Indira Gandhi National Open University
School of Sciences

MMTE-003
PATTERN RECOGNITION AND IMAGE PROCESSING

Block
3
IMAGE IMPROVEMENT II

Curriculum Design Committee

Dr. B.D. Acharya
Dept. of Science & Technology, New Delhi

Prof. Adimurthi
School of Mathematics
TIFR, Bangalore

Prof. Archana Aggarwal
CESP, School of Social Sciences
JNU, New Delhi

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Dr. Atul Razdan
Prof. Parvin Sinclair
Prof. Sujatha Varma
Dr. S. Venkataraman

Course Design Committee

Dr. A.K Sinha
DRDO, Delhi

Prof. Vipula Singh
RNSIT, Bangalore

Prof. Muthukumar Subramanyaom
NIT, Puducherry

Faculty Members
School of Sciences, IGNOU
Prof. Sujatha Varma
Dr. S. Venkataraman
Dr. Deepika

Block Preparation Team

Prof. K.K. Biswas (Retd.) (Editor)
IIT, Delhi

Prof. Vipula Singh
RNSIT, Bangalore

Dr. Deepika (Tranformation of all units)
School of Sciences, IGNOU

Course Coordinator: Dr. Deepika

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BLOCK 3

In Block 2 of this course, we have explained spatial domain operations on images. Various point operations such as Contrast stretching, Clipping and thresholding, Digital Negative, Intensity levels slicing, Bit extraction for image enhancement have been explained. Image histogram and its importance is demonstrated. Image enhancement techniques based on Histogram modeling such as Histogram equalization and Histogram specification were explained. In the last unit of the block, neighbourhood operations for image enhancement were explained. Various neighbourhood operations discussed were spatial averaging, low pass and high pass filtering, Median, Min, Max filtering etc. The working operation, their applications and advantages are discussed at length.

In this block, image enhancement in frequency domain is discussed. Further we also discuss image restoration techniques.

In Unit 8, image enhancement in frequency domain is discussed. The process of frequency domain filtering is explained with steps. Low pass and high pass filters are explained with their transfer functions, applications and advantages.

In Unit 9, difference between image restoration and enhancement is elaborated. Image degradation models are explained as how a mathematical model can be created to investigate various noisy situations. Different noise models such as additive, Gaussian, Rayleigh, uniform, gamma, impulse noise are explained with their expression and probability density function.

In Unit 10, various image restoration techniques are explained. Inverse filtering and wiener filtering is discussed to remove noise and other degradations from the input image.

UNIT 8

IMAGE ENHANCEMENT IN FREQUENCY DOMAIN

Structure **Page No.**

8.1 INTRODUCTION

In the previous unit, we have considered an image in the spatial domain which is the form in which camera captures it. In this unit, we shall view the image as a signal and apply the well known filtering techniques used in signal processing. The only difference here will be that image would be considered as 2-D signal (along x and y axes). We shall see that this view of the image results in number of benefits over the spatial domain treatment.

In this unit, we will discuss various enhancement techniques in frequency (fourier) domain. We discuss the basic issues associated with frequency domain filtering. We also discuss various low pass and high pass filters in frequency domain with their applications and advantages in image enhancement.

Now, we shall list the objectives of this unit. After going through the unit, please read this list again and make sure that you have achieved the objectives.

Objectives

After studying this unit, you should be able to

- define images in frequency domain
- perform filtering in frequency domain
- apply different types of image smoothing filters
- apply different types of image sharpening filters

Let us begin with shifting the centre of the spectrum.

8.2 SHIFTING THE CENTRE OF THE SPECTRUM

To start with we note that any signal (periodic or non periodic) can be expressed as the summations of sines and/or cosines multiplied by a weighting function. This is carried out by applying Fourier transform on the image.

In 1D signal, the fourier transform takes the form

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux} dx$$

$F(u)$ can be expressed in polar coordinates:

$$F(u) = |F(u)| e^{j\phi(u)}$$

where $|F(u)| = \left[R^2(u) + I^2(u)\right]^{\frac{1}{2}}$ (magnitude or spectrum)

$$\phi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right] \quad \text{(phase angle or phase spectrum)}$$

$R(u)$: the real part of $F(u)$

$I(u)$: the imaginary part of $F(u)$

The various benefits of frequency domain analysis are the following:

1) It is convenient to design a filter in frequency domain. As filtering is more intuitive in frequency domain, designing an appropriate filter is easier.

2) Implementation is very efficient with fast DFT via FFT.

3) Convolution in spatial domain reduces to multiplication in frequency domain which is a much simpler process.

However, the image in spatial domain is not continuous but consists of discrete values. The discrete version of fourier transform is

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x) e^{\frac{-2\pi ux}{N}}, u = 0, 1, ..., N-1$$

Also, since the image is two dimensional signal, we need 2D Fourier transform. For a $N \times N$ image it takes the form:

$$F(u, v) = \frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x, y) e^{-j2\pi\left(\frac{ux+vy}{N}\right)},$$

where u and v are the frequencies along x and y axes and take the values $0, 1, 2, ..., N-1$.

In the spatial domain we consider the origin to be located at top left corner of the image. For better display in the frequency domain, it is common to shift the origin to centre of the image.

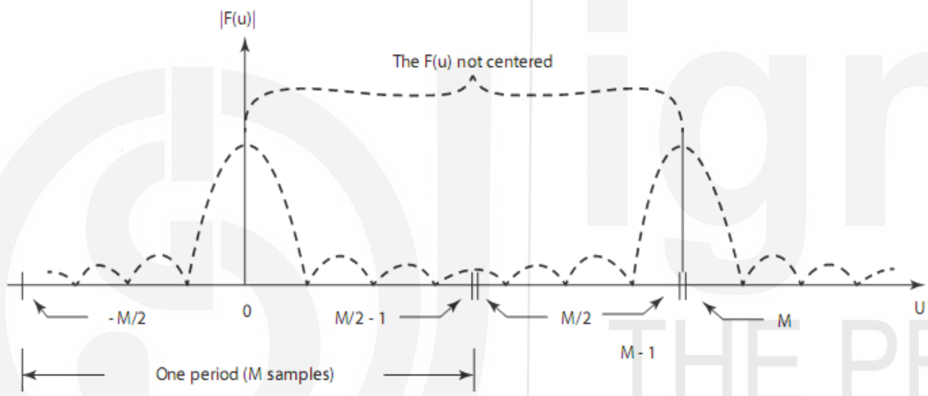
Periodicity of Fourier transform is given by

$$v(k,l) = v(k+M,l) = v(k,l+N) = v(k+M,l+N) \quad (1)$$

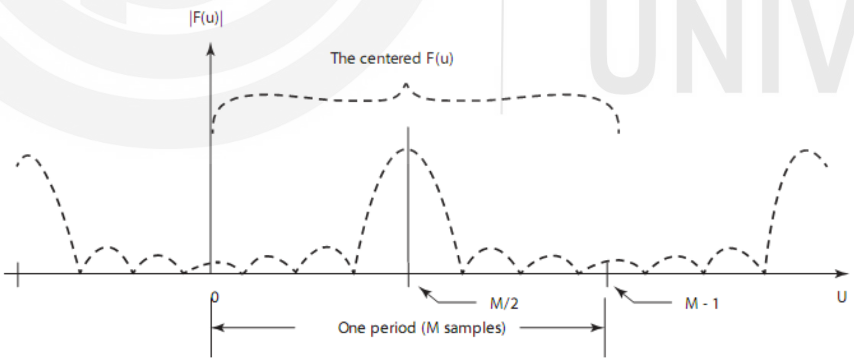
$$u(m,n) = u(m+M,n) = u(m,n+N) = u(m+M,n+N) \quad (2)$$

Fig 1(a) shows that the values from $N/2$ to $N-1$ are the same as the value from $N-1$ to 0. As DFT has been formulated for value of k in the interval $[0, N-1]$, the result of this formulation yield two back to back half periods in this interval. To display one full interval between 0 to $N-1$ as shown in Fig. 1(b), it is necessary to shift the origin of transform to the point $k = N/2$. To do so we have to take advantage of translation property of Fourier transform.

$$v(m,n)(-1)^{m-n} \xleftrightarrow{FT} v\left(k-\frac{M}{2}, l-\frac{N}{2}\right) \quad (3)$$



(a) Spectrum of f(x) without shifting centre.



(b) Spectrum of f(x) after shifting centre.

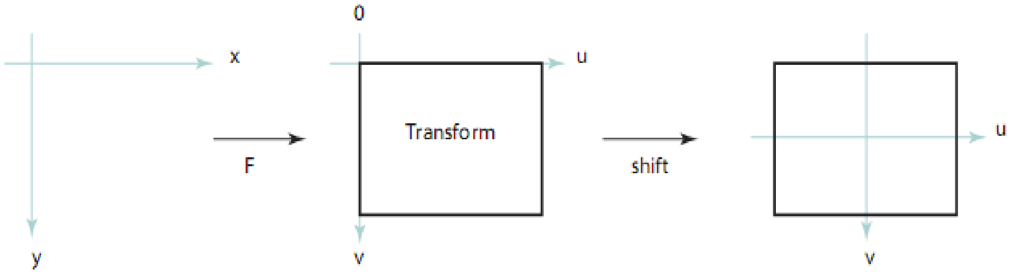
Fig. 1

Fig. 2 (a) and (b) show how the origin shifts from left corner of the image to centre of the image.

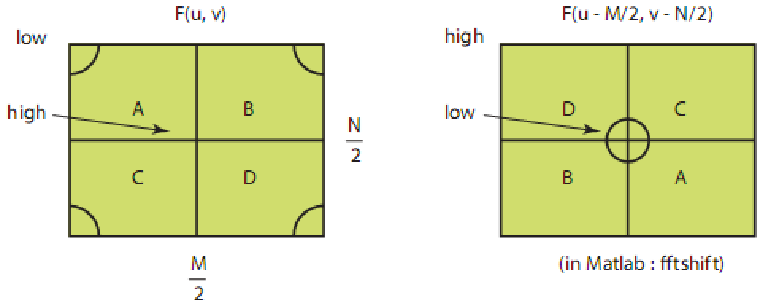
Basic Property of images in Frequency Domain

The forward transform of input image $u(m,n)$ is given by

$$v(k,l) = \frac{1}{N}\sum_{m=0}^{N-1}\sum_{m=0}^{N-1} u(m,n)\, W_N^{km}\, W_N^{ln} \qquad 0 \le k,l \le N-1 \quad (4)$$



(a) Change of centre in the spectrum of an image.



(b) Change of centre in the spectrum of an image.

Fig. 2

Following properties of the Fourier transform are observed

i) Each term of $v(k,l)$ contains all the values of $u(m,n)$ modified by the values of exponential terms.

ii) Frequency is directly related to the rate of change of grey level values.

iii) DC value or the average grey level value in an image is the slowest varying components corresponding to $u = 0, v = 0$. It is also the largest component in the frequency domain.

iv) Smooth variation of grey levels corresponds to low frequency components. Slow varying components can be the background of an image, hair of a person, skin, or texture etc.

v) Faster grey level changes correspond to high frequency components. These can be the edges/boundary of the objects or noise present in the image.

vi) As we move away from origin, higher frequencies are encountered.

Fig. 3 shows the variation in frequency of a centred spectrum.

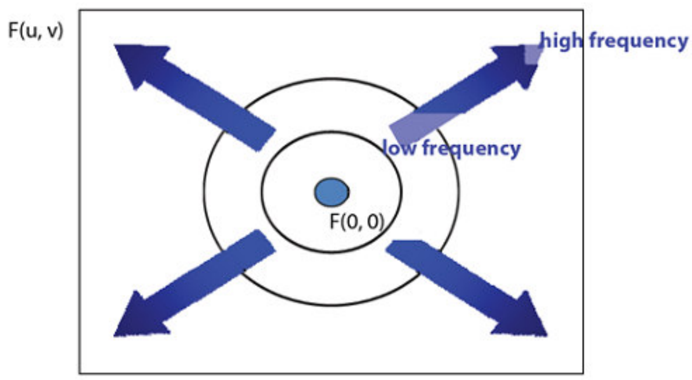


Fig 3: Frequency variation in an image.

Also, note that the rotation of an image in spatial domain causes exactly same rotation in frequency domain.

- Rotating $f(x, y)$ by θ rotates $F(u, v)$ by θ.

Once the image is transformed in frequency domain, it is easy to carry out image processing operations on it. We apply a low pass filter, if we are interested in only slowly varying components of the image (like object shapes), and we wish to suppress high frequency components (like noise). If we are interested in highlighting the edges or special textures, we can employ high pass filters, which will allow high frequency components to be displayed. Filtering in frequency domain is multiplication of a suitable filter $H(u, v)$ by image in Fourier domain $F(u, v)$ to result in $G(u, v)$. By taking inverse Fourier Transform of $G(u, v)$ we get the image back in spatial domain.

Generally, the filters are centred and are symmetric about the centre. Input image should also be centred. Following steps are followed to carry out filtering in frequency domain (Fig. 4):

Step 1: Multiply input image $f(x, y)$ by $(-1)^{x+y}$ to move the origin in the transformed image to

$$u = \frac{M}{2} \text{ and } v = \frac{N}{2}$$

Step 2: Compute $F(u, v)$, Fourier transform of the output of step 1.

Step 3: Multiply filter function $H(u, v)$ to $F(u, v)$ to get $G(u, v)$.

