UNIT 1 PROVING VALIDITY USING RULES OF INFERENCE

Contents

1.0 OBJECTIVES

The principal objective of this unit is to illustrate the Rules of Inference with examples and to make explicit the art of testing arguments, though these are dealt in the unit 3 of previous block. Here the focus is on formulating verbal form of arguments in their symbolic form and then prove the validity of argument. Such an exercise exposes the students to various Rules of Inference in proving validity of given argument.

1.1 INTRODUCTION

Logic is a science of right reasoning. All type of reasoning may not be valid. Hence, testing the rightness and wrongness of an argument becomes necessary. Logic classifies arguments as valid or invalid. We need a set of criteria for testing the validity of certain arguments. The symbolic logicians have spelt out certain Rules of Inference which would help us to see the correctness of certain arguments. In modern logic an argument is regarded as a sequence of statements. When proof is constructed to test the argument, the proof also takes the same form, which the argument takes. In this type of proof there is correspondence between the scheme of the given argument and the scheme of the proof. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as 'Rules of Inference' in modern logic. Many of these Rules have their origin in traditional logic.

1.2 NECESSITY OF RULES OF INFERENCE

As we have already seen that there are various types of syllogism, disjunctive, hypothetical etc., the Rules of Inference have classified them into various forms of criteria using symbols.

However, only limited types of arguments are covered by classical logic. All types of arguments is not alike in all respects. Some are Simple enough so that the truth-table technique is adequate for the purpose of testing. Generally, any argument, which consists of two or three simple but different propositions, can be easily put to test by truth-table method. If the argument consists of more than three different propositions, then the truth-table method is bit difficult and confusing. In those cases, the symbolic logicians have proposed an alternative.

Look at the following examples that makes this point clear. An argument, which is complex in this sense, is nothing but an aggregate of several simple (by simple, in this context, we mean short) arguments.

1)	2)	3)
$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow q$
p	q	$q \Rightarrow r$
$\therefore q$	$\therefore r$	$\therefore$

(3) is the sum of (1) and (2) in which the conclusions, q & r is hidden. In classical logic we have 'complex' type of argument in the form of sorites. (We should remember that complex, simple, etc. are relative). An example for sorites is given:

All Indians are Asians.
All Dalits are Indians.
All Mahars are Dalits.
∴ All Mahars are Asians.

In this poly-syllogistic arrangement of argument there are three premises and a conclusion. Usually any sorites consists of at least two syllogistic arguments and therefore, two conclusions. So it is more complex than an ordinary syllogism. This point becomes clear when we break sorites into constituent syllogisms.

All Indians are Asians.
All Dalits are Indians.] → All Dalits are Asians.
All Mahars are Dalits → All Mahars are Dalits
∴ All Mahars are Asians

1.3 MEANING OF PROOF OF VALIDITY

The Rules of Inference in the sentential logic are based on the classical traditional logic. In symbolic or modern logic, an argument is regarded as a sequence of statements. While constructing the proof to test the argument, the proof has to take the same form of the argument to which it corresponds. So there is a correspondence between these two, the scheme of the argument and the proof to test it. Every step, which is adduced while constructing proof, is the conclusion of the preceding statements, and in turn, becomes the premise for statements, which follow it (if not all, at least to some). Rules, which govern the process of deducing hidden conclusion, constitute what are known as 'Rules of Inference' in modern logic.

A particular pattern of proof construction is devised by modern logic. Discarding more descriptive method, which consumes both space and time, modern logic has discovered much shorter and simpler method. Whatever conclusion can be drawn from any two given premises is written on left had side (LHS), while the Rule and the premises to which this particular Rule applies to derive the conclusion used in further proof, are written on the right hand side (RHS). The serial numbers are used instead of premises to make the procedure simpler and more economical in terms of time and effort to grasp the argument. Yet, one must ensure that the premises, the conclusion drawn from them and corresponding Rule are always juxtaposed.

For instance let us look at the following argument.:

p => (q v r)
¬ r
¬ q

∴ ¬ p

There is a standard form in which we write the argument. After we write down the last premise we use a slash on the RHS which is followed by the conclusion. The given premises are numbered and the subsequent conclusions which are drawn also are progressively numbered. We enter the numbers on the RHS accordingly, we shall rewrite the argument.

1).	p => (q v r)	
2).	¬ r	
3).	¬ q / ∴ ¬ p	
4).	¬ r Λ ¬ q	2, 3, Conj.
5).	∴ ¬ p	1, 4, M.T.

M.T. stands for *modus tollens*. The Rule, which is familiar to those who have studied traditional logic. 1 and 4 signify 1^{st} and 4^{th} lines to which this Rule is applied. We need not mention which is the premise and which is a conclusion because except the last line all other lines consist of statements, which are regarded as premises.

It is not necessary that the premises should be written in the given order only. Care should be taken to omit conclusion from numbering. Finally, a word about symbols: We need not stick on to proposition form only. Hence when arguments are symbolized we use only uppercase letters.

1.4 NINE RULES OF INFERENCE

This section may be a repetition of a unit in the previous block which also lists out the same Rules of Inference. Modern logic considers nine Rules of Inference. It is sufficient to know them and how and where they should be applied. There is no need to prove them.

1). *Modus Ponens* (M.P.)

p => q
P
∴ q

2). *Modus Tollens* (M.T.)

$$p \Rightarrow q$$
$$\neg q$$
$$\therefore \neg p$$

3). *Hypothetical Syllogism (H.S.)*

$$p \Rightarrow q$$
$$q \Rightarrow r$$
$$\therefore p \Rightarrow r$$

4). *Disjunctive Syllogism D.S.)*

$p \vee q$		$p \vee q$
$\neg p$	Or	$\neg q$
$\therefore \neg q$		$\therefore \neg p$

5). *Constructive Dilemma (C.D.)*

$$(p \Rightarrow q) \wedge (r \Rightarrow s)$$
$$P \vee r$$
$$\therefore q \vee r$$

6). *Destructive Dilemma (D.D.)*

$$(p \Rightarrow q) \wedge (r \Rightarrow s)$$
$$\neg q \vee \neg s$$
$$\therefore \neg p \vee \neg r$$

7). *Simplification (Simp.)*

$$p \wedge q$$
$$\therefore p$$

8). *Conjunction (Conj.)*

p
q
∴ p Λ q

9). *Addition (Add.)*

P
∴ p v q

Absorption Rule was added by Copi to the remaining Rules. In these Rules, the first six Rules are standard Rules of traditional logic. The last three Rules need a little explanation. In simplification, since p Λ q is given to us, we accept that p is true, and q is true as well. So there is no harm in dropping any of them. The case of conjunction is slightly different. p is given to us, so we take it as true; q is given to us. So we take q also as true. Since both are taken as true we can conveniently conjoin them. The case of addition is slightly different. Suppose that we have only p in the premises. Since it is a premise, we take it as true. Suppose that we require q to be added to p. We do not know whether q is true or not. There is no harm in adding q to p because even if q is false p v q still remains true because p is true. After all, one true component can make disjunction true. But what is important is that conjunction does not mean addition. In logical language, addition means disjunction but not conjunction.

1.5 USAGE OF RULES OF INFERENCE TO TEST VALIDITY

1). p => (q v r)
2). ¬ r
3). ¬ q / ∴ ¬ p
4). ¬ r Λ ¬ q 2, 3 Conj.
5). ∴ ¬ p 1, 4, M.T.

Since this is the first argument, let us elaborate the process.

¬ r appears in 2nd line and ¬ q appears in 3rd line. Therefore the Rule of 'Conjunction' is applied. We get ¬ r Λ ¬ q

This forms the fourth line in the sequence. Now we shall consider 1st and 4th line together.

1). p => (q v r)
4). ¬ r Λ ¬ q

$\therefore \neg p$

In (1), (q v r) is the consequent. q v r being a disjunction, when it is denied, in accordance with de Morgan's law, it becomes a conjunction with original disjuncts being replaced by their respective negation. Since consequent is denied in the second premise, the antecedent has to be denied in the conclusion and it is done. Therefore in traditional logic it is a valid mood, viz., *modus tollendo tollens*, which has become a Rule of Inference in modern logic. For brevity, we say *modus tollens.* The form of (1) and (4) corresponds exactly to the form of Rule 2. The conclusion, which we obtained through formal proof, is the same as the conclusion of the given argument. This is how an argument is tested for validity. This is a model of explanation, which suits any argument.

1.6 CONVERTING VERBAL FORMS OF ARGUMENT INTO SYMBOLS

For change, let us start with verbal form of argument and symbolize the statements and logical constants before proceeding to test the validity of the arguments.

Example 1

I). If Raja joins, then the club's social prestige will rise; and if Pandiyan joins, then the club's financial position will be more secure. Either Raja or Pandiyan joins. If the club's social prestige rises, then Pandiyan will join; and if the club's financial position becomes more secure, then Suresh will join. Therefore either Pandiyan or Suresh will join.

1	Raja joins	=R
2	The club's social prestige will rise	= S
3	Pandiyan joins	= K
4	The club's financial position rises	= F
5	Suresh will join	= G

Now the argument becomes:

1	(R=>S) Λ (K=>F).	
2	R v K.	
3	(S=>K) Λ (F=>G)/∴ K v G.	
4	S v F	1, 2, C.D.
5	∴ K v G	3, 4, C.D.

Answer to the first argument makes one point very clear. Verbal expression is naturally very long and tedious, whereas symbolic representation is short and clear.

Example 2

II). If Arivazhagan received the wire, then he took the plane; and if he took the plane, then he will not be late for the meeting. If the telegram was incorrectly addressed, then Arivazhagan will be late for the meeting. Either Arivazhagan received the wire or the telegram was incorrectly addressed. Therefore either Arivazhagan took the plane or he will be late for the meeting.

1.	Arivazhagan received the wire	=V
2.	He took the plane	= P
3.	He will not be late of the meeting	= ¬L
4.	Telegram was incorrectly addressed	= ¬T
5.	Arivazhagan will be late for the meeting	=L

Now the arguments becomes:

1	(V=>P) Λ(P=>¬L)	
2	(¬T=>L)	
3	V v ¬T	/∴ P v L
4	V=>P	1, Simp.
5	(V=>P) Λ (¬T=>L)	4, 2, Conj.
6	P v L	5, 3, C.D.

Check your progress I.

Construct the symbolic forms of the following arguments.

1. If Nallan buys the plot, then an office building will be constructed; whereas if Mahesh buys the plot, then it quickly will be sold again. If Ponvanan buys the plot, then a store will be constructed; and if the store is constructed, then Lakshmi will offer to lease it. Either Nallan or Ponvanan will buy the lot. Therefore either an office building or a store will be constructed.

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2. If Arumugam goes to the meeting, then a complete report will be made; if Arumugam does not go to the meeting, then a special election will be required. If a complete report is made, then an investigation will be launched. If Arumugam going to the meeting implies that a complete report will be made, then if the making of a complete report implies that an investigation will be launched, then either Arumugam goes to the meeting and an investigation is launched or Arumugam does not go to the meeting and no investigation is launched. If Arumugam goes to the meeting and an investigation is launched, then some members will have to stand trial. But if Arumugam does not go to the meeting and no investigation is launched then the organization will disintegrate very rapidly. Therefore either some members will have to stand trial or the organization will disintegrate very rapidly.

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3. If Mr. Vetriselvan is the Kumar's next-door neighbour, then Mr. Vetriselvan's annual earnings are exactly divisible by three. If Mr. Vetriselvan's annual earnings are exactly divisible by 3, then Rs.20, 000/= is exactly divisible by 3. But Rs.20,000/= is not exactly divisible by 3. If Mr. Karuppaiah is Kumar's next-door neighbour, then Mr. Karuppaiah lives half way between Bengaluru and Chennai. If Mr. Karuppaiah lives in Bengaluru, then he does not live half way between Bengaluru and Chennai. Mr. Karuppaiah lives in Bengaluru. If Mr. Vetriselvan is not Kumar's next-door neighbour, then either Mr. Karuppaiah or Mr. Andiappan is Kumar's next-door neighbour. Therefore Mr.Andiappan is Kumar's next-door neighbour.

1.7 EXAMPLES FOR USING RULES OF INFERENCE

1)
1. (B v N) => (K Λ L)
2. ¬K
3. ¬M / ∴ ¬B Λ ¬M
4. ¬K v ¬L 2, Add.
5. ¬B Λ ¬N 1, 4, M.T.
6. ¬B 5, Simp.
7. ¬N Λ ¬M 6, 3, Conj.

2)
1. (M v N) => (P Λ Q)
2. N /∴ P
3. M v N 2, Add.
4. P Λ Q 1, 3, M.P.
5. ∴ P 4, Simp.

