UNIT 1 INTRODUCTION TO THE FORM OF ARGUMENTS IN MODERN LOGIC

Contents

1.0 OBJECTIVES

The purpose of this unit is to introduce the students to the importance of argument forms in modern logic. Arguments and argument forms are not the same. In our study of modern logic this distinction turns out to be crucial. This is so, mainly because all the Rules of Inference refer to argument forms rather than to arguments. This will become evident as we go along. In short, this block is designed to introduce the students of logic to the symbols and techniques of modern logic.

1.1 INTRODUCTION

Since this is the first unit dealing with symbolic logic, we confront a number of questions before we deal with the forms of argument in particular. What is the main concern of logic? What is the context in which symbolic logic was born? What is the history of logic? How do the old and new systems of logic differ? Why do we use symbols? What are the advantages of using symbols? What is the nature of arguments? We shall briefly deal with questions of this kind before we speak of argument forms proper.

The main question in logic, whether modern or ancient, has always been:

> Does the conclusion *follow from, (= a consequence of)*, the premise(s)?

We need a general theory of inference to answer effectively this prime question of deductive reasoning. Explanation of the relation between the premises and the conclusions in deductive arguments, on the one hand and discovery of techniques to distinguish between valid and invalid deductive arguments, on the other, constitute such general theory. Two great logical theories have emerged in order to achieve these goals: The first is Aristotelian (classical) Logic, and the second is symbolic (modern) logic. The latter is an extension of the former. Though both these bodies of logical theory aim at the discovery of truth, they adopt different techniques. This is one difference. The second difference consists in the concept of proposition itself. Aristotle restricted the meaning of proposition to *subject-predicate form*, which is why it is called *predicate logic* whereas the second extends the concept of proposition to include simple and what is ordinarily understood in grammar as compound sentence. Inclusion of relational proposition brought logic and mathematics much closer. Since the technique invented by modern logic is much more advanced than the technique adopted by Aristotelian system in terms of performance, modern logic achieved what Aristotelian system could not. In this unit and in the units to follow a brief exposition of methods will find the place they richly deserve.

The answer to the question whether the conclusion *follows* from the premise(s) is not at all easy to deal with. This is so because of the *linguistic fallacies* such as equivocation, amphiboly, metaphorical styles, and so on. That is to say, there are peculiarities in natural language (English or any other) that make exact logical analysis rather difficult: Words may be vague or equivocal, the construction of arguments may be ambiguous, metaphors and idioms may be confusing or misleading, and emotional appeals may distract. Modern logic overcomes these difficulties with the introduction of an *artificial language.* The symbols themselves are artificial in the sense that they do not belong to any natural language. Symbolic logic achieved the breakthrough when it formulated sentences of arguments in this language. This is the context in which symbolic logic was born.

Symbols help us to get to the heart of the argument, unlike the natural language. It makes our logical operations mechanical and easy. (It is like using Indo-Arabic numbers 1, 2, 3, … instead of the Roman numerals I, II, III, etc. It is easier to multiply 113 by 9 than to multiply CXIII by IX). Symbols thus greatly facilitate our thinking about arguments and argument forms. Though it may sound paradoxical, symbolic language helps us to accomplish some intellectual tasks without even having any knowledge of the content of statements. Even if linguistic difficulties are thus overcome, the question of deciding the validity or invalidity of arguments remains, of course.

1.2 A SHORT STORY OF LOGIC

The second point of clarification has to do with the history of Logic. Logic was first systematized into a science by Aristotle, the Greek philosopher, in the fourth century BC. His contribution had been regarded as the last word in logic for several centuries. Aristotle's contemporaries and the medieval scholastic thinkers followed him with some cosmetic changes here and there. But nothing significant was added to the Aristotelian (Traditional) logic.

In the seventeenth century G. W. Von Leibniz, a mathematician and philosopher, however, felt that Aristotelian logic needed some modification. His suggestion was a prelude to the path which the development of logic took during subsequent centuries. It was only in the nineteenth century that the logicians began actualizing the ideas conceived by Leibniz. Then on, the development of logic has been unprecedented. This was due to the fast development in mathematics and its extensive dependence on logic, thanks to a host of philosophers of mathematics like Boole, Russell, Frege, etc. to name a few. It must be remembered that dissent voice was not absent. Philosophers like Poincare did oppose this influential school of thought.

This particular revelation may be surprising, not just interesting for us. They established that logic is the foundation of mathematics. This is the principal thesis of *Principia Mathematica* written by Bertrand Russell and A. N. Whitehead. Emphasis on this relation between logic and mathematics may induce the feeling that one should have a mathematical background in order to understand symbolic logic. But this belief is not really well-founded. However, sound knowledge of mathematics is, surely, most useful.

1.3 CLASSICAL LOGIC AND SYMBODLIC LOGIC

Though these two systems differ with regard to the meaning of proposition which is of fundamental importance, the fact is that modern logic makes explicit what was implicit in Aristotelian logic. So, it may be said that the difference between the old and the new is one of *degree* rather than of *kind*. Some examples will easily make this disclosure clear. The difference between an adult and infant is a matter of degree. But the difference between a boy and a girl is in kind. But the difference in degree, between the ancient and modern logic, is enormous since particular class of symbols used by symbolic logic makes logic an immeasurably more powerful tool for analysis and deduction. This class of symbols is what is known as sentential connective. Of course, there are other symbols too which played crucial role in the development of logic.

How did it happen? Surely, it did not happen overnight. It is not revolution, but evolution that took place. As mentioned earlier, the performative ability of symbols achieved this feat. The special symbols in modern Logic permit us (i) to exhibit with greater clarity the logical structures of arguments obscured by their formulation in ordinary language, (ii) to divide more easily arguments into valid and invalid, for in it the peripheral problems of vagueness do not arise; and (iii) establish the nature of deductive argument. Though we have said that both symbolic logic and traditional logic are basically the same, these differences have to be taken note of. a) Traditional logic takes the *terms* (in a proposition) as the basic unit of analysis and is concerned with their relation. Symbolic logic takes proposition as the basic unit and is concerned with the relation between propositions. b) Symbolic logic, while dealing with argument forms, uses symbols instead of propositions which made the task much easier. It may be noted that we do not, for practical purposes, make a distinction between sentences, statements, and propositions. We use them interchangeably, though a clear distinction can be made between them since a sentence is always a part of language while a proposition is what a sentence in any language means.

1.4 WHY USE SYMBOLS?

As we have already said, the use of symbols is helpful i) to avoid peripheral linguistic difficulties, ii) to economize space and time needed for writing, iii) to restrict the attention needed for grasping the meaning of long sentences or equations, and so on. This distinct advantage explains why various sciences have developed their own symbolic language. Thus, for example, in mathematics the equation

$$A \times A \times A\ A \times A \times A \times A \times A \times A \times A \times A \times A = B \times B \times B \times B \times B \times B \times B$$

is expressed more briefly and intelligibly as $A^{12} = B^7$. Logic too has evolved technical notations to achieve the goal. Aristotle used certain abbreviations to facilitate his own investigations. But then these are symbols which can perform only at elementary level. For that matter all terms are symbols only. Therefore what matters is the performative ability of symbols.

Modern Logic, however, introduced many more symbols. Such a step enabled logicians to simplify the most complex argument. Simplicity does not mean that something is devoid of content. It only means that an argument is capable of being tested with minimum number of Rules and within shortest possible time. In fact accomplishment of this task requires something like creativity. What is the value of all this exercise, it may be asked. The answer is simple. When mistakes are easily detected, they are less likely to be made.

1.5 THE NATURE OF ARGUMENTS

Having clarified some of the issues raised at the beginning of this unit, we now pass on to the nature of arguments. This is better understood when it is contrasted with argument forms. What is an *Argument?* An argument is a group of sentences where one sentence is claimed to follow from others, which are regarded as supplying conclusive evidences for its truth. Every argument has a *structur*e, viz. premises and conclusion. Premises provide support to the conclusion. Therefore premises can be regarded as evidences based on which conclusion is accepted. All arguments involve the claim that their premises provide evidence for the truth of conclusions. But it is important to note that only *deductive argument* claims that the premises provide *absolutely conclusive evidences* for the truth of the conclusion. This is the reason why deductive arguments are characterized as 'valid' or 'invalid.' However, *inductive argument* claims that the premises constitute *some* evidences for the conclusion. Therefore, the characterization 'valid' & 'invalid' cannot properly be applied to inductive arguments. Here our main concern is with deductive arguments. A deductive argument is valid when the premises and the conclusion are so related as to make it absolutely impossible for the premises to be true unless the conclusion is true too. The task of deductive logic is a) to clarify the nature of the relation which holds between premises and the conclusion in a valid argument, and b) to provide techniques for distinguishing valid from invalid arguments.

1.6 TRUTH AND VALIDITY

Truth and falsity are properties of propositions whereas validity and invalidity are properties of arguments. This leads us to an important question; what is the relation between the validity or invalidity of an argument and the truth or falsity of its premises and the conclusion? The answer to this question has two parts.

A) Valid arguments with true propositions:

Here is an example.

All mammals have lungs.

All bats are mammals.

∴ All bats have lungs.

An argument may contain only false propositions and be still valid.

All mammals have wings.
All trout are mammals.
∴ All trout have wings.

This is valid. For, what it affirms is; if the premises are true, then the conclusion must be true, even though, as a matter of fact, they are all false.

Note: These two examples of arguments show that valid arguments may or may not have true conclusions. Therefore the validity of an argument does not guarantee the truth of its conclusion. However, the truth of the conclusion does guarantee the validity.

B) Invalid arguments with true propositions:

If I am the prime minister of India, then I am famous.
I am not the prime minister of India.
∴ I am not famous.

Here it is clear that although both premises and conclusion are true the argument is invalid. This can be shown to be invalid by comparing it with another argument of the same form.

If Amitabh Bachchan is the prime minister of India, then he is famous.
Amitabh Bachchan is not the prime minister of India.
∴ Amitaba Bachchan is not famous.

This is invalid since its premises are true but its conclusion is false.

Last two examples show that although some invalid arguments have false conclusions, not all of them are so. The falsehood of its conclusion does not guarantee the invalidity of an argument. However, the invalidity of an argument does guarantee the falsehood of the conclusion. But the falsehood of conclusion does guarantee that either the argument is invalid or at least one of its premises is false. There is asymmetry involved between validity and invalidity which must be noticed. Hence there are two conditions that an argument must satisfy to establish the truth of its conclusion: (a) it must be valid. In this case logician is concerned with validity even for arguments whose premises might be false; (b) all premises must be true. (It is the task of scientific inquiry to determine the last condition).

Check your progress I.

Note: Use the space provided for your answers.

Examine the following arguments.

1) Analyze the features of traditional logic and modern logic.

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....
....

2) Distinguish between truth and validity.

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1.7 ARGUMENT FORMS

Argument forms are important in modern logic. For the Rules provided by the modern logicians to test the validity of arguments are based primarily on the *form* of the argument and not on its content. As we have already said, the primary task of deductive logic is to distinguish valid arguments from invalid ones. We have already shown above that if the premises of a valid argument are true, then the conclusion *must* be true. We have also seen that if the conclusion of a valid argument is false, at least one of the premises must be false. In other words, the premises of a valid argument give *incontrovertible proof* of the conclusion drawn. Now we need to make this formal account of validity more precise. In order to do this we introduce the concept of *argument form.* Let us consider two examples, which evidently have the *same* form:

1.
If Tagore wrote the plays attributed to Shakespeare, then Tagore is a great writer.
Tagore is a great writer.
Therefore Tagore wrote the plays attributed to Shakespeare.

Even if we agree with the premises of this hypothetical syllogism, we cannot agree with its conclusion. For, we can clearly see that this argument is invalid. One of the ways of proving its invalidity is to use the method of logical analogy. For we can as well argue:

2. If Nehru was assassinated, then Nehru is dead.
Nehru is dead.
Therefore Nehru was assassinated.

No one will seriously defend this argument because its premises are known to be true and the conclusion is known to be false. Therefore, this argument is obviously invalid. The *form* of this argument is the same as that of the first argument which is invalid. This way of refuting an argument is very effective. This way of refutation is known as *refutation by logical analogy*. This method points to an excellent technique of testing arguments. That is, to prove the invalidity of an argument it is enough to formulate an argument that (i) has exactly the same form as the first and (ii) has true premises and a false conclusion. This method is based on the fact that validity and invalidity are purely *formal* characteristics of arguments. In other words, any two arguments that have exactly the same form are either both valid or both invalid, no matter what the differences are in the subject matter with which they are concerned.

A given argument becomes clear when the simple sentences in it are abbreviated by capital letters. We may thus abbreviate the statements, 'Tagore wrote the plays attributed to Shakespeare,' 'Tagore was a great writer,' 'Nehru was assassinated,' and 'Nehru is dead,' by the letters, T, G, A, and D respectively. In this manner we can easily symbolize the two sample arguments given above as

$$\begin{array}{ll} T => G & \qquad A => D \\ \ \ G & \qquad \ \ D \\ \therefore T & \qquad \therefore A \end{array}$$

Comparison of these two examples illustrates what we mean by argument form.

How do we obtain argument forms? We need a method to obtain them because we base our study on the forms of arguments only rather than on particular arguments having those forms. What applies to the form applies equally well to what conforms to such form. This is a sort of generalization very much akin to mathematical induction. This is made feasible by way of introducing the notion of *variables.* Modern logicians use, to avoid confusion, small or lowercase letters from the middle part of the alphabet, p, q, r, s, … as *statement variables.* A statement variable is simply a letter for which, we substitute a statement. Not only simple sentences but compound sentences also can be substituted for sentence (statement) variables. In the light of these considerations, we can now define what an argument form is: An *argument form* is any array of symbols which contains *sentence* variables (p, q, r, s, t) such that when sentences are substituted for the sentence variables – the same sentence replacing the same sentence variable throughout – the result is an argument. The form of two arguments is as follows:

$$\begin{array}{l} p => q \\ \ \ q \\ \therefore p \end{array}$$

This is an example for argument form. For, when the sentences T and G are substituted for the sentence variables p and q respectively, the result is the first argument given above. Similarly, if we substitute A and D for the sentence variables p and q, the result is the second argument given above. This leads us to the idea of what logicians mean by a *substitution instance:* Any argument that results from the substitution of statements for statement variables in an argument form is called a substitution instance of that argument form. Therefore if the form is invalid, then any argument which subscribes to this form is invalid.