Step 4: Take inverse Fourier transform of $G(u, v)$ to get $g(x, y)$.

Step 5: Take the real part of $g(x, y)$ to get $g_r(x, y)$

Step 6: Multiply the result of step 5 by $(-1)^{x+y}$ to shift the centre back to origin and enhanced image is generated.

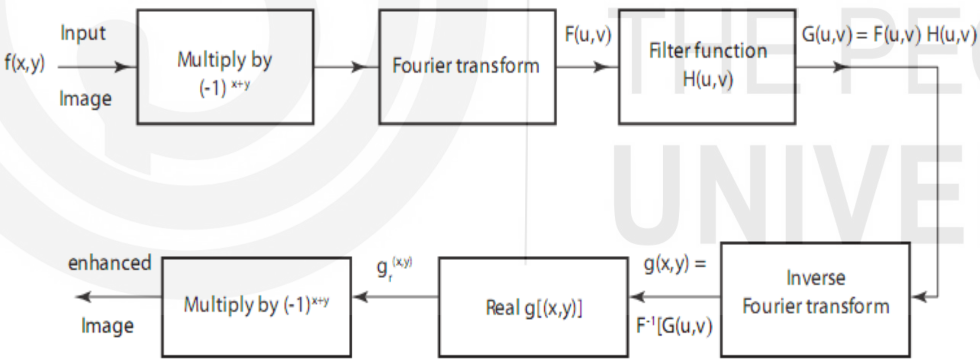


Fig. 4: Block Diagram of Filtering in Frequency Domain.

Types of Frequency Domain Filters

Frequency domain filters are categorized into three types.

1. Smoothing filters
2. Sharpening filters
3. Homomorphic filters

Smoothing filters are low pass filters and are used for noise reduction. It blurs objects. Sharpening filters are high pass filters and produce sharp images with dark background. Laplacian and high boost filters are used to produce sharp images. Homomorphic filters are based on illumination and reflectance model, and create a balance between smoothing and sharpening filtering effect. This classification is shown in Fig. 5.

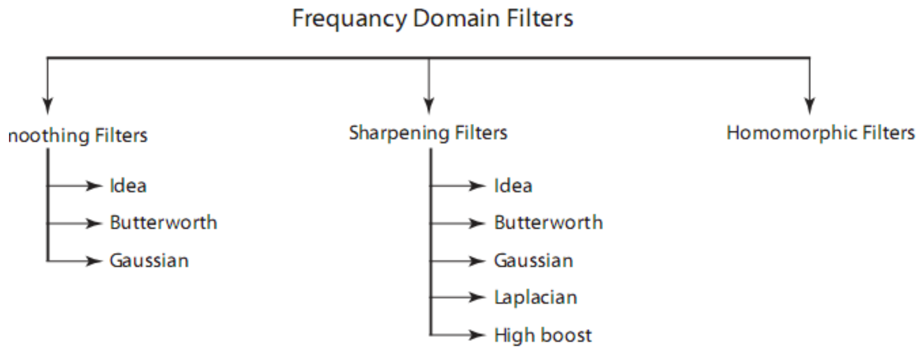


Fig. 5: Types of Frequency Domain Filters.

Try the following exercises.

E1) Write the steps involved in frequency domain filtering with the help of block diagram.

E2) Explain how image enhancement is better in the frequency domain as compared to spatial domain.

In the following section, we will discuss smoothing filters in the frequency domain.

8.3 SMOOTHING FILTERS

Smoothing filters are low pass filters (LPF). Edges, sharp transitions and noise in the grey levels contribute to high frequency contents in an image. A low pass filter only passes low frequency and blocks the high ones. It removes noise but in the process introduces blurring as a side effect in the image. The basic model of filtering is

$$G(u, v) = H(u, v) F(u, v) \tag{5}$$

where $F(u, v) =$ Fourier transform of the image to the filtered, $H(u, v) =$ Transfer function of the filter, and $G(u, v) =$ Enhanced image where high frequency components have been alternated.

The transfer function $H(u, v)$ is of three types

a) Ideal LPF
b) Butterworth LPF
c) Gaussian LPF

Ideal filters has sharp slope in transition band whereas Gaussian filter has smooth slope in transition. Butterworth filter has a parameter called filter order which controls the slope of transition band. Higher value of filter order leads to ideal filter.

8.3.1 Ideal Low Pass Filters (ILPF)

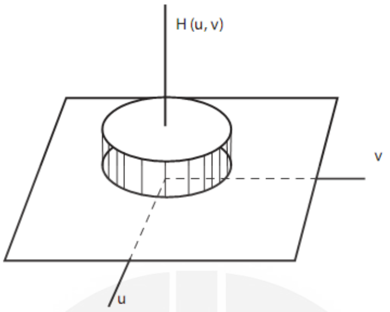
Low pass filter removes all frequencies above a certain frequency components D_0. Ideal low pass filter is defined by the transfer function

$$H(u, v) = \begin{cases} 1 & D(u, v) \le D_0 \\ 0 & D(u, v) > D_0 \end{cases}$$

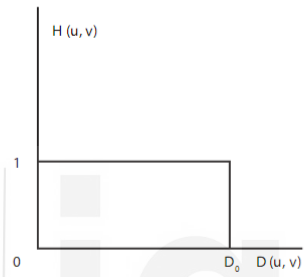
Where $D(u,v)=\left[\left(u-\frac{M}{2}\right)^{1/2}+\left(v-\frac{N}{2}\right)^{1/2}\right]$

$D(u,v)$ is the distance from point (u,v) to the centre $\left(\frac{M}{2},\frac{N}{2}\right)$. If size of an image is $M\times N$, then the centre is at $\left(\frac{M}{2},\frac{N}{2}\right)$. Filter transfer function is symmetric about the midpoint.

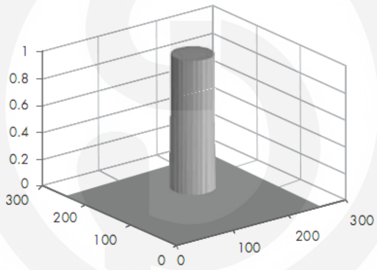
D_0 is non negative quantity specifying the frequency content to be retained. It is also called cut off frequency. Fig. 6(a), Fig. 6(b) is the plot of ILPF and Fig. 6 (c) is perspective plot and Fig. 6(d) is filter displayed as an image.



(a) Transfer Function



(b) Plot of Ideal LPF



c) Prospective plot of ILPF

(d) ILPF displayed as an image with D_0 = 10

Fig. 6

Choice of cut off frequency in an ideal LPF

1. The cutoff frequency D_0 decides the amount of frequency components passed by the filter.

2. Smaller the value of D_0, more are the number of frequency components eliminated by the filter.

3. In general, D_0 is chosen such that most of the frequency components of interest are passed while unnecessary components are eliminated.

4. A useful way to establish a set of standard cut off frequencies is to compute circles having a certain percentage of the total image power.

$$P_T=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}P(u,v)$$

Here $P(u,v) = |F(u,v)|^2$ is the total image power.

5. Consider a circle of radius $D_0(\alpha)$ as the cut off frequency with respect to a threshold a such that

$$\sum_u \sum_v P(u,v) = \alpha P_T \,.$$

6. Thus, we can fix a threshold α which tells how much of the total energy is retained and obtain an appropriate cut off frequency $D_0(\alpha)$.

Properties of ILPF

As it is an ideal filter, it is non-real, non-actual and non-physical. But it can be stimulated in computers. Fig 7 and 8(b) to (c) show low pass filtered images with different cut off frequencies. As the filter radius increases, less and less power and information is removed which resulted in less blurring. A very noticeable effect that can be seen in the output image is ringing.

Now, the question arises that what is ringing?

Ringing is undesirable and unpleasant lines around the objects present in the image Fig. 7 (b). As the cut of frequency D_0 increases, effect of ringing reduces. Ringing is a side effect of ideal lpf .

Why is there Ringing in Ideal LPF?

Ideal LPF function is a rectangular function as shown in Fig. 6. The inverse Fourier transform of a rectangular function, is a sinc function. We can observe two distinctive characteristic of $\sin c$ function:

1. A dominant component at the origin which is responsible for blurring.
2. Concentric circular components are responsible for ringing which is the characteristic of an ideal LPF.

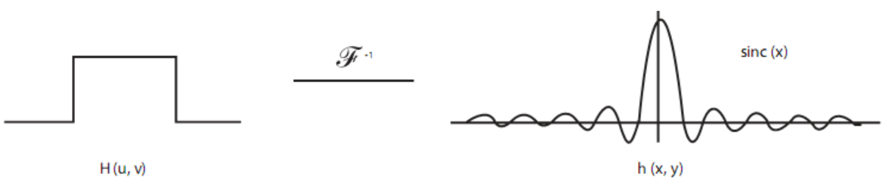


Fig. 6: Fourier Inverse of Rectangular Transfer Function

Radius of the centre component $\propto \dfrac{1}{\text{cut off frequency}}$

Number of circles per unit distance from origin $\propto \dfrac{1}{\text{cut off frequency}}$

Thus, as the cut off frequency (D_0) is increased, blurring as well as ringing reduces. The examples are given in Fig. 7 and Fig. 8.

(a) Original image (b) Output of ILPF with D_0=30 (c) Output of ILPF with D_0= 50

Fig. 7

(a) Original image (b) Output of ILPF with D_0=50 (c) Output of ILPF with D_0=80

Fig. 8

8.3.2 Butterworth Low Pass Filters (BLPF)

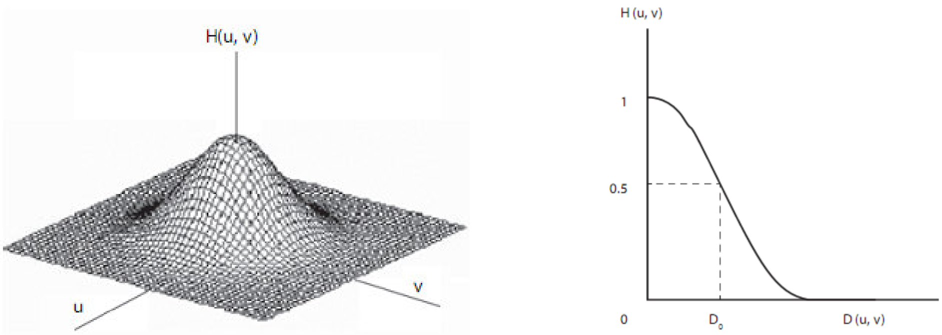
The Butterworth filter replaces the sharp cutoff of Ideal LPF by a smooth cutoff. Frequency response of BLPF does not have a sharp transition between pass band and stop band. It is more appropriate for image smoothing and does not introduce ringing effect for lower order filters. Transfer function of BLPF is given by

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}},$$

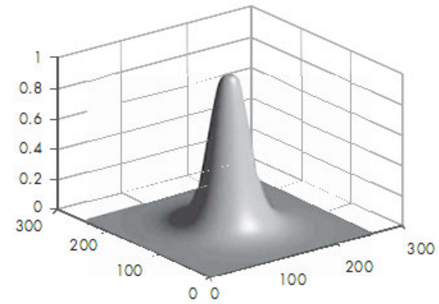
where the cut off frequency or distance from the centre $D_0 = \left(\frac{M}{2}, \frac{N}{2}\right)$, and

the filter order is n, and $D(u,v) = \left[\left(u-\frac{M}{2}\right)^2 + \left(u-\frac{N}{2}\right)^2\right]^{1/2}$

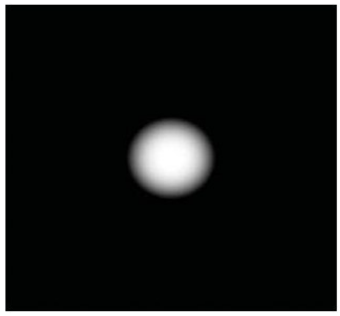
Fig. 9 (a) and Fig. 9 (b) show the transfer function of BLPF. Fig. 9 (c) is the plot of BLPF and Fig. 9 (d) is BLPF displayed as an image.



(a) Plot of BLPF (b) Plot of BLPF transfer function



(c) Plot of BLPF



(d) BLPF displayed as an image

Fig. 9

Transfer function of BLPF does not have sharp transition near the cut off. For $n = 1$, the transition is very smooth. As the filter order increases, the transfer function approaches towards ideal LPF. No ringing is visible on the image filtered by BLPF for $n = 1$. Noise is reduced and blurring is observed in all the images. For $n = 2$, ringing is un-noticeable, but it can becomes more significant for higher values of n. Fig. 10 shows the increasing effect of ringing as n increases from 1 to 20.

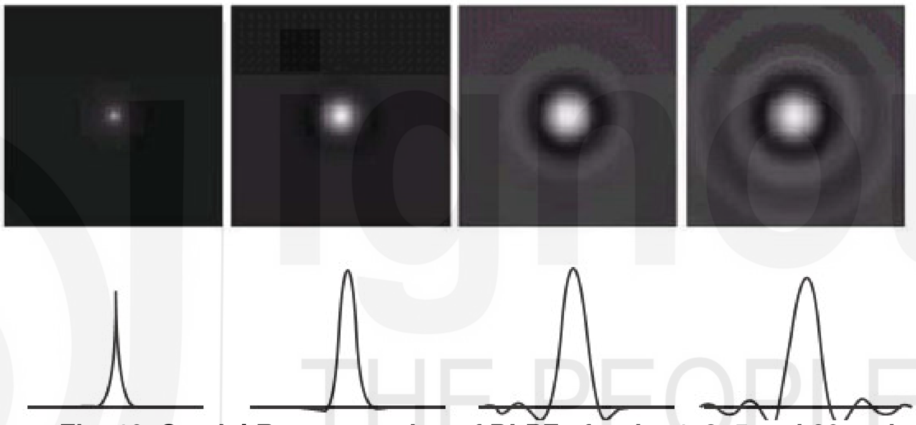


Fig. 10: Spatial Representation of BLPF of order 1, 2, 5 and 20 and Corresponding Intensity Profile

The output corresponding to the change in the values of D_o and n are shown in Fig. 11.

