attainment of Pareto optimum solution in a number of markets. The outcome of insurance market is illustrated where insurers may have less information about the potential risk than that of insurance purchasers. A net result of such type of situation is individuals holding private information manipulating it to their advantage. To analyse the market outcomes, we have introduced the principal-agent framework in which the principal wants to delegate a task to the agent, which has costs to the agent. Due to such a feature the agent chooses to act in a manner maximising her utility in the process affecting the welfare of the agent. Anticipating such a course of action, the principal makes provision in the contract, which may include incentive for the agent to put in best efforts. The problem that has been addressed is related to solutions when information is hidden or action is hidden by the agent prior to the contract. In case of hidden action the principal cannot perfectly observe the agent's move and is required to design incentive schemes. In case of hidden information, the principal knows the type of agent and her utility function. However, there may be many different types of agents with different utility functions. Incentive designs have to be worked out differently as the principal cannot observe the objective function of the agent perfectly. Such problems are examined through discussion in moral hazard and adverse selection. In case of moral hazard, the principal tries to influence the action of the agent by considering agent's utility function on the only observable variable of outcome. We have analysed the problem by considering symmetric option that helps derive the first best option and bringing in its contrast with asymmetric information.

The problem of adverse selection arises from the fact that the principal has less information than the agent about the risk involved. We have seen two examples of this. Akerlof's model of the market for lemons is discussed. When buyers are uncertain about the quality of used cars and offer to pay only a price for average quality, the better quality cars disappear from the market. This happens as the seller of these cars do not get a fair price. In the second example, an insurance company is uncertain about the risk quality of the applicants and offers an insurance premium appropriate for average risk. As a result, individuals with lower risk drop out. They consider the premium to be very high. Thus, only the individuals with high risk get insured. As a solution

to these problems we have discussed, how individuals can convey information about their product quality and risk involved through signaling and screening.

The models on hidden information is discussed keeping in view the efficient use of economic information. It is seen that the principal has to pass on additional incentive to agent as a rent for holding information that induces her for revelation. In the process, we seek to find mechanism of designing contract that offers the agent incentives subject to incentive and participation constraints. The outcomes of such solutions are pointed out to be impeding efficiency of trade.

In the last part of the unit, we have seen another application to asymmetric information that explains the prevalence of unemployment. Efficient wage model by Shapiro-Stiglitz is discussed. We have seen that with imperfect information where participation constraints of employees are slack because of limited liability constraints efficiency wage model can shed interesting insight.

19.7 KEY WORDS

Adverse Selection: Transaction in a market characterised by asymmetric information going in favour of better informed party.

Hidden Action Model: Model dealing with a situation where symmetric information exists prior to contracting but asymmetry arises at post-contracting stage.

Hidden Information Model: Model dealing with a situation where asymmetric information exists prior to contracting.

Incentive Compatibility: An incentive scheme that influences the action of the agent to pick up the best as desired by the principal.

Individual Rationality: An incentive scheme in which individual agent is given an income level equal to her reservation level utility which can be achieved from alternative opportunities.

Intertemporal Decision: Current decisions affecting also the choices available in the future.

Iso-utility Curve: Another name of indifference curve. In a graphical representation of a utility function, an indifference curve plots all combinations of goods and services, which provide the same utility.

Participation Constraint: See individual rationality.

Pooling Equilibrium: A situation in a market with asymmetric information where different types of agents are induced to choose same effort level.

Screening Consumer: Initiative taken by individuals of uniform category in markets characterised by adverse selection to create a situation in which information will be revealed.

Second Best Outcome: The best allocation of resources which can be obtained when various constraints preclude attaining true economic efficiency.

Separating Equilibrium: A situation in a market with asymmetric information, where different types of agents are induced to choose different effort levels.

Signaling: Initiative taken by individuals of informed category in markets characterised by adverse selection to help identify their true risk category.

Single Crossing Properly: The fact of indifference curve on individual of higher ability crossing that of lower ability only once.

19.8 SOME USEFUL BOOKS

Meunier, Valerie (2005). Lecture notes from *Chapter 6: Moral Hazard, Strategic Behavior, Information and Contractual Relationships*, available in Internet site, www.econ.au.dk/fag/7010/F2005/default.htm

Pindyck, Robert S. And Daniel, L. Rubinfeld (1992&1995), *Microeconomics*, Prentice Hall of India, New Delhi.

Sen, Anindy (1999), *Microeconomics: Theory and Application*, Oxford University Press, New Delhi.

Tian, Guoqiang (2005), Lecture Notes: Microeconomic Theory, available on Internet address econweb.tamu.edu/tian/micro

Varian, Ha R. (1996), *Intermediate Microeconomics*, (6th Edition), W. W. Norton & Company, New York.