We have become familiar with one argument form. It corresponds to what traditional logic calls mixed hypothetical syllogism. Since we have already considered invalid argument form, it is desirable that we consider now a valid argument form.

3.

If there is nuclear warfare, then humanity perishes.
There is nuclear warfare.
Therefore the humanity perishes.

It is obvious that this argument is valid. If this aspect is not clear, then we shall resort to logical analogy and consider another example.

4.

If there is inflation, then the cost of living goes up.
There is inflation.
Therefore the cost of living will go up.

A cursory look at these arguments shows that they have identical structure. Therefore if one of them is valid, then the other one also must be valid. It is obvious that (4) is valid. It means that (3) also must be valid. Logical analogy is helpful only for a beginner who is not familiar with the irrelevance of matter of argument in a study of deductive logic.

This is not the only argument form we come across. Traditional logic considered several other types of arguments; categorical syllogism, mixed disjunctive syllogism and pure hypothetical syllogism are the other types. All these types correspond to argument forms. Since in the last unit we had studied exhaustively several aspects of categorical syllogism, we can restrict ourselves to conditional arguments. Let us recall these arguments.

Mixed Disjunctive Syllogism:

a) $p \vee q$
$\neg p$
$\therefore\ q$

A disjunctive syllogism which conforms to this form is valid. Otherwise, it is invalid. So we shall consider the form of an invalid argument.

b) $p \vee q$
p
$\therefore\ q$

c) $p \vee q$
q
$\therefore\ p$

In our study of logic we must have a clear perception of what we have to do. We may have to examine several arguments and arguments with plurality of structures. In such a situation we have to reduce every argument to the form to which it corresponds. Proper identification or matching of argument and argument form is necessary. Any mismatch will lead us astray. Therefore this is an important step in our endeavour. This will also explain why we should be familiar with argument forms.

Against this background, we shall examine examples. Though this is a repetition of what we studied earlier, considering the importance of argument form it is useful to do so.

5.

Voters are either indifferent or ignorant.
Voters are not indifferent.

∴ Voters are ignorant.

6.

Men are either humane or boorish.
Men are boorish.
∴ Men are humane.

How should we match? A little care reveals that (a) matches 9 (it is unimportant whether first alternative is denied or second alternative is denied) whereas (b) and 10 match. Any argument is examined in this fashion only.

Consider the form of pure hypothetical syllogism.

a)	b)	c)
$p \Rightarrow q$	$p \Rightarrow q$	$p \Rightarrow q$
$q \Rightarrow r$	$\neg p \Rightarrow r$	$p \Rightarrow r$
$\therefore p \Rightarrow r$	$\therefore \neg q \Rightarrow r$	$\therefore p \Rightarrow r$

We have three argument forms. Among them first two are valid whereas the last form is invalid. In fact except first two forms any other form is invalid. Let us construct arguments which match these forms.

7.

If atmosphere is polluted, then life on this planet becomes extinct.
If life on this planet becomes extinct, then God does not exist.
∴ If atmosphere is polluted, then God does not exist.

8.

If scientists are honest, then science will progress.
If scientists are not honest, then religion is strong.
∴ If science does not progress, then religion is strong.

9.

If politicians are patriots, then the country becomes prosperous.
If politicians are patriots, then democracy does not fail.
∴ If the country prospers, then democracy does not fail.

7 and 8 are valid. Therefore corresponding argument forms are also valid. 9 is invalid. Therefore corresponding argument form also is invalid. In other words, every argument has matter (content) and form. But it is the form that is fundamental from the point of view of validity. Modern logicians usually classify arguments according to the forms the arguments exhibit. Since it is possible, theoretically, to construct any number of arguments it is impossible to consider matter as the theme of deductive logic. Therefore form is the parameter to examine arguments.

As is clear from the above description, variables are s*ymbols which can be replaced***.** There are three types of replaceable variables: class, individual, and sentential.

Class variables are those replaceable by names of classes. For example, cats, horses, mangoes etc. are class variables.

Individual variables are those that are replaceable by the names of individuals. Newton, the tallest animal in the world, the most massive star, etc. are individual names. In other words, general terms signify class variables whereas singular terms signify individual variables. Later we will understand that the difference between these variables contributes to the difference in notations, a point which becomes clear when we undertake a study of quantification.

Sentential variables are those replaceable by the names of sentences. x is regarded as sentential variable and when a proposition consists of 'x', it is called propositional function. Propositional function is neither true nor false. It takes truth-value only when it is given some value. Value is given when the propositional variable is replaced by a proposition. All propositions are constants in contrast with propositional function. It is an accepted practice to represent constants with upper case letters while propositional function is always represented with lower case x.

Similarly, we have already seen what is meant by a substitution instance. Let us clarify it a little more now. A substitution instance is any argument, which results from the substitution of sentences for the sentential variables of an argument form, is said to have that form, or to be a substitution instance of that form. For example, the argument 'U v W and ¬U; ∴W' has the form

p v q and ¬ p, ∴q

1.8 TRUTH-TABLE

The simplest way of understanding argument forms is through truth-table. This is important for one more reason. Construction of truth-table is basic to our study of symbolic logic. We shall construct truth-tables to distinguish between valid and invalid forms.

Mixed hypothetical syllogism: p => q; p; ∴q

p	q	p => q	p	q
1	1	1	1	1
1	0	0	1	0
0	1	1	0	1
0	0	1	0	0

5th column stands for the conclusion. This table discloses that the conclusion is false only when one of the premises is false. In all other circumstances the conclusion is true. This is the condition of any valid inference. The same explanation holds good for the remaining argument forms.

Mixed disjunctive syllogism: p v q; ¬ p, ∴q

p	q	p v q	¬ p	q
1	1	1	0	1
1	0	1	0	0
0	1	1	1	1
0	0	0	1	0

Pure hypothetical syllogism: p => q; q => r; p => r

p	q	r	p => q	q => r	p => r
1	1	1	1	1	1
0	0	0	1	1	1
1	0	0	0	1	0
1	1	0	1	0	0
0	0	1	1	1	1
0	1	1	1	1	1
0	1	0	1	0	1
1	0	1	0	1	1

It is obvious that the conclusion is false only when at least one premise is false. In this particular instance the last but one column shows the truth-values of the conclusion. Therefore this form is a valid form.

Testing an argument form containing n distinct sentential variables requires a truth-table having 2^n rows. How do we construct such a truth-table? It is convenient to do it by following the cyclic method, i.e. practice of simply alternating pairs of 1s and 0s down the extreme right hand initial column, alternating pairs of 1s with the pairs of 0s down the column directly to its left, next alternating quadruples of 1s with quadruples of 0s… and so on. It is important to remember that truth-values do not belong to sentence forms (propositional function) but to sentences.

1.9 KINDS OF SENTENCE FORMS AND SENTENCES

These are of three kinds: tautology, contradiction, and contingent. First, let us consider

(i) *Tautology:* It is a sentence form having only true substitution instance.

Example: p v ¬p

p	¬ p	p v ¬ p
1	0	1
0	1	1

This can be known to be true without empirical investigation. That is, it is necessarily true. Any sentence, which is a substitution instance of a tautologous sentence form, is *formally* true; and is itself said to be a tautology. Example: Balboa discovered the Pacific Ocean (B). In this case, B is to be known empirical, whereas the truth of 'B v ¬ B' is necessarily known.

(ii) *Contradiction:* It is a kind of sentence form having only false substitution instances. (Its substitution instances are also called 'contradictory' or a 'contradiction'). Example: Cortex discovered the pacific (C). This is known to be false empirically (*happens* to be) false whereas 'C and ¬ C' is *formally* false.

'Contradiction' may mean (a) *relation between sentences* (impossible for two sentences both to be true); (b) *Self-contradictory* sentence: (logically impossible for a particular sentence to be true, the sense in which it is presently used by us).

p	¬ p	p and ¬ p
1	0	0
0	1	0

(iii) *Contingent:* These are sentences or sentence forms that are neither tautologous nor contradictory. p, ¬p, p v q, p => q, are all contingent. Their truth-values are not formally determined but depend on what happens to be the case.

1.10 TESTING THE VALIDITY OF ARGUMENT FORMS

One of the methods of testing the validity of an argument is provided: An argument is valid if and only if the conclusion is a consequence of the premises. In other words, it is valid if and only if whenever the premises are true, so is the conclusion. So in order to determine whether a given argument is valid or not, we must reduce the sentences of the argument to their logical forms and run a joint truth - table for both the premises and the conclusion. If the truth - table shows that whenever the premises are true, the conclusion is also true, then the argument is valid, not otherwise.

Check your progress II.

Note: Use the space provided for your answers.

1 Distinguish between tautology and contradiction with examples.

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....
....

2) Examine the role of truth-table in modern logic.

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1.11 EXERCISES

Use truth-table to determine the validity or invalidity of each of the following argument forms:

1) p Λ q / ∴ p
2) p / ∴ p Λ q
3) p v q / ∴ p
4) p / ∴ p v q
5) p / p => q

1.12 LET US SUM UP

Logic has its beginning in the works of Aristotle. Leibniz laid the foundation for modern logic. Aristotelian and modern logic differ with respect to method. Mathematicians contributed to the evolution of modern logic. Symbols play a major role in modern logic. So it is also called symbolic logic. Truth characterizes statements and validity characterizes argument. Argument form and arguments are different. Tautology, contradiction and contingent are the forme of sentences. Truth-table is the most convenient method of determining the equivalent forms of compound propositions.

1.13 KEY WORDS

Tautology: An expression is said to be tautologous if it is true in all circumstances.
Contradiction: An expression is said to be a contradiction if it is false in all circumstances.

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UNIT 2 CONJUNCTION, DISJUNCTION, CONDITIONAL AND BICONDITIONAL

Contents

2. 0 OBJECTIVES

The purpose of this module is to introduce students to the classification and symbolization of sentences in symbolic logic. In this context it may be noted that we use sentences, statements, and propositions interchangeably. Recognition of compound sentence as a distinct class of proposition is a sort of line of demarcation between traditional logic and modern logic, which places symbolic logic on a different pedestal. Therefore through this unit we intend to introduce you to the elements of symbolic logic. Various species of compound proposition are introduced which serve as spring board for further study of logic. Thereby another objective is served. You will become familiar with different techniques which help you to test more complex arguments.

2.1 INTRODUCTION

We have already discussed in the previous unit the importance of argument forms in modern logic. Several classes of proposition constitute arguments. The complexity of arguments is just without bounds. A good deal of groundwork is required before we confront such arguments which is, undoubtedly, an intellectually challenging task. In order to achieve this task we have to familiarize ourselves with distinction between kinds of proposition and several ways in which sentences are combined and more importantly we have to determine the truth-conditions of such sentences accurately. These are the pre-requisites for further analysis.

Modern logic recognizes three kinds of proposition; simple, compound and general. Let us deal with the last kind first. Propositions recognized by classical logic, viz. A, E, I and O are called general in modern logic. Simple sentence in logical sense is equivalent to what is simple in grammar. In other words, a simple sentence consists of one clause only and singular term in the place of subject. Consider these statements.

1 . Rathi is neat.
2 . Rathi is neat and Rathi is sweet.

1 is a simple sentence whereas 2 is a compound sentence. A compound sentence consists of at least two components. Hence compound sentence in logical sense is equivalent to what grammar regards as compound sentence. Of course, the components of compound statement may themselves be compound.

It is important to notice one subtle distinction between compound sentence in grammatical sense and compound sentence in logical sense. This distinction has nothing to do with the structure of sentences but with our perception of sentences. In grammar we are not concerned with the truth-conditions of compound proposition. But in logic it is our primary task. The truth-value of a true compound proposition is TRUE and the truth-value of a false compound proposition is FALSE. There is a technique of determining the truth-value of compound proposition. In effect, the truth-value of a compound proposition is a function of the truth-value of its constituents. Logic which deals with this particular function is called truth-functional logic. Barring a few cases, which are exceptions, in all other cases the truth-value of compound proposition is functional. “A compound proposition is said to be truth-functionally compound if and only if its truth-value is a function of the truth-value of its components.” In some exceptional cases we find compound propositions which are not truth-functionally compound. Consider this proposition, “John believes that lead is heavier than zinc.” This is a *non-truth-functionally compound sentence.* Its truth-value is completely independent of its component simple sentences. Such propositions are not significant in logic. Hence we shall ignore such propositions.

There are different kinds of compound sentences, each requiring its own logical notation. Negation, conjunction, disjunction, conditional (implication), and biconditional are the kinds of compound sentence with which we are concerned. In this module an exhaustive description of the method of determining the truth-condition is attempted.

How are compound propositions formed? Sentences are conjoined using connectives called *sentential connectives.* Symbolic logic has recognized five such connectives; *not, and, or, if…then,* and *if and only if.* These five connectives generate respectively *negation, conjunction, disjunction, implication,* and *biconditional (bicondition)* or *double implication.* Propositions are replaced by lower case letters like p, q, r, etc. or simply p_1, p_2, p_3, etc. Since proposition is the central theme of our study, this is called propositional calculus or calculus of propositions. While propositions are called variables, sentential connectives are called logical constants. Later we will understand that these constants determine the truth-values of compound propositions.

2.2 NEGATION

Negation deserves our special attention because it is a compound proposition in a unique sense though grammatically it is simple only. It is also a pointer to the exact meaning of and also the sense in which we use the term *compound.* Let us consider the following sentence.

3. India is not a member of the UN Security Council.

This is a negative sentence. It is evident that only in grammatical sense it is simple. Why does logic understand this sentence as compound? This way of understanding stands in need of clarification. Negation is not merely a compound sentence. It is truth-functionally compound, i.e. the truth of 3 depends upon the truth of some other statement. We must find out what that statement is.

4. India is a member of the UN Security Council.

Suppose that 4 is true. Then 3 is false. If we suppose that 4 is false, then 3 is true. Therefore we say that 3 is truth-functionally dependent upon 4 and vice versa. Earlier we said that p, q, r,…. are variables. This is so because in their places we can insert any propositions of our choice. If a sentence replaces the variable, then such proposition is replaced by the first letter of first word or any subsequent word. While doing so, we disregard articles or verbs. Further, we must ensure that the same letter is not repeated.