3)
1. (T => K) Λ (R => S)
2. S => D
3. D => T
4. R / ∴ T
5. R => S 1 Simp.
6. S 5,4, M. P.
7. D 2, 6, M. P.
8. ∴T 3, 7, M.P.

4)
1. (P =>Q) Λ(R=>S)

6)
1. (K => A) Λ (M => D)
2. ¬A /∴ ¬K v ¬M
3. ¬A v ¬D 2, Add.
4. ¬K v ¬M 1, 3 M.T.

7)
1. (A Λ B) => (C v D)
2. A
3. B / ∴C v D
4. A Λ B 2, 3, Conj.
5. C v D 1,4, M.P.

8)
1. (A v B) Λ(¬D ΛE)
2. A v B =>K/∴K Λ(¬D ΛE)
3. A v B) 1, Simp.
4. K 2, 4, M.P.
5. ¬D ΛE 1, Simp.
6. K Λ(¬D ΛE) 4, 5, Conj.

9)
1. Av(B ΛC)

2. ¬A=> ¬Q
3. A => ¬B
4. ¬B /∴ ¬PV ¬S
5. ¬A 3,4, M.T.
6. ¬Q 2,5, M.P.
7. P=>Q 1, Simp.
8. ¬P 7, 6, M.T.
9. ¬P v ¬S 9, Add.

5)
1. A Λ (B v C)
2. A=>P
3. Q / ∴PΛQ
4. A 1, Simp.
5. P 2,4, M.P.
6. P Λ Q 5,3, Conj.

11)
1. (B≡K)=>(Z ΛD)
2. ¬(Z ΛD) /∴¬(B≡K)
3. ∴¬ (B≡K) 1,2, M.T.

2. A=>P
3. ¬P / ∴C Λ (¬Λ)
4. ¬A 2,3, M.T.
5. B ΛC 1,4, D.S.
6. C 5, Simp.

10)
1. ¬B
2. ¬D
3. (A=>B) Λ(C=>D)
4. K / ∴ C (¬K Λ¬A)
5. A=>B 3, Simp.
6. ¬A 5, 1, M.T.
7. C=>D 3, Simp.
8. ¬C 7, 2, M.T.
9. ¬CΛ(¬K Λ¬A) 8,4,6, Conj.