Varian, Hal R. (1992), *Microeconomic Analysis*, (3rd Edition), W. W. Norton & Company, Inc., New York.

19.9 ANSWER OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

- 1) Read Section 19.2 and answer
- 2) Read Sections 19.2 and answer
- 3) Read Sections 19.2 and answer

Check Your Progress 2

- 1) Read Sections 19.3 to 19.4 and answer
- 2) Read Section 19.3 and answer
- 3) Read Sections 19.3 and 19.4 and answer

Check Your Progress 3

- 1) Read Section 19.5 and answer
- 2) See equilibrium condition in 19.5 and answer

19.10 EXERCISES

- 1) (Due to Varian) Consider the hidden action principal-agent problem described with $f = u^{-1}$. Assume that $u(s)$ is increasing and concave; show that f is an increasing, convex function.
- 2) (Due to Varian) Let $V(c_a, c_b)$ be the utility received by the principal using the optimal incentive scheme when the costs of actions a and b are c_a and c_b , respectively. Derive an expression for $\partial V / \partial c_a$ and $\partial V / \partial c_b$ in terms of the parameters appearing in the fundamental condition, and use these expressions to interpret those parameters.
- 3) (Due to Varian) Prove the following implication of the single-crossing property: If $c'_2(x) > c'_1(x)$ for all x , then for any two distinct levels of

output x_1 and x_2 , for which $x_2 > x_1$, we must have $c_2(x_2) - c_2(x_1) > c_1(x_2) - c_1(x_1)$.

- 4) (Due to Nicholson, (1992) The used car supply in Metropolis consists of 10,000 cars. The value of these cars ranges from Rs.5,000 to Rs.15,000 with exactly one car being worth each dollar amount between these two figures. Used car owners are always willing to sell their cars for what they are worth. Demanders of used cars in Metropolis have no way to telling the value of a particular car. Their demand depends on the average value of cars in the market ($\bar{P}$) and on the price of the cars themselves (P) according to the equation

$$Q = 1.5\bar{P} - P.$$

- a) If demanders base their estimate of $\bar{P}$ on the entire used car market, what will its value be and what will be the equilibrium price of used cars?
 - b) In the equilibrium described in part (a), what will be the average value of used cars actually traded in the market?
 - c) If demanders revise their estimate of $\bar{P}$ on the basis of the average value of car actually traded, what will be the new equilibrium price of used cars? What is the average value of cars traded now?
 - d) Is there a market equilibrium in this situation at which the actual value of $\bar{P}$ is consistent with supply-demand equilibrium at a positive price and quantity?
- 5) (Due to Nicholson, (1995) Two used car dealerships compete side by side on a main road. The first, Harry's Cars, sells high-quality cars that it carefully inspects and, if necessary, services. It costs Harry's, on average, \$8,000 to buy and service each car that it sells. The second dealership, Lew's Motors, sells lower-quality cars. It costs Lew's on average only \$5,000 for each car that it sells. If consumers knew the quality of the used cars they were buying, they would gladly pay \$10,000 on average for cars Harry's sells and pay only \$7,000 on average for the cars Lew's sells.

Unfortunately, the dealerships are too new to have established reputations so consumers don't know the quality of each dealership's cars. Consumers shopping at these dealerships figure that they have a 50-50 chance of ending up with a high-quality car, no matter which dealership they go to, and hence are willing to pay \$8,500 on average for a car.

Harry's has an idea – it will offer a bumper-to-bumper warranty for all the cars it sells. It knows that a warranty lasting Y years will cost \$500 Y on average, and it also knows that if Lew's tries to offer the same warranty, it will cost Lew's \$2000 Y on average.

- a) Suppose Harry's offers a one-year warranty on all the cars it sells. Will this generate a credible signal of quality? Will Lew's match the offer, or will it fail to match it so that consumers can correctly assume that because of the warranty, Harry's cars are high quality and hence worth \$10,000 on average?
- b) What if Harry's offers a two-year warranty on its cars? Will this generate a credible signal of quality? What about a three-year warranty?

- c) If you were advising Harry's, how long a warranty would you urge it to offer? Explain why.
- 6) (Due to Nicholson, (1995) A firm's short-run revenue is given by $R = 10e - e^2$, where e is the level of effort by a typical worker (all workers are assumed to be identical). A worker chooses his level of effort to maximise his wage net of effort $w - e$ (the per-unit cost of effort is assumed to be 1). Determine the level of effort and the level of profit (revenue less wage paid) for each of the following wage arrangements. Explain why these different principal-agent relationships generate different outcomes.
- a) $w = 2$ for $e \geq 1$; otherwise $w = 0$.
 - b) $w = R/2$.
 - c) $w = R - 125$.



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