This is one step in the process of symbolization. Second step is very important. We symbolize connectives too.

Connective	symbol
Not	¬

Now 3 and 4 become ¬ I and I respectively.

Since the negation of a true sentence is false and the negation of a false sentence is true, the following truth-table defines the symbol '¬' thus:

Table 1:

p	¬p
1 0	0 1

Hence the Rule of negation is:

> A negation is true if what is negated is false, and is false if what is negated is true.

In the symbolization of negation, it is important to remember that there are other words and phrases besides 'not' such as 'it is false,' 'it is untrue,' and so on. So there are different natural ways of writing negation, such as:

- It is not the case that indiscipline is tolerated.
- It is false that an honest man is a man of millions.
- It is untrue that that Indians are lazy.

2.3 CONJUNCTION

In conjunction sentences are joined by 'and'. The sentences so combined are called *conjuncts.* Sometimes propositions are misleading. The statement given below illustrates the point.

5. Shasi is intelligent and a hardworking student.

You may be tempted to think that 5 is a simple sentence. In reality, it is a compound sentence. The break-up is as follows.

5a. Shasi is intelligent.
5b. Shashi is a hardworking student.

The symbol we use for conjunction in this module is ‘Λ’ (and), and not the dot ‘.’as is customary. There are many other words besides ‘and’ for which the symbol ‘Λ’ is used. Some of these words are; but, yet, both, although, however, moreover, as well as, while, etc. Some examples are given below:

6 Hari is poor, but he is honest. P Λ H
7 It is hot, yet tolerable H Λ T
8 Shasi is intelligent although not very careful. I Λ ¬ C
9 Both Mohan and Mini are students of Logic. M Λ R

Since conjunction is a truth-functionally compound sentence, its symbol **A** is a truth-functional connective. The truth-table of conjunctive proposition provides the truth-condition of conjunctive proposition.

Table 2:

p	q	p Λ q
1	1	1
1	0	0
0	1	0
0	0	0

Since this truth-table specifies the truth-value of p Λ q in every possible case it can be taken as defining the symbol ‘Λ ’
We may take note of one important aspect: Conjunction has mathematical properties.
That is, a conjunctive function is *commutative, associative* (or *distributive) idempotent.*

p Λ q if and only if q Λ p Commutative
p Λ (q Λ r) if and only if (p Λ q) Λ r Associative
p Λ p if and only if p Idempotent

In terms of relation the properties of conjunction can be stated as follows. Symmetry, transitivity and reflexive are the relations which conjunction satisfies. It is symmetric in virtue of its commutative property. It is transitive because if the statement ‘A and B’ is true and the statement C and D’ is true, then the statement ‘A and D’ is true. It is reflexive because if A is true, then A and A is also true. A relation which is symmetric, transitive and reflexive is known as equivalence relation. Therefore conjunction can be said to satisfy the parameters of equivalence relation.

2.4 DISJUNCTION

Disjunction (also called *alternation*) is a combination of two sentences with connective *or* linking the sentences. The two sentences so combined are called *disjuncts* (or *alternatives*). The symbol used for disjunction is 'v' (called wedge). Here are some examples and their symbolic representation:

10 . Either I will send him an email or I will telephone him. M v T
11 . Either it rains or we shall not go for an outing. R v ¬ G
12 . Either A is not honest or B is not telling the truth. ¬ H v ¬ T

The sentential connective 'v' can be used in two senses:

a) Weak or inclusive sense. In this sense it means *not only either-or, but can be both*. Examples:

13. Either Ramu is a cynic or he is a liar.
14. Either Sita is poor or she is sincere.

Let us make this inclusive sense more concrete: We can think of a mother asking her daughter to choose one of the two dresses A or B. The mother wants her daughter to choose one and reject the other. But we will not be surprised if the daughter says that she would take both. In contracts and other legal documents, this weak sense is made explicit by the use of the phrase 'and/or.'

b) Strong or exclusive sense. In this sense disjunction excludes third possibility. This is possible only when the alternatives stated are mutually exclusive and totally exhaustive. Consider this example.

15. Either a line is straight or it is curved.

Let us take a concrete example to explain the exclusive sense of 'or'. Suppose you have a guest and you ask him or her 'what will you have, tea or coffee?' You will be certainly surprised if your guest says 'both.' This means that we use both the inclusive and exclusive senses of 'v' not only in logic but also in our ordinary life.

Latin has two different Words corresponding to the two different senses of 'or': *vel* (inclusive) and *aut* (exclusive). We use 'p v q' in its inclusive sense. A weak disjunction is false only if both of its disjuncts are false. Therefore the Rule is:

a) At least one disjunct is true (weak).
b) One disjunct is true and the other disjunct is false (strong).

Exclusive disjunction is symbolized by A. H. Lightstone in his work '*Set Theory and Real Number System'* in this manner.

$$p \underline{v} q$$

This notation helps us to distinguish strong from weak.
Consider the following disjunctive syllogism:

Arg. 1

U.N.O. will be strengthened or there will be the III World War.
U.N.O. will not be strengthened.
Therefore, there will be the III World War.

It is clear that this argument is valid on *either* interpretation of 'or'. So, we symbolize 'or' by '∨' (wedge) regardless of which sense of 'or' is intended. We can therefore write 'p ∨ q' and define it as:

Table 3:

p	q	p v q
1	1	1
1	0	1
0	1	1
0	0	0

It may be noted here that, like conjunction, disjunction too has the very same mathematical properties. The disjunctive function has *commutative, associative* as well as *idempotent* properties.

p v q if and only if q v p	Commutative
p v (q v r) if and only if (p v q) v r	Associative
p v p if and only if p	Idempotent

Idempotent property is the same as reflexive property in mathematical language. Therefore disjunction satisfies the parameters of equivalence relation.

Now a word about correct *punctuation;* Punctuation becomes relevant when two or more than two sentential connectives are involved. As in mathematics, in logic also *parentheses* perform the function of punctuation. Since mathematics is more familiar to you, we shall begin with an example from mathematics.

a) (5+6) 4
b) 5+ (6) 4

It is obvious that a) and b) are not equal. This is because the positions of parentheses determine the meaning of the given expression. Similarly, in symbolic logic parentheses play a decisive role in determining the exact meaning of expressions. When two or more than two connectives are involved, we should first understand the exact meaning of expression and then use parentheses properly. Otherwise, we will go wrong.

2.5 EXERCISES

So far we have considered negation, conjunction, and disjunction. Before we go further, let us work out some exercises.

1. If A and B are true sentences and X and Y are false, discover the truth-value of the compound sentence $\neg [(\neg A \vee X) \vee \neg (B \wedge Y)]$.

This is how it can be worked out: Since A is true, $\neg A$ is false, and since X is false also, the disjunction $(\neg A \vee X)$ is false. Since Y is false, the conjunction $(B \wedge Y)$ is false and so its negation $\neg(B \wedge Y)$ is true. Hence the disjunction $(\neg A \vee X) \vee \neg (B \wedge Y)$ is true and its negation, which is the original sentence, is false. (We always begin with the innermost component).

Check your progress I.

Note: Use the space provided for your answers.

a) If p is true and q is false, then work out the value of the following expressions:

1. $\neg (p \wedge q)$
2. $(p \vee q) \wedge (p \wedge q)$
3. $\neg (p \wedge q) \vee (p \wedge q)$
4. $\neg (p \vee \neg q) \wedge (q \wedge \neg q)$
5. $\neg (p \wedge p) \vee \neg (q \vee p)$

b) If A and B are true sentences and X and Y are false sentences, which of the following compound sentences are true?

1. $\neg (A \vee X)$
2. $\neg A \vee \neg X$
3. $\neg B \wedge \neg Y$
4. $\neg (B \wedge Y)$
5. $A \vee (X \wedge Y)$
6. $(A \wedge X) \vee Y$
7. $(A \vee B) \wedge (X \vee Y)$
8. $(A \wedge B) \wedge (B \vee Y)$
9. $(A \vee X) \wedge (B \vee Y)$
10. $A \wedge \{X \vee (B \wedge Y)\}$
11. $A \vee (X \wedge (B \vee Y)$
12. $X \wedge (A \vee (Y \vee B))$
13. $\neg (\neg (\neg (A \wedge \neg X) \vee \neg A) \wedge \neg X)$
14. $\neg (\neg (\neg (A \vee \neg B) \wedge \neg A) \vee \neg A)$
15. $\{(X \vee A) \wedge \neg Y)\} \vee \neg \{(X \vee A) \wedge \neg Y\}$
16. $\{A \vee (X \wedge Y)\} \vee \neg \{(A \vee X) \wedge (A \wedge Y)\}$
17. $\{(X \wedge (A \wedge Y)\} \wedge \neg \{(X \vee A) \wedge (X \vee Y)\}$
18. $\{(X \vee (A \vee B)\} \vee \neg \{(X \wedge A) \vee (X \wedge B)\}$
19. $\{X \wedge (A \vee Y)\} \wedge \neg \{(X \vee A) \wedge (X \vee Y)\}$
20. $\{X \vee (A \vee Y)\} \vee \neg \{(X \wedge A) \vee (X \wedge Y)\}$

2.6 IMPLICATION

Conditional statement is a compound statement of the form 'if the train is late, then we will miss the connection.' In the strict sense of the term disjunction also is a conditional statement. However, for the time being we shall restrict ourselves to the form mentioned above. Other words for a conditional statements are implication and hypothetical proposition. Here we will be using them interchangeably. The component before 'then' is called *antecedent* (implicant, or rarely *protasis*) and the component after 'then' is called *consequent* (implícate, or rarely *apodosis*). In an implication, antecedent is said to imply the consequent, or the consequent is said to be implied by the antecedent.

A conditional asserts that *if* its antecedent is true, then its consequent must be true.

While symbolizing implication, it is important to identify antecedent and the consequent correctly. Some examples will be of help:

a. If it rains, then we shall go for a picnic. R => G
b. If it does not rain, then we shall not go for a picnic. ¬ R => ¬ G
c. You will get the job only if you pass the test. P => G

All the following sentence forms are symbolized as
p => q

q only if p
q if p
q provided that p
q on condition that p
q in case p
p hence q
p implies q
Since p, q
p is a necessary condition for q
p is a sufficient condition for q

There are at least four senses in which 'if-then' is used:

- *Logical:* For example, 'If all cats like liver and Dinah is a cat, then Dinah likes liver.'
- *Definitional:* In this sense, the consequent follows from by the very definition of a Word. For example, 'If the figure is a triangle then it has three sides.'
- *Causal:* In this sense, there is a causal connection between and the consequent. For example, 'If gold is placed in *aqua ragia,* then gold dissolves.'
- *Decisional*: 'If you shave your head, then I will change my name.'

How do we find out the meaning which is common to these four senses of 'if-then'? In order to find out an answer to this question, we must ask: What circumstances would suffice to establish the *falsehood* of a conditional? Hence the Rule: A conditional is false only in one case:

Any conditional with a true antecedent and a false consequent is false.

Incidentally, it is important to note that this is the chief characteristic of deductive logic. If all the premises are true and the conclusion is false, then the deductive argument is invalid. A true proposition can imply only a true proposition.

The truth - condition of implication is represented in the form of a table.

Table 4:

p	q	p => q
1	1	1
0	1	1
0	0	1
1	0	0

This table shows that implication is false under only one circumstance i. e., when a false conclusion is derived from a true premise. Under all other circumstances it is true.

Compare any truth-table with any other truth-table. You will notice that the truth-value of any two compound propositions differ though variables remain the same. Compound propositions differ only with regard to sentential connectives. This is an important aspect which proves that only sentential connectives determine the truth-value of compound propositions independent of variables.

Here are some examples worked out. In all cases assume that p is1 and q is 0.

a. (p => q) v p
= (1 => 0) v 1
= 0 v 1
= 1

b. q => (p Λ ¬ q)
= 0 => (1 Λ ¬ 0)
= 0 => (1 Λ 1)
= 0 => 1
= 1

c ¬ (p Λ q) => ¬ q
= ¬ (1 Λ 0) => ¬ 0
= ¬ 0 => 1
= 1 => 1
= 1

Material Implication

We have considered the different senses of ‘if …. then.’ Not all conditional statements need assert one of the four kinds of implication mentioned earlier. Material implication constitutes a fifth type that may be asserted in ordinary discourse as follows: ‘If Gandhiji was a military genius, then I am a monkey’s uncle.’ Conditional proposition of this sort is often used as an emphatic or humorous way of denying its antecedent. So the full meaning of this conditional seems to be the denial that ‘Gandhiji was a military genius’ is true when ‘I am a monkey’s uncle’ is false. What it means is simply this: since the consequent is so obviously false, the conditional must be understood as denying the antecedent.

The use of material implication (=>) as the common and partial meaning is justified on the ground that the validity of valid arguments involving conditionals is preserved when the conditionals are regarded as asserting material implication only. (It must, of course, be admitted that such symbolizing abstracts from or ignores part of the meaning of most conditional sentences. But the justification for doing so is demonstrated by some of the Rules of Inference such as Modus Ponens, Modus Tollens and Hypothetical Syllogism.

However, ‘if … then’ relation, like ‘either or’, is not as simple as it appears. For, the ordinary people and the logicians do not look at it in the same way. For an ordinary person two simple sentences are related conditionally only under two conditions: i) Their meanings must be related; and ii) the consequent must follow from antecedent. However, a logician is not particular about these conditions provided antecedent should not be true if the consequent is false. The logician’s main focus is on the logical properties of an implicative relation.

Though the truth-value of implication is thus worked out indirectly, the problem is not over. For, if we look at the truth-value of implication carefully, there appears to be a paradox:
The first three rows in it are saying that any sentence, true or false, can imply a true proposition. Further, only a false conclusion from a true premise is not admissible. This is the result of the logical properties of an implication. But this is not acceptable to an ordinary person: How can any sentence, true or false, imply any true proposition? This is paradoxical. Similarly, the third row is saying that any sentence, true or false, can be implied by a false proposition. This again is paradoxical.

But we must look at the reason behind these paradoxes: In the cases of conjunction, disjunction, negation, (and, also, biconditional or equivalence, as we shall see later), the ordinary language helps the logicians in framing their truth-values. But they could establish the truth of implication only indirectly. This, however, does not fit in with the ordinary use of ‘if then’, and hence the paradox of material implication. This problem can be solved if we keep the logical standpoint and the ordinary standpoint separate. For logical purposes, we take the former standpoint of ‘if …. then’ called *material implication.*

It is quite rewarding to consider a few examples as exercises. This is the only way to learn logic.