18)
1. (K ΛT)=>(A v B)
2. (A v B)=>(P Λ¬L)
3. (P Λ¬L)=>D

``` ¬(D) /∴¬(K ΛT)
5. (K ΛT)=>(P Λ¬L) 2, H.S.
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6. (K ΛT)=>D 5,3, H.S.
7. ∴¬(K ΛT) 6,4 M.T.

12)
1. (K ΛA)=>(¬ B v C)
2. M=>(K ΛA)
3. M /∴¬B v C
4. M=> (¬B v C) 2,1. H.S.
5. ∴¬B v C 4,3, M.P.

13)
1. A=>D
2. B=>C
3. ¬D v ¬C/∴¬A v ¬B
4. (A=>D)Λ(B=>C)1,2,Conj.
5. ∴¬Av ¬B 4,3, D.D.

14)
1. JV(K Λ L)
2. J=>D
3. ¬D/∴K Λ L
4. ¬J 2,3, M.T.
5. ∴ (K ΛL) 1,4 D.S.

15)
1. A Λ(B=>C)
2. B /∴C
3. B=>C 1, Simp.
4. ∴C 3,2, M.P.

16)
1. Av(B ΛC)
2. A=>D
3. ¬D /∴B
4. ¬A 2,3, M.T.
5. B ΛC 1,2, D.S.
6. ∴B 5, Simp.

17)
1. (A v B)=>C
2. D=>¬C
3. D /∴¬(A v B)
4. ¬C 2,3, M.P.
5. ∴¬(A v B) 1,4, M.T.

19)
1. A=>D
2. B=>C
3. A v B /∴ D v C
4. (A=>D)Λ(B=>C)1,2, Conj.
5. ∴D v C 4,3, C.D.

20)
1. (A=>G)=>(K v ¬D)
2. ¬(K v ¬ D) /∴¬(A=>G)
3. ∴¬(A=>G) 1,2, M.T.

21)
1. D v (A=>B)
2. (A=>B)=>(C v K)
3. ¬(C v K) /∴D
4. ¬(A=>B) 2,3, M.T.
5. ∴D 1,4, D.S.

22)
1. (A=>B) Λ(C=>D)
2. A/∴B v D
3. A v C 2, Add.
4. ∴B v D 1,3, C.D.

23)
1. A=>B
2. B=>C
3. ¬C /∴¬A
4. ¬B 2,3, M.T.
5. ∴¬A 1,4, M.T.

24)
1. (A=>C) Λ(B=>D)
2. K=>A
3. K /∴C v D
4. A 2,3, M.P.
5. A v B 4, Add.
6. ∴C v D 1,5, C.D.

1.8 RULES OF REPLACEMENT

Not all arguments can be tested if restricted to Rules of Inference only, though as shown above somewhat complex and diverse arguments succumb to these Rules. Just as modern logic tried to supplement traditional logic, within modern logic, the need was felt to supplement the Rules of Inference. Hence we have the Rules of Replacement. The structure of argument may be such that it may require only one of the kinds or both. We have ten such Rules, which are called the Rules of Replacement. The difference between these two sets of Rules is that the Rules of Inference are themselves inferences whereas Rules of Replacement are not. This is because the Rules of Replacement are restricted to change or changes in the form of statements. For example, if A or B is changed to B or A, then such change is governed by one Rule. Similarly, if A Λ (B v C) is changed to (A Λ B) v (A Λ C), then this change is governed by some other Rule. Also, in the mode of application of Rules there is a restriction. Unlike Rules of Inference which should be applied to the whole line only any Rule of Replacement can be applied to any part of the line. This is because all Rules of Replacement are, logically, equivalent expressions.

Now let us list Rules of Replacement.

1. De Morgan's Law (De.M.) — $\neg (p \land q) \equiv \neg p \lor \neg q$
   $\neg (p \lor q) \equiv \neg p \land \neg q$

2. Commutation Law (Com.) — $p \lor q \equiv q \lor p$
   $p \land q \equiv q \lor p$

3. Double Negation(D.N.) — $\neg (\neg p) \equiv p$

4. Transposition (Trans.) — $(p \Rightarrow q) \equiv (\neg q \Rightarrow \neg p)$

5. Material Implication (Impl.) — $(p \Rightarrow q) \equiv \neg p \lor q$

6. Material Equivalence (Equiv.) — $(p \equiv q) \equiv (p \Rightarrow q) \land (q \Rightarrow p)$
   $\{(p \equiv q) \equiv \{(p \land q) \lor (\neg p \land \neg q)\}$

7. Exportation (Exp.) — $\{(p \land q) \Rightarrow r\} \equiv \{p \Rightarrow (q \Rightarrow r)\}$

8. Tautology (Taut.) — $p \equiv p \lor p$
   $p \equiv p \land p$

9. Association (Ass.) — $\{p \lor (q \lor r)\} \equiv \{(p \lor q) \lor r\}$
   $\{p \land (q \land r)\} \equiv \{p \land q) r\}$

10. Distribution (Dist.) $\{p \wedge (q \vee r)\} \equiv \{(p \wedge q) \vee (p \wedge r)\}$
$\{p \vee (q \wedge r)\} \equiv \{p \vee q) \wedge (p \vee r)\}$

[Note: When the expression includes both ‘Λ’ and ‘v’ only distribution law can be applied but not association law.]

Let us restate de Morgan’s law differently. Negation of a conjunction is equivalent to the disjunction of the negation of components and negation of disjunction is equivalent to the conjunction of negation of components. Similarly, in the case of implication the expression is equivalent to the transposition of the negation of components. Likewise the rest of the Rules can be interpreted. This relation becomes further clear if we construct truth-table for any Rule (for both sides of the equation). The student is advised to construct truth-table for any Rule of Replacement to verify the equivalence. Our immediate task is to become familiar with the technique of testing arguments.

(1).

1. $\{I \Rightarrow (J \Rightarrow K)\} \wedge (J \Rightarrow \neg I)$ $/ \therefore \{(I \wedge J) \Rightarrow K\} \wedge (J \Rightarrow \neg I)\}$
2. $\{(I \wedge J) \Rightarrow K\} \wedge (J \Rightarrow \neg I)$ 1, Exp.

(2.)

1. $(R \wedge S) \Rightarrow (\neg R \vee \neg S)$ $/ \therefore \neg (\neg R \vee \neg S) \Rightarrow \neg (R \wedge S)$
2. $\neg (\neg R \vee \neg S) \Rightarrow \neg (R \wedge S)$ 1, De.M.

(3.)

1. $(T \vee \neg U) \wedge \{(W \wedge \neg V) \Rightarrow \neg T\}$ $/ \therefore (T \vee \neg U) \wedge \{(W \Rightarrow (\neg V \Rightarrow \neg T)\}$
2. $(T \vee \neg U) \wedge \{(W \Rightarrow (\neg V \Rightarrow \neg T)\}$ 1, Exp.

(4.)

1. $X \vee Y) \wedge (\neg X \vee Z)$ $/ \therefore (X \vee Y \wedge \neg X) \vee \{(X \vee Y) \wedge Z\}$
2. $(X \vee Y \wedge \neg X) \vee \{(X \vee Y) \wedge Z\}$ 1,Dist.

(5.)

1. $Z \Rightarrow (A \Rightarrow B)$ $/ \therefore Z \Rightarrow \neg\{ \neg (A \Rightarrow B\}$
2. $Z \Rightarrow \neg\{ \neg (A \Rightarrow B\}$ 1,D.N.

(6.)

1. $(\neg F \vee G) \wedge (F \Rightarrow G)$ $/ \therefore F \Rightarrow G.$).
2. $(F \Rightarrow G) \wedge (F \Rightarrow G)$ 1, Impl.
3. $F \Rightarrow G$ 2, Taut.

Now we shall consider different types of arguments, which may involve both kinds of Rules.

(1.)

1.	(O => ¬P) Λ (P => Q)		
2.	Q => O		
3.	¬R => P /∴ R		
4.	¬ Q v O	2,	Impl.
5.	O v ¬ Q	4,	Com.
6.	(O => ¬ P) Λ (¬ Q => ¬P)	1,	Trans,
7.	¬P v ¬P	6, 5,	C.D.
8.	¬P	7,	Taut.
9.	¬ ¬ R	3, 8,	M.T.
10.	∴ R	9,	D.N.

(2.)

1.	X => (Y => Z)		
2.	X => (A => B)		
3.	X Λ (Y v A)		
4.	¬Z /∴ B		
5.	(X Λ Y) => Z	1,	Exp.
6.	(X Λ A) => B	2,	Exp.
7.	(X Λ Y) v (X Λ A)	3,	Dist.
8.	{(X Λ Y) => Z} Λ {(X Λ A) => B}	5,6,	Conj.
9.	Z v B	8, 7,	C.D.
10.	∴ B	9, 4,	M.T.

(3.)

1.	C => (D => ¬ C)		
2.	C ≡ D /∴ ¬C v ¬D		
3.	C => (¬ ¬C => ¬D)	1,	Trans.
4.	C => (C => ¬D)	3,	D.N.
5.	(C Λ C) => ¬D	4,	Exp.
6.	C => ¬D	5,	Taut.
7.	¬C v ¬D	6,	Impl.

(4.)

1.	E Λ (F v G)		
2.	(E Λ G) => ¬ (H v I)		
3.	¬ (¬H v ¬I) => ¬ (E Λ F) / ∴H ≡ I		
4.	(E Λ G) => (¬H Λ ¬I)	2,	De.M.
5.	¬ (H Λ I) => ¬ (E Λ F)	3,	De.M.
6.	(E Λ F) => (H Λ I)	5,	Trans.
7.	{(E Λ F) => (H Λ I)} Λ {(E Λ G) => (¬H Λ ¬I)}	6,4,	Conj.
8.	(E Λ F) v (E Λ G)	1,	Dist.
9.	(H Λ I) v (¬H Λ ¬I)	7,8,	C.D.
10.	∴ H ≡ I	9,	Equiv.

(5.)

1. J v (¬K v J)			
2. K v (¬J v K)	/ ∴ J ≡ K		
3. (¬K v J) v J		1,	Com.
4. ¬K v (J v J)		3,	Ass.
5. ¬K v J		4,	Taut.
6. K => J		5,	Impl.
7. (¬J v K) v K		2,	Com.
8. ¬J v (K v K)		7,	Ass.
9. ¬J v K		8,	Taut.
10. J => K		9,	Impl.
11. (J => K) Λ (K => J)		10, 6,	Conj.
12. ∴ J ≡ K		11,	Equi.

Check Your Progress II

Note: Use the space provided for your answers.

1

1)
1. Av(B=>C)
2. A=>D
3. ¬D /∴B=>C
4. ¬A
5. ∴B=>C

2)
1. (K≡L)=>A ΛB
2. D=>(K≡L)/ ∴A
3. A Λ B
4. ∴A

3)
1. I=>J
2. J=>K
3. L=>M
4. I v L /∴ K v M
5. I=>K 1,2 HS
6. (I=>K)Λ(L=>M)
7. ∴K v M

4)
1. (A=>B)=>(C=>D)
2. (E=>F)=>(A=>B)
3. ¬(C=>D) /∴¬ (E=>F)
4. (E=>F)=>(C=>D)
5. ∴¬(E=>F)

5)
1. A ΛB
2. (A v C)=>D/∴A ΛD
3. A
4. A v C
5. D
6. A Λ D

6)
1. (E v F) Λ(G v H)
2. (E=>G) Λ(F=>H)
3. ¬G /∴H
4. E v F
5. G v H
6. ∴H

1.9 LET US SUM UP

Modern Logic is an extension of traditional logic. However, there is qualitative difference in testing. Difference consists in accuracy and clarity of proof. Nine Rules of Inference include many Rules from traditional logic like modus ponens. All nine Rules are not required always. Only some Rules are required. There is no Rule, which says that one line must be considered only once.

1.10 KEY WORDS

Modus Ponens (MP) : It is a valid and simple argument form sometimes referred to as affirming the antecedent or the law of detachment.

Modus Tollens (MT): It is a valid and simple argument form that is denying the consequent.

Validity: **A**n argument is valid if and only if the truth of its premises entails the truth of its conclusion.

1.11 FURTHER READINGS AND REFERENCES

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UNIT 2 CONDITIONAL PROOF

Contents

1.0 OBJECTIVES

There are as many kinds of techniques as there are arguments. This unit is designed to introduce you to two new Rules which help you to compare the new technique or techniques with the earlier techniques. This comparative appraisal is possible only when you use both methods and consider suitable examples. This is the main objective of this unit. This unit enables you to understand that what is regarded as elementary valid argument forms or Rules of Inference and Rules of Replacement are also fundamental and hence indispensable. This is the second objective intended to be achieved.

2.1 INTRODUCTION

It is wrong to think that there is a single technique which is applicable on all occasions irrespective of the structure of arguments. On some occasions Rules of Inference and Replacement are useful and on some other occasions the technique known as Conditional Proof (C. P.) is useful. But there is no single method which is indispensable for all occasions. It is important to recognize this simple fact which prevents us from following a particular method blindly. When the advantage of new technique becomes clear, the significance of new Rule or Rules also becomes clear and it will be put into use. When we know that it is disadvantageous, we refrain from using that particular method. This means that a method is not to be used just because it has worked on earlier occasions. Instead, we try to apply a different method which is thought to be useful. The method of Conditional Proof (C.P.) is different in kind from the Rules of Inference or Replacements in one sense. There are a certain types of arguments, which can be tested with any of the Rules discussed in the previous chapters only with great difficulty or it may be practically impossible to test them at all. In all such cases C. P. comes to our rescue. Let us apply these methods and compare the results in order to understand the fact that the relevance of method is determined by the structure of argument.

1).

1	(A v B) => (C Λ D)	
2	(D v E) => F/ ∴ A => F	
3	¬ (A v B) v (C Λ D)	1, Impl.
4	(¬ A Λ ¬ B) v (C Λ D)	3, De. M.
5	(¬ A v C) Λ (¬ A v D) Λ (¬ B v C) Λ (¬ B v D)	4, Dist.
6	¬ (D v E) v F	2, Impl.
7	¬ A v D	5, Simp.
8	(¬D Λ ¬ E) v F	6, De. M.
9	(¬D v F) Λ (¬E v ¬ F)	8, Dist.
10	¬D v F	9, Simp.
11	A => D	7, Impl.
12	D => F	10, Impl.
13	∴ A => F `	11, 12, H. S.

It is obvious that we have used at present only Rules of Inference and Rules of Replacement. What is the position? It may be noted that from 3rd line to 13th line there are one hundred and twelve words and five Rules are used on eleven occasions. These figures become significant when we use new set of Rules and then calculate the length of proof construction which helps us to compare the length, number of words and others involved in these methods mentioned above. An important restriction is that the Rules of Inference are useful generally only when the arguments, have unconditional conclusions. So an argument, which has conditional conclusion, falls out of their purview on many occasions if simplicity is the yardstick. The most familiar example for conditional proposition is implicative proposition. Since implicative propositions have equivalent disjunctive and negation forms, they are also to be regarded as conditional propositions. Therefore if the conclusion is any one of these forms, then either the construction of proof may be very long and complex or may even be impossible. On such occasions we are likely to err. Therefore in order to insulate ourselves against highly probable errors we have to look for safer routes. C. P. is one such path. Again, C.P is not a system of proof, which is absolutely independent of nineteen Rules. Only, the number increases to twenty. Among them one Rule is called Rule of C. P. which is compulsorily used to test the validity when the conclusion is conditional. This Rule is unique in the sense that nowhere else it is used.

There are two Rules under this category. They are known as C. P., and the Strengthened Rule of C. P. This classification does not imply that the former is weak. The difference lies only in scope. We shall begin with the former first.

2.2 CONDITIONAL PROOF (C.P.)

Any deductive argument, whether valid or invalid, can be expressed in the form of a conditional proposition. What is more important to know is that the original argument is valid only when the corresponding conditional statement satisfies a condition known as 'tautology'. Otherwise, the argument is invalid. Consider this example:

2).

All A are B

All B are C / ∴ All A are C

Its corresponding conditional form is as follows:
"If all A are B and all A are C, then all A are C". (1)
Let the first premise be symbolized as P_1 and second as P_2. Conclusion is symbolized as C. Now (1) becomes:
$(P_1 \wedge P_2) \Rightarrow C$ (2)
(2) is said to be tautologous because its corresponding proposition form is tautologous. A proposition form is said to be tautologous when it has only true substitution. There are two conditions to be satisfied if C. P. should be used to show that the argument is valid.
1) Conclusion must be a conditional proposition.
2) It should be possible to deduce a conditional proposition from a conjunction of premises by a sequence of elementary valid arguments, which satisfy the relevant Rules of Inference. That is, all premises in C.P. should be supported by these Rules. The additional premise, which is a characteristic mark of C. P., is always the antecedent of the conclusion and the construction of proof always begins with antecedent of the conclusion as the premise. This premise itself is called C.P. An example of argument, which requires C.P., is given below.

$P \Rightarrow (A \Rightarrow B)$ (3)
When P stands for the conjunction of premises, one of the Rules of Replacement, i.e., Exportation Rule permits us to rewrite (3) as:
$(P \wedge A) \Rightarrow B$ (4)