(a) Output of BLPF for D_0= 30

(b) Output of BLPF for D0= 40

(c) Output of BLPF for n = 4, D_0 = 30

(d) Output of BLPF for n = 20 D_0 = 30

Fig. 11

8.3.3 Gaussian Low Pass filters (GLPF)

Still a better variant of the low pass filter is the Gaussian Low Pass filter which have smooth transition between pass band and stop band. It does not introduce any ringing in the output image. The transfer function of GLPF is given by

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2},$$

where $D(u,v)$ is the distance from the origin of Fourier transform, and σ is the measure of spread/dispersion of the Gaussian curve.

Larger the value of σ, larger is the cut off frequency and the filter is milder. Let $\sigma = D_0$ then transfer function is given by

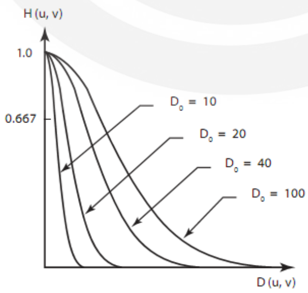
$$H(u,v) = e^{-D^2(u,v)/2D_0^2},$$

where D_0 is the cut off frequency.

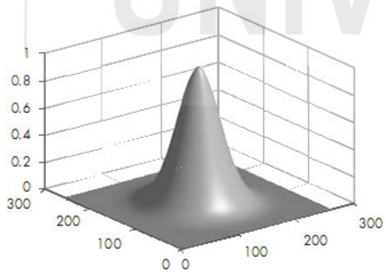
When $D(u,v) = D_0$, the amplitude of transfer function is down to 0.607 of its maximum value of 1.

$D_0 \uparrow$ = Milder filter, more components are passed
$D_0 \uparrow = \downarrow$ blurring

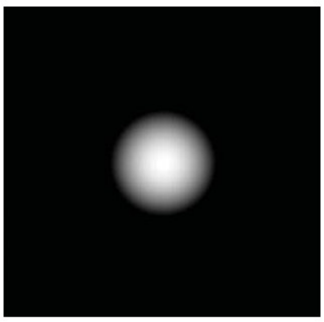
Fig. 12 (a) is GLPF transfer function, Fig. 12 (b) is plot of GLPF and Fig. 12 (c) is GLPF displayed as an image. Fig. 13 (a) to Fig. 13 (c) are GLP filtered images. No ringing is observed in the output, but only blurring is visible. As the cut off frequency increase, blurring reduces. No ringing in the output is a very big advantage of GLPF. These filters can be used in situations where no artifacts are desirable (eg. medical imaging). In medical imaging, GLPF is preferred over ILPF/ BLPF.



(a) GLPF Transfer Function for Various Values of D



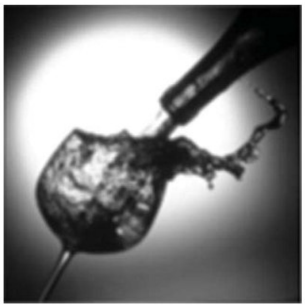
(b) Plot of GLPF



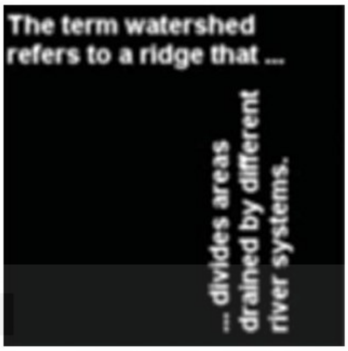
(c) GLPF Displayed as an Image

Fig. 12

(a) Output of GLPF for D_0 = 10



(b) Output of GLPF for D_0 = 300



(c) Output of GLPF with D_0 = 50

Fig. 13

	Ideal	**Butter worth**	**Gaussian**
Transfer Function	$H(u,v) = \begin{cases} 1, D(u,v) \le D_0 \\ 0, D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1+[D(u,v)/D_0]^{24}}$	$H(u,v) = e^{-D^2(u,v)/2D^2}$
Applications	Reduce noise	Reduce noise	Reduce noise
Problems	Blurring Ringing	Blurring, Ringing for higher order filters $(n > 2)$	Blurring, No ringing

Let us discuss some of the applications of Low pass filters in frequency domain.

8.3.4 Applications of Low Pass Filters

LPF are generally used as a preprocessing step before an automatic recognition algorithm. It is also used to reduce noise in images. Few examples are listed below.

1. **Character Recognition:** Input to an automatic character recognition system is generally of poor quality. Input may contain noise due to improper acquisition system or there may be gaps in the alphabets. (broken alphabets). Because of these problems, character recognition system fails to give expected results consistently. Hence, LPF is used as a preprocessing step to blur the image. Blurring is used to bridge small gaps in the alphabets. This is done using GLPF with D_0 = 80. This increases chances of getting correct result from automatic character recognition system.

2. **Object Counting:** Object counting is to count the number of objects in an image. The output of an object counting algorithm may give wrong output because of poor quantity of input image. If there are small gaps in the boundary of objects, automatic algorithm will not give expected results.

Blurring is used to fill in small gaps in the boundary of objects which helps in producing correct results.

3. **Printing and Publishing Industry:** Unsharp masking is used in publishing industry to sharpen image where blurred version of an image is subtracted from the image itself to get sharpen image.

4. **“Cosmetic”** processing is another use of low pass filter prior to printing. Blurring is used to reduce the sharpness of fine skin lines and small blemishes on human face. Smoothened images look very soft and pleasing and face looks younger.

Try the following exercises.

E3) Give the formula for transform function of a Butterworth low pass filter.

E4) Explain and compare ideal low pass filter and Butterworth filter for image smoothing.

E5) Explain smoothing frequency domain filters. What is ringing effect?

E6) Discuss the applications of image smoothing filters.

In the following section we will discuss sharpening filters.

8.4 IMAGE SHARPENING IN FREQUENCY DOMAIN

In the Fourier transform of an image, high frequency contents correspond to edges, sharp transition in grey levels and noise. Low frequency contents correspond to uniform or slowly varying grey level values.

High pass filtering is achieved by attenuating low frequency components without disturbing high frequency components. High pass filter (HPF) can also be viewed as reverse operation of low pass filter. Transfer function of HPF is given by

$$H_{hp}(u, v) = 1 - H_{lp}(u, v)$$

Here, $H(u, v)$ is the transfer function of a LPF. Thus

Smoothing $\rightarrow$ LPF $\rightarrow$ attenuates high frequency components
Sharpening $\rightarrow$ HPF $\rightarrow$ attenuates low frequency components

Here, we discuss only real and symmetric filters. Following sharpening filters are discussed in this section:

1. Ideal high pass filter
2. Butterworth high pass filter
3. Gaussian high pass filter

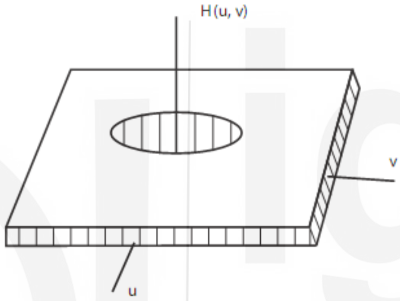
High pass filters are used for enhancing edges. These filters are used to extract edges and noise is enhanced, as a side effect.

8.4.1 Ideal High Pass Filter (IHPF)

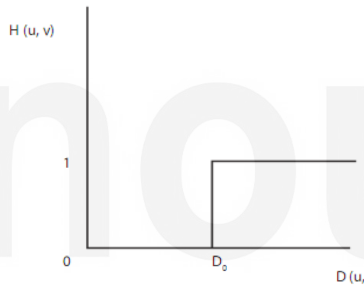
Transfer function of a 2D IHPF is given by

$$H(u,v) = \begin{cases} 1, & \text{if } D(u,v) \geq D_0 \\ 0, & \text{if } D(u,v) < D_0 \end{cases}$$

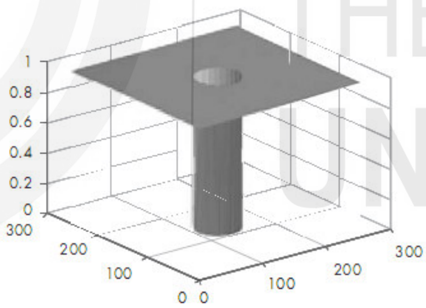
Here, D_0 is the cut off frequency and $D(u,v)$ is the distance from the origin of the Fourier transform. Fig. 14 (a) and Fig. 14 (b) is the IHPF and its transfer function respectively. Fig. 14 (c) is plot of IHPF and Fig. 14 (d) is IHPF as an image. Note that the origin (0,0) is at the centre and not in the corner of the image. The abrupt transition from 1 to 0 of the transfer function $H(u,v)$ cannot be realized in practice. However, the filter can be simulated on a computer. This filter sets to all frequencies inside the circle of radius D and passes all frequencies above D_0 without any attenuation. Ringing is clearly visible in the output (Fig. 15 (b), and Fig. 16(c)) other than sharp edges and boundaries. Output image looks very dark and dull as the high value DC component $G(0,0)$ is eliminated.



(a) Plot of IHPF



(b) Transfer function of IHPF



(c) Plot of IHPF

(d) IHPF displayed as an image

Fig. 14

(a) Output of IHPF for D_0= 50

(b) Output of IHPF for D_0 = 60

Fig. 15

8.4.2 Butterworth High Pass Filter (BHPF)

Butterworth filter does not have sharp transition between passband and stop band. The slope depends on the order of the filter. Transfer function of BHPF is

$$H(u,v) = \frac{1}{1+\left[\frac{D_0}{D(u,v)}\right]^{2n}},$$

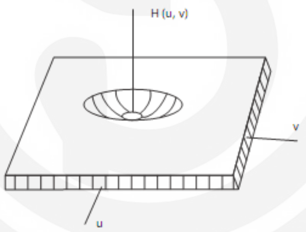
where n is the order of the filter, D_0 is the cut off frequency and $D(u,v)$ is the distance from the origin of Fourier transform.

Fig. 16 (a) and Fig. 16 (b) are BHPF transfer function and Fig. 16 (c) and Fig. 16 (d) are plot and image display of BHPF.

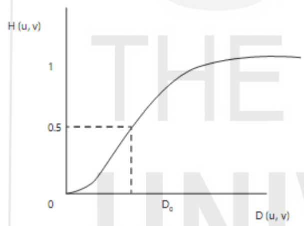
Frequency response does not have a sharp transition as in the ideal HPF.

Thus, less distortion is seen in the output with no ringing effect even for smaller values of cut off frequencies. This filter is more appropriate for image sharpening than ideal HPF as there is no ringing in output.

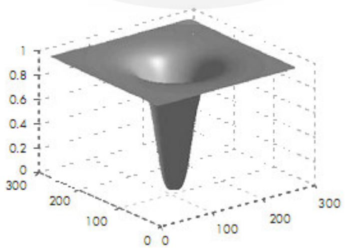
Fig. 16(b) is the plot of GHPF for $D_0 = 30, n = 2,$ and Fig. 16 (c) GHPF displayed as an image. Fig. 17(a) and Fig.17 (b) are the output of GHPF for D_0 = 30 and 130 respectively for $n = 2$. It is clear from the output, as D_0 increases, more and more power is removed from the output image. Thus, output looks sharper for higher value of D_0. Fig. 17(d) is the output for $D_0 = 30\ n = 20,$ ringing is clearly visible in the output. As n increases, ringing in butterworth filter increases.



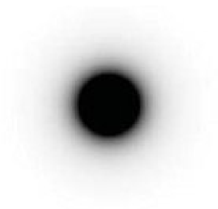
(a) Transfer function of BHPF



(b) Transfer function of BHPF

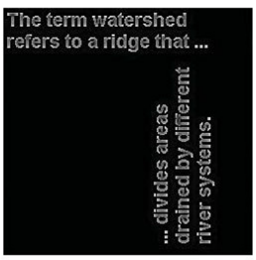


(c) Plot of BHPF



(d) BHPF displayed as an image

Fig. 16



(a) Output of BHPF with D_0 = 130, n = 2 (b) Output of BHPF with D_0= 30, n = 2

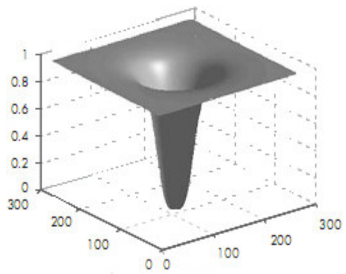
Fig. 17

8.4.3 Gaussian High Pass Filter (GHPF)

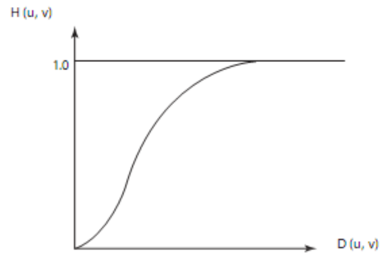
Gaussian high pass filters have smooth transition between passband and stopband near cutoff frequency. The parameter D is a measure of spread of the Gaussian curve. Larger the value D_0, larger is the cut off frequency. Transfer function of GHPF is

$$\mathrm{H}(u,v) = 1 - e^{\frac{-\mathrm{D}^2(u,v)}{2\mathrm{D}_0^{\,2}}},$$

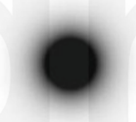
where D_0 is the cut off frequency and $\mathrm{D}(u,v)$ is the distance from origin of Fourier transform.