If A and B are true sentences and X and Y are false sentences, which of the following compound sentences are true?

1. X => (X => Y)
2. (X => X) => Y
3. (A => X) => Y
4. (X => A) => Y
5. A => (B => Y)
6. A => (X => B)
7. (X => A) => (B => Y)
8. (A => X) => (Y => B)
9. (A => B) => (¬A => ¬B)
10. (X => Y) => (¬X => ¬Y)
11. (X => A) => (¬X => ¬A)
12. (X => ¬Y) => (¬X => Y)
13. ((A Λ X) => Y) => (A => Y)
14. ((A v B) => X) => (A => (B => X))
15. ((X Λ Y) => A) => (X=> (Y => A))
16. ((A Λ X) => B) => (A => (B=>X))
17. (X => (A => Y)) => ((X => A) => Y
18. (X => (X => Y)) => ((X =>X) => X)
19. ((A => B) => A) => A
20. {(X=>Y) => X} => X

2.7 BICONDITIONAL

A biconditional is a compound statement which is a combination of two sentences. The connective used to obtain this proposition is *if and only if*. For example, 'you will catch the train if and only if you reach the station on time.' is a biconditional statement. This is symbolized as C <=> R. Such sentences are true when both the components have the same truth-value. A biconditional is true in two cases only. Both the components must be either true or false, as is clear from the following truth-table:

Table 5:

p	q	p <=> q
1	1	1
1	0	0
0	1	0
0	0	1

It must be noted that bi-conditional is a conjunction of two implicative propositions.

p if and only if q is logically equivalent to (p => q) Λ (q => p). Truth-table assists us to find out how it is so.

Table 6:

p	q	p <=> q	p => q	Λ	q => p
1	1	1	1	1	1
1	0	0	0	0	1
0	1	0	1	0	0
0	0	1	1	1	1

Compare the truth-values of columns 3 and 5 in each row. Since the values have remained the same, we conclude that p if and only if q is logically equivalent to 'if p, then q and if q, then p'.

Here are some more examples that have similar but logically different forms; and therefore we have to be very careful in symbolizing them:

i) Mr. X will catch the bus if he reaches on time. R => C
ii) Mr. X will catch the bus only if he reaches on time. C => R
iii) Mr. X will catch the bus if and only if he reaches on time. C <=> R

Equivalent Forms:
It is possible to transform compound proposition without changing the meaning of proposition. Such transformation yields equivalent propositions. Let us construct a truth-table for each compound proposition to understand how this works.

Implication:

Table: 7

				Implication	Disjunction	Negation
p	q	¬ p	¬q	p => q	¬ p v q	¬ (p Λ ¬ q)
1	1	0	0	1	1	1 1 0 0
1	0	0	1	0	0	0 1 1 1
0	1	1	0	1	1	1 0 0 0
0	0	1	1	1	1	1 0 0 1

The advantage of truth-value method is obvious. Without any verbal explanation and with least effort it is possible to identify equivalence between propositions. The equivalence relation, which exists between implication and disjunction, is self-explanatory. However, relation with negation requires some clarification. There are two columns under negation, which reflect truth-values. Suppose that we ignore negation sign and corresponding truth-values and consider the last column then we are not considering negation but conjunction. The last column in the absence negation preceding first bracket is the same as the following one:

Table 8

p Λ ¬ q
0
1

0
0

However, the required form is not conjunction but negation. The truth-value of negation form, of course, truth-functionally depends upon the truth-value of conjunction form. Therefore while selecting the column, which corresponds to negation form, we should exercise a little caution.

It is necessary to consider another form of equivalence relation and this is relevant only with respect to implication. Examine this table.

Table: 9

				Implication	Contraposition
p	q	¬ p	¬ q	p => q	¬ q => ¬ p
1	1	0	0	1	1
1	0	0	1	0	0
0	1	1	0	1	1
0	0	1	1	1	1

In this transformation the components are replaced by their complements and transposed simultaneously. It will be an error to just transpose without effecting the change in quality because implication does not satisfy the symmetric property. What happens exactly when p => q becomes q => p? The reader is advised to construct truth-table and see the result.

The reader must be in a position to discover the equivalent form for disjunction. It is plain from table7 that if p, then q (p => q) is logically equivalent to ¬ p or q (¬ p v q). We shall express in the form of equations.

Disjunction Implication Negation of conjunction

(¬ p v q) ≡ (p => q) ≡ ¬ (p Λ ¬q)

The formula is simple. Replace the first disjunct by its negation and simultaneously ‘v’ by ‘=>’. We get implication. Therefore the equivalent form for ‘p v q’ must be ‘¬ p => q.’ Contraposition for disjunctive proposition is superfluous because it satisfies the symmetric property. Let us construct truth-table for the equivalent forms for this proposition.

Table 10

p	q	¬ p	¬ q	p v q	¬ p => q	¬ (¬ p Λ ¬q)	
1	1	0	0	1	1	1	0
1	0	0	1	1	1	1	0
0	1	1	0	1	1	1	0
0	0	1	1	0	0	0	1

It is quite interesting to note that conjunction and biconditional do not have equivalent forms. Truth-table again comes to our rescue to know why it is so. It is sufficient if we consider any one-form, say, implication to know why it is so. If one equivalent form is absent, it is imperative that other forms are also absent.

Table 11

p	q	¬ p	¬ q	p Λ q	p => q	¬ p => q	p => ¬ q	¬ p => ¬q
1	1	0	0	1	1	1	0	1
1	0	0	1	0	0	1	1	1
0	1	1	0	0	1	1	1	0
0	0	1	1	0	1	0	1	1

Except that truth-values of conjunction do not tally with any possible arrangement in implication form, no other explanation is conceivable for the absence of equivalent forms to conjunction. The students are advised to test other forms with respect to disjunctive syllogism to convince themselves of the veracity of this statement.

Biconditional proposition also does not have any equivalent form. The reason is very simple. Biconditional is, in reality, a conjunction two implications. First we shall know why it is a conjunction.

Table 12

p	q	¬ p	¬ q	p <=> q	(p => q) Λ (q => p)
1	1	0	0	1	1 1 1
1	0	0	1	0	0 0 1
0	1	1	0	0	1 0 0
0	0	1	1	1	1 1 1

The method of computing is as follows; first, we shall compute the truth-values of first implication (p => q) and then we will compute the truth-values of second implication (q => p). These two sets of truth-values together determine the truth-value of conjunction. When we compare columns 5 and 7, we will come to know that these two expressions have identical truth-values in all instances. It shows that biconditional is also a conjunctive proposition where the conjuncts themselves are compound propositions. Therefore what applies to conjunction naturally, applies to biconditional also.

Check your progress II.

Note: Use the space provided for your answer.

1. If p is true and q is false, then work out the values of the following statements:

   1. p => (q <=> p)
   2. ¬ p <=> (p => p)
   3. (p => q) <=> (¬ p v q)

2. Which of the following sentences are true?

   a) (New Delhi is the capital of India) or (Rome is the capital of Italy).

   b) (New Delhi is the capital of Spain) and ¬ (Paris is the capital of France).

   c) (New Delhi is the capital of India) Λ ¬ (Paris is the capital of France v New Delhi is the capital of India).

   d) ¬ { ¬ (Stockholm is the capital of Norway Λ Paris is the capital Spain)} v ¬{ ¬(London is the capital of England) Λ ¬ (New Delhi is the capital of Spain)}.

   e) (Paris is the capital of France) v ¬{(New Delhi is the capital of Spain) Λ ¬ (¬Paris is the capital of France Λ ¬ New Delhi is the capital of Spain)}.

3. If A, B, and C are true statements and X, Y, and Z are false statements, which of the following are true?

   1 ¬A v B

   2. (A Λ X) v (B v Y)

   3 ¬(X Λ ¬Y) Λ (B v ¬C)

   4. ¬(X v Y) Λ (¬X Λ Y)

   5. ¬ ((A v B) v ¬ (B Λ A))

4. If A, B, and C are true statements and X, Y, and Z are false statements, determine which of the following are True.

   1. A => B
   2. (A => B) => Z
   3. X => (Y => Z)
   4. ((X => Z) => C) => Y
   5. {(A v X) ¬Y} => ((X => A) => (A => Y))

2.8 LET US SUM UP

There are five kinds of compound propositions. Compound proposition in grammatical sense is different from compound sentence in logical sense. All compound propositions do not have equivalent forms. The truth of any compound proposition is determined by the truth-values of components. Sentential connectives determine the truth-value of compound propositions.

2.9 KEY WORDS

Truth-table: It is a technique with the help of which we determine the truth- values of compound propositions.

Biconditional proposition: biconditional is a conjunction of two implicative propositions.

2.10 FURTHER READINGS AND REFERENCES

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UNIT 3 RULES OF INFERENCE AND THE NATURE OF VALIDITY OF ARGUMENTS

Contents

3.0 OBJECTIVES

The purpose of this module is to help students to familiarize with the Rules of Inference which can be regarded as the pivot of the study of logic. The central theme of logic consists in the classification of arguments. Through this unit this main aim is achieved. In the previous unit we learnt the technique of determining the truth of compound sentences and the decisive role played by sentential connectives. Second objective of this unit is to demonstrate the latent relation between Rules of Inference and the truth of compound sentences. This method is devised to show that the Rules of Inference are demonstratively certain. If the Rules are demonstratively certain, then scrupulous adherence to these Rules produce arguments with unquestionable validity. The aim of this unit is to discover such arguments.

3.1 INTRODUCTION– TOOLS OF TESTING ARGUMENTS

In modern logic an argument is regarded as a sequence of statements. When proof is constructed to test the argument, the proof also takes the same form, which the argument takes. In this type of proof there is correspondence between the scheme of the given argument and the scheme of the proof. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as 'Rules of Inference' in modern logic. Many of these Rules have their origin in traditional logic.

There is a certain way of constructing proof in modern logic. More descriptive method, which consumes both space and time, has given way to much shorter and simpler method. Whatever conclusion can be drawn from any one or two given premises is written on the left hand side (LHS) while the Rule and the premises to which this particular Rule is applied to derive the conclusion used in further proof are written on the right hand side (RHS). Quite often, Rule of Inference is applied to one line only. As an economy measure, instead of premises, corresponding serial numbers are written. Thereby we save time. We must ensure that drawn conclusion, the respective premises and the Rule applied are always juxtaposed. This procedure is the simplest and most economical in terms of time and effort to grasp the argument.

We have learnt in the previous unit the technique of conjoining simple sentences to generate compound sentences, and also we learnt the method of fixing the truth or falsity of such sentences. This knowledge is the pre-requisite for our further study.

We make use of twenty two Rules. Out of them nine are called Rules of Inference. There are ten Rules which are called Rules of Replacement (also can be called Transformation or Equivalence Rules). Reductio ad absurdum or indirect proof, Conditional Proof and the Strengthened Rule of Conditional Proof are the other Rules. The application of Rules is not at random. The unique composition of argument determines the kind of Rule to be applied. We will begin with Rules of Inference and also we shall examine the logical status of two Rules later.

Rules of Inference:

1 *Modus Ponens* (M.P.)

$$\begin{array}{l} p \Rightarrow q \\ p \\ \therefore q \end{array}$$

2 *Modus Tollens* (M.T.)

$$\begin{array}{l} p \Rightarrow q \\ \neg q \\ \therefore \neg p \end{array}$$

3 *Hypothetical Syllogism* (H.S.)

$$\begin{array}{l} p \Rightarrow q \\ q \Rightarrow r \\ \therefore p \Rightarrow r \end{array}$$

4 *Disjunctive Syllogism* (D.S.)

$$\begin{array}{l} p \vee q \\ \neg p \\ \therefore q \end{array}$$

5 *Constructive Dilemma (C.D.)*

$$\begin{array}{l} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ p \vee r \\ \therefore q \vee s \end{array}$$

6 *Destructive Dilemma (D.D.)*

$$\begin{array}{l} (p \Rightarrow q) \wedge (r \Rightarrow s) \\ \neg q \vee \neg s \\ \therefore \neg p \vee \neg r \end{array}$$

7 *Simplification (Simp.)*

$p \wedge q$

$\therefore p$

8 *Conjunction (Conj.)*

p

q

$\therefore p \wedge q$

9 *Addition* (Add.)

p

$\therefore p \vee q$

Copi I. M. has replaced D. D. with another Rule called Absorption.

10. *Absorption* (Abs.)

$p \Rightarrow q$

$\therefore p \Rightarrow (p \wedge q)$

Since it is not possible to dispense with either of these Rules, Copi's decision to replace D.D. with Absorption is not clear. Hence it is obvious that nine becomes ten. A Rule becomes indispensable only because arguments, more often than not, consist of steps with diverse structure, which demand acceptance of different Rules. Of course, law of parsimony stipulates that not a single superfluous law is admissible. Therefore in accordance with this stipulation it must be maintained that if any new Rule is added, it is presumed that the addition is necessitated by the complexity of the argument.

We may have nine or ten Rules. The number is really immaterial. This is so because they are insufficient to test arguments with every conceivable structure. Therefore Rules of Replacement are added. Before listing these Rules we must become familiar with a subtle distinction between these two sets. Suppose that a premise (also understood as line) in an argument consists of several connectives. In such a case, Rule of Inference must be applied to only *whole line.* It will be a mistake to apply to a part of the line. Why is it a mistake? Consider this example.

$$(A \wedge B) \Rightarrow (C \vee D)$$

According to the Rule of Inference even if A or B or both are false, 1st line remains true only irrespective of the truth-value of the second component. Suppose that the Rule of Simplification is applied to the first component. Then suppose that A Λ B becomes A. If A is false, then we are including a false proposition which does not guarantee the truth of the conclusion because when a premise is false the conclusion can as well be false though the argument is valid. Rule of M. P. or M. T. to the 1st line can be applied provided one of the lines consists of either affirmation of the 1st component or denial of the 2nd component without omitting any proposition within parentheses. However, this restriction does not apply to the second set of Rules which will be considered later.

Before we proceed further let us examine the logical status of these Rules. What applies to one Rule also applies to any Rule of Inference or Replacement. So we can restrict ourselves to just two Rules. Consider M.P. The Rule can be put in the form of an expression in this way.