It is obvious that the conclusion of (4) is the consequent of the conclusion of (3). Since you start with an assumed premise, the proof is known as C.P. Here is the difference. All other premises are taken as true. The assumption should not really matter. Even if the assumed premise is false, it is possible to deduce a valid conclusion. If B can be validly drawn from P Λ A, then not only (4) is valid its corresponding original argument (3) also must be valid because (3) and (4) are logically equivalent.

Now consider the argument considered above.

3)

1. (A v B) => (C Λ D)
2. (D v E) => F / ∴ A => F
3. A /∴ F   C. P.
4. A v B   3,   Add.
5. C Λ D   1, 4,   M.P.
6. D   5,   Simp.
7. D v E   6,   Add.

8. ∴ F 2, 7, M.P.

Now compare the lengths of 1 and 3. In 3 there are thirty five words whereas in 1 there are one hundred and twelve words and in 3 four Rules are used on six occasions whereas in 1 five Rules are used on eleven occasions. This comparison helps us to know the advantage of new technique.

A brief explanation of steps involved in proof construction in this method is necessary. You should start from assuming A which is the antecedent of the conclusion. Always the first line must have this structure in C. P. Slash against line 3 in, ∴ and (C.P) indicate that the method of C. P. is being used.

If there is only one condition in the conclusion, then C.P is used once. If there are two conditions in the consequent component of the conclusion, then C.P. is used twice. It means that the number of times the C. P. has to be used is equivalent to the number of conditions that appear in the consequent of the conclusion. Now it is plain that the complexity of argument increases with the increase in the number of conditions in the conclusion. The following example illustrates the procedure to be followed in such cases.

4)

1	A => (B => C)		
2	B => (C => D) /∴ A => (B => D)		
3	A	/∴ B => D	(C.P.)
4	B	/∴ D	(C.P.)
5	B => C	1, 4,	M.P.
6	C	5, 4,	M.P.
7	C => D	2, 4,	M.P.
8	∴D	7, 6,	M.P.

The student is advised to use the Rules of Inference or Replacement or both depending upon the need and compare the lengths of proof construction.

Consider an argument with a disjunctive conclusion and use both the methods without making any presumption to compare the lengths and complexity of proof construction. we shall begin with earlier method.

5)

1	A = > B	
2	C = > D	
3	(¬ B v ¬ D) (¬ A v ¬ B) /∴ (¬ A v ¬ C)	
4	(A = > B) Λ (C = > D)	1, 2, Conj.
5	¬ B v ¬ D	3, Simp.
6	∴ ¬ A v ¬ C	4, 5, D. D.

A = > ¬ C is equivalent to the original conclusion. Therefore if C. P. method has to be used, then A = > ¬ C must replace ¬ A v ¬ C. Now you shall construct proof using C. P. method.

4	A	/∴ ¬ C (C.P.)
5	¬ B v ¬ D	3, Simp.
6	B	1, 4, M. P.
7	¬ D	5, 6, D.S.
8	∴ ¬ C	2, 7, M.T.

In the former method thirty three words, three lines and three Rules are involved whereas in the latter thirty four words, five lines and four Rules are involved. This comparison illustrates the fact that just because the conclusion is conditional the method of C. P. is not necessarily preferable.

Now let us consider a more complex example with multiple variables and more number of premises. Again we will begin with the Rules of Inference.

1	H => (I => J)	
2	K => (I => J)	
3	(¬ H Λ ¬ K) => (¬ L v ¬ M)	
4	(¬ L => ¬ N) Λ (¬ M => ¬ O)	
5	(P => N) Λ (Q => O)	
6	¬ (I => J)	/∴ ¬ P v ¬ Q
7	¬ H	1,6, M.T.
8	¬ K	2,6, M.T.
9	¬ H Λ ¬ K	7,8, Conj.
10	¬ L v ¬ M	3,9, M.P.
11	¬ N v ¬ O	4,10, C.D.
12	∴ ¬ P v ¬ Q	5,11, D.D.

In this method there are forty three words, six lines and five Rules. Now apply the method of C.P. Before we do so the conclusion must be transformed into implicatory form. Hence the conclusion is P => ¬ Q.

6)	¬ (I => J) / P => ¬ Q	
7)	P /∴ ¬ Q (C. P.)	
8)	P => N	5, Simp.
9)	N	5, 7, M. P.
10)	¬ L => ¬ N	4, Simp.
11)	L	10, 9, M.T.
12)	¬H	1, 6, M.T.
13)	¬K	2, 6, M.T.
14)	¬H Λ ¬K	12, 13, Conj.
15)	¬ L v ¬ M	3, 14, M.P.
16)	L => ¬ M	14, Impl.
17)	¬ M	16, 11, M.P.
18)	¬ M => ¬ O	4, Simp.
19)	¬ O	18, 17, M.P.
20)	Q => O	5, Simp.

21) ∴ ¬ Q 20, 19, M.T.

We know prima facie that this method is very long with ninety seven words, fifteen lines and five Rules and we know that with the exception of Conjunction other Rules recur again and again. Therefore C.P. must be used only when it is economical in terms of space and effort.

2.3 EXERCISES I

I. Some arguments are considered below which are tested using the method of C. P.

1)

1. P => (Q => R) /∴ (P => Q) => (P => R)
2. P => Q /∴ P => R (C.P.)
3. P /∴ R (C.P.)
4. (P => Q) => R 1, Exp.
5. ∴R 4, 2, M.P.

2)

1. P => (Q => R) / ∴ Q => (P => R)
2. Q /∴ P => R (C.P.)
3. P /∴ R (C.P.)
4. Q => R 1, 3, M.P.
5. ∴R 4, 2, M.P.

3)

1. P => Q / ∴ ¬ Q => ¬ P
2. ¬ Q / ∴ ¬ P (C.P.)
3. ∴¬P 2, 1, M.T.

4)

1. P => ¬ ¬ P
2. ¬ ¬ P /∴ P (C.P.)
3. ∴ P 2, D.N.

5)

1. A => B / (B => C) => (A => C)
2. B => C /∴ A => C (C.P.)
3. A /∴C (C.P.)
4. B 1, 3, M.P.
5. ∴C 2, 4, M.P.

6)

1. (A => B) Λ (A => C) /∴ A => (B v C)

2. A	/∴B v C		(C.P.)
3. A => B		1,	Simp.
4. B		3, 2,	M.P.
5. ∴B v C		4,	Add.

7)

1. (A => B) Λ (A => C)	/ ∴ A => (B Λ C)		
2. A	/∴B Λ C		(C.P.)
3. A => B		1, Simp.	
4. B		3, 2,	M.P.
5. A => C		1,Simp.	
6. C		5, 2,	M.P.
7. ∴B Λ C		4, 6,	Conj.

8)

1. A => B	/∴ (¬ A => B) => B		
2. (¬ A => B)	/∴B	(C.P.)	
3. ¬ A	/∴B	(C.P.)	
4. ∴B		2, 3,	M.P.

9)

1. (A => B)	/∴ (A Λ C) => (B Λ C)		
2. A Λ C	/∴B Λ C		(C.P.)
3. A		2,	Simp.
4. B		1, 3,	M.P.
5. C		2,	Simp.
6. ∴B Λ C		4, 5,	Conj.

10)

1. B => C	/∴ (A v B) => (C v A)		
2. A v B	/∴C v A	(C.P.)	
3. ¬ A => B		2,	Impl.
4. ¬ A => C		3, 1,	H.S.
5. A v C		4,	Impl.
6. ∴C v A		5,	Com.

11)

1. (A v B) => C	/∴ [(C v D) => E] => (A => E)		
2. (C v D) => E	/ ∴ A => E	(C.P.)	
3. A	/∴ E	(C.P.)	
4. A v B		3,	Add.
5. C		1, 4,	M.P.
6. C v D		2,	Add.
7. ∴E		2, 6,	M. P.

12)

1. (P Λ Q) => P

2. P / ∴ Q => P
3. Q /∴P (C.P.)
4. P Λ Q 2, 3, Conj.
5. ∴P 4, Simp.

[Note: You can apply M. P. to 1 and 4 to obtain the same result.]

13)

1. P => Q /∴ (¬ Q v R) => ¬ (R Λ P)
2. (R Λ P) => (Q Λ R) 1, Trans.
3. R Λ P /∴Q Λ R (C.P.)
4. P 3, Simp.
5. Q 1, 4, M.P.
6. R 3 Simp.
7. ∴Q Λ R 5, 6, Conj.

Check your progress I.

Note: Use the space provided for your answer.

1. Explain the significance of Conditional Proof.

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2. What is the advantage of C. P. method?

2.4 THE STRENGTHENED RULE OF C. P.

In Conditional Proof method, the conclusion depends upon the antecedent of the conclusion. There is another method, which is called the Strengthened Rule of Conditional Proof. In this method, the construction of proof does not necessarily assume the antecedent of the conclusion. The structure of this method needs some elaboration. An assumption is made initially. There is no need to know the truth-status of the assumption because an assumption may be false, but the conclusion can still be true. Further, the assumption can be any component of any premise or conclusion. This method is called the Strengthened Rule because we enjoy more freedom in making assumption or assumptions which means that plurality of assumptions is allowed. It strengthens our repertoire of testing equipments. In this sense, this method is called the Strengthened Rule of C.P. Another feature of this method is the limit of assumption. The last step is always outside the limits of assumption. If there are two or more than two assumptions in an argument, then there will be a separate last step for each assumption. This last step can be regarded as the conclusion relative to that particular assumption. It shows that the last step is deduced with the help of assumption in conjunction with the previous steps in such a way that the Rules of Inference permit such conjunction. Before the conclusion is reached the function of assumption also ceases. Then it will have no role to play. Then, automatically, the assumption is said to have been discharged. When the Strengthened Rule of C. P. is used adjacent to the line of assumption, the word assumption is not mentioned unlike as in the case of C.P. Here the head of the bent arrow points to ‘assumption’. In case of the Strengthened Rule of C.P., the conclusion is always a conditional statement which consists of statements from the sequence itself.

Thus the range of the application of condition is defined. In order to easily identify the range of its application, a slightly different method is used. An arrow is used to indicate what is assumed and with the help of the same arrow its range also is defined. The application of C.P. is restricted to the space covered by the arrows. All steps, which are outside this arrow, are also independent of the condition. While the head of the arrow makes the assumption, its terminus separates the lines, which depend upon the condition from the line, which does not depend. Since the conclusion does not depend upon its own antecedent, it has to depend upon the first assumption only. In this sense, it is a strengthened condition. In this case there is no reason to mention C.P. because the arrow helps us to identify the assumption. Consider this example:

1).

1	(A v B) =>{(C v D) =>E} /∴ A=>[(CΛD) => E]		
2	A		
3	A v B	2,	Add.
4	(C v D)=>E	1, 3,	M.P.
5	(CΛD)		
6	C	5,	Simp.
7	C v D	6,	Add.
8	E	4, 7,	M.P.
9	(C v D)=>E	5, 8,	C.P.
10	∴ A=> [CΛD) =>E]	2, 9,	C.P.

Rules mentioned on the RHS make it clear that all lines from 3 to 9 depend on A either directly or through lines which depend A. In lines 9 and 10 implication makes them C.P.

One advantage of C.P. in its strengthened form is that it has an extended application. It can be used in all those cases where conclusions are conditional but do not appear to be so. Using the strengthened Rule of C. P. let us solve some problems.

2).

1. (E v F) = > G
2. H => (I ∧ G) / ∴ (E => G) ∧ (H ∧ I)
3. E
4. E v F 3, Add
5. G 4, 3, M. P.

6. E => G 3, 5, C. P.
7. H
8. I ∧ G 2, 7, M. P.
9. I 8, Simp.

10. H => I 7, 9, C. P.
11. ∴ (E => G) ∧ (H => I) 6, 10, Conj.

3).

1. Q v (R => S)
2. {R => (R ∧ S)} => (T v U)
3. (T => Q) ∧ (U => V) / ∴ Q v V

In this argument, in reality, the conclusion is conditional. We know that disjunction can be translated to implication form. When it so translated, the conclusion becomes

¬ Q => V

Now let us construct proof for this argument.

4. ¬ Q
5. R => S 1, 4, D. S.

6. R
7. S 5, 6, M. P.
8. R ∧ S 6, 8, Conj.

9. R => (R ∧ S) 6, 8, C. P.
10. T v U 2, 9, M. P.
11. ∴Q v V 3, 10, C. D.

4).

1.	$(C \lor D) \Rightarrow (E \Rightarrow F)$	
2.	$\{E \Rightarrow (E \land F)\} \Rightarrow G$	
3.	$G \Rightarrow \{(\lnot H \lor \lnot\lnot H)\} \Rightarrow (C \land H)\}$ / $\therefore C \equiv G$	
4.	C	
5.	$C \lor D$	4, Add
6.	$E \Rightarrow F$	1, 5, M. P.
7.	E	
8.	F	6, 7, M. P.
9.	$E \land F$	7, 8, Conj.
10.	$E \Rightarrow (E \land F)$	7, 9, C. P.
11.	G	2, 10, M. P.
12.	H	
13.	$\lnot\lnot H$	12, D. N.
14.	$\lnot\lnot H \lor \lnot H$	13, Add
15.	$\lnot H \lor \lnot\lnot H$	14, Com.
16.	$G \Rightarrow (\lnot H \lor \lnot\lnot H)$	11, 15, C. P.
17.	$C \land H$	3, 16, M. P.
18.	C	17, Simp.
19.	$C \Rightarrow G$	18, 11, C. P.
20.	$G \Rightarrow C$	11, 18, C. P.
21.	$(C \Rightarrow G) \land (G \Rightarrow C)$	19, 20, Conj.
22.	$\therefore C \equiv G$	21, Equiv.

5)

1.	$(E \lor F) \Rightarrow G$	
2.	$H \Rightarrow (I \land G)$ / $\therefore (E \Rightarrow G) \land (H \land I)$	
3.	E	
4.	$E \lor F$	3, Add
5.	G	4, 3, M. P.
6.	$E \Rightarrow G$	3, 5, C. P.
7.	H	
8.	$I \land G$	2, 7, M. P.
9.	I	8, Simp.
10.	$H \Rightarrow I$	7, 9, C. P.
11.	$\therefore (E \Rightarrow G) \land (H \Rightarrow I)$	6, 10, Conj.

6)

1. (C v D) => (E => F)
2. {E => (E $\wedge$ F)} => G
3. G => {(¬ H v ¬ ¬ H)} => (C $\wedge$ H)} / ∴ C ≡ G
4. C
5. C v D 4, Add
6. E => F 1, 5, M. P.
7. E
8. F 6, 7, M. P.
9. E $\wedge$ F 7, 8, Conj.
10. E => (E $\wedge$ F) 7, 9, C. P.
11. G 2, 10, M. P.
12. H
13. ¬ ¬ H 12, D. N.
14. ¬ ¬ H v ¬ H 13, Add
15. ¬ H v ¬ ¬ H 14, Com.
16. G => (¬ H v ¬ ¬ H) 11, 15, C. P.
17. C $\wedge$ H 3, 16, M. P.
18. C 17, Simp.
19. C => G 18, 11, C. P.
20. G => C 11, 18, C. P.
21. (C => G) $\wedge$ (G => C) 19, 20, Conj.
22. ∴ C ≡ G 21, Equiv.

7).

1 ¬ P∨ (Q => R)

¬ Q ∨ (¬ R ∨ D) / ∴ P => (Q => D)

1.	¬ P ∨ (Q => R)	
2.	¬ Q ∨ (¬ R ∨ D)	/ ∴ P => (Q => D)
3.	P	/ ∴ (Q => D)
4.	Q	/ ∴ D
5.	¬ ¬ Q	D.N 4.
6.	¬ R ∨ D	D.S. 2 &5
7.	¬ ¬ P	D.N 3
8.	Q => R	D.S 1&7
9.	R	M.P. 8&4
10.	¬ ¬ R	D.N. 9
11.	D	D.S.6&10
12.	Q => D	C.P. 4-11
13.	∴ P => (Q => D)	C.P. 3-12

8)

(P => ¬ Q) ∨ R

(R => S)

¬ S / ∴ Q => ¬ P

1.	(P => ¬ Q) ∨ R	
2.	(R => S)	
3.	¬ S	/ ∴ Q => ¬ P
4.	Q	/ ∴ ¬ P
5.	¬ R	M.T 2&3.
6.	R ∨ (P => ¬ Q)	Com 1.
7.	(P => ¬ Q)	D.S 6&5
8.	¬ ¬ Q	D.N 4.
9.	¬ P	M.T. 7&8
10.	∴ Q => ¬ P	C.P 4&9

9)

$\neg A \vee (B \Rightarrow C)$
$\neg B \vee (\neg C \vee D)$ $Q \Rightarrow \neg P$ / $\therefore$ $A \Rightarrow (B \Rightarrow D)$

1.	$\neg A \vee (B \Rightarrow C)$	
2.	$\neg B \vee (\neg C \vee D)$	/ $\therefore$ $A \Rightarrow (B \Rightarrow D)$
3.	A	/ $\therefore$ $(B \Rightarrow D)$
4.	B	/ $\therefore$ D
5.	$\neg \neg B$	D.N 4.
6.	$\neg C \vee D$	D.S. 2 &5
7.	$\neg \neg A$	D.N 3
8.	$B \Rightarrow C$	D.S 1&7
9.	C	M.P. 8&4
10.	$\neg \neg C$	D.N. 9
11.	D	D.S.6&10
12.	$B \Rightarrow D$	C.P. 4-11
13.	$\therefore \Rightarrow (B \Rightarrow D)$	C.P. 3-12

10)

$\neg P \vee (Q \Rightarrow R)$
$\neg Q \vee (\neg R \vee S)$ / $\therefore P \Rightarrow (Q \Rightarrow S)$

1.	$\neg P \vee (Q \Rightarrow R)$	
2.	$\neg Q \vee (\neg R \vee S)$	/ $\therefore$ $P \Rightarrow (Q \Rightarrow S)$
3.	P	/ $\therefore$ $(Q \Rightarrow S)$
4.	Q	/ $\therefore$ S
5.	$\neg \neg Q$	D.N 4.
6.	$\neg R \vee S$	D.S. 2 &5
7.	$\neg \neg P$	D.N 3
8.	$Q \Rightarrow R$	D.S 1&7
9.	R	M.P. 8&4
10.	$\neg \neg R$	D.N. 9
11.	S	D.S.6&10
12.	$Q \Rightarrow S$	C.P. 4-11
13.	$\therefore P \Rightarrow (Q \Rightarrow S)$	C.P. 3-12

Check your progress II.

Note: Use the space provided for your answer.

1. Write a brief note on the salient aspects of the Rule of Strengthened Proof.

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2. What do you mean by ‘discharging of assumption’?

2.5 EXERCISES II

1. $(A \vee B) \Rightarrow (C \wedge D)$
   $(D \vee E) \Rightarrow F$ / $\therefore$ $(A \Rightarrow F)$

2. $[A \Rightarrow (B \Rightarrow C)]$
   $[B \Rightarrow (C \Rightarrow D)]$ / $\therefore$ $[A \Rightarrow (B \Rightarrow D)]$