(a) GHPF transfer function



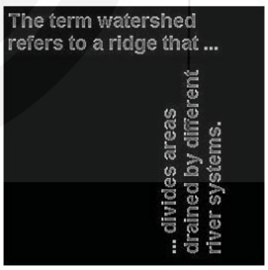
(b) Plot of GHPF



(c) GHPF displayed as an image

Fig. 18

Fig. 18(a) is GHPF Transfer function filter. Plot and image are displayed in Fig. 18 (b) and Fig. 18 (c). Output in Fig. 19 is much smoother than previous two filters with no ringing effect.



(a) Output of GHPF with D_0=30

(b) Output of GHPF with D_0=120

Fig. 19

Let us compare these three high pass filters in frequency domain filters in the following table.

Table 1

	Ideal	Butterworth	Gaussian
Transfer function	$\mathrm{H}(u,v) = \begin{cases} 1, & \mathrm{D}(u,v) \le \mathrm{D}_0 \\ 0, & \mathrm{D}(u,v) > \mathrm{D}_0 \end{cases}$	$\mathrm{H}(u,v) = \dfrac{1}{1+\left[\dfrac{\mathrm{D}}{\mathrm{D}(u,v)}\right]^{2n}}$	$\mathrm{H}(u,v) = 1 - e^{\frac{\mathrm{D}^2(u,v)}{2\mathrm{D}_0^{\,2}}}$
Application	**Edge enhancement**	**Edge enhancement**	**Edge enhancement**
Problems	**Ringing**	**No Ringing**	**No Ringing**

Try the following exercises.

E7) How many types of high pass filters are there in frequency domain? List them.

E8) Give the formula for transform function of a Gaussian high pass filter.

Now, we summarise what we have studied in the unit.

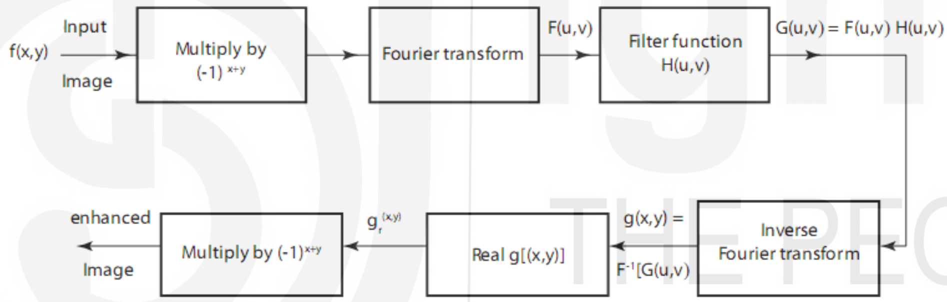
8.5 SUMMARY

In this unit, we have discussed the following points.

1. Image characteristics in frequency domain
2. Filtering in frequency domain
3. Basic steps of frequency domain filtering
4. Various low pass and high pass filters
5. Various image smoothing filters in frequency domain
6. Various image sharpening filters in frequency domain

8.6 SOLUTIONS AND ANSWERS

E1)



$$u = \frac{M}{2} \text{ and } v = \frac{N}{2}.$$

1. Multiply input image $f(x, y)$ by $(-1)^{x+y}$ to centre the transform to
2. Compute $F(u, v)$, Fourier transform of the output of step 1.
3. Multiply filter function $H(u, v)$ to $F(u, v)$ to get $G(u, v)$.
4. Take inverse Fourier transform of $G(u, v)$ to get $g(x, y)$.
5. Take the real part of $g(x, y)$ to get $g_r(x, y)$
6. Multiply the result of step 5 by $(-1)^{x+y}$ to shift the centre back to origin and enhanced image is generated.

E2) Image enhancement can be done very effectively in frequency domain. High frequency noise, undesirable breakages in the edges and other imperfections can be taken care by filtering in frequency domain. Low pass and high pass filters are implemented with ease and perfection in frequency domain.

E3) $$H(u, v) = \frac{1}{1 + [D(u, v) / D_0]^{2n}},$$

where D_0 = Cut off frequency or distance from the centre $n =$ filter order $\left(\frac{M}{2}, \frac{N}{2}\right)$

E4)

	Ideal	**Butterworth**	**Gaussian**
Transfer function	$H(u,v) = \begin{cases} 1, & D(u,v) \le D_0 \\ 0, & D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{24}}$	$H(u,v) = e^{-D^2(u,v)/2D^2}$
Application	**Reduce noise**	**Reduce noise**	**Reduce noise**
Problems	**Blurring Ringing**	**Blurring, Ringing for higher order filters**	**Blurring no ringing**

E5) Smoothing filters are low pass filters (LPF). Edges, sharp transitions and noise in the grey levels contribute to high frequency contents in an image. A low pass filter only passes low frequency and blocks the high ones. It removes noise and introduces blurring as a side effect in the image.

Ringing is undesirable and unpleasant lines around the objects present in the image. As the cut of frequency D_0 increases, effect of ringing reduces. Ringing is a side effect of ideal *lpf*.

E6) LPF are generally used as a preprocessing step before an automatic recognition algorithm. It is also used to reduce noise in images. Few examples are listed below.

Character Recognition, Object counting, Printing and publishing industry, “Cosmetic” processing etc.

E7) The sharpening filters are listed as follows:

1. Ideal high pass filter
2. Butterworth high pass filter
3. Gaussian high pass filter

High pass filters are used for enhancing edges. These filters are used to extract edges and noise is enhanced, as a side effect.

E8) Gaussian high pass filters have smooth transition between passband and stop band near cut off frequency. The parameter D is a measure of spread of the Gaussian curve. Larger the value D_0, larger is the cut off frequency. Transfer function of GHPF is

$$H(u,v) = 1 - e\frac{-D^2(u,v)}{2D_0^2},$$

where $D_0 =$ cut off frequency and $D(u,v)$ is the distance from origin of Fourier transform.

UNIT 9

IMAGE RESTORATION-I

Structure **Page No.**

9.1 INTRODUCTION

In the previous unit, we have looked at image improvement without bothering about source which caused a degradation in the quality of the image. If the source is known to us, it is possible to improve the quality of the image in a better way. We shall discuss this in this unit.

Image restoration is a pre-processing method that suppresses a known **degradation**. Image acquisition devices introduce degradation because of defects in optical lenses, non- linearity of sensors, relative object camera motion, blur due to camera mis-focus, atmospheric turbulence etc. Restoration tries to reconstruct an image that was degraded by a known degradation function. Iterative restoration techniques attempt to restore an image by minimizing some parameter of degradation, whereas blind restoration techniques attempt improve the image without knowing the degradation function. Like image enhancement, image restoration also aims to improve image quality, but it is more objective process where as enhancement is a subjective process. Noise is visually unpleasant, it is bad for compression and bad for analysis.

Restoration involves modeling of these degradations and applying inverse process to recover the original image.

Objectives

After studying this unit, you should be able to

- describe images degradation models
- state difference between restoration and enhancement
- apply different noise models

We shall begin the unit by discussing image degradation.

9.2 IMAGE DEGRADATION

Overall our objective is to improve image. For that it is important to understand image degradation if we want to remove it. Degradations are of three types

a) Noise
b) Blur
c) Artifacts

Let us define these one by one.

a) **Noise** is a disturbance that causes fluctuations in pixel values. Pixel values show random variations and can cause very disturbing effects on the image. Thus suitable strategies should be designed to model and remove/ reduce noise. Original image is shown in Fig. 1(a) and noisy image with added Gaussian noise is shown in Fig. 1 (b).

(a) Original **(b) Noisy Image**

Fig. 1

b) **Blur** is a degradation that makes image less clear.This makes image analysis and interpretation very difficult. Motion blur is a very common cause of blurring where blur occurs due to the movement of object or camera. Fig. 2 (a) shows original image and Fig. 2 (b) shows blurred image.

(a) Original Image **(b) Blurred image**

Fig. 2

c) **Artifacts** or distortions are extreme intensity or color fluctuations that can make image meaningless. Distortions involve geometric transformations such as translation, rotation or change in scale.

Now, the question arises that what are the sources which contribute to image degradation. Image degradation (as shown in Fig. 3) can happen due to

a) **Sensor Distortions:** Involves quantization, sampling, sensor noise, spectral sensitivity, de-mosaicking, non linearity of sensor etc.

b) **Optical Distortions:** are geometric distortion, blurring due to camera mis-focus.

c) **Atmospheric Distortions:** are haze, turbulence etc.

d) **Other Distortions:** Low illumination, relative motion between object and camera etc.

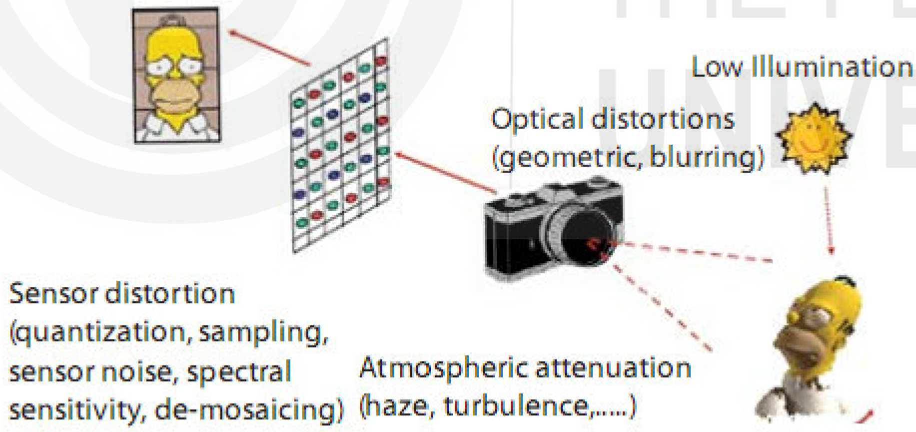


Fig. 3: Typical Degradation Sources

The processes that are used to remove degradation are mainly image enhancement and image restoration.

Restoration is the process of inverting a degradation using knowledge about its nature, whereas enhancement is a process that aims to improve 'bad' quality image so that it looks better. Restoration is distinguished from enhancement, as degradation can be considered as an external influence. Table 1 lists the differences between enhancement and restoration.

Table 1: Enhancement v/s Restoration

	Enhancement	Restoration
1.	It gives better visual representation	It remove effects of sensing environment
2.	No model required	Mathematical model of degradation
3.	It is a subjective process	It is an objective process
4.	Contrast stretching, histogram equalization etc are some enhancement techniques	Inverse filtering, wiener filtering, denoising are some restoration techniques.

Try the following exercises.

E1) What are the factors that can cause image degradation.

In this section we will discuss image degradation/restoration model.

9.3 IMAGE DEGRADATION/RESTORATION MODEL

Consider the block diagram given in Fig. 4 shows the block diagram of degradation/restoration model. Degradation function $h(x, y)$ and noise $n(x, y)$, operate on input image $f(x, y)$ to generate a degraded and noisy image $g(x, y)$.