Argument form compound statement

p => q [(p => q) Λ p] => q

p

∴ q

Construct truth-table for R.H.S.

Table1:

1	2	3	4	5	6	7	8	9
p	¬ p	q	¬ q	(p => q)	Λ	p	=>	q
1	0	1	0	1	1	1	1	1
1	0	0	1	0	0	1	1	0
0	1	1	0	1	0	0	1	0
0	1	0	1	1	0	0	1	0

Consider the truth-value of implication on the extreme right (column 8 which is actually called main column in propositional calculus) to determine the truth-value of compound expression. Since it takes the value 1 in all cases, the compound proposition and its corresponding Rule of Inference is a tautology which means that it is true always.

Now examine the expression for C.D.: [(p => q) Λ (r => s) Λ (p v r)] => (q v s).

The truth-table is constructed as follows. Negation of proposition is not considered in this case in order to reduce the number of columns and they are not required in this case.

Table2:

1	2	3	4	5	6	7	8	9	10	11	12
Sl. No.	p	q	r	s	{(p => q)	Λ	(r => s)	Λ	(p v r)}	=>	(q v s)
1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	0	0	1	0
3	1	1	1	0	1	0	0	0	1	1	1
4	1	0	1	1	0	0	1	0	1	1	1
5	0	1	1	1	1	1	1	1	1	1	1
6	1	1	0	0	1	1	1	1	1	1	1
7	1	0	0	1	0	0	1	0	1	1	0
8	1	0	0	0	0	0	1	0	1	1	0
9	0	1	0	0	1	1	1	0	0	1	1
10	1	0	1	0	0	0	0	0	1	1	0
11	0	1	1	0	1	0	0	0	1	1	1
12	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	1	1	1	1	0	0	1	1
14	0	0	1	0	1	0	0	0	1	1	0
15	1	1	0	1	1	1	1	1	1	1	1
16	0	1	0	1	1	1	1	0	0	1	1

There are sixteen rows and in all these rows the truth-value in the 11th column, i.e. main column, is 1 only and hence it is a tautology. Since we have made a random selection of Rules, this conclusion must hold good for all Rules of Inference and Rules of Transformation. The student is advised to test this property in other cases to satisfy natural curiosity

3.2 METHODS OF TESTING THE VALIDITY OF ARGUMENTS

Why Look for an Alternative to the Truth-table Method?

Any attempt to test an argument by the use of logical analogy or by the method of truth-table to see whether there is any substitution instance with true premises and a false conclusion is fraught with difficulties. A given argument is proved invalid if a refuting analogy can be found for it. But discovering such refuting analogies is not always that easy. Fortunately, it is not necessary. For, there is a simpler, and a purely mechanical test for arguments of this kind based on the same principle.

Similarly, testing the validity of arguments by using the truth-table method is simple and convenient only when there are two or three variables. But the situation is very different when there are several variables. Suppose that there are two variables. We require only four rows and eight columns (see table 1). If there are three variables, then we require eight rows and eight columns (provided we do not include the negation of variables). It means that if there are 'n' number of variables, then the length of the rows is given by the formula

$$2^n$$

So if there are five variables, we have to construct thirty two rows. Then proof construction becomes highly complex. One of the priorities during construction of proof is economy in time and effort. What is brief is simple. Economy, simplicity clarity are the parameters of accepted proof construction. This is what is called law of parsimony. Therefore an alternative is required.

3.3 APPLICATION OF ELEMENTARY RULES OF INFERENCE

Let us consider an example:

p => (q v r)
¬ r
¬ q

∴ ¬ p

The meaning of proof construction should become clear now. It is a sequence of statements each of which is either a premise of that argument or follows from preceding statements of the sequence by an elementary valid argument, and the last statement in the sequence is the conclusion of the argument whose validity is being proved.

1.

1	p => (q v r)	
2	¬ r	
3	¬ q / ∴ ¬ p	
4	¬ q Λ ¬ r	3, 2, Conj.
5	∴ ¬ p	1, 4, M.T.

Since this is the first argument, let us elaborate the process.

$\neg$ r appears in 2nd line. Therefore the Rule of 'Conj. permits us to include $\neg$ r . We get
$\neg q \wedge \neg r$.

This forms the fourth line in the sequence. Now we shall consider 1st and 4th line together.

2.

1). $p \Rightarrow (q \vee r)$

2). $\neg q \wedge \neg r$

$\therefore \neg p$

In (1), (q v r) is the consequent. q v r being a disjunction, when it is denied, it becomes a conjunction with original disjuncts being replaced by their respective negation. This is a law called de Morgan's law about which more will be said later. Since consequent is denied in the second premise, the antecedent has to be denied in the conclusion and it is done. Hence the Rule followed is M. T. It is clear that the form of (1) and (2) corresponds exactly to the form of Rule 2. The conclusion, which we obtained through formal proof, is the same as the conclusion of the given argument. This is how an argument is tested for validity. This is a model of explanation, which suits any argument.

Consider an argument in natural language.

1.

If Ravi is nominated, then she will go to Delhi.
If she goes to Delhi, then she will campaign there.
If she campaigns there, then she will meet Shibu.
Ravi will not meet Shibu.
Either Ravi will be nominated or someone more eligible will be selected.
Therefore someone more eligible will be selected.'

The validity of this argument may be intuitively obvious. But we must prove it. In order to do so, we must first translate the argument into our symbolism which takes this form.

1 $A \Rightarrow B$
2 $B \Rightarrow C$
3 $C \Rightarrow D$
4 $\neg D$
5 $A \vee E$ / $\therefore$ E
6 $A \Rightarrow C$ 1,2, H.S.
7 $A \Rightarrow D$ 6,3, H.S.
8 $\neg A$ 7.4, M.T.
9 $\therefore E$ 5, 8, D.S.

From these examples it is clear that in this method the conclusion always follows '/' and '/' Immediately succeeds the last premise. This is an important aspect because all premises are numbered whereas the conclusion is not numbered. Therefore it is written adjacent to the last premise. Steps involved in the construction of proof are also the elements of the set of premises. This method requires very few steps and Rules. On the L. H. S., steps, which are involved in the construction of proof, are written. Every line after the last premise stands in need of justification and the justification is provided by one or the other Rule listed above. It must be noted that the same Rule can be applied any number of times. This method of proof is called formal method of proof which becomes clear very shortly.

From the first two premises A => B and B => C we validly infer A => C using the Rule of Hypothetical Syllogism. From A => C and the third premise, C => D we validly infer A => D, using again the Rule of Hypothetical Syllogism. From A => D and the fourth premise, ¬D we validly infer ¬A using the Rule of *Modus Tollens*. We apply the Rule of Disjunctive Syllogism to ¬A and the fifth premise A v E and validly infer E, the conclusion of the original argument. That the conclusion can be deduced from the five premises of the argument with the help of three elementary Rules proves that this method is the most useful method. From the description given above it becomes evident that the Rule of H. S. is applied twice.

[The rest of the arguments are worked out for which explanation is not provided. The student is expected to construct the same.]

2	1	(B v N) => (K Λ L)	
	2	¬K	
	3	¬M / ∴ ¬B Λ ¬M	
	4	¬K v ¬L	2, Add.
	5	¬B Λ ¬N	1, 4, M.T.
	6	¬B	5, Simp.
	7	∴ ¬B Λ ¬ M	6, 3, Conj.

3	1	(K => A) Λ (M => D)	
	2	¬A /∴ ¬K v ¬M	
	3	¬A v ¬D	2, Add.
	4	∴ ¬K v ¬M	1, 3 D.D.

4	1	(M v N) => (P Λ Q)	
	2	N /∴ P Λ Q	
	3	M v N	2, Add.
	4	∴ P Λ Q	1, 3, M.P.

5	1	(A Λ B) => (C v D)	
	2	A	
	3	B / ∴ C v D	
	4	A Λ B	2, 3, Conj.
	5	∴ C v D	1,4, M.P.

6	1	(T => K) Λ (R => S)	
	2	S => D	
	3	D => T	
	4	R / ∴ T	
	5	R => S	1 Simp.
	6	S	5,4, M. P.
	7	D	2, 6, M. P.
	8	∴ T	3, 7, M.P.

7	1	(A v B) Λ(¬D ΛE)	
	2	A v B =>K /∴ KΛ(¬D ΛE)	
	3	A v B)	1, Simp.
	4	K	2, 3, M.P.
	5	¬D ΛE	1, Simp.
	6	∴ K Λ (¬D Λ E)	4, 5, Conj.

8	1	(P =>Q) Λ(R=>S)	
	2	¬A=> ¬Q	
	3	A => B	
	4	¬B /∴ ¬P V ¬ S	

9	1	A v (B Λ C)	
	2	A=>P	
	3	¬P / ∴ C	
	4	¬A	2,3, M.T.

	5	¬A	3,4, M.T.
	6	¬Q	2,5, M.P.
	7	P=>Q	1, Simp.
	8	¬P	7, 6, M.T.
	9	∴ ¬ P v ¬ S	8 Add.

	5	B Λ C	1,4, D.S.
	6	∴ C	5, Simp.

10	1	A Λ (B v C)	
	2	A=>P	
	3	Q / ∴ PΛQ	
	4	A	1, Simp.
	5	P	2,4, M.P.
	6	∴ P Λ Q	5,3, Conj.

11	1	¬B	
	2	¬D	
	3	(A=>B) Λ(C =>D)	
	4	K / ∴ C (¬K Λ¬A)	
	5	A=>B	3, Simp.
	6	¬A	5, 1, M.T.
	7	C=> D	3, Simp.
	8	¬ C	7, 2, M.T.
	9	(¬ K Λ ¬A)	4,6, Conj.
	10	∴¬CΛ (¬KΛ¬A)	8, 9, Conj.

12	1	(B ≡ K) => (Z Λ D)	
	2	¬ (Z Λ D) / ∴ ¬ (B ≡ K)	
	3	∴ ¬ (B ≡ K)	1,2, M.T.

13	1	(K Λ T) => (A v B)	
	2	(A v B) => (P Λ ¬ L)	
	3	(P Λ¬L)=>D	
	4	¬(D) / ∴ ¬(K ΛT)	
	5	(KΛT)=>(P Λ¬L)	1,2, H.S.
	6	(K Λ T) => D	5,3, H.S.
	7	∴ ¬ (K Λ T)	6,4 M.T.

14	1	(K Λ A) => (¬ B v C)	
	2	M=> (K Λ A)	
	3	M / ∴ ¬ B v C	
	4	M => (¬B v C)	2,1, H.S.
	5	∴ ¬ B v C	4,3, M.P.

15	1	A=>D	
	2	B=>C	
	3	A v B / ∴ D v C	
	4	(A=>D)Λ (B=>C)	1,2,Conj.
	5	∴ D v C	4,3, C.D.

16	1	A => D	
	2	B => C	
	3	¬ D v ¬ C/ ∴ ¬A v ¬ B	
	4	(A=>D)Λ(B=>C)	1,2,Conj.
	5	∴ ¬Av ¬B	4,3, D.D.

17	1	(A=>G) => (K v ¬ D)	
	2	¬ (K v ¬ D) / ∴ ¬(A => G)	
	3	∴ ¬ (A=> G)	1,2, M.T.

18	1	J V (K Λ L)	
	2	J=>D	
	3	¬D / ∴ K Λ L	
	4	¬J	2,3, M.T.
	5	∴ (K Λ L)	1,4 D.S.

19	1	D v (A=> B)	
	2	(A=> B) => (C v K)	
	3	¬(C v K) / ∴ D	
	4	¬(A=>B)	2,3, M.T.
	5	∴ D	1,4, D.S.

20	1	A Λ (B => C)	
	2	B / ∴ C	

21	1	(A => B) Λ (C=>D)	
	2	A/ ∴ B v D	

	3	B => C	1, Simp.		3	A v C	2, Add.
	4	∴ C	3,2, M.P.		4	∴ B v D	1,3, C.D.
22	1	A v (B Λ C)		23	1	A => B	
	2	A=> D			2	B => C	
	3	¬D / ∴ B			3	¬ C / ∴ ¬ A	
	4	¬A	2,3, M.T.		4	¬ B	2,3, M.T.
	5	B Λ C	1,4 , D.S.		5	∴ ¬A	1,4, M.T.
	6	∴ B	5, Simp.				
24	1	(A v B) => C		25	1	(A=> C) Λ (B => D)	
	2	D => ¬ C			2	K => A	
	3	D / ∴ ¬ (A v B)			3	K / ∴ C v D	
	4	¬ C	2,3, M.P.		4	A	2,3, M.P.
	5	∴ ¬ (A v B)	1,4, M.T.		5	A v B	4, Add.
					6	∴ C v D	1,5, C.D.

Not all arguments can be tested only with the Rules of Inference, though as shown above somewhat complex and diverse arguments succumb to these Rules. Just as modern logic tried to supplement traditional logic, within modern logic, the need was felt to supplement the Rules of Inference. Hence we have the Rules of Replacement. The structure of argument is such that it may require only the Rules of Replacement or only the Rules of Inference or both. We have ten such Rules, which are called the Rules of Replacement. The difference between these two sets of Rules is that the Rules of Inference are themselves Inferences whereas Rules of Replacement are not. This is because the Rules of Replacement are restricted to change or changes in the form of statements. For example, A or B is changed to B or A; A Λ (B v C) is changed to (A Λ B) v (A Λ C). Also, in the mode of application of Rules there is a restriction. Unlike Rules of Inference which should be applied to the whole line only any Rule of Replacement can be applied to any part of the line a difference pointed out earlier. All Rules of Replacement are, logically, equivalent.

Check your progress I

Note: Use the space provided for your answers.

Examine the following arguments.

1) Analyze the significance of Rules of Inference.

....
....
....

2) Distinguish between the application of Composition and Simplification.

....
....

Now let us list Rules of Replacement.