3. $C \Rightarrow D$ / $\therefore$ $[(D \Rightarrow E) \Rightarrow (C \Rightarrow E)]$

4. $(N \Rightarrow P) \wedge (B \Rightarrow S)$ / $\therefore$ $[(N \wedge B) \Rightarrow (P \wedge S)]$

5. $(E \Rightarrow F$
   $[E \Rightarrow (F \Rightarrow G)]$
   $[E \Rightarrow (G \Rightarrow H)]$ / $\therefore$ $(E \Rightarrow H)$

6. $[\neg (P \vee Q) \vee (R \wedge S)]$
   $[\neg (S \vee T) \vee U]$ / $\therefore$ $(P \Rightarrow U)$

7. $[P \Rightarrow (\neg Q \vee R)]$
   $[Q \Rightarrow (\neg S \Rightarrow \neg R)]$ / $\therefore$ $[P \Rightarrow (Q \Rightarrow S)]$

8. $\neg C \vee D$ / $\therefore$ $[(\neg D \vee E) \Rightarrow (C \Rightarrow E)]$

9. $(M \Rightarrow Q) \wedge (B \Rightarrow S)$ / $\therefore$ $[(M \wedge B) \Rightarrow (Q \wedge S)]$

10. $(p \Rightarrow q)$
   $[P \Rightarrow (Q \Rightarrow R)]$
   $[P \Rightarrow (R \Rightarrow S)]$ / $\therefore$ $(P \Rightarrow S)$

11. $(\neg A \vee B) \Rightarrow (C \wedge D)$
   $(D \vee E) \Rightarrow \neg F$ / $\therefore$ $(\neg A \Rightarrow \neg F)$

12. $[A \Rightarrow (\sim B \Rightarrow C)]$
    $[\neg B \Rightarrow (C \Rightarrow D)]$ $/\therefore [A \Rightarrow (\neg B \Rightarrow D)]$

13. $\neg X \Rightarrow \neg D$ $/\therefore [(\neg D \Rightarrow E) \Rightarrow (\neg X \Rightarrow E)]$

14. $(B \Rightarrow S) \wedge (N \Rightarrow P)$ $/\therefore [(N \wedge B) \Rightarrow (P \wedge S)]$

15. $(X \Rightarrow F)$
    $[X \Rightarrow (F \Rightarrow G)]$
    $[X \Rightarrow (G \Rightarrow H)]$ $/\therefore (X \Rightarrow H)$

16. $[\neg (V \vee Q) \vee (R \wedge X)]$
    $[\neg (X \vee T) \vee U]$ $/\therefore (V \Rightarrow U)$

17. $[\neg P \Rightarrow (\neg X \vee R)]$
    $[X \Rightarrow (\neg S \Rightarrow \neg R)]$ $/\therefore [\neg P \Rightarrow (X \Rightarrow S)]$

18. $\neg Y \vee Z$ $/\therefore [(\neg Z \vee E) \Rightarrow (Y \Rightarrow E)]$

19. $(B \Rightarrow S) \wedge (M \Rightarrow Q)$ $/\therefore [(M \wedge B) \Rightarrow (Q \wedge S)]$

20. $(P \Rightarrow M)$
    $[P \Rightarrow (M \Rightarrow R)]$
    $[P \Rightarrow (R \Rightarrow \neg S)]$ $/\therefore (P \Rightarrow \neg S)$

2.6 LET US SUM UP

In this unit we have tried to understand the significance of Conditional Proof. Its advantages and limitations were assessed in comparison with Rules of Inference and Replacement. A distinction was made between C. P. and the Strengthened Rule of C. P. We have learnt that logicians allow us to assume or introduce any proposition and not necessarily the antecedent of a conditional as a conditional assumption. But all assumptions must be discharged by applying the Rule of conditional proof.

2.7 KEY WORDS

Discharging the assumption: Ending an assumption when its truth is no longer being assumed as a maneuver within the proof.

2.8 FURTHER READINGS AND REFERENCES

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UNIT 3 INDIRECT PROOF

Contents

3.0. OBJECTIVES

The central theme of this unit is to provide an exposition of Indirect Proof. This will bring us to the last Rule with which we are concerned in our analysis of arguments which comprise of compound and simple propositions. As mentioned in the previous units, this is another tool devised to test arguments in as simple manner as possible.

3.1.INTRODUCTION

Indirect Proof, as its name suggests, is a proof procedure that establishes the validity of an argument indirectly when it is either very difficult to prove directly or just impossible. Modern logic adapted this method from geometry when arguments with which it is concerned were of varying complexities. This addition rendered the task of logician much easier.

3.2. THE MEANING OF INDIRECT PROOF

The method of indirect proof is often called *Reductio ad Absurdum* (R A A), a method quite common in the construction of proof of geometrical theorems. This method is characterized by a special feature. In order to prove a certain statement, its contradiction is assumed to be true from which the conclusion is logically deduced which in turn contradicts our assumption. Suppose that A is derived from certain premises. If A contradicts ¬ B, then either A must be false or ¬ B must be false. A cannot be false because it is logically deduced from what is purported to be true. Therefore ¬ B must be false, which means that A must be true. This is how a theorem in geometry or an argument in logic is, sometimes, proved.

This method has a distinct advantage. Sometimes the length of proof is too long. In logic it is important that we use least number of steps. Second requirement is clarity. Combination of these two is what is most desired and also desirable. In such circumstances this method is most useful.

The use of this method consists in beginning with the contradiction of what is to be proved. A point to be noted here is that, the contradiction of what has to be proved is marked by writing I.P. or R. A. on the right hand side (R.H.S.) just adjacent to the assumption. In this unit both abbreviations are used for the sake of familiarity. Here, the expression R.A. and R.P. stand for '*Reductio* assumption' and '*Reductio* proof' respectively. The denial of conclusion is named as Reduction Proof (R.P.).

In C.P. also we begin with an assumption. The difference is that in I. P. what is assumed is the contradiction of the conclusion whereas in the case of C. P. it is the antecedent of the conclusion. Consider this argument.

1)

1.	$A \Rightarrow (B \land C)$		
2.	$(B \lor D) \Rightarrow E$		
3.	$D \lor A$	$/ \therefore E$	
4.	$\neg E$		I.P. (R. A.)
5.	$\neg (B \lor D)$	2, 4,	M.T.
6.	$\neg B \land \neg D$	5,	De. M.
7.	$\neg D$	3,	Simp.
8.	A	3, 7,	D.S.
9.	$B \land C$	1, 8,	M.P.
10.	B	9,	Simp.
11.	$B \lor D$	10,	Add.
12.	E	2, 11,	M.P.
13.	$E \land \neg E$	12, 4,	Conj.

From 10th Step onwards the problem can be reworked in this manner.

11.	$\neg B$	6,	Simp.
12.	$B \land \neg B$	10, 11,	Conj
13.	$\therefore E$		R. P.

Whether we get $E \land \neg E$ or $B \land \neg B$, the result remains the same. In both the cases there are two steps in the argument whose conjunction leads to contradiction. Whenever there is contradiction one conjunct must be false so that the other one has to be true.

3.3. APPLICATION OF INDIRECT PROOF

We learnt the application of Rules on earlier occasions when we considered arguments with varied structure. On similar lines, Indirect Proof is applied to arguments and thereby validity of arguments is tested.

2).

1.	$(A \lor C)$	
2.	$(A \Rightarrow C)$ $/ \therefore C$	
3.	$\neg C$	R.A
4.	$\neg A$	2, 3, M.T.

5.	$C \vee A$	1, Com.
6.	A	5, 3, D.S.
7.	$A \wedge \neg A$	6, 4, Conj.
8.	C	R.P.

7th step involves contradiction. The final step in which the conclusion is repeated is redundant, but it is permitted for the sake of comprehensiveness. The names '*reductio* assumption' and 'reduction proof' are not very frequent and we have several other usages. Yet the names introduced here will serve the purpose. Accomplishing an explicit contradiction itself is more than adequate to show that the preferred conclusion is derivable, because $(A \wedge \sim A) \supset C$ is tautology. The unique conclusion can be derived after accomplishing an explicit contradiction by taking the negation of the conclusion in question as a conditional assumption.

3)

1.	$A \vee (B \wedge C)$		
2.	$A \Rightarrow C$	$/\therefore C$	
3.	$\neg C$		I.P.
4.	$\neg A$	2, 3,	M.T.
5.	$B \wedge C$	1, 4,	D.S.
6.	C	5,	Simp.
7.	$C \wedge \neg C$	6, 3,	Conj.
8.	$\therefore C$	R. P.	

7th step involves contradiction; therefore $\neg C$ is false which means that C is true.

4)

1.	$(D \vee E) \Rightarrow (F \Rightarrow G)$		
2.	$(\neg G \vee H) \Rightarrow (D \wedge F)$ $/\therefore G$		
3.	$\neg G$		I.P.
4.	$\neg G \vee H$	3,	Add.
5.	$D \wedge F$	2, 4,	M.P.
6.	D	5,	Simp.
7.	$D \vee E$	6,	Add.
8.	$F \Rightarrow G$	1, 7,	M.P.
9.	$\neg F$	8, 3,	M.T.
10.	F	5,	Simp.
11.	$F \wedge \neg F$	10, 9,	Conj.
12.	$\therefore G$	R. P.	

11th step is contradiction. Therefore $\neg G$ is false; which means that G is true

5).

1.	$(H \Rightarrow I) \wedge (J \Rightarrow K)$		
2.	$(I \vee K) \Rightarrow L$		
3.	$\neg L$ $/\therefore \neg (H \vee J)$		
4.	$H \vee J$		I.P.
5.	$I \vee K$	1, 4,	C.D.

6.	L	2, 5,	M.P.
7.	L Λ ¬ L	6, 3,	Conj.
8.	∴¬ (H v J)	R. P.	

7^{th} step involves contradiction. Therefore 4 is false; which means that ¬ (H v J) is true.

6).

1.	(M v N) => (O Λ P)		
2.	(O v Q) => ¬ R Λ S		
3.	(R v T) => (M v N) /∴¬R		
4.	R		I.P.
5.	R v T	4,	Add.
6.	M v N	3, 5,	M.P.
7.	O Λ P	1, 6,	M.P.
8.	O	7,	Simp.
9.	O v Q	8,	Add.
10.	¬ R Λ S	2, 9,	M.P.
11.	¬ R	10,	Simp.
12.	R Λ ¬ R	4, 11,	Conj.
13.	∴¬ R	R. P.	

12^{th} step involves contradiction. Therefore R is false which means that ¬ R is true.

7).

1.	(V => ¬ W) Λ (X => Y)		
2.	(¬ W => Z) Λ (Y=> ¬ A)		
3.	(Z => ¬ B) Λ (¬A => C)		
4.	VΛ X / ∴¬ B Λ C		
5.	¬ (¬ B Λ C)		I.P.
6.	B v ¬ C	5,	De. M.
7.	¬ Z v A	3, 6,	D.D.
8.	W v ¬Y	2, 7,	D.D.
9.	¬V v ¬X	1, 8,	D.D.
10.	(VΛ X) Λ (¬ V v ¬ X)	4, 9,	Conj.
11.	∴¬ B Λ C	R. P.	

10^{th} Step involves contradiction. According to de Morgan's law (VΛ X) and (¬ V v ¬ X) are contradictories. Therefore ¬ (¬ B Λ C) is false, which means that (¬ B Λ C) is true. We can also prove these arguments using formal proof of validity. Consider the 5^{th} argument.

8).

1.	(H => I) Λ (J => K)	
2.	(I v K) => L	
3.	¬ L	/∴¬ (H v J)

4. ¬(I v K) 2, De. M.
5. ¬ I Λ ¬ K 4, De. M.
6. ¬ I 5, Simp.
7. ¬ I v ¬ K 6, Add.
8. ∴ ¬ H v ¬ J 1, 7, D. D.

When the 5th argument was solved using IP method, it involved thirty three words and five steps, whereas formal proof required forty words and nine steps. Therefore the former is shorter and preferable.

Now consider the seventh argument.

9).

1. (V => ¬ W) Λ (X => Y)		
2. (¬ W => Z) Λ (Y => ¬ A)		
3. (Z=> ¬B) Λ (¬A=> C)		
4. V Λ X /∴ ¬ B Λ C		
5. V => ¬ W	1,	Simp.
6. V	4,	Simp.
7. ¬ W	5, 6,	M.P.
8. X => Y	1,	Simp.
9. X	4,	Simp.
10. Y	8, 9,	M.P.
11. ¬ W=> Z	2,	Simp.
12. Z	11, 7,	M.P.
13. Y => ¬ A	2,	Simp.
14. ¬ A	13, 10,	M.P.
15. Z => ¬ B	3,	Simp.
16. ¬ B	15, 12,	M.P.
17. ¬ A => C	3,	Simp.
18. C	17, 14,	M.P.
19. ∴ ¬ B Λ C	16, 18,	Conj.

When the 7th argument was solved using I.P. method, it involved fifty seven words and seven steps, whereas formal proof required ninety words and nineteen steps. Therefore the former is shorter and preferable.

We learnt in the earlier unit that sometimes C. P. is shorter than formal proof and sometimes it is longer than formal proof. Same situation prevails in the case of I. P. also. Consider the following example.

1 H => (I => J) `
2 K => (I => J)
3 (¬ H Λ ¬ K) => (¬ L v ¬ M)
4 (¬ L => ¬ N) Λ (¬ M => ¬ O)
5 (P => N) Λ (Q => O)

1	¬ (I => J)	/∴ ¬ P v ¬ Q
2	¬ H	1,6, M.T.
3	¬ K	2,6, M.T.
9	H Λ ¬ K	7,8, Conj.
10	¬ L v ¬ M	3,9, M.P.
11	¬ N v ¬ O	4,10, C.D.
12	∴ ¬ P v ¬ Q	5,11, D.D.

This proof construction consists of forty two words, six lines and five Rules used six times. Let us use Indirect Proof method to know which method is shorter and simpler.

1	H => (I => J)	`
2	K => (I => J)	
3	(¬ H Λ ¬ K) => (¬ L v ¬ M)	
4	(¬ L => ¬ N) Λ (¬ M => ¬ O)	
5	(P => N) Λ (Q => O)	
6	¬ (I => J)	/∴ ¬ P v ¬ Q
7	¬ (¬ P v ¬ Q)	I.P.
8	P Λ Q	7, De. M.
9	P	8, Simp.
10	P v Q	9, Add.
11	N v O	5, 10, C.D.
12	L v M	4, 11, D.D.
13	¬ H	1, 6, M. T.
14	¬ K	2, 6, M.T.
15	¬ H Λ¬ K	13, 14, Conj.
16	¬ L v ¬ M	3, 15, M. P.

It must be more than obvious that even after ten steps and sixty nine words and the application of nine Rules I. P. did not yield the expected results. It must be noted that ¬ L v ¬ M does not negate the twelfth step i.e. L v M is not the contradictory of the sixteenth step i.e., ¬ L v ¬ M. Therefore even if subsequent steps yield the desired result, it is, surely, not profitable to follow the I. P. method in this case.

3.4 EXAMPLES

1).

1	(P v Q)	
2	(P = > Q)	/ ∴ Q
3	¬ Q	R.A
4	¬ P	2, 3, M.T.
5	Q ∨ P	1, Com.
6	P	5, 3, D.S.

7 P $\Lambda \neg$ P   6 , 4, Conj.
8 Q   R.P.

2).

1 (A$\vee$ B)
2 (A = > B)   /$\therefore$ B
3 $\neg$ B   R.A
4 $\neg$ A   2, 3,M.T.
5 B $\vee$ A   1, Com.
6 A   5, 3, D.S.
7 A Λ $\neg$ A   6 , 4, Conj.
8 B   R.P.

3).

1. P => (Q => R)
2. S $\vee$ (P $\vee$ R)
3. P => Q   /$\therefore$ (S $\vee$ R)
4. $\neg$ (.S$\vee$ R)   R.A.
5. $\neg$ S Λ $\neg$ R   4, De .M.
6. $\neg$ S   5, Simp.
7. P $\vee$ R   2, 6, D.S
8. (P Λ Q) => R   1, Exp.
9. P => (P Λ Q)   3, Abs.
10 P => R   9, 8, H.S.
11 $\neg$ R Λ $\neg$ S   5, Com.
12. $\neg$ R   11, Simp.
13. $\neg$ P   10, 12, M.T.
14. R $\vee$ P   7, Com.
15. P   14, 12, D.S .
16. P Λ $\neg$ P   15, 13, Conj.
17. $\therefore$S $\vee$ R   R.P.

Check your progress I.

Note: Use the space provided for your answer.

1. Explain the scope of Indirect Proof.

----

2. Give the meaning of R.A and R.P?

3.5 EXERCISES ON INDIRECT PROOF

Evaluate the relative advantages and disadvantages of formal proof and I. P. methods with the help of following arguments.

1.
(B v N) => (K Λ L)
¬ K
¬ M / ∴ ¬ B Λ ¬ M

2.
(M v N) => (P Λ Q)
N /∴ P

3. A=> (B Λ C)
¬ B / ∴ ¬ A

4. P ∨ Q
P ∨ ¬ Q /∴ P

5. A => B
A ∨ B /∴B

6. (M ∨ ¬ M) => ¬ (¬ N Λ ¬ O)
(N ∨ O) => ¬ P /∴ ¬ P

7. [(W ∨ X) => (Y Λ W)]
(X => Y)
[¬ Z => (W ∨ X)] / ∴ (Z ∨ W)

8. P => (Q Λ R)
¬ Q / ∴ ¬ P

9. ¬ (¬ P Λ ¬ Q)
¬ P => ¬ Q /∴ P

10. ¬ B => ¬A
¬ (¬ A Λ ¬ B) /∴B

11. (A ∨ ¬ A) => ¬ (¬ B Λ ¬ O)

$(B \vee O) \Rightarrow \neg Q \quad / \therefore \neg Q$

12. $[(X \vee W) \Rightarrow (W \wedge Y)]$
$(\neg Y \Rightarrow \neg X)$
$[\neg Z \Rightarrow (X \vee W)] \quad / \therefore (W \vee Z)$

13. $\neg A \vee (C \wedge X)$
$\neg X \quad / \therefore \neg A$

14. $B \vee A$
$\neg B \vee A \quad / \therefore A$

15. $\neg A \vee D$
$A \vee D \quad / \therefore D$

16. $(P \vee \neg P) \Rightarrow \neg (\neg N \wedge \neg O)$
$(N \vee O) \Rightarrow \neg Q \quad / \therefore \neg Q$

17. $[\neg (\neg W \wedge \neg X) \Rightarrow \neg (\neg Y \vee \neg W)]$
$(\neg Y \Rightarrow \neg X)$
$[\neg Z \Rightarrow \neg (\neg W \wedge \neg X)] \quad / \therefore \neg (\neg Z \wedge \neg W)$

18. $A \text{ v } (B \wedge C)$
$A \Rightarrow C \ / \therefore C$

19. $(D \text{ v } E) \Rightarrow (F \Rightarrow G)$
$(\neg G \text{ v } H) \Rightarrow (D \wedge F) \ / \therefore G$

20. $(G \Rightarrow H) \Rightarrow (I \text{ v } J)$
$K \text{ v } \neg (L \Rightarrow M)$
$(G \Rightarrow H) \text{ v } \neg K$