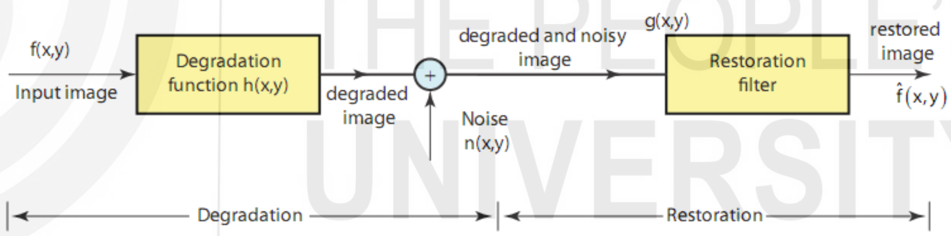


Fig. 4: Block diagram of degradation/restoration model

The notations used in the block diagram are

$f(x, y) =$ original image

$h(x, y) =$ degradation function

$n(x, y) =$ additive noise

$g(x, y) =$ degraded and noisy image

$\hat{f}(x, y) =$ restored image

The objective of restoration process is to estimate $\hat{f}(x, y)$ from the degraded version $g(x, y)$, when some knowledge of degradation function H and noise n is available. The degraded image $g(x, y)$ as shown in Fig. 5 can be expressed mathematically as

$$g(x, y) = h(x, y) * f(x.y) + n(x, y)$$

This equation is in spatial domain and * represents convolution operation. An equivalent frequency domain representation is graphically shown in Fig. 6 and expressed as

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Here $G(u,v) = F[g(x,y)]$

$H(u,v) = F[h(x,y)]$

$F(u,v) = F[g(x,y)]$

$N(u,v) = F[n(x,y)]$

$F(u,v) = H^4(u,v)[G(u,v) - N(u,v)]$

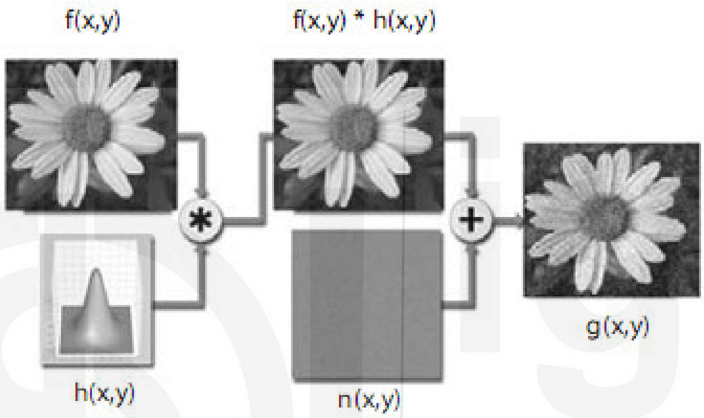


Fig. 5: Image Degradation Model (Spatial Domain)

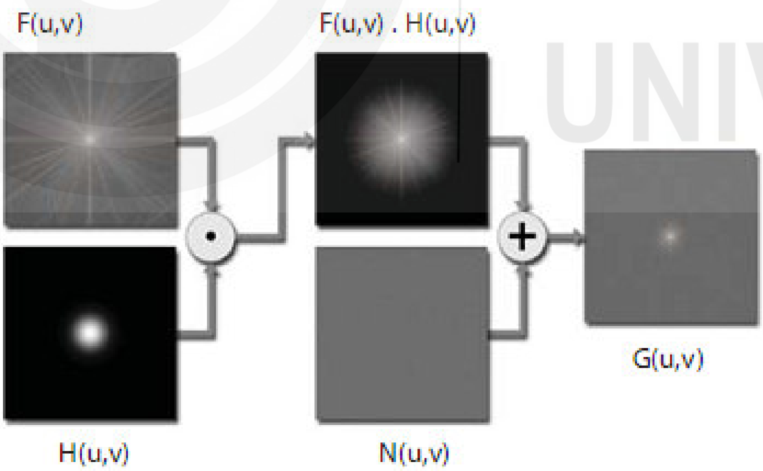


Fig. 6: Image Degradation Model (Frequency Domain)

Restored image can be obtained by the above equation. The problems in implementing this equation are

1) The noise N is unknown. Only the statistical properties of noise can be known.

2) The operation H is singular or ill posed. It is very difficult to estimate H.

Try an exercise.

E2) Explain in detail an image degradation model.

In the following section, we shall discuss noise models in detail.

9.4 NOISE MODELS

Major source of noise in digital images is during image acquisition. Non-ideal image sensors and poor quality of sensing elements contribute to majority of noise. Environmental factors such as light conditions, temperature of atmosphere, humidity, other atmospheric disturbances also account for noise in images. Transmission of image is also a source of noise. Images are corrupted with noise because of interference in the channel, lightning and other disturbances in wireless network. Human interference also plays a part in addition of noise in images.

Properties of Noise

Spatial and frequency characteristics of noise are as follows:

1) Noise is assumed to be 'white noise' (it could contain all possible frequency components), as such, fourier spectrum of noise is constant.

2) Noise is assumed to be independent in spatial domain. Noise is '**uncorrelated**' with the image, that is, there is no correlation between pixel value of image and value of noise components.

The spatial noise descriptor is the statistical behavior of the intensity values in the noise component. Noise intensity is considered as a random variable characterized by a certain probability density function (PDF).

Restoration techniques are oriented towards modeling the degradation (noise in this case) and restore an image to the original state. Most types of noise are modeled as known PDFs. Based on the estimated parameters from the noisy image, a particular noise PDF is chosen. Noise models are divided into two categories:

a) Noise which is independent of spatial location: Gaussian, Rayleight, Gamma, Exponential, Uniform noise are examples of this category.

b) Noise which is dependent on spatial location: Periodic noise is example of this type of noise.

Following are the most commonly occurring noise models:

Gaussian Noise

Gaussian noise model is most frequently used in practice. The PDF of a Gaussian random variable 'z' is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}, \qquad (2)$$

where $z =$ intensity/grey level value

$\mu =$ mean (average) value of z

$\sigma =$ standard deviation

Plot of $p(z)$ with respect to z is shown in Fig. 7. 70% of its values are in the range $[(\mu-\sigma),(\mu-\sigma)]$ while 95% of the values are in the range $[(\mu-2\sigma),(\mu+2\sigma)]$. DFT of **Gaussian (normal) noise is another Gaussian process**. This property of Gaussian noise makes it most often used noise model. Some examples where Gaussian model is the most appropriate model are electronic circuit noise, sensor noise due to low illumination or high temperature, poor illumination.

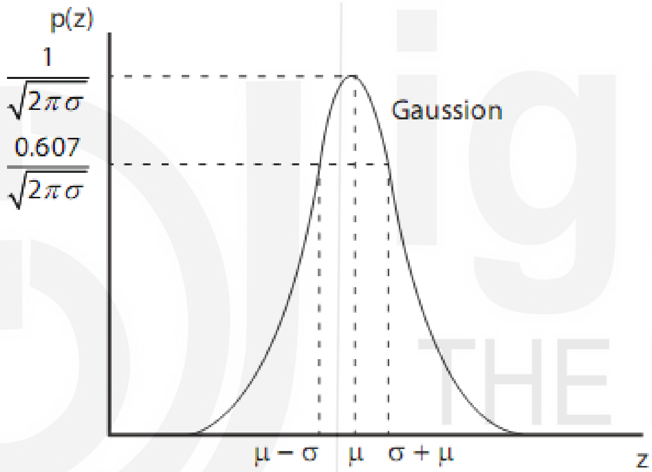


Fig. 7: PDF of Gaussian Noise Model

Gaussian noise is useful for modeling natural processes which introduce noise (e.g. noise caused by the discrete nature of radiation and the conversion of the optical signal into an electrical one – detector/shot noise, the electrical noise during acquisition – sensor electrical signal amplification, etc.).

Rayleigh Noise

Radar range and velocity images typically contain noise that can be modeled by the Rayleigh distribution. Rayleigh distribution is defined by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}, \qquad (3)$$

Mean density is given as $\mu = a + \sqrt{\pi^{b/4}}$ and variance is given by $\sigma^2 = \frac{b(4-\pi)}{4}$.

Plot of PDF is shown in Fig 8. As it is clear that the curve doesn't start from origin and is not symmetrical with respect to the centre of the curve. Thus, Rayleigh density is useful for approximating skewed (non-uniform) histograms. This is mainly used in range imaging.

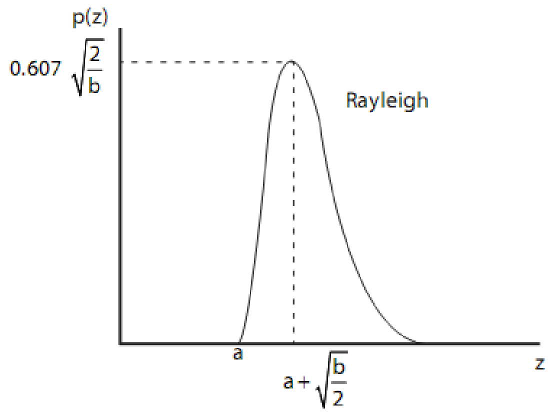


Fig. 8: PDF of Rayleigh Noise

Erlang (Gamma) Noise

Erlang noise is given by

$$p(z)=\begin{cases}\dfrac{a^b z^{b-1}}{(b-1)!}e^{-nz} & z \geq 0 \\ 0 & z < 0\end{cases}, \qquad (4)$$

Where a and b are positive integers, mean density is given by $\mu = \dfrac{b}{a}$ and variance is $\sigma^2 = \dfrac{b}{a^2}$.

When the denominator is a gamma function, the PDF describes the gamma distribution. Plot is shown in Fig. 9.

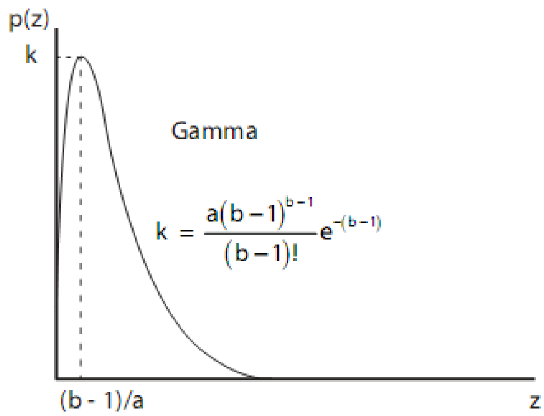


Fig. 9: PDF of Erlang Noise

Uniform Noise

Uniform noise is specified as

$$p(z)=\begin{cases}\frac{1}{b-a} & \text{if} \quad a\le z\le b\\ 0 & \text{otherwise}\end{cases}$$

Then mean and variance of uniform noise is given by

$$\mu=\frac{a+b}{2},\ \ \sigma^2=\frac{(b-a)^2}{12}$$

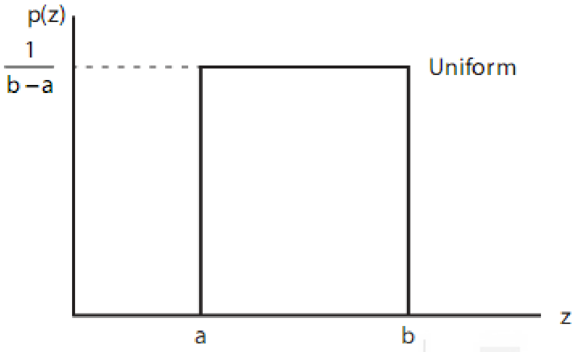


Fig. 10: PDF of Uniform Noise.

Fig. 10 shows the plot of PDF of uniform noise. Uniform noise is least used in practice.

Impulse (Salt and Pepper) Noise

Impulse (salt and pepper) noise is specified as

$$p(z)=\begin{cases}P_a & z=a\\ P_b & z=b\end{cases}, \tag{6}$$

Fig. 11 shows the plot of PDF of impulse noise. If $b>a$, intensity (grey level) 'b' will appear as a light dot on the image and 'a' appears as a dark dot. This is a **'bipolar'** noise, If $P_a=0$ or $P_b=0 \Rightarrow$ **unipolar noise** Generally, a and b values are saturated (very high or very low value), resulting in positive impulses being white (salt) and negative impulses being black (pepper). If $P_a=0$ and P_b exists, this is called '**pepper noise**' as only black dots are visible as noise. If $P_b=0$, only P_a exists, this is called '**salt noise**' as only white dots are visible on the image as noise.

Impulse noise occurs when quick transitions happen, such as faulty switching takes place. Noise parameters are generally estimated based on histogram of small flat area of noisy image.

The salt & pepper noise is generally caused by malfunctioning of camera's sensor cells, by memory cell failure or by synchronization errors in the image digitizing or transmission.

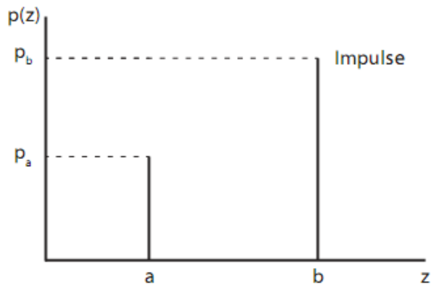


Fig. 11 PDF of Uniform noise.

Fig. 12 shown an example of impulse noise with $p = 0.1$ added to input image and togenerate a noisy image g. Noise level $p = 0.1$ means that approximately 10% of pixelsare contaminated by salt or pepper noise (highlighted by box)

128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128

f(x,y)

$\Rightarrow$

128	128	$\boxed{255}$	$\boxed{0}$	128	128	128	128	128	128
128	128	128	128	$\boxed{0}$	128	128	128	128	0
128	128	128	128	128	128	128	128	128	128
128	128	$\boxed{0}$	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
128	128	128	128	128	128	128	128	128	128
$\boxed{0}$	128	128	128	128	$\boxed{255}$	128	128	128	128
128	128	128	128	128	128	128	128	128	$\boxed{255}$
128	128	128	128	128	128	128	$\boxed{255}$	128	128

g(x,y)

Fig. 12: Numerical Example of Adding Impulse Noise with P = 0.1.

Fig. 13 shows the flower image with different types of noise. It is very easy to identify the effect of different types of noise on the images. Fig. 13 (a) shows original image, Fig. 13 (b) shows image with Gaussian noise. Fig. 13 (c) shows image with salt and pepper noise and Fig. 13 (d) shows image with uniform noise. The amount of noise added can also vary. If the amount of noise added is more, it becomes very difficult to remove it.

(a) Original image

(b) Image with Gaussian noise

(c) Image with salt & pepper noise

(d) Image with uniform noise

Fig. 13

Let us discuss an important type of noise.

Periodic noise is a spatially dependent noise. During Image acquisition, electrical or electromechanical interference may cause such type of periodic noise. A strong periodic noise can be seen in the frequency domain as equi-spaced dots at a particular radius around the centre (origin) of the spectrum. Fig. 14 shows image with periodic noise.

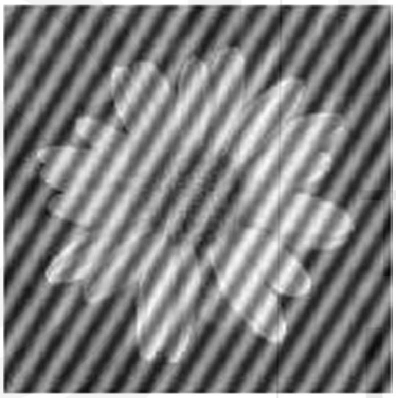


Fig. 14: Image with periodic noise

Now, you may like to try the following exercises.