1 De Morgan's Law (De.M.) ¬ (p Λ q) ≡ ¬ p v ¬q
¬ (p v q) ≡ ¬ p Λ ¬q

2 Commutation Law for addition (Com.) p v q ≡ q v p
Commutation Law for multiplication p Λ q ≡ q v p
3 Double Negation(D.N.) ¬ (¬ p) ≡ p
4 Transposition (Trans.) (p => q) ≡ (¬q => ¬ p)
5 Material Implication (Impl.) (p => q) ≡ ¬ p v q
6 Material Equivalence (Equiv.) (p ≡ q) ≡ (p => q) Λ (q => p)
{(p ≡ q) ≡ {(p Λ q) v (¬ p Λ ¬q)}
≡ p Λ p
7 Exportation (Exp.) {(p Λ q) => r} ≡ {p => (q => r)}
8 Tautology (Taut.) p ≡ p v p
9 Association (Ass.) {p v (q v r)} ≡ {(p v q) v r}
{p Λ (q Λ r)} ≡ {p Λ q) r}

10 Distribution (Dist.) {p Λ (q v r)} ≡ {(p Λ q) v (p Λ r)}
{p v (q Λ r)} ≡ {p v q) Λ (p v r)}

[Note: When the expression includes both 'Λ' and 'v' only distribution law can be applied but not association law.]

If we construct truth-table for any Rule (for both sides of the equation), we will understand that they are equivalent expressions. The student is advised to construct truth-table for any Rule of Replacement to verify the same. Our immediate task is to become familiar with the technique of testing arguments. Wherever Rule of Replacement applies to a part of the line the transformed part is italicized.

26. 1 {I => (J => K)} Λ (J => ¬I) / ∴{(I Λ J) => K} Λ (J => ¬I)}

2 ∴{(I Λ J) => K} Λ (J => ¬I) 1, Exp.

27.

1 (R Λ S) => (¬R v ¬S) / ∴ (¬R v ¬S) => (R Λ S)

2 ∴ (¬R v ¬S) => (R Λ S) 1,De.M.

28.

1 (T v ¬U) Λ {*(W Λ ¬V) => ¬T*}/ ∴(T v ¬U) Λ {(W=>(¬V=>¬T)}

2 ∴(T v ¬U) Λ {(W=>(¬V=>¬T)} 1, Exp.

29.

1 X v Y) Λ (¬X v Z)/ ∴ (X v Y Λ ¬X) v {(X v Y) Λ Z}

2 ∴ (X v Y Λ ¬X) v {(X v Y) Λ Z} 1,Dist.

30.

1 Z => (*A* => *B*) / ∴ Z => ¬{ ¬ (A => B}

∴Z => ¬{ ¬ (A => B} 1,D.N.

31.

1 (¬F v G) Λ (F => G) / ∴F => G.).

2 (F => G) Λ (F => G) 1, Impl.

3 ∴ F => G 2, Taut.

Now we shall consider different types of arguments, which may involve both kinds of Rules.

32.

1 (O => ¬P) Λ (P => Q)

2 Q => Q ∴

3 ¬R => P / R

4 ¬Q v Q 2, Impl.

5 O v ¬ Q 4, Com.

6 (Q => ¬P) Λ (¬ Q => ¬P) 1, Trans,

7 ¬P v ¬P 6, 5, C.D.

8 ¬P 7, Taut.

9 ¬ ¬ R 3, 8, M.T.

10 R 9, D.N.

33.

1 X => (Y => Z)

2 X => (A => B)

3 X Λ (Y v A)

4 ¬Z / ∴B

5 (X Λ Y) => Z 1, Exp.

6 (X Λ A) => B 2, Exp.

7 (X Λ Y) v (X Λ A) 3, Dist.

8 {(X Λ Y) => Z} Λ {(X Λ A) => B} 5,6, Conj.

9 Z v B 8, 7, C.D.

10 ∴B 9, 4, M.T.

34

1 C => (D => ¬ C)

2 C ≡ D / ∴ ¬C v ¬D

3 C => (¬ ¬*C* => ¬*D*) 1, Exp.

4 C => (C => ¬D) 3, D.N.

5 (*C Λ C*) => ¬D 4, Exp.

6 C => ¬D 5, Taut.

7 ∴ ¬C v ¬D 6, Impl.

35

1 E Λ (F v G)

2 (E Λ G) => ¬ (H v I)

3	¬ (¬H v ¬I) => ¬ (E Λ F) / ∴ H ≡ I	
4	(E Λ G) => (¬H Λ ¬I)	2, De.M.
5	¬ (H Λ I) => ¬ (E Λ F)	3, De.M.
6	(E Λ F) => (H Λ I)	5, Trans.
7	{(E Λ F) => (H Λ I)} Λ {(E Λ G) => (¬H Λ ¬I)}	6,4,Conj.
8	(E Λ F) v (E Λ G)	1, Dist.
9	(H Λ I) v (¬H Λ ¬I)	7,8, C.D.
10	H ≡ I	9, Equiv.

36

1	J v (¬K v J)		
2	K v (¬J v K) / ∴ J ≡ K		
3	(¬K v J) v J	1,	Com.
4	¬k v (*J v J*)	3,	Ass.
5	¬K v J	4,	Taut.
6	K => J	5,	Impl.
7	(¬J v K) v K	2,	Com.
8	¬J v (*K v K*)	7,	Ass.
9	¬J v K	8,	Taut.
10	J => K	9,	Impl.
11	(J => K) Λ (K => J)	10, 6,	Conj.
12	∴ J ≡ K	11,	Equi.

37

1	(E Λ F) Λ G		
2	(F ≡ G) => (H v I) / ∴ I v H		
3	E Λ (F Λ G)	1,	Ass.
4	(F Λ G) Λ E	3,	Com.
5	(F Λ G)	4,	Simp.
6	(F Λ G) v (¬ F Λ ¬ G)	5,	Add.
7	F ≡ G	6,	Equiv.
8	H v I	2, 7,	M.P.
8	∴I v H	8,	Com.

38

1	(M => N) Λ (¬ O v P)	
2	M v ¬ O / ∴ N v P	
3	∴ N v P	1, 2,C. D.

39

1	(L v M) v (N Λ O)	
2	(¬ L Λ O) Λ ¬ (¬ L Λ M) / ¬L Λ N	
3	¬ L Λ [O Λ ¬ (¬ L Λ M)]	2,Ass.
4	¬ L	3,Simp.
5	L v {(M v (N Λ O)} ∴	1, Ass.
6	M v (N Λ O)	5,4, D.S.
7	¬ (¬ L Λ M)	2, Simp.
8	Lv ¬ M	7, De. M.
9	¬ M	8, 4, D.S.
10	N Λ O	6, 9, D.S.
11	N	10, Simpl.
12	∴ ¬ L Λ N	4,11, Conj.

40

1	E => (F => G) / ∴ F => (E => G)		
2	(*E Λ F*) => G	1,	Exp.
3	(F Λ E) => G	2,	Com.
4	∴F => (E => G)	3,	Exp.

41

1	H => (I Λ J) / ∴H=>I		
2	¬ H v (I Λ J)	1,	Impl.
3	(¬ H v I) Λ (¬ H v J)	2,	Dist.
4	¬ H v I	3,	Simp.
5	∴ H => I	4,	Impl.

42

1	N => Q / ∴ (N Λ P) => O		
2	¬ N v O	1,	Impl.
3	¬ P v ¬ N v O	2,	Add.
4	¬ (P Λ N) v O	3,	De.M.
5	(*P Λ N*) => O	4,	Impl.
6	∴ (N Λ P) => O	5,	Com.

43

1	(Q v R) => S / ∴Q => S		
2	(¬ Q Λ ¬R) v S	1,	Imp.
3	(¬ Q v S) Λ (¬ R v S)	2,	Dist.
4	¬ Q v S	3,	Simp.
5	Q => S	4,	Impl.

44

1	T => ¬ (*U* => *V*) / ∴T => U		
2	T => ¬ {¬ *U v V*}	1,	Impl.
3	¬T v (U Λ ¬V)	2,	De. M.

4	(¬T v U) Λ (¬T v ¬V)	3,	Dist.
5	¬T v U	4,	Simp.
6	∴T => U	5,	Impl.

45

1	W => (*X v* ¬*Y*)	/ ∴W => (Y => X)	
2	W => (¬Y v X)	1,	Com.
3	∴W => (Y => X)	2,	Exp.

46

1	H => (I v J)		
2	¬I	/ ∴H => J	
3	¬H v (*I v J*)	1,	Impl.
4	¬H v (J v I)	3,	Com.
5	(¬H v J) v I	4,	Ass.
6	¬H v J	5, 2,	D.S.
7	∴H=> J	6,	Impl.

47

1	(*K v L*) => ¬ (M Λ N)		
2	(¬M v ¬N) => (O ≡ P)		
3	(O ≡ P) => (Q Λ R) / ∴ (L v K) => (R Λ Q)		
4	(L v K) => ¬ (M Λ N)	1,	Com.
5	(L v K) => (¬M v ¬N)	4,	De.M.
6	L v K) => (O ≡ P)	5, 2,	H.S.
7	(L v K) => (*Q Λ R*)	6, 3,	H.S.
8	∴ (L v K) => (R Λ Q)	7,	Com.

48

1	(*D Λ E*) => F		
2	(D => F) => G	/ ∴E=>G	
3	(E Λ D) => F	1,	Com.
4	E => (D => F)	3,	Exp.
5	∴E => G	4,2,	H.S.

49

1	(H v I) => {J Λ (K Λ L)}		
2	I	/ ∴J Λ K	
3	I v H	2,	Add.
4	H v I	3,	Com.
5	J Λ (K Λ L)	1, 4,	M.P.
6	(J Λ K) Λ L	5,	Ass.

7	∴ J Λ K	6,	Simp.

50

1	(M v N) => (O Λ P)		
2	¬ O / ∴ ¬M		
3	¬ O v ¬ P	2,	Add.
4	¬ (O Λ P)	3,	De.M.
5	¬ (M v N)	1, 4,	M.T.
6	¬M Λ ¬N	5,	De.M.
7	∴¬M	6,	Simp.

51

1	T Λ (U v V)		
2	T => {U => (W Λ X)}		
3	(T Λ V) => ¬ (W v X) /∴ W ≡ X		
4	(T Λ U) => (W Λ X)	2,	Exp.
5	(T Λ V) => (¬W Λ ¬X)	3,	De.M.
6	{(T Λ U) => (W Λ X)} Λ {(T ΛV) => (¬W Λ ¬X)}	4, 5,	Conj.
7	(TΛU) v (TΛV)	1,	Dist.
8	(W Λ X) v (¬WΛ¬X)	6,7,	C.D.
9	∴ W ≡ X	8,	Taut.

52

1	Y => Z		
2	Z => {Y => (R v S)}		
3	¬ (R Λ S) / ∴ ¬Y		
4	Y => {Y => (R Λ S)}	1,2,	H.S.
5	(*Y Λ Y*) => (R Λ S)	4,	Exp.
6	Y => (R Λ S)	5,	Taut.
7	∴¬ Y	6, 3,	M.T.

53

1	A v B		
2	C v D / ∴{(A v B) Λ C} v {((A v B) Λ D)		
3	(A v B) Λ (C v D)	1, 2,	Conj.
4	∴ {(A v B) Λ C} v {(A v B) Λ D}	3,	Dist.

54

1	(I v ¬ ¬*J*) Λ K		
2	{¬ *L* => ¬ *(K Λ J)*} Λ {K => (I=> ¬M)} /∴ M Λ ¬ L		
3	{(K Λ J) => L} Λ {*K* => *(I* => ¬ *M)*}	2,	Trans.
4	{(K Λ J) => L} Λ {(K Λ I) => ¬ M}	3,	Exp.
5	(I v J) Λ K	1	D.N.
6	K Λ (I v J)	5	Com.
7	(K Λ I) v (K Λ J)	6	Dist.
8	(K Λ J) v (K Λ I)	7	Com.
9	L v ¬M	4, 8	C. D.

10	¬ M v L	9,	Com.
11	∴ M Λ ¬ L)	10,	De. M.

3.4 EXERCISES

A. For the following valid arguments, state the Rule of Inference by which its conclusion follows from their premise(s):

1 (A Λ B) => C
∴ ¬ ¬ (A Λ B) = > C

2 [N => (O Λ P)] Λ [Q => (O Λ R)]
N v Q
∴ (O Λ P) v (O Λ R)

3 ¬ (H Λ ¬I) => (H => I)
(I < => H) => ¬ (H Λ ¬I)
∴(I <=> H) => (H => I)

4 (J => K) Λ (K => L)
L=> M
∴J => K) Λ (K => L)] Λ (L => M)

5 [(O => P) => Q] => ¬ (C v D)
∴C v D) =>¬ [(O => P) => Q]

B. Construct a formal proof of validity for the following arguments by adding additional premise or premises.

1 M v N
¬M Λ ¬N
∴N

2 A => B
∴ A => C

3 (P => Q) Λ (R => S)
∴ Q v S

4 ¬ (H v I) v J
¬(¬H v I)

∴ J v ¬H

5 (W Λ X) => (Y Λ Z)

∴ ¬ (W Λ X)

6 Q => (R v S) (T Λ U) => R

(R v S) => (T Λ U)

∴ Q => R

Check your progress II.

Note: Use the space provided for your answers.

1) Distinguish between distributive law and commutative law.

....
....
....

2) Explain de Morgan's law with example.

....
....

3.5 LET US SUM UP

There are nine Rules of Inference. Six of them are those accepted by traditional logic. Remaining three Rules are the inventions of modern logic. Rules of Inference must be applied to the whole line only whereas the Rules of Replacement can be applied to a part of the line. Any complex argument can be examined easily with the help of these Rules.

3.6 KEY WORDS

Replacement: It means that there is internal change within a line which does not alter the meaning of the compound proposition.

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UNIT 4 FALLACY

Contents

4.0 OBJECTIVES

The purpose of this unit is to enable the students to recognize fallacies. This is necessary because it is possible to distinguish valid arguments from invalid arguments only when we can identify invalid arguments. In other words, you can know what is good only when you know what is bad and if you know what is bad you, surely, will avoid the same. This is what, precisely, Socrates said. Fallacy is the class name given to bad (illogical) arguments. Fallacies are like plastic flowers: They give you the impression that they are valid. But, a student of logic, like a bee, should be able to distinguish between the real (valid) and the artificial (invalid).

4.1 INTRODUCTION

There are as many types of fallacies as there are types of errors in arguing. Falsehood has many faces whereas truth has only one. Therefore our task is clear. What do we mean by fallacy? How do they arise? How are they classified? How can we avoid them? These are some of the questions to which we turn now.