$N \Rightarrow (L \Rightarrow M)$
$\neg (I \text{ v } J) \ / \therefore \neg N$

3.6 INDIRECT PROOF AND PROOF OF TAUTOLOGY

Just as arguments are classified as valid and invalid, statements are classified as tautologous and nontautologous. Under the latter category there is further classification into contingent and contradiction. All conditional arguments can be transformed into statement forms. If an argument is valid, then its corresponding statement form is tautologous and if a statement form is tautologous, then its corresponding conditional argument is necessarily valid. Such statement form also is conditional whose premise is the antecedent and conclusion is the consequent of the original argument. We must remember that disjunctive form also is conditional.

Consider the simplest conditional argument with two variables; p and q.

$p \Rightarrow q$
p
$\therefore q$

Since this argument is valid we should first determine the truth-value of the premises and conclusion in order to ensure that false conclusion is not derived from true premises. This can be achieved with the help of truth-table.

p	q	¬ p	¬ q	p	=>	q
1	1	0	0	1	1	1
1	0	0	1	1	0	0
0	1	1	0	0	1	1
0	0	1	1	0	1	1

There are four rows in which the truth-value of p => q needs to be determined. If there are three variables, then we will have eight rows. It means that the number of rows is expressed in the form of formula

$$2^n$$

where n stands for the number of variables. Against this background, we shall consider statement form for the conditional argument of the form mentioned above.

(For the sake of simplicity we can omit negations of p and q since they are not required). We obtain the statement form by conjoining premises to which the conclusion is connected using implication again.

p	q	p => q	[{(p => q) Λ p} => q]
1	1	1 1 1	1 1 1 1 1
1	0	1 0 0	0 0 1 1 0
0	1	0 1 1	1 0 0 1 1
0	0	0 1 1	1 0 0 1 0

It should be noticed that in the last but one column the truth-value obtained is 1 in all instances. Therefore we say that the statement form is tautologous. If the value happened to be 0 in all instances, then, the statement form becomes contradictory. On the other hand, if the statement form takes 1 and 0 in different instances, then the statement form is said to be contingent. Let us restrict ourselves to tautology. When there are many variables truth-table method becomes quite complex and not viable on practical grounds. In such circumstances I.P. method becomes useful. Consider an example.

1).

1	(A => B) v (A => ¬ B)	
2	¬ {(A => B) v (A => ¬ B)}	I. P.
3	¬ (A => B) Λ ¬ (A => ¬ B)	2, De. M.
4	¬ (¬ A v B) Λ ¬ (¬ A v ¬ B)	3, Impl.
5	(A Λ ¬ B) Λ (A Λ B)	4, De. M.
6	A Λ ¬ B	5, Simp.
7	¬ B	6, Simp.
8	A Λ B	5, Simp.
9	B	8, Simp.
10	B Λ ¬ B	9, 7, Conj.

11 In tenth step there is contradiction. Therefore ¬ {(A => B) v (A => ¬ B)} is false which shows that (A => B) v (A => ¬ B) is tautologous.

2).

1	(A => B) v (B => A)	
2	¬ [(A => B) v (B => A)]	I. P.
3	¬ (A => B) Λ ¬ (B => A)	2, De. M.
4	¬ (¬ A v B) Λ ¬ (¬ B v A)	3, Impl.
5	(A Λ ¬ B) Λ (B Λ ¬ A)	4, De. M.
6	A Λ ¬ B	5,Simp.
7	A	6, Simp.
8	B Λ ¬ A	5, Simp.
9	¬ A	8, Simp.
10	A Λ ¬ A	8, 9, Conj.

Result is similar to the first argument.

3).

1	(A => B) v (¬ A => B)	
2	¬ [(A => B) v (¬ A => B)]	I. P.
3	¬ (A => B) Λ ¬ (¬ A => B)	2, De.M.
4	¬ (¬ A v B) Λ ¬ (A v B)	3, Impl.
5	(A Λ ¬ B) Λ (¬ A Λ ¬ B)	4, De. M.
6	A Λ ¬ B	5, Simp.
7	A	6,Simp.
8	¬ A Λ ¬ B	5, Simp.
9	¬ A	8, Simp.
10	A Λ ¬ A	7, 9, Conj.

In this proof system also we have obtained the same result. It means that the second and third statements are tautologous.

Combination of *Reductio ad absurdum* and truth- table methods is another technique of testing arguments. While doing so we have to make two assumptions. In the first place we must assume that all premises are true. Secondly, the conclusion must be assumed to be false. If this combination can be achieved, then the given argument is invalid. Otherwise, the argument must be valid. Examine this argument.

1).

1 (A v B) => (C Λ D)

2 (D v E) => F/ ∴ A => F

The conclusion is false only if A is true and F is false. The second premise can be true provided D and E are false because F is false. If the first premise must be true then false consequent should not be implied by true antecedent. In the first premise the consequent is false because D is false and conjunction of which D is a component is false. Since the consequent is false the antecedent must be false so that the implication is true. But the antecedent is true because one of the components of the antecedent is true. Therefore antecedent fails to satisfy the requirement. This shows that the consequent cannot be false.

Now consider an argument with conjunctive conclusion.

2).

$(B \vee N) \Rightarrow (K \wedge L)$

$\neg K$

$\neg M \quad / \therefore \neg B \wedge \neg M$

Assume that the conclusion is false. Then at least one component must be false. Let us assume that ¬ B is false. Then B must be true. Therefore (K Λ L) must be true. This is possible when both K and L are true. But when K is true ¬ K is necessarily false. We have assumed that every premise must be true. The obtained result contradicts our assumption. Therefore the conclusion must be true. Therefore the argument is valid.

Now consider an argument with simple conclusion.

3).

$(M \vee N) \Rightarrow (O \wedge P)$

$(O \vee Q) \Rightarrow \neg R \wedge S$

$(R \vee T) \Rightarrow (M \vee N) \quad / \therefore \neg R$

Let ¬ R be false. Then ¬ R Λ S is false. Therefore the second premise can be true only if (O v Q) is false. (O v Q) can be false only when both the components are false. Since O is false, (O Λ P) is also false. Therefore the first premise can be true only if (M v N) is false. Now we shall examine the last premise. Since the consequent is false, the antecedent also must be false if the premise must be true. However, R is true since ¬ R is false. (R v T) is true since R is true. When we derive false conclusion from true antecedent the premise becomes false which contradicts our assumption which states that all premises must be true. Therefore the conclusion must be true.

If verbal explanation is replaced by assigning of truth-values to all the variables and sentential connectives, then the determination of validity becomes far simpler than one can imagine. This is left for the reader as an exercise.

Check your progress II.

Note: Use the space provided for your answer.

Use I. P. and formal methods to test the following arguments.

1 $\neg A \vee B$

$\neg(\neg A \wedge \neg B) \quad / \therefore B$

----

2 $\neg (P \land \neg Q)$

$\neg (\neg Q \land \neg P)$ $/\therefore Q$

3.7 LET US SUM UP

In this unit we have provided an exposition of indirect proof, by giving definition and principles of this method. The conclusion ought to be negated and assumed as an additional premise (R.A.). Once it is reduced to absurdity, we restated the conclusion and named it as Reduction proof (R.P.). With the help of this method we have learnt that logicians use indirect method to prove an argument with least effort.

3.8 KEY WORDS

Theorem: A logical truth.
Valid argument form: An argument form which has no invalid substitution instance.
Contradiction: An obvious contradiction that is a substitution instance of the statement form $(A \land \neg A)$
Reductio Ad Absurdum (R.A.A): Proof technique of reducing an assumption to absurdity by deriving explicit contradiction from it.

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UNIT 4: PROVING INVALIDITY

Contents

UNIT 4.0 OBJECTIVES

The central endeavor of this unit is to explicate the significance of proving invalidity. Proving invalidity is significant not in negative sense, but in positive sense. The singular objective of this unit is very clear. If we know what is wrong, we will know what is right in the *right* sense of the word and we will avoid consciously the pitfalls of illogical ways of arguing. Otherwise, we may walk into the trap of fallacies. Thus by the end of this unit one should be able to establish invalidity of seemingly valid arguments. Further, one should be able to identify the difference or differences between proving tautology and proving invalidity.

4.1 INTRODUCTION

Building up of proof system is an efficient mode to show that an argument is valid. Suppose that an argument happens to be invalid. Then it is not possible to construct proof of its invalidity using any rule applied so far. If it is not possible to show that given argument is not valid, then it should be possible to demonstrate its invalidity. Let us consider two cases to make the point clear. Suppose that we fail to demonstrate that God exists. Then half the battle is won (or half the battle is last). Next stage is to demonstrate that God does not exist. In contrast consider this case. Suppose that the prosecution fails to establish that the accused has committed the crime. The court does extend the benefit of doubt and acquits the accused. But this is not an accepted position in logic. The function of logic is two-fold; prove a certain proposition and disprove some other. Inability to prove (or disprove) is not tantamount to contradiction. At worst, it can only be a contrary. In modern logic contrary is not an accepted relation. Only contradiction is logically sound. When the logician accomplishes both the tasks, his victory is complete. Against this background, we must regard the relevance of proof of invalidity.

The foregoing discussion makes one point clear. If the method of proving invalidity is construed as a rule, then in addition to the rules we have become familiar with, we are in possession of one more tool to test arguments.

UNIT 4.2 PROVING INVALIDITY

Construction of truth-table is the foundation of propositional calculus. It is not possible to prove invalidity without its help. The method is quite simple. Irrespective of the number of propositions, the entire operation can be completed in one straight line. We devise a single, row represented by a straight line. All variables, their negations and all compound propositions are arranged horizontally above the straight line. The truth-value is entered exactly below the respective proposition. It is not necessary that the truth-value of every variable has to be entered. It depends upon the situation. The straight line separates the variables and the respective truth-values. Calculation of truth-value of propositions follows the elementary principles of propositional calculus. Since we have to prove that the argument is invalid, '0' is the truth-value to be assigned necessarily to the conclusion. This is the first step to be followed. If the conclusion is a compound proposition, then the sentential connective must be assigned the value '0'. Suppose that in the conclusion there are multiple sentential connectives. Then the sentential connective which has maximum range must be assigned the designate value. If implication links the antecedent and the consequent, then implication will have maximum range. Therefore in such case the implication must be assigned this value. In all other cases a little examination is adequate to identify the connective with maximum range. In the next step, all premises must be assigned the truth-value '1' only. Again, if any premise is a compound proposition then same method which is applicable to compound conclusion also applies to the premise or premises. A word of caution is required. It may not be possible to assign the required truth-value always in the very first attempt. We may have to take to trial-and-error method at times. This is more so when the premises are quite lengthy with multiple sentential connectives. It is also possible that more than one combination of truth-values for the variables of the premises may yield the desired result. Suppose that it is impossible to assign the truth-values in any the manner described above. Then the argument must be regarded as valid. Therefore this method can be used to prove validity also. However, we are concerned with proving invalidity at present.

At this stage one clarification is required. Unlike validity, invalidity is not governed by any rules. Of course, it is more than obvious that errors do not have any rules, which govern. On the other hand, only violation is possible. Hence the method of proving invalidity is different. The principle of inference dictates that a true premise and a false conclusion together result in invalidity. This is the reason why in order to determine invalidity we should assign truth-values in such a way that the premise or premises are true and the conclusion is false. If we succeed in doing so then the argument is invalid. This method is so simple that the test can be completed in one line (or two lines depending upon the number of variables and constants) as it happens in the case of truth-table.

So far we concentrated on the theoretical aspect. We shall apply now this method to an argument. The conclusion finds place at the end of the line always.

I

1

1. E => (F v G)
2. G => (H Λ I)
3. ¬ H /∴ E => I

					4				7	5				8	6	3	1	2
E	F	G	H	I	{E =>	(F	v	G)}	Λ	{G =>	(H	ΛI)}		Λ	¬H	E=> I		
1	1	0	0	0	1 1	1 1	0	1	0 1	0 0 0	1	1	1 0 0					

While following this method '0' should be assigned to the conclusion making the premises true. If this combination cannot be achieved, then the argument is valid, i.e., after making the conclusion 0 if the conjunction of premises cannot take the value 1, then the argument is valid. There is no need to look for too many false premises. It is enough if one premise is false. The components of conclusion and the components of the premises should be paired properly to carry out the test.