E3) What is noise? How noise can be eradicated?

E4) Explain the types of noises based on its probability distribution.

In the following section, we will discuss restoration in the presence of noise only-spatial filtering.

9.5 RESTORATION IN THE PRESENCE OF NOISE ONLY – SPATIAL FILTERING

If H is an identity operator and degradation is only due to additive noise,

$$g(x,y) = f(x,y) + n(x,y)$$
$$G(u,v) = F(u,v) + N(u,v)$$

As noise is unknown, $f(x,y) = g(x,y) - n(x,y)$ is not realistic. Thus, spatial filtering is used when additive random noise is present. Mean and median filters are used for noise removal. Band reject and band pass filters are used for periodic noise removal.

9.5.1 Mean Filters

Smoothing spatial are explained in unit 7 section 7.2 in detail. Consider Fig. 15, $S_{x,y} = \text{Sub}$ image window of size $m \times n$ centred at (x,y). Fig. 16 shows 3×3 and 5×5 sub images. Mean filter computes average value of the corrupted image $g(x,y)$ in the area defined by $S_{x,y}$

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{s,t \in S_{xy}} g(s,t)$$

Such filter smooths local variations in an image, thus reducing noise and introducing blurring. This filter is well suited for random noise like Gaussian, uniform noise.

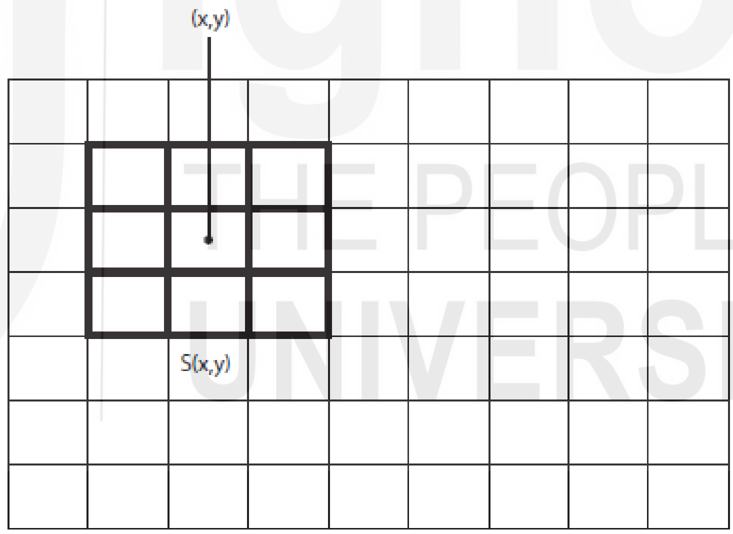
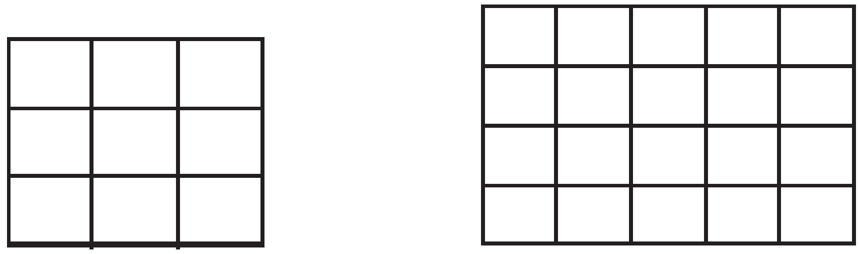


Fig. 15: Graphic Illustration of Sub-Image in An Image



(a) $S(x,y)$ of size 3×3 (b) $S(x,y)$ of size 5×5

Fig. 16: Sub-Image of Various Sizes

Thus, new value at (x,y) in image in Fig. 17 is

$$\{g(s,t)\} = 19 = [30+10+20+10+250+25+20+25+3] = 46.7 \approx 47$$

30	10	20
10	250	25
20	25	30

→

×	×	×
×	$46.7 \approx 47$	×
×	×	×

Fig. 17: Example of Mean Filtering.

Let us apply this in the following example.

Example 1: Show effect of 3×3 mean filter on a simple image in Fig. 18 (a) and Fig. 19 (a)

Solution: As explained in Sec. 7.2, a 3 × 3 mean filter is overlapped with image and output for that particular pixel is derived and then filter centre is moved to the next pixel. We generate a lower size image because the filter mask doesn't overlap fully on first and last row and column.

(a)

0	0	0	0	0
0	0	0	1	1
0	0	1	1	1
0	0	1	20	1
0	0	1	1	1

Mean filter →

(b)

	$1/9$	$3/9$	$5/9$	
	$2/9$	$24/9$	$27/9$	
	$3/9$	$25/9$	$28/9$	

Fig.18: Input and Output Image.

Mean filter removes random noise by introducing blurring. Random noise value of 20 in Fig. 18 (a) is removed from the resultant image (b). But, importantly the edge is also diluted and blurred. In the second image Fig. 19 (a), which has fairly constant values, the pixel values remain more or less same in Fig. 19 (b).

(a)

0	1	2	3	4
5	6	7	8	9
5	5	5	9	9
5	5	5	9	9
5	5	5	9	9

Mean filter →

(b)

	4	5	6	
	4	6.5	8	
	5	6	8	

Fig.19 Input and Output Image.

Gaussian noise is added to the input image Fig. 20 (a). $3\times3, 5\times5$ and 7×7 mean filters are applied to the noisy image and the output images are displayed in Fig. 20 (b), (c), (d). As it is clear from the output, 3×3 filter (Fig. 20 (b)) does not remove the noise completely. Noise is still seen in the image but blurring is less. In 5×5 (Fig. 20 (c)) filtering more noise is removed but image gets blurred. In 7×7 (Fig. 20(d)), too much blurring is seen in the output.

(a) Original Image

(b) Filtered Image by Mean Filter 3×3

(c) Filtered Image by Mean Filter 5×5

(d) Filtered Image by Mean Filter 7×7.

Fig. 20

Let us discuss median filter.

9.5.1 Median Filters

Median filter replaces the pixel value by the median of the pixel values in the neighbourhood of the centre pixel (x, y). The filtered image is given by

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s, t)\}$$

Fig. 21 shows the procedure of applying 3×3 median filter on an image. As impulse noise appears as black (minimum) or white (maximum) dots, taking median effectively suppresses the noise.

Thus, median filter provides excellent results for salt and pepper noise with considerably less blurring than linear smoothing filter of the same size. These filters are very effective for both bipolar and unipolar noise. But, for higher noise strength, it affects clean pixels as well and a noticeable edge blurring exists after median filtering.

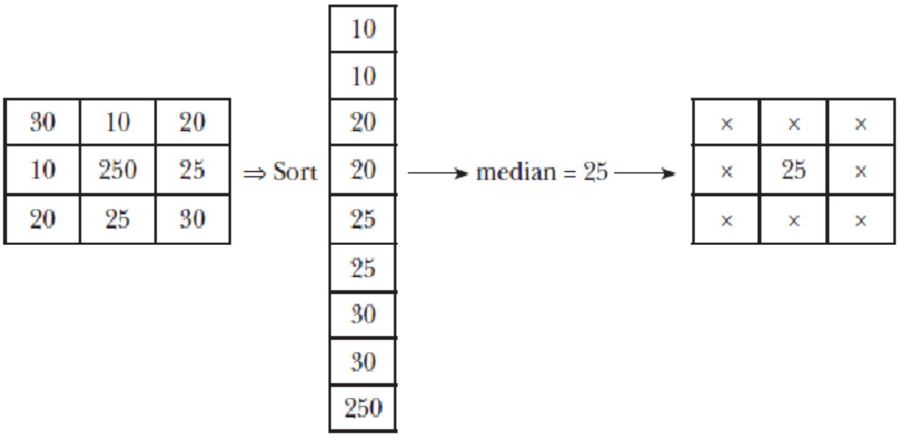


Fig. 21: Example of Median Filtering.

To understand this clearly, see the following example.

Example 2: Show the effect of 3×3 median filter on a simple image given in Fig. 22 (a) and Fig. 22 (b).

128	128	128	128	128
128	0	128	128	128
128	128	128	128	128
128	128	128	128	128
128	128	128	128	128

(a) Input image

Median filter →

	128	128	128	
	128	128	128	
	128	128	128	

(b) Output image

Fig. 22

Solution: When a 3×3 median filter is implemented, all 9 pixels around 'hotspot' are arranged in ascending/ descending order. Center pixel is taken as output and center pixel is replaced by the output. This process is repeated for the entire image.

128	128	128	0	128
128	0	128	128	128
0	0	255	255	255
0	0	128	255	0
128	0	0	0	128

(a) Input image

Median filter →

	128	128	255	
	0	128	255	
	0	0	128	

(b) Output image

Fig. 23

***

It is clear from Example 2, Fig. 22 (a) and Fig. 22 (b) that if noise strength is low in noisy image, output is completely clean. But if noise strength is more (more number of noisy pixels in the image), output is not completely noise free as can be seen in Fig. 23 (a) and Fig. 23 (b).

Let us see the effect of the median filter.

Salt and pepper noise is added to an image given in Fig. 24 (a), noisy image is shown in Fig. 24 (b). 3×3 mean filter and 5 × 5 median filter is applied on it. As it is clear from the result, (Fig. 24 (c) and Fig. 24 (d)) mean filter is not effective in removing salt and pepper noise. But median filter completely removes salt and pepper noise without introducing any blur.

(a) Original Image

(b) Noisy Image

(c) Filtered Image with Mean Filter

(d) Filtered Image with Media Filter

Fig. 24

Salt and pepper noise with density of 0.3 is added to an image. The noisy image (Fig. 25 (a)) is filtered using $3\times3, 5\times5$ and 7×7, median filter. The results in Fig. 25 (b), (c), (d) show that 3×3 median filter is unable to remove the noise completely as the noise density is high. But 5×5 and 7×7 median filters remove noise completely but some distortions are seen specially in Fig. 25 (d).

(a) Noisy Image

(b) Filtered Image with 3×3 Median Filter

(c) Filtered Image with 5×5 Median Filter

(d) Filtered Image with 7×7 Median Filter

Fig. 25

Now, in the following section, we shall discuss noise reduction.

9.6 PERIODIC NOISE REDUCTION

Periodic noise is spatially dependent noise and it occurs due to electrical or electromagnetic interference. It gives rise to a regular noise pattern in an image. Frequency domain (fourier domain) techniques are very effective in removing periodic noise. Basic steps in frequency domain filtering remain same as discussed in Unit 8. Here, we are discussing two frequency domain filters; namely band reject filter and band pass filter.

9.9.1 Band Reject Filter

Removing periodic noise from an image involves removing a particular range of frequencies from the image. Transfer function of ideal band reject filter is

$$H(u,v) = \begin{cases} 1 & D(u,v) < D_0 - \frac{W}{2} \\ 0 & D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2}, \\ 1 & D(u,v) > D_o + \frac{W}{2} \end{cases} \qquad (7)$$

where W is the width of the band (band width), D_0 is its radial centre and $D(u,v)$ is the distance from the origin and is given by

$$D(u,v) = \left[\left(u - \frac{M}{2} \right)^{1/2} + \left(v - \frac{N}{2} \right)^{1/2} \right]$$

$D(u,v)$ is the distance measured from the point (u,v) to the centre $\left(\frac{M}{2}, \frac{N}{2} \right)$. If size of an image is $M \times N$, then the centre is at $\left(\frac{M}{2}, \frac{N}{2} \right)$.

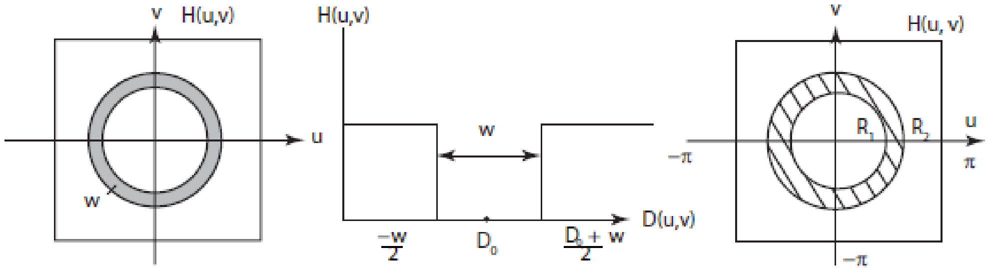


Fig 26: Frequency response of ideal band reject filter

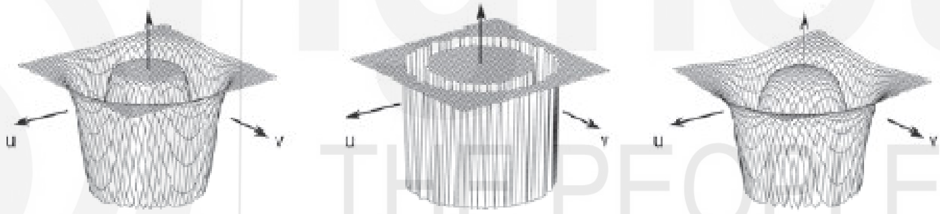
Transfer function of butter worth band reject filter of order 'n' is given by

$$H(u,v) = \frac{1}{1 + \left[\dfrac{D(u,v)W}{D^2(u,v) - D_0^2}\right]^{2n}} \tag{8}$$

Gaussian band reject filter is given by

$$H(u,v) = 1 - e^{\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)W}\right]} \tag{9}$$

Fig. 27 gives the plots of ideal, butterworth and gaussian band reject filters.