Logic deals with the Rules of correct thinking. Hence fallacy arises when we violate any of these Rules. Strictly speaking, *a fallacy is a type of arguing which appears to be valid, but actually invalid.* The term fallacy comes from the Latin word 'fallo,' meaning 'I deceive.'

We reason *in*correctly when the premises of an argument fail to support its conclusion. Every fallacy can be a Non-sequitur ('it does not follow'). Examples: 'This man is not clever because he cannot talk fast' or 'He is not a patriot because he does not wear khadi.' Any argument of this sort is fallacious.

Therefore any error in reasoning is a fallacy. Logicians use the term ‘fallacy’ to mean *typical* errors, that is, mistakes in reasoning that exhibit a pattern that can be identified and named. The great logician Gottlob Frege, regarded as the father of Modern Logic, has made this observation: One of the tasks of a logician is to ‘indicate the *pitfalls* laid by language in the way of the thinker.’ Every fallacy is a *type* of incorrect argument. The particular argument that violates some known or unknown Rule is commonly said to *be* a fallacy because it is an individual example of that typical mistake. When the Rule is unknown, it is the business of logician to discover or frame the Rule.

Illustration: Suppose one argues, ‘All science is essentially materialistic; Karl Marx who was certainly a materialist, must therefore have been scientific.’ This is a bad reasoning, fallacious, because conclusion does not *follow* from the premise. If every P is a Q, it does not follow from the fact that one is a Q that he is a P. All cats are mammals, but not every mammal is a cat. Here we can easily identify a *pattern* of mistake. We call this pattern ‘the fallacy of affirming the consequent.’

This is a *formal* fallacy that violates one of the Rules of mixed hypothetical syllogism. Examples of some other formal fallacies are: undistributed middle, illicit major and illicit minor. Most fallacies, however, are not formal but *in*formal; they are patterns of mistakes that arise from confusions concerning the *content* of the language used. Such informal fallacies arise in very many ways and they are often more difficult to detect than formal ones because language is slippery and imprecise, and can set traps. In this unit, we consider only informal fallacies, since the students are already familiar with the formal fallacies in their study of classical logic.

A word of caution: In ordinary speaking and writing people sometimes make mistakes without being aware of them. Therefore, though our logical standards should be high, in their application to arguments in ordinary life we should also be generous and fair. The sources of fallacies in our daily life are misinterpretations, false assumptions, lack of knowledge, distraction of the mind, prejudices, and so on.

4.2 CLASSIFICATION OF FALLACIES

Fallacies are numerous. Logicians are not unanimous as to the number and manner of the classification of fallacies. Some logicians have distinguished up to 112 different fallacies! Different logicians classify them differently. So, there is no correct taxonomy of fallacies. Often their grouping is arbitrary.

In our discussion we follow the latest classification done by Copi (2010, 395-461). He groups them into four main categories: fallacies of (i) relevance, (ii) induction, (iii) presumption, and (iv) ambiguity. What follows is an outline of these fallacies:

4.3 FALLACIES OF RELEVANCE

Fallacies of relevance are so called because relevance is ignored in such cases. Six of such mistakes are of interest here.

R1. APPEAL TO EMOTION (*ARGUMENTUM AD POPULUM*) is making use of the feelings and prejudices of people ('to the populace') rather than their reason. This is perhaps the most common of fallacies. Example: In campaigning for election in India one might ask: 'Should you not vote for the Congress? Did not the congress men suffer imprisonment for the sake of the country?' Thus, the speaker (or writer) appeals to patriotism, an honorable emotion, without clear evidence to appeal to the feelings of his audience. The oratory of Adolph Hitler, whipping up the racist enthusiasm of his German listeners, is another classic example.

Besides politicians, many others, like advertisers, commit this fallacy. Advertising industry has raised it almost to the status of a fine art. Example: beauty products are associated with youth and captivating personalities, and self-confidence; men depicted using the products are generally handsome and famous, the women graceful and charming. They arrest our attention by relentless appeals to our emotions of every kind. Their suggestion is that the product – say soft drinks, or soap,– is sexy, or is associated with wealth or power, or some other admired traits, and therefore we, in purchasing it, will acquire some of that same merit.

Sometimes they use what is known as 'bandwagon fallacy' – to imitate others in action or thought or speech (fashion also can be included) because so many others are doing it. This is very common in media. Example: look at the following advertisement: 'Smooth sailing is all that's there to LIFE with LIC's JEEVAN SARAL (easy life). (*The Week*, 22 Feb. 2005).

A variant of this frequent fallacy is called *argumentum ad misericordiam* (appeal to pity). This is used very much in criminal trials. There may be cases where it may be justified. But, when the argument boils down to no more than an appeal to 'merciful heart' it is plainly fallacious. All sections of people use this fallacy some time or other. Very often parents use this sort of argument to secure their child's admission in prestigious schools. This is a familiar instance of this fallacy. Though often successful, the appeal to pity is ridiculed in the story of the trial of a youth accused of murdering his parents with an axe. Confronted with overwhelming proof of his guilt, his attorney appealed for leniency on the grounds that his client is now an orphan!

R2. THE RED HERRING is distracting the attention of listeners from the topic under discussion. As the story goes, red herring is used to distract or confuse dogs. It means a *trail* which is left to *mislead deliberately*. So whatever can keep the listener off the track may serve as a red herring. In a popular novel and movie, *The Da Vinci Code,* one of the characters, a Catholic Bishop, enters the plot in ways that very cleverly mislead. His name aptly suits the mission; Bishop Aringarosa – meaning 'red herring' in Italian.

R3. THE STRAW MAN is a way of arguing against some view by presenting an opponent's position as one that is easily torn apart. That is, it is very much easier to win a fight against a man made of straw than against one made of flesh and blood. To argue that one should not join the civil services since some civil servants are corrupt and by joining the service one would be supporting this systematic corruption is an example of a straw man argument. But this argument is not justifiable because someone may decide to join administration with the laudable intention of eradicating corruption in public life. This fallacy results when we adopt the most extreme view possible – that *every* act or policy of a certain kind is to be rejected. This argument is easy to win, but not relevant to the conclusion originally proposed.

R4. ARGUMENT AGAINST THE PERSON (ARGUMENTUM AD HOMINEM) consists in attacking the character of the opponent instead of proving or disproving the point at issue. Instead of proof, the argument merely refers to his conduct. The thrust of the argument which commits the fallacy of *ad hominem* is not on the disputed conclusion, but on some person who defends it. This kind of personal attack is hurting, and might be conducted in either of two ways: One is *abusive* and the other *circumstantial.*

Abusive attack means 'questioning the integrity of the opponent'. But the character of an adversary is logically irrelevant to the truth or falsity of the reasoning employed. A proposal may be attacked as unworthy because it is supported by 'extremists,' or by 'fundamentalists.' But such allegations, even when plausible, are not relevant to the merit of the proposal itself. Socrates was convicted of impiety partly because of his long association with persons known to have been disloyal to Athens and rapacious in conduct.

Circumstantial *ad hominen* is to argue that you are as bad as I am; just as guilty of whatever it is that you complained about. Example: A hunter, accused of needless slaughter of harmless animals, sometimes replies by noting that his critics eat the flesh of harmless cattle. But the fact that the critics eat meat is totally irrelevant to the question raised, viz. whether needless killing is ethical. Another example: it may be unfairly suggested that a member of the clergy must accept a given proposition because its denial would be incompatible with the Scriptures.

The circumstances of an opponent are not properly the issue in serious arguments. It is the substance of what is claimed, or denied, that must be addressed. It may be rhetorically effective but that does not make up for its error. However, sometimes in court room proceedings, for example, it is acceptable, and often effective to call a judge's attention to the unreliability of a witness, and by so doing undermine the claims upheld by the testimony of that witness. But even this attack on the person of the witness does not establish the *falsehood* of what had been asserted.

R5. APPEAL TO FORCE (*ARGUMENTUM AD BACULUM*) consists in appealing to physical force to make the opponent to submit. 'Appeal to the stick' is hardly logic, though sometimes very effective, for example, in making the criminals confess their crimes. However, no one would agree that 'might is right.' The threat of force in any form is unreasonable and therefore fallacious. It is indeed very odd if someone says, 'When you have no case, well get angry and threaten.' This is equal to saying: the best policy of defending yourself is to become offensive.

Right from the domestic front to the international forum, threat, a 'subtle' weapon, is used as a powerful instrument of persuasion. Many powerful nations are using 'arm twisting' policy like reducing financial aid, cutting the technical assistances, and so on, if the opponent countries do not sign a particular treaty. Though threats are used implicitly on a large scale by all sections of society, accepting a conclusion merely on the basis of threat is not at all sound from a logical point of view.

R6. MISSING THE POINT (*IGNORATIO ELENCHI*) is diverting attention from the real point at issue. It is arguing beside the point. This applies to many kinds of arguments where the conclusion does not *follow* from the premises. Example: 'The object of war is peace; therefore, soldiers are the best peacemakers.' Even if it is assumed that the object of war is peace, still it does not imply that soldiers are the best peacemakers.

Of the various informal fallacies of relevance, *ignoratio elenchi* is perhaps the most difficult to describe with precision. It is confusion in reasoning that the speaker does not fully recognize. Aristotle, who classified fallacies, explained it as a *mistake* made in trying to refute another's argument; defender tries to argue for P and the opponent counters it with an irrelevant Q.

In a sense, every fallacy is an *ignoratio elenchi* because there is a gap between the premises and the conclusion, and thus the debator misses the point. So, *ignoratio elenchi* is a catch-all class of fallacies: *Non sequitur* is similar fallacy in which the conclusion does not simply follow from the premises. *Non sequitur* is more often applied when the failure of argument is obvious. 'A great, rough *non sequitur,'* Abraham Lincoln observed in a speech in 1854, 'was sometimes twice as dangerous as a well polished fallacy.'

R7 ARGUMENT FROM IGNORANCE (ARGUMENTUM AD IGNORANTIAM) is taking advantage of the ignorance of the opponent. Example: 'there is neither heaven nor hell because no one has seen it.' Or, 'Ghosts do not exist because no one has proved its existence so far.' If a proposition has not yet been disproved, can we conclude that it is true? If we say yes, then we are arguing from the *supposed* absence of disproof to the presence of proof. This is what the examples given above show. Or, if some proposition has not yet been proved true, then can we conclude that it is false? If we can, then we are arguing form the absence of disproof to the presence of proof. The following example falls under this category. 'Since no one has disproved a supernatural being, the supernatural being exists.'

Both these inferences are defective. For, ignorance or absence of evidence is taken as evidence for the conclusion. Unfortunately, our daily life is sprinkled with this fallacy. The customer asks the shopkeeper concerning the quality of an item like cloth. The spontaneous reply is that so far no one has complained against it. Similarly, a customer asks the manufacturer about the quality of the glass he is buying. The standard answer is: 'I have been making and selling glass for nearly ten years and since then I have not heard any complaint against it.'

Even in science this fallacy crops up. In archeology, for example, evidences mighty have been destroyed over a period of time. But we cannot, therefore, conclude that an otherwise plausible claim is false. This fallacy has been very attractive to pseudo-scientists who make unverifiable claims about psychic phenomena. Their claims regarding telepathy or clairvoyance are examples of this sort of fallacy. They justify their proposition by arguing that critics have been unable to disprove it.

This fallacy can be a major hindrance to progress. Galileo, whose newly invented telescope revealed the mountains and valleys of the moon, was confronted with this sort of fallacious argument by his opponents. Similarly, whenever some change is proposed, within an institution or in society at large, those threatened by it are likely to counter it with this type of fallacy thus: 'How can we know whether it will work? How can we know whether it is safe? We do not know. So, we should not adopt the change proposed.' True, it is often impossible to prove the workability or safety in advance. But it is not the ground for rejection.

R8 THE APPEAL TO INAPPROPRIATE AUTHORITY (*ARGUMENTUM AD VERECUNDIAM*) is also known as 'ipse dixit' (the master has said it). If I argue that 'the soul is immortal because Plato says so, then I commit this fallacy. This fallacy occurs when the premises appeal to the judgment of some person(s) or text(s) that have no *legitimate* claim to authority in the matter at hand. This is a very common and crafty fallacy because a person who is an expert in one field is taken as an expert in some other, comparatively, unrelated field. If, for example, we take Bertrand Russell, a great authority on philosophy, as an authority on the matter of shoes, we commit this fallacy.

The most blatant examples of misplaced appeals to authority appear in advertising 'testimonials.' Since Sachin Tendulkar, an authority in cricket, says that a particular cool drink is good, we should accept that drink as superb.' This is fallacious. If Sachin had recommended a particular brand of cricket bat, then his words would have been authoritative. But when it comes to the matter of drinks he is no better than any other. Likewise, we cannot take a scientist, an expert in making nuclear weapons, as an authority on international economical or political matters. Nor can we consider a great religious leader as an expert on financial matters

R9 MISERICORDIAM: This is an appeal to pity. From Plato's dialogues we understand that in ancient Greece, the criminals followed this method to escape punishment. It is doubtful whether this was followed by one who was not guilty.

Of course, no fallacy is committed when we are guided by the judgment of acknowledged experts. In fact, such recourse to authority is necessary for most of us on very many matters. Taking expert opinion is surely one of the reasonable ways of supporting a conclusion though not conclusive.

Check Your Progress I

Note: Use the space provided for your answers.

1 Explain the meaning of fallacy? Distinguish between formal and informal fallacies..............
....
....
....

2) Give examples for ARGUMENT AGAINST THE PERSON (ARGUMENTUM AD HOMINEM) and APPEAL TO INAPPROPRIATE AUTHORITY (ARGUMENTUM AD VERECUNDIAM)
.

....
....

4.4 FALLACIES OF INDUCTION

In the strict sense of the term there is nothing like inductive fallacy though several fallacies are included under this category. A fallacy arises when a Rule of inference is violated. This can be put in this way also. An argument is invalid if and only if it is fallacious. Distortion of meaning in the process of understanding can also be regarded as a fallacy because distortion of meaning can be construed as violation of grammatical or syntactical Rules. On the contrary, inductive inference is not governed by any logically certain Rule as such. In other words, inductive argument is neither valid nor invalid. It is either probable or improbable. Further, probability or improbability is a matter of degree. An argument may be very highly improbable. In terms of truth-value its value can never reach '0' when the truth-value of premises is '1'. In other words, regarding improbable argument as fallacious is itself fallacious. Therefore fallacy of an inductive argument stands or falls with disputed Rule or Rules. Without going into the merits or demerits of this criticism, let us use the term fallacy in a loose sense and consider briefly some supposed inductive fallacies.