We know the way of filling up the truth-values. Since the conclusion is a compound proposition the conclusion is false only when the sentential connective of the conclusion is false. Therefore this column is filled up first. Since the truth-values of the components determine the truth-value of any compound proposition, those respective columns are filled up next. Exactly on similar lines, the truth-values of premises are filled up. Accordingly, last but one column shows that the conclusion is false. 4, 5 and 6 show that first, second and third premises are true. 7 and 8 show that the conjunction of all the premises is true. We have begun our job with assigning the truth-value to the conclusion and the rest of the steps logically followed the first one. This being the case, false conclusion should not be derivable from the conjunction of true premises in the case of a valid argument. Since this has happened the argument is invalid. We shall consider some more examples. The order of filling up of truth-values is not given for the remaining arguments. The student is advised to find out the same. From now on conjunction linking the premises must be treated as implicit.

2

1. J => (K => L)
2. K => (¬ L => M)
3. (L v M) => N /∴ J => ¬N

Note that J => (K => L) ≡ (J Λ K) => L according to the Rule Material Equivalence. Therefore the truth-value is fixed for the second implication in similar cases.

J K L ¬L M N ¬N {J => (K => L)} {K => (¬L => M)}(L v M) => N} (J => ¬N)
1 0 1 0 1 1 0 1 0 1 1 0 0 1 1 1 1 1 1 1 1 0 0

Here the conclusion is '0' whereas the combination of premises is 1. Hence the argument is invalid.

3

1. (O v P) => Q
2. Q => (P v R)
3. O => (¬ S => P)
4. (S => O) => ¬ R /∴P ≡ Q

In accordance with the Rule of Material Equivalence, the conclusion can be restated as follows.

(P ≡ Q) ≡ {(P => Q) Λ (Q => P)}. In other words, equivalence relation satisfies the truth-condition of biconditional proposition. For the sake of convenience we shall use the statement on R. H. S. This expression is very long. A little care while assigning the truth-values to the propositions is required.

Suppose that the first component of the conclusion is taken as false. That is sufficient for the entire conclusion to become false since the conclusion is a conjunction. There is no need to fill up the truth-value of the last component. Let us assume, therefore, that P => Q is false. We shall work out the rest as per the procedure and find out the result. The truth-values for the individual variables are omitted for the remaining examples. The student is advised to fill up the same for the sake of practice.

{(O v P) => Q} {Q => P v R}{O=>¬S=>P}{S=>O=>¬R}(P => Q) Λ(Q => P)
1 1 1 0 0 1 0 0 0

Note that O => ¬ S => P ≡ (O Λ ¬ S) => P. Accordingly fix the truth-value.

The very first premise is false when the conclusion is false (when the premise turns out to be 0, there is no need to assign the truth-values to the remaining premises since the result remains unaltered). Therefore according to the assumption we made the conclusion must be valid. However, there is one more component which has to be examined before arriving at final judgment. Let us assume that Q => P is false and then proceed to show the invalidity.

{(O v P) => Q}{Q => (P v R)}{O => (¬S=>P)}S => (O=> ¬R)}(P => Q)Λ(Q => P)
0 0 0 1 1 1 1 0 1 1 0 0 1 0 1 0 1 0 0 1 0 0

4

1. X ≡ (Y => Z)
2. Y ≡ (¬ X Λ ¬ Z)
3. Z ≡ (X v ¬ Y)
4. Y /∴ X v Z

We should be in a position now to deal with the premises. Since components on L.H.S. and R.H.S. of all the premises become consequent all of them must have same truth-value. Keeping this point in mind let us assign truth-values.

{X≡ (Y => Z)}{Y≡ (¬XΛ¬Z)} {Z≡ (Xv¬Y)} Y (X v Z)
0 1 1 0 0 1 1 1 1 1 0 1 0 0 0 1 0 0 0

It is important to note that the truth-values entered below equivalence relations correspond to the truth-values of the premises.

The explanation remains the same in all cases.

5

1. T ≡ U
2. U ≡ (V Λ W)
3. V ≡ (T v X)
4. T v X /∴T Λ X

The conclusion is conjunction. Therefore, again, we have two options which are required to be tested. Suppose that T takes the value 0 and X takes 1. Then we will get the following result.

(T≡U) (U ≡ V ΛW) {V≡ (T v X)} (T v X) (TΛX)
0 1 0 0 1 0 0 0 0 0 0 1 1 0 0 1

The third premise is false when the conclusion is false. Hence according to this assumption the argument turns out to be valid. Let us make second assumption. Accordingly, X takes the value 0. Then we will get the following result.

(T≡U) (U≡ VΛW) {V≡ (T v X)} (T v X) (TΛX)
1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 0 0

One aspect becomes clear. When the conclusion is a compound proposition, we have to try all possibilities till we get the desired result. Even after exhausting all possibilities if a premise cannot become false, only then can we conclude that the argument is valid. The student is advised to find out the result when T and X are 0.

6

1. (A => B) Λ (C => D)
2. A v C
3. (B v D) => (E Λ F)
4. E => (F => G)

5. G => (A => H) /∴H

{A =>B)Λ(C =>D)}{(A v C){(B v D =>(EΛ F)}{(E=> (F =>G)}{G => (A => H) H
0 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0

The argument is invalid.

7

1. I v (J Λ K)
2. (I v J) => (L ≡ ¬ M)
3. (L => ¬ M) => (M Λ ¬ N)
4. (N => O) Λ (O => M)
5. (J => K) => O /∴O

{I v (JΛK)}{(I v J)}=> (L≡¬M)}{(L=>¬M) => (MΛ¬N)}{(N => O) Λ
11 0 0 0 1 1 0 1 0 1 0 0 1 0 1 1 1 1 0 1 0 1
(O=>M)} {J => (K => O) O
0 1 1 0 0 1 0 0

8

1. P ≡ (Q ≡ ¬ R)
2. Q => (¬R v ¬ S)
3. {R => (Q v ¬ T} Λ (P => Q)
4. {U => (S Λ T)} Λ (T => ¬ V)
5. {(Q Λ R) => ¬ U} Λ {U => (Q v R)}
6. (Q v V) Λ¬ V /∴ ¬ U Λ ¬ V

{P≡(Q≡¬R)}{Q => (¬Rv¬S)}{R => (Qv¬T)}Λ(P => Q)}[{U=>(SΛT) }
1 1 1 1 1 1 1 1 1 0 0 1 1 1 0 1 1 1 1 1 1 1 1

Λ(T=>¬V)] [{(Q ΛR) => ¬U} Λ{U = > (Q v R)}]{(Q v V) Λ ¬V(¬UΛ¬V)
1 1 1 1 1 0 0 1 0 1 1 1 1 1 0 1 1 0 1 1 0 0 1

9

1. A=> B
2. C=> D
3. B v C /∴ A v D

{A=> B}	{C=> D}	{B v C}	{A v D}
0 1 1	0 1 0	1 1 0	0 0 0

II

It is a very good exercise to consider arguments in verbal form and then translate them to the usual symbolic form.

10

If pressure increases, then volume decreases.

Pressure has not increased.

Therefore volume does not decrease.

First we shall symbolize the variables.

a) pressure increases ---------- P

b) pressure has not increased ------ ¬ P

c) volume decreases ------- V

d) volume does not decrease ------- ¬ V

In the form of symbols the argument is represented as follows.

P = > V

¬ P

∴ ¬ V

As mentioned earlier we shall assign the truth-values as follows.

{P = > V}	¬P	¬V
0 1 1	1	0

Consider now a polysyllogistic argument.

2

1 All minisers are politicians. ----------- M

2 All politicians are voters.---------- P

3 All voters are educated.-------- V

4 No educated persons are honest.-------- ¬E

Therefore no ministers are honest. ------- ¬ M

We must remember that all universal propositions are hypothetical in nature. Therefore the first proposition actually means that if there is anyone who is a minister, then he must be a politician. Accordingly, we shall symbolize the statements.

1 M = > P

2 P = > V

3 V = > ¬ E

∴ ¬
E

{M = > P}	{P = > V}	{V=>¬E}	¬E
0 1 0	0 1 0	0 1 0	0

Since all three implications are true, when the conclusion is false, the argument is invalid. It is advantageous to mark the conclusion at the end of R. H. S. and enter its truth-value first followed by the truth-values of premises. Only then we will be in a position to fix without confusion the truth-values of component propositions.

One distinct advantage of this method must now be more than obvious. There is no need to remember the distribution pattern of terms in categorical proposition. When this is abandoned the laws of syllogism or polysyllogism are rendered superfluous. It is sufficient if we are familiar with the truth-conditions of compound propositions.

Consider now a slightly complex argument.

3 If tax evaders are not punished, then either development takes back seat or the government is compelled to keep tax slab high. If the government is compelled to keep tax slab high, then common man becomes the victim of inept administration in democratic set-up. Common man does not become a victim of inept administration in a democratic set-up. Therefore either development does not take back seat or the tax evaders are punished in a democratic set-up.

This argument deserves to be split into individual components and then symbolized.

1 tax evaders are not punished --------- $\neg$ T

2 development takes back seat -------- D

3 the government is compelled to keep tax slab high ------- H

4 common man becomes the victim of inept administration in democratic set-up ---- C

5 common man does not become the victim of inept administration in democratic set-up ------- $\neg$ C

6 development does not take back seat --------- $\neg$ D

7 the tax-evaders are punished in a democratic set-up ------- T

We can now symbolize each proposition.

1st premise: If tax evaders are not punished, then either development takes back seat or the government is compelled to keep tax slab high. $\neg T \Rightarrow (D \vee G)$

2nd premise: If the government is compelled to keep tax slab high, then common man becomes the victim of inept administration in democratic set-up. $G \Rightarrow C$

3rd premise: Common man does not become a victim of inept administration in a democratic set-up. $\neg C$

Conclusion: Therefore either development does not take back seat or the tax evaders are punished in a democratic set-up. $\neg D \vee T$

Even a professional cannot discover easily fallacy in this argument. This argument can be easily shown to be invalid if the method of assigning the truth-value is followed.

1 $\neg T \Rightarrow (D \vee G)$

2 $G \Rightarrow C$

3 $\neg C$

$\therefore \neg D \vee T$

$\underline{\{\neg T \Rightarrow (D \vee G)\} \quad \{G \Rightarrow C\} \quad \neg C \quad \{\neg D \vee T\}}$

1 1 1 1 0 0 1 0 1 0 0 0

There is no other way of showing the invalidity of this argument. For confirmation let us use I. P. method.

1 ¬ T = > (D v G)

2 G = > C

3 ¬ C / ∴¬ D v T

4 ¬ (¬ D v T) I. P.

5 D Λ ¬ T 4, De. M.

6 ¬ T 5, Simp.

7 D v G 1, 6, M.P.

8 ¬ G 2, 3, M.T.

9 D 7, 8, D. S.

10 D Λ ¬ T 9, 6, Conj.

We only succeeded in returning to the fifth step. Since the argument is supposed to be invalid, instead of showing contradiction we should have succeeded in showing consistency with the help of I. P. method. We could show neither validity nor invalidity of this argument since we encountered sort of stalemate.

4 Let us slightly alter Berkeley's argument on immaterialism and find out its logical status.

'If things are material, then they are bundle of qualities. All qualities are ideas. All ideas are equivalent to mental entities. Therefore there is no matter'.

We shall split the argument into its components.

1st premise: If things are material, then they are bundle of qualities. --------- M = > Q

2nd premise: All qualities are ideas. -------- Q = > I

3rd premise: All ideas are equivalent to mental entities. ----- I = > E

Conclusion: Therefore there is no matter. ----- ¬ M

We shall put the argument in formal manner.

1 M = > Q

2 Q = > I

3 I = > E / ∴ ¬
M

To an unsuspecting mind no error is noticeable in this argument unless the argument is expressed in formal way. It is easy to conclude that the valid conclusion should have been M = > E. But if we consider what M and E stand for in Berkeley's system, then it becomes a different story altogether. That is not our concern now. It is enough if we mention that if H. S. is applied twice (to 1 and 2 and later to 2 and 3), we obtain M = > E. But this is not the conclusion which is required to be tested.

{ M = > Q}	{Q = > I}	{I = > E}	¬M
1 1 1	1 1 1	1 1 1	0

Check Your Progress I

Note: Use the space provided for your answer.

1. Define and explain the proof of Invalidity.

----

2. How do we confirm an argument as Invalid?

------------------------------------ --

4.3 ADVANTAGES OF THE METHOD OF PROVING INVALIDITY

Suppose that we opt for truth-table method. What will be the situation? Consider, for example, the following argument:

P => (Q v R)
S => (T v U)
¬ Q => (U v V)
(U => S) Λ (¬ T => ¬ S)
¬ V

∴P => (Sv U)

A truth-table for this argument will have 128 rows according to the formula 2^n where n= 7 and n is the number of variables and 10 columns of truth-values. Therefore if the truth-table method has to be followed, then the number of boxes to be filled up it is, incredibly, 1280. The distinct advantage of this method lies precisely here. Let us begin by assuming that P=> (Sv U) is false. Given that the only way for a conditional statement to be false is; its antecedent must be true and its consequent must be false which means that 'P' must be true and (S v U) must be false, and since a disjunction is false only when both of its disjuncts are false, 'S' and 'U' must both be false on our crucial line of the truth-table. Notice how far we have come already:

	P	Q	R	S	T	U	V
P => (Q v R)							
S => (Tv U)	1			0			0
¬ Q => (U v V)							
(U => S) Λ (¬ T => ¬ S)							
¬ V							

P => (S v U)

At the second stage, we consider true premises. The fifth premise is simple for our purpose. '¬ V' is true if and only if 'V' is false. And, since 'U' and 'V' are both false, the consequent of the third premise is false; in order to make that premise true, its antecedent, '¬ Q' must also be made false, which entails that 'Q' must be true. Thus, we have narrowed our search even further:

	P	Q	R	S	T	U	V
P => (Q v R)							
S => (T v U)	1	1		0		0	0
¬ Q => (U v V)							
(U => S) Λ (¬ T => ¬ S)							
¬ V							

P => (S v U)

We have restricted ourselves only to the required propositions. That is the way to economize time and effort.

Thus, we have proved that the argument is invalid. When 'P', 'Q', 'R', and 'T' are true and 'S', 'U', and 'V' are false, the premises are true and the conclusion is false. We have discovered the easiest way of proving invalidity, no matter how complex is the argument.

4.4 ASSUMPTIONS OF PROVING INVALIDITY

Let is restate the assumptions to be followed.

i. Assume the conclusion as False and at the same time given premises as True.
ii. Using Basic Truth-Table Method to substitute the truth-values for all variables.
iii. Proceed from conclusion to premises.
iv. If we find no false premise, then the given argument is invalid.

Check Your Progress II

Note: Use the space provided for your answer.

1. Elucidate the advantages of proving invalidity.

----

2. Write down the assumptions of proving invalidity.

4.5 SECOND METHOD OF PROVING INVALIDITY; EXAMPLES

We shall consider a second method of proving invalidity.

1