(a) Ideal **(b) Butterworth (order 1)** **(c) Gaussian**

Fig. 27: Plots of Band Reject Filters

9.9.2 Band Pass Filter

Band pass filter performs just opposite to band reject filter. The transfer function of band pass filter can be obtained from band pass filters.

$$H_{bp}(u,v) = 1 - H_{br}(u,v), \tag{10}$$

Where, H_{bp} is transfer function of band pass filter and H_{br} is transfer function of band reject filter.

Ideal band pass filter is given by

$$H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 - \dfrac{W}{2} \\ 1 & D_0 - \dfrac{W}{2} < D(u,v) < D_0 + \dfrac{W}{2}, \\ 0 & D(u,v) \ge D_0 + \dfrac{W}{2} \end{cases} \tag{11}$$

Where $D(u,v)$ is the distance from origin, W is the band width
D_0 is the radial centre or the cut off frequency.

Fig. 28 shows the transfer function of ideal band pass filter.
Butterworth band pass filter of order 'n' is given by

$$H(u,v) = 1 - H(u,v)_{\text{butterworth band reject}} \tag{12}$$

$$H(u,v) = 1 - \frac{1}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}} = \frac{\left[\dfrac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}{1+\left[\dfrac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}} \tag{13}$$

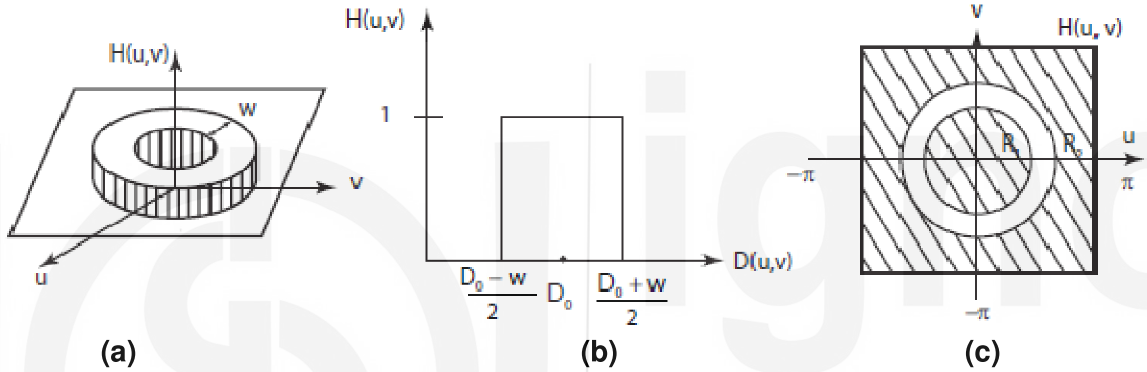


Fig. 28: Frequency Response of Ideal Band Pass Filter

Similarly, Gaussian band pass filter is given by

$$H(u,v) = 1 - H(u,v)_{\text{gaussian band pass}}$$

$$= 1 - \left[1 - e^{-1/2\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}\right]$$

$$= e^{-1/2\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

Now, let us summarise what we have discuss in this unit.

9.7 SUMMARY

We have discussed following points in this unit:

1. Sources of degradation.
2. Difference between enhancement and restoration.
3. Image degradation/restoration model.
4. Various types of noises with their pdfs.
5. Mean and median filters for noise reduction

6. Band reject and band pass filters for periodic noise reduction.

9.8 SOLUTIONS AND ANSWERS

E1) Image degradation can happen due to

a) **Sensor distortions:** Involves quantization, sampling, sensor noise, spectral sensitivity, de-mosaicking, non linearity of sensor etc.

b) **Optical distortions:** are geometric distortion, blurring due to camera mis-focus.

c) **Atmospheric distortions:** are haze, turbulence etc.

d) **Other distortions:** Low illumination, relative motion between object and camera etc.

E2) Fig shows the block diagram of degradation/restoration model. Degradation function $h(x, y)$ and noise $n(x, y)$, operate on input image $f(x, y)$ to generate a degraded and noisy image $g(x, y)$.

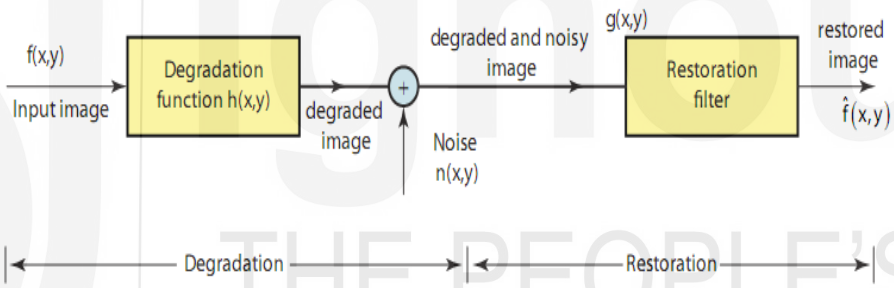


Fig. 29: Block diagram of degradation/restoration model

$f(x, y) =$ original image

$h(x, y) =$ degradation function

$n(x, y) =$ additive noise

$g(x, y) =$ degraded and noisy image

$\hat{f}(x, y) =$ restored image

E3) Noise is a disturbance that causes fluctuations in pixel values. Pixel values show random variations and can cause very disturbing effects on the image. Thus suitable strategies should be designed to model and remove/ reduce noise. Major source of noise in digital images is during image acquisition. Non-ideal image sensors and poor quality of sensing elements contribute to majority of noise. Environmental factors such as light conditions, temperature of atmosphere, humidity, other atmospheric disturbances also account for noise in images. Transmission of image is also a source of noise. Images are corrupted with noise because of interference in the channel, lightning and other disturbances in wireless network. Human interference also plays a part in addition of noise in images.

Properties of Noise

Spatial and frequency characteristics of noise are as follows:

1) Noise is assumed to be 'white noise' (it could contain all possible frequency components), as such,Fourier spectrum of noise is constant.

2) Noise is assumed to be independent in spatial domain. Noise is '**uncorrelated**' with the image, that is, there is no correlation between pixel value of image and value of noise components.

Based on noise properties and types of noise, different filters are used to reduce/remove noise.

E4) **Gaussian Noise**

Gaussian noise model is most frequently used in practice. The PDF of a Gaussian random variable 'z' is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(z-u)^2}{2\sigma^2}},$$

where $z =$ intensity/grey level value
$\mu =$ mean (average) value of z
$\sigma =$ standard deviation

Rayleigh Noise

Radar range and velocity images typically contain noise that can be modeled by the Rayleigh distribution. Rayleigh distribution is defined by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

Mean density is given $\mu = a + \sqrt{\pi b/4}$ as $\sigma^2 = \frac{b(4-\pi)}{4}$

Erlang (Gamma) Noise

Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-z} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

a and b are positive integers. Mean density is given by

and variance $\sigma^2 = \frac{b}{a^2}$ is $\mu = \frac{b}{a}$

Uniform Noise

Uniform noise is specified as

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Then mean and variance of uniform noise is given by

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Impulse (Salt and Pepper) noise

Impulse (salt and pepper) noise is specified as

$$p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \end{cases}$$

UNIT 10

IMAGE RESTORATION-II

Structure **Page No.**

10.1 INTRODUCTION

Image restoration is the process of retrieving an original image from degraded image. The idea is to obtain an image as close to the original image as possible. This is possible by removing or minimizing degradations. This is often difficult in case of extreme noise and blurs, and often called an **inverse problem**. An inverse problem aims to find the cause and extent of degradation.

In the previous unit, we have understood about the degradation in images. Restoration involves modeling of these degradations and applying inverse process to recover the original image. For the restoration process, it is mandatory that we estimate the degradation process accurately. Else we will not be able to remove it.

Now, we shall list the objectives of this unit. After going through the unit, please read this list again and make sure that you have achieved the objectives.

Objectives

After studying this unit, you should be able to

- estimate degradation function

- apply Inverse filtering
- apply Wiener filtering

10.2 ESTIMATION OF DEGRADATION FUNCTION

In order to restore the image, we need to estimate the degradation function. There are three principal ways to estimate the degradation function to be used in restoration:

1) Observation
2) Experimentation
3) Mathematical modelling

Once the degradation function has been estimated, then, restoration is a de convolution process also called **Blind Deconvolution.**

10.2.1 Observation

In restoration using observation, we assume that an image $g(x, y)$ is degraded with an unknown degradation function H. We try to estimate H from the information gathered from the image itself. For example, in case of burred image, a small rectangular section of image containing a part of object and background is taken (Fig. 1). To reduce the effect of noise, the chosen part should be such that it shows presence of a strong signal. We try to un-blur that sub-image manually as much as possible and generate $f_s(x, y)$ from $g_s(x, y)$.

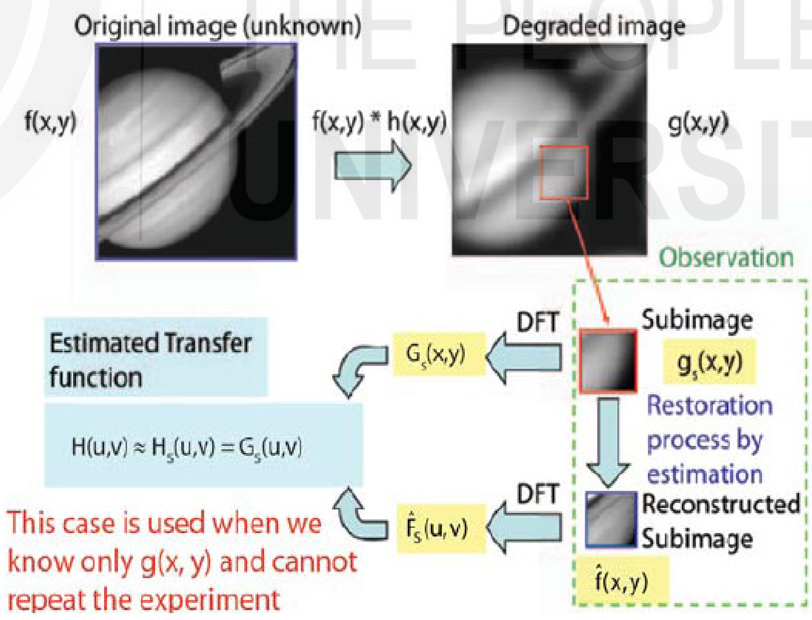


Fig. 1: Estimation by image observation

Here, $g(x, y)$ is the original sub image

$\hat{f}_s(x, y)$ is the restored version of $g_s(x, y)$

Thus, degradation can be estimated for the sub image by

$$H_s(u,v)=\frac{g_s(u,v)}{\hat{F}_s(u,v)}$$

From the characteristics of $H_s(u,v)$, we try to deduce the complete degradation function $H_s(u,v)$ based on the assumption of position invariance. For example, if $H_s(u,v)$ has a Gaussian shape, we can construct $H(u,v)$ on a larger scale with the same (Gaussian)shape. This is a very involved process and is used in very specific situations.

10.2.2 Experimentation

It is possible to estimate the degradation function accurately if the equipment used to acquire the degraded image is available. The processes is shown in Fig. 2.

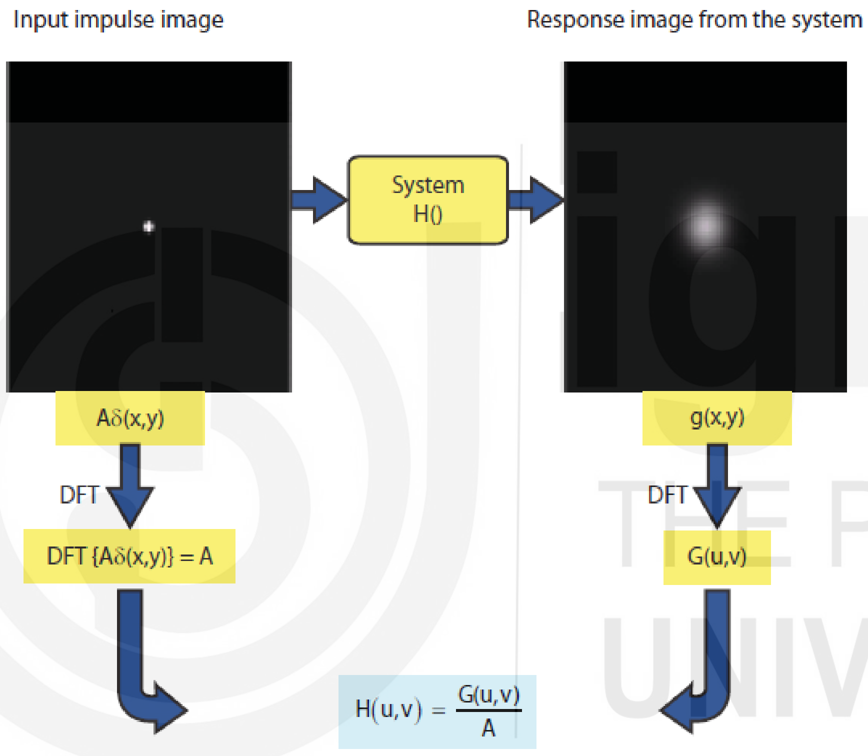


Fig. 2: Estimation by experimentation

We now list the steps performed for estimation.