D1 ANALOGY: An argument which infers unobserved similarity from observed similarity or similarities between two or more than two objects or persons is known as analogy. It is true that Rules are laid down to distinguish strong analogy from weak analogy. But the point is that the logical status of strong and weak arguments remains the same. Therefore if one of them is fallacious, then the other one also is fallacious. Only pragmatic considerations help us to retain one at the cost of the other.

D3. FALSE CAUSE (*ARGUMENTUM NON CAUSA PRO CAUSA*): The fallacy of false cause is committed when two events are causally connected when, in reality, such connection does not exist. This is a very common mistake. Superstition, for example, suffers from this fallacy. Suppose that someone says that a black cat crossed the path of a traveler and shortly afterwards he broke his head and therefore the black cat crossing the path is cause. This is an example of this fallacy.

Post hoc ergo propter hoc ('after this, therefore because of this') is another form of this fallacy. Every antecedent of an event is not necessarily the cause of the consequent event. Example: 'thunder is heard after the lightning. Therefore lightning is the cause of thunder.' Mistakes of this kind are rather common. Unusual weather conditions are blamed on some unrelated celestial phenomenon that happened to precede them; an infection caused by a virus is thought to be caused by a chill wind, or wet feet, and so on.

False cause is also the fallacy committed when one mistakenly argues against some proposal on the ground that any change in a given direction is sure to lead to further changes in the same direction – and thus to grave consequences. Taking this step, it may be said, will put us on a slippery slope to disaster – and such reasoning is therefore called the fallacy of the *slippery slope.*

A brief criticism is required. Identification of causal necessity with logical necessity is now history. Failure at two levels accounts for this sort of identification; one, failure to distinguish sequence from consequence and second, failure to recognize the dominant role played by the way in which human mind can perceive and cannot perceive the pattern of events. Again, causal necessity has only pragmatic significance. Mistaking pragmatic necessity to logical necessity is the real fallacy.

D4. HASTY GENERALIZATION: In reality, the term *hasty generalization* is a misnomer. Generalization, whether or not hasty, does not conform to any Rule. In the strict sense of the word, generalization becomes fallacious when association of events within fair sample is taken to represent association within the larger population. Quite often, generalization raises its head, sometimes ugly, when we pass judgment on humans divided by creed or nationality. Unless it is proved that dividing factors are the defining elements of character or personality no judgment can be accepted as authoritative. This explains why widespread stereotypes about people, who come from certain countries, with certain ethnic background, are and commonly mistaken and also why hasty generalizations about foreign cultures is illogical.

4.5 FALLACIES OF PRESUMPTION

These are fallacies that assume the truth of some unproved propositions. Such presumption often goes unnoticed. It is, therefore, usually sufficient to call attention to the smuggled assumption and to its doubtfulness or its falsity to expose such a fallacy. There are three common fallacies of this kind.

P1. ACCIDENT arises due to a lack of clarity regarding the meaning of terms used. It has two forms: (i) ***direct or simple fallacy of accident*** consists in arguing that what is true of a thing under normal circumstance is also true of it under special circumstances. Consider this example: 'Freedom is the birth right of man; so no one should be imprisoned.' This is ordinarily true but it is not applicable to a man who has committed a serious crime. Another example is more educative. 'Such and such a person should be fined for ignoring a 'No Swimming' sign when the purpose of jumping into water is to rescue some one from drowning.'

(ii) **The Converse fallacy of Accident** is the opposite of the direct fallacy of accident. It occurs when we argue that what is true of a thing under special circumstances is also true under normal circumstance. Consider this example. 'Liquor is beneficial in certain cases of diseases; they must, therefore, be beneficial for all persons and so its prohibition must be lifted.' This is similar to *hasty generalization.*

In the realm of morals, the fallacy of accident occurs if one is not careful in applying the general moral dictum. For instance, it is true that telling a lie is wrong but in order to save one's life, it is not wrong to tell lie. We have to consider the distinction between general Rule and special circumstances with greater care. Logician's job is to warn the arguer that the fallacy can creep in reasoning when we argue from an unqualified statement to a qualified one. We cannot simply assume that a generalization applies universally. For instance, there is a general principle in law, ‘hearsay Rule,’ that hearsay evidence may not be accepted in court. But this does not apply when the person, whose communications are reported, is dead.

P2. COMPLEX QUESTION is also called *fallacy of many questions.* It is a deceitful device. This fallacy consists in asking a question in such a way as to presuppose the truth of some proposition buried in the question. This is a favorite device of layers. For instance, a lawyer asks a defendant: ‘have you stopped beating your wife.’ It assumes that you are married, and that your wife is alive, and that you used to beat your wife, and so on. But none of these may be the case. The truth may be that you are a bachelor. The best way to face this fallacy is to refute all the presuppositions hidden in the question one by one, instead of giving a straight yes or no answer which might land you in trouble.

The appearance of a question in an editorial or headline often has the purpose of suggesting the truth of the unstated assumptions on which it is built: ‘Judge Took Bribe?’ This technique is a common mark of what is known as ‘yellow journalism.’ And in debate, whenever a question is accompanied by the aggressive demand that it be answered ‘yes or no,’ there is reason to suspect that the question is ‘loaded’ – that it is unfairly complex.

P3. BEGGIGNG THE QUESTION (*PETITIO PRINCIPII)* consists in cleverly assuming the conclusion in the premises instead of proving it. Example: ‘I should not do this because it is wrong.’ This argument does not prove why the action is wrong but merely assumes it to be evil. Thus, if we assume what needs proof, then we are mere beggars, begging what we ought to earn by proof. This fallacy ends where it begins.

A celebrated example of the fallacy which occurs in the philosophical writings is the argument that everything in the world has a cause, since if it did not; we should have effects without causes. This fallacy is a subtle one because it assumes rather than restate the conclusion.

J. S. Mill argued that categorical syllogism commits the fallacy of *petitio principii.* For example, consider this argument. ‘All men are mortal; Ram is a man; Therefore, Ram is mortal.’ Here while establishing the truth of the premises, the conclusion is already taken into account. Without disputing this comment, let us take a non-syllogistic argument committing this fallacy: A man registered a woman in a hotel as his wife and replied, when asked for proof, ‘Certainly she’s my wife because I am her husband.’

This fallacy is not limited to common man. Sometimes even powerful minds are snared by this fallacy. For instance, logicians have, for long, sought to prove the reliability of *the law of uniformity of Nature.* This is an inductive principle which says that the laws of nature will operate tomorrow as they do today and therefore in basic ways nature is uniform, ‘That the future will essentially be like the past’ is the claim at stake. This is never doubted in ordinary life, but it turns out to be very difficult to justify on philosophical grounds. If we ask ‘why conclude that the future will be like the past?’, then the answer would be, ‘because it always has been like the past.’

4.6 FALLACIES OF AMBIGUITY

These fallacies arise as a result of the shift of meaning of words and phrases, shift from the meanings that they have in the premises to different meanings *ascribed* to them in the conclusion. Such mistakes are called *fallacies of ambiguity.* The deliberate use of such devices is usually crude and readily detected; but at times the ambiguity may be obscure, the error accidental, and the fallacy subtle. Five varieties are distinguished blow.

A1. EQUIVOCATION is the fallacy which consists in using words or phrases with two or more meanings, deliberately or accidentally, while formulating an argument. Fallacies related to ambiguous terms discussed in categorical syllogism serve as examples in this context also. The Rule is: 'In a categorical syllogism there should be three and only three terms, each used twice in the same sense.' The violation of this Rule is the simplest way of committing this fallacy.

A2. AMBHIBOLY consists in a misunderstanding due to the ambiguous grammatical construction of propositions. Many advertisements inadvertently commit this fallacy. 'Why go elsewhere to be cheated? Come to us.' Consider another example: Suppose we are told that a number is equal to three times five plus four. This is an example of amphiboly since the answer might be either [(3x5) + 4 = 19] or [3 x (5 + 4) = 27].

A3. ACCENT consists in lifting a word out of context resulting in illegitimate emphasis upon it. For example consider this sermon. 'Thou shall not bear false witness against your neighbour.' If we emphasize unduly the word *neighbour,* then it would mean that you are free to bear false witness against others. On the other hand, if *against* is stressed, then it would mean that you may bear false witness *for* your neighbor, and so on.

The accent may be oral, or may make use of italics, or other devices in the language. Often much of what we mean depends on the accent of words in an argument. Propagandists deliberately distort statements to mislead audiences. Example: tabloid newspapers often use bold and large print to attract the reader's attention. Advertising relies on this device heavily. Consider this advertisement in a newspaper: '**TAKE ABSOLUTELY FREE**' the contest entry form. If we consider the extent of the uses of *emphasis* in various forms and the use of meanings deliberately taken out of context, we can rename accent as 'the fallacy of emphasis.'

A4. COMPOSITION occurs when what is true of the parts taken separately is said to be true of the whole taken collectively. Example: 'cotton cannot be strong enough to make clothes of; for, look, I can break this cotton thread quite easily.' It is true that when each thread is taken separately, but not true when they are taken together. Another example: J. S. Mill commits this fallacy when he argues that the general happiness is the greatest good because each individual desires happiness.

A5 DIVISION is the opposite of composition: what is true of the whole is taken to be true of its parts. Here is an example. 'He must be a catholic, for he is an Italian and Italy is a catholic country.' True, Italy as a whole is Catholic but the same need not be true of every Italian. Another example is given below. Some people argue that what is best for the nation must necessarily be advantageous for each citizen.

Check Your Progress II

Note: Use the space provided for your answers.

1 Explain the meaning of composition and division.................... ....
....
....
....

2) Give examples for amphiboly and accent.
....
....

4.7 EXERCISES

Name the fallacies in the following:

R1. (i) One of Patrick Henry's famous speeches (in Virginia on 23 March 1775) concludes with this appeal: …There is no retreat but in submission and slavery. The next gale that sweeps from the north will bring to our ears the clash of resounding arms. …Is life so dear, or peace so sweet, as to be purchased at the price chains and slavery? Forbid it, Almighty God! I know not what course others may take; but as for me, give me liberty or give me death.' It is reported that the crowd, upon hearing his speech, jumped up and shouted: 'To arms! To arms!'

(ii) At his trial in Athens, Socrates referred with disdain to other defendants who had appeared before their jury accompanied by their children and families, seeking acquittal by evoking pity. Socrates continued: 'I, who am probably in danger of my life, will do none of these things. …and I have a family, yes, and sons, O Athenians, three in number, one almost a man, and two others who are still young; and yet I will not bring any of them here to petition you for acquittal. (Plato, *Apology*. 34)

R2. In the world of finance, a prospectus is issued to attract investors in a company about to go public, which speaks much about the company but does not mention the price of its shares.

R4. Mr. X has no appreciation of music; for, has he ever purchased a ticket for a musical performance?

R5. (i) The father tells his son, 'Sunny, next time before giving you pocket money, I will see your report card.'

(ii) The union workers threaten the establishment that if their demands are not met, they will go on a strike.

R6. Suppose a person emphasizes the importance of increasing funding for the public schools. His opponent responds by insisting that a child's education involves much more than schooling and gets underway long before its formal schooling begins.

D1. (i) 'All experiments with recombinant DNA must be stopped immediately,' said one scientist, who asked: 'If Dr. Frankenstein must go on producing his little biological monsters … how can we be sure of what would happen once the little beasts escape from the laboratory?'

(ii) Another scientist who wanted to block any further experimentation with DNA made his appeal explicitly: 'Can we predict the consequences? We are ignorant of the broad principles of evolution … We simply do not know. We are ignorant of the various factors we currently perceive to participate in the evolutionary process. We are ignorant of the depth of security of our own environmental niche … We do not know.'

D2. 'We admire the depth and insight of great fiction, say, in the novels of R. K. Narayan. Therefore we resort to his judgment in determining the real culprit in some political dispute.'

D3. (i) 'The death penalty in the U.S. has given us the highest crime rate and greatest number of prisoners per 100,000 populations in the industrialized world.' Therefore death penalty is the cause of the highest crime rate.

(ii) 'The slippery slope argument, although influential, is hard to deal with rationally. It suggests that once we allow doctors to shorten the life of patients, who make such request, doctors could and would wantonly kill burdensome patients who do not want to die. This suggestion is not justified….'

D4. The owner of a 'fish and chips' shop in England defended the healthfulness of his deep-fried cookery with this argument: 'Take my son, Martin. He's been eating fish and chips his whole life, and he just had a cholesterol test, and his level is below the national average. What better proof could there be than a fryer's son?'

P1. i) 'To charge interest on the money loaned is quite legitimate. Therefore to take interest loaned to a friend in distress is quite legitimate.'

ii) 'All killers of humans are murderers. Soldiers are killers of humans in war. Therefore the soldiers are murderers.'

iii) 'To give charity to young, healthy beggar is wrong. Therefore, charity is bad.'

P2. i). 'How exactly did you feel when you murdered your brother?'
ii) 'Why are our politicians so corrupt?'
iii) 'How can we change our education system to make our studies more effective?'

P3. 'You ought to give alms because it is a duty to be charitable.'

A1. 'Idle men are inefficient. Idle men are incapable. Therefore, idle men are invaluable.'

A2. 'The farmer blew out his brains after receiving an affectionate farewell of his family with a shotgun.

A3. ‘Since every part of a certain machine is light in weight the machine as a whole is light in weight.’

A4. i) ‘That cricket team is a good team. So, each of its players must be good.’
ii) This corporation is very important and Mr. Das is an official of this corporation and therefore Mr. Das is very important.’

4.8 LET US SUM UP

Fallacy is an illogical way of arguing. Formal informal fallacies are two kinds of fallacy. A formal fallacy is committed when a Rule of inference is committed. Ambiguity in expression results in informal fallacy. If we know when a fallacy is committed we are less likely to err. Therefore knowledge of erroneous thinking has positive advantage. Inductive inference is not governed by any Rule of inference. Therefore in the strict sense of the word there is nothing like inductive fallacy.

4.9 KEY WORDS

Fallacy: An argument which violates a Rule of inference is called a fallacy.

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