```
  0   0   1        1   0
(¬A  Λ   B )    =>   C
  1   0   0       1    0
 ( A  =>  C )    =>  D
    1   1     1
    B   =>  E
________________________
  ∴B  =>   (D Λ  E )
     1  0       0  0  1
```

2

```
  1   1   1    1   0
( A  => B )  v   C
  1   0     0   1   0
( A  =>  C) =>  D
  1   1     1
  B  =>   E
________________________
 ∴ B  => (D Λ  E)
     1  0    0  0  1
```

Same type of method is adopted in this argument also and found that there is no contradiction, in the given premises. Hence the argument is invalid.

3

0 0 1 1 0
(¬ P Λ Q) => R
1 0 0 1 0
(P => R) => S
1 1 1
Q => T

∴Q => (S Λ T)
1 0 0 0 1

4

1 1 1 1 0
(P Λ ¬ Q) v R
1 0 0 1 0
(P => R) => S
1 1 1
¬ Q => ¬ T

∴¬ Q => (S Λ ¬ T)
1 0 0 0 1

4.6 EXERCISES

1. P => Q
   R => S
   Q v R / ∴ P v S
2. E => (F v G)
   G => (H Λ I)
   ¬ H /∴ E => I
3. J => (F => L)
   K => (¬ L => M)
   (L v M) => N / ∴J => N
4. (A v ¬ A) => ¬ (¬ B Λ ¬ C)
   (B v C) => ¬ D / ∴ ¬ D v A
5. [(¬ W v X) => (Y Λ W)]
   (X => Y)
   [¬ Z => (¬ W v X)] / ∴ (Z v ¬ W)
6. A => B

C => D

B v C / ∴ A v D

7. ¬ E v (F v G)
¬ G v (H Λ I)
¬ H / ∴ E v I

8. ¬ J v (¬ F v L)
¬ K v (L v M)
¬ (L v M) vN /∴ ¬ J v N

9. (X v ¬ X) => ¬ (¬ B Λ ¬ C)
(B v C) => ¬ Y / ∴ ¬ Y v X

10. [(X v W) => (¬ W Λ Y)]
(¬ X v Y)
[¬ Z => (X v W)] / ∴ (W v Z)

11. ¬ P => ¬ Q
¬ R => ¬ S
¬ Q v ¬ R / ∴ ¬ P v ¬ S

12. (Fv G) v ¬ E
¬ (H Λ I) => ¬ G
¬ H / ∴ ¬ I => ¬ E

13. (¬ F v L) v ¬ J
(L v M) v ¬ K
¬ (L v M) v N / ∴ N v ¬ J

14. (P v ¬ P) => ¬ (¬ Q Λ ¬ R)
(Q v R) => ¬ S /∴ ¬ S v P

15. [(¬ P v Q) => (R Λ P)]
(Q => R)
[¬ Z => (¬ P v Q)] / ∴ (Z v ¬ P)

4.7 LET US SUM UP

In this Unit we have tried to give an idea about proving invalidity, by giving definition, importance, and principles of this method. This new method of proving invalidity is shorter than writing out a complete truth-table, and the amount of time and work saved is proportionally greater for more complicated arguments. In proving the invalidity of more extended arguments, a certain amount of trial and error may be needed to discover a truth-value assignment, which works. But even so, this method is quicker and easier than writing out the entire truth-table. It is obvious that the present method will suffice to prove the invalidity of any argument, which can be shown to be invalid by a truth-table.

4.8. KEY WORDS

Argument: A structure composed only of statement variables and symbols such that all

its substitution instances will be arguments.
Invalid argument form: An argument form, which has at least one false substitution instance.

4.9 FURTHER READINGS AND REFERENCES

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