Step 1: First step is to adjust the equipment by varying the system setting such that the image obtained is similar to the degraded image that needs to be restored.

Step 2: Second step is to obtain the impulse response of the degradation by imaging an impulse using the same system setting, since a linear space – invariant system is completely characterized by its impulse response. An impulse is simulated by a maximally bright dot of light. As shown in the Fig. 2, impulse response is given by

$$H(u,v)=\frac{G(u,v)}{A},$$

where $G(u,v) = DFT[g(x,y)] = DFT$[degraded impulse], and A is the constant describing the strength of the impulse.

10.2.3 Modelling

Modelling is used to estimate the degradation function. Scientists have studied several environmental conditions and other processes which cause degradation, and have formulated several fundamental models for the degradation functions. Degradation model based on atmospheric turbulence blur is given as

$$H(u,v) = e^{-k(u^2+v^2)5/6}$$
$$h(x,y) = e^{-k(x^2+y^2)5/6},$$

where k is a constant that depend on the nature of blur. Various values used for the constant k along with their type of turbulence are given as

$k = 0.0025$ for server turbulence
$k = 0.001$ for wild turbulence
$k = 0.00025$ for low turbulence

This is commonly used in remote sensing and axial imaging applications. Degradation model for uniform out of focus blur (optical blur).

$$h(x,y) = \begin{cases} \frac{1}{L^2}; & \frac{-L}{2} \le x^0, y^0 \le \frac{L}{2} \\ 0\;; & \text{otherwise} \end{cases} \quad (1)$$

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin(ua+vb)\, e^{-j\pi(ua+vb)} \quad (2)$$

Blur can be due to motion (camera and object moving with respect to each other). With suitable values of T, a and b, blurred image can be generated using this transfer function. Fig. 3(a) shows original image and Fig. 3(b) shows blurred image.

(a) Original Image **(b) Blurred Image**

Fig. 3

Try an exercise.

E1) What are the different methods of estimation of image degradation function?

In the following section, we shall discuss inverse filtering.

10.3 INVERSE FILTERING

Inverse filter is also known as reconstruction filter. Deblurring is very important in restoration applications because blurring is visually annoying. It is bad for analysis, and de-blurred images have plenty of applications. Applications include astronomical imaging, law enforcement (identifying criminals), biometrics etc. In this unit we will discuss Inverse filtering, pseudo-inverse filtering and Wiener filtering for deblurring.

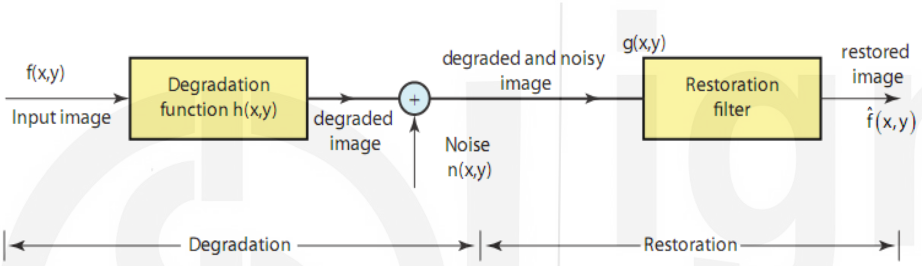


Fig. 4: Block diagram of degradation/restoration model

As in the absence of noise, degradation model becomes,

$$G(u,v) = F(u,v)H(u,v) \tag{3}$$

Simplest approach to restoration is direct inverse filtering, where we can compute an estimate $\hat{F}(u,v)$, of the transform of the original image simply by dividing transform of the degraded image by the degradation function.

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} \tag{4}$$

In presence of noise, degradation model as shown in Fig. 4 becomes

$$G(u,v) = F(u,v)H(u,v) + N(u,v) \tag{5}$$

After applying inverse filtering

$$\hat{F}(u,v) = H_R(u,v)G(u,v) \tag{6}$$

Substituting the values of $H_R(u,v)$ and $G(u,v)$, we get

$$\hat{F}(u,v) = \frac{1}{H(u,v)}[F(u,v)H(u,v) + N(u,v)]$$

$$= \frac{F(u,v)H(u,v)}{H(u,v)} + \frac{N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)} \tag{7}$$

Thus, in case of noisy degraded images, output is also noisy.

If $H(u,v) \approx 0 \frac{N(u,v)}{H(u,v)} \to \infty$

noise is amplified and it dominates the output.

Limitations of inverse filtering are:

1) It is an unstable filter
2) It is sensitive to noise. In practice, inverse filter is not popularly used.

To remove the limitations of inverse filter, pseudo inverse filters are used. Pseduo Inverse filter is defined as,

$$H_R(u,v) = \begin{cases} \frac{1}{H(u,v)} & |H(u,v)| \geq \in \\ 0 & |H(u,v)| < \in \end{cases}$$

where, $\in$ is a small value.

Pseudo-inverse filters eliminates the first problem of inverse filters (un-stability).

As $H(u,v) \to 0, H_R(u,v) = 0.$

Hence, it does not allow $H_R(u,v) \to \infty.$

It is a stable filter. This filter also has a problem of noise amplification. Inverse filtering is applied to the image blurred with a Gaussian (Fig. 5(b)). The output image (Fig. 5(c)) is very close to the original image (a). Then same inverse filtering is used when the blurred image is subjected to additive noise with different strengths. Outputs are shown in Fig. 5(d), (e) and (f). As noise increases, the output of filter goes down.

(a) Original Image

(b) Image Blurred with a Gaussian

(c) Inverse Filter Applied to Noiseless Blurred

(d) Inverse Filter Applied to Blurred Image Plus

(e) Inverse Filter Applied to Blurred Image Plus Noise (0.1)

(f) Inverse Filter Applied to Blurred Plus Noise (0.5)

Fig. 5

Try an exercise.

E2) Explain in brief the inverse filtering approach and its limitations in image restoration.

In the following section, we shall discuss wiener filter.

10.4 WIENER FILTER

Wiener filter is also known as minimum mean square error. This approach includes both the degradation function and power spectrum of noise characteristics in developing the restoration filter. Wiener filter restores the image in the presence of blur as well as noise.

This method is founded by considering image and noise as random variables and objective is to find as estimate $\hat{f}$ of the uncorrupted image f such that the mean square error between them is minimized. This error is given by

$$e^2 = E\{(f - \hat{f})^2\}), \qquad (9)$$

where, $E\{.\}$ is the expected value of the argument. Noise and image are assumed to be uncorrelated. Filter transfer function is given by

$$H_R(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)}, \quad (10)$$

where, $S_{fg}(u,v)$ is the power spectral density of recovered image and noisy image and $S_{gg}(u,v)$ is the power spectral density of noisy image

$$H_R(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)} = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{nn}(u,v)}, \quad (11)$$

where, $H(u,v)=$ degradation function.

$|H(u,v)|^2 = H(u,v)H^*(u,v)$

$S_{nn}(u,v) =$ power spectral density of noise

$S_{ff}(u,v) =$ power spectral density of undergraded image.

If $S_{ff}(u,v), S_{nn}(u,v)$ and $H(u,v)$ is known $H_R(u,v)$ is completely known. Wiener filter works very well for specific applications and is not suitable for general images. For example, if a wiener filter $H_R(u,v)$ is working well for faces, same filter would not work for landscapes etc. Now we discuss several cases to test wiener filter.

Case 1: When, there is no noise $S_{nn}(u,v)=0$

$$H_R(u,v) = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + 0}$$

$$H_R(u,v) = \frac{1}{H(u,v)} = \text{inverse filter}$$

Thus, if there is no noise, Wiener filter = Inverse filter.

Case 2: Noise is present, but there is no degradation, $H(u,v)=1$

$$\therefore H_R(u,v) = \frac{1.S_{ff}(u,v)}{1.S_{ff}(u,v) + S_{nn}(u,v)}$$

$$= \frac{S_{ff}(u,v)/S_{nn}(u,v)}{\frac{S_{ff}(u,v)}{S_{nn}(u,v)} + 1}$$

$$= \frac{SNR}{SNR+1}$$

$$SNR = \text{Signal to noise ratio} = \frac{S_{ff}(u,v)}{S_{nn}(u,v)} = \frac{\text{PSD of signal}}{\text{PSD of noise}}$$

$$\Rightarrow SNR >> 1 \;\; H_R(u,v) \approx \frac{SNR}{SNR} = 1$$

a) If signal to noise ratio is high

Thus, if SNR high, wiener filter acts like pass band and allows all the signal to pass through without any attenuation.

b) If $SNR << 1$, if Signal to noise ratio is low, then

$$H_R(u,v) = \frac{SNR}{1} = SNR$$

$$= \text{a very low value}$$

$$\approx 0$$

Thus, if SNR is low and noise level very high, $H_R(u,v) \approx 0$, acts as a stop band for signal and doesn't allow signal to pass, thus attenuating noise. If noise is high in the signal, wiener filter reduces it after filtering.

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2}$$

SNR gives a measure of the level of information bearing signal power (i.e. of the original, undegraded image) to the level of noise power. Images with low noise tend to have high SNR and conversely, the same image with higher noise level has a high SNR.

The mean square error is given by

$$MSE = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - \hat{f}(x,y)]^2$$

Here, $f(x,y)$ is the original image and $\hat{f}(x,y)$ is the restored image.

Wiener filter is also called minimum mean square error (MMSE) or Least square (LS) filtering because it minimizes the error between the image and its estimate.

$$H_R(u,v) = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{nn}(u,v)}$$

$$= \frac{H^*(u,v)}{|H(u,v)|^2 + \left(\frac{S_{nn}(u,v)}{S_{ff}(u,v)}\right)} \longrightarrow \text{difficult to estimate}$$

Wiener filter in this form is not very useful, as it is difficult of estimate noise power spectrum S_{nn} and undergraded image power spectrum S_{ff},

$S_{nn}(u,v) \rightarrow$ difficult to estimate

$S_{ff}(u,v) \rightarrow$ difficult to estimate

To solve this, we can do an approximation

$\frac{S_{nn}(u,v)}{S_{ff}(u,v)} \approx k$ (approximated by constant k)

$$H_R(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + k}$$

And estimated restored image

$$\hat{F}(u,v) = H_R(u,v)G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + k}\right] G(u,v)$$

$$= \left[\frac{1}{H^*(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + k}\right] G(u,v)$$

k is chosen experimentally and iteratively for best results. In Fig. 6, small noise is added to a blurred image, which is restored by wiener filter in Fig. 6(b).If the amount of added noise is increased Fig. 6(c), the restored image by wiener filter (Fig. 6(d)) is not good. Thus, it is apparent that the wiener filter only works well when the noise is small.

(a) Blurred Image with Small Additive noise

(b) Image Restored by by Wiener Filter

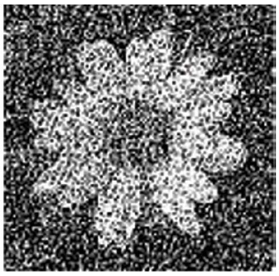
(c) Blurred Image with Increase Additive Noise

(d) Image Restored by Wiener Filter

Fig. 6

(a) Original Image

(b) Blurred Image



(c) Restored image

Fig.7: Applying Wiener Filter

Image is blurred using linear motion = 15, angle = 5 shown in Fig. 7(b). Wiener filter is used to deconvolve the blurred image. The output (Fig. 7(c)) is not clear as the wiener filter does not use any prediction about noise density.

Now, try an exercise.

E3) Discuss the minimum mean square error (Wiener) filtering.

Now, we summarise what we have studied in the unit.

10.4 SUMMARY

We have discussed the following points:

1. Methods of estimation of degradation function.
2. Inverse filtering.
3. Wiener filtering.

10.5 SOLUTIONS AND ANSWERS

E1) There are three methods of estimation of degradation function:
  a) Observation
  b) Experimentation
  c) Modelling

E2) In presence of noise, degradation model as shown in figure 4 becomes

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

After applying inverse filtering

$$\hat{F}(u,v) = H_R(u,v)G(u,v)$$

Substituting values of $H_R(u,v)$ and $G(u,v)$

$$\hat{F}(u,v) = \frac{1}{H(u,v)}[F(u,v)H(u,v) + N(u,v)]]$$
$$= \frac{F(u,v)H(u,v)}{H(u,v)} + \frac{N(n,v)}{H(u,v)}$$
$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Limitations of inverse filtering are:

1) It is an unstable filter
2) It is sensitive to noise. In practice, inverse filter is not popularly used.

E3) This approach includes both the degradation function and power spectrum of noise characteristics in developing the restoration filter. Wiener filter restores the image in the presence of blur as well as noise.

This method is founded by considering image and noise as random variables and objective is to find as estimate $\hat{f}$ of the uncorrupted image f such that the mean square error between them is minimized. This error is given by

$$e^2 = E\{(f - \hat{f})^2\})$$

Where $E\{.\}$ is the expected value of the argument. Noise and image are assumed to be uncorrelated.

$$\Rightarrow \boxed{H_R(u,v) = \frac{S_{fg}(u,v)}{S_{gg}(u,v)}}$$

$$\begin{aligned}\hat{F}(u,v) &= H_R(u,v)G(u,v)\\ &= \left[\frac{H^*(u,v)}{|H(u,v)|^* + k}\right]G(u,v)\\ &= \left[\frac{1}{H^*(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + k}\right]G(u,v)\end{aligned}$$

k is chosen experimentally and iteratively for best results.

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