
UNIT 1 INTRODUCING THE QUANTIFIERS 'ALL' AND 'SOME' AND THEIR SYMBOLIC REPRESENTATION

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1.0 OBJECTIVES

One of the principal objectives of this unit is to bring to the fore the limits of Aristotelian logic and the way in which modern logic succeeded in extending the boundaries of Aristotelian logic. Against this background, a brief reference to historical element in 'Introduction' becomes significant. It is not only significant but also necessary. Second objective of this unit is to show how in the absence of new set of rules the Rules discussed in the previous units do not help us to test a particular class of argument. In the third place, this unit aims at explaining the role played by quantifiers in the restructuring of traditional square of proposition. Thus by the end of this unit we should be able to have a basic understanding of predicate logic. Secondly, we must be in apposition to symbolize the proposition using quantifiers. Finally, we must be able to identify the internal structure of the proposition.

1.1 INTRODUCTION

Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with *things or objects to which the predicates may be ascribed*. This statement, in general, and the words in italics, in particular, must act as springboard for our further study and its significance becomes evident shortly. Quantification logic has its roots in 'Set Theory'. Set theory itself is of recent origin. This theory took its birth in the nineteenth century in the works Georg Cantor. However, other mathematicians, most notably, Boole, Venn and de Morgan meant it though they did not develop the theory. Nor did they use the term in the sense in which Cantor used. Such things are not uncommon in the development of science. For example, Michael Faraday did not use the word *field* in his work on physics though he meant it. This piece of fact from the History of Mathematics must be borne in mind in order to pay what is due to Aristotle. Prior to this, we must understand that the important concepts developed by

Cantor viz. subset, proper subset, the difference between the two, null set (empty set), denumerable and equivalence are some of the key concepts which play key role in our study.

Where did Aristotle err? This question needs to be addressed. As a matter of fact, Aristotle did not err. That is the reason why the word *defect* is not the right word to be used while assessing Aristotelian system. Instead, limitation is the apt word to be used in our analysis of Aristotelian system. Aristotle had an idea of class at elementary level. He evolved the concepts of class inclusion (total or partial) and class exclusion (total or partial), but could not proceed further. This explains the limits of his analysis of categorical proposition based on quality and quantity of proposition and the outcome of his analysis. Since Set theory in the sense in which Cantor developed was unknown during Aristotle's age, it, surely, would be anachronistic to criticize Aristotle for his limited perspective of predicate logic. Therefore let us first identify the loose ends in Aristotelian system. This will help us to understand the significance of 'Quantification Logic' in particular and modern logic in general.

Aristotle did not differentiate between universal proposition and singular proposition. A proposition is singular when the subject is a proper name. In this respect, singular proposition differs from particular proposition, though later we understand that both are existential propositions. In his analysis these two are, more or less, the same. An understanding of subtle difference and its consequences is quite illuminating. Any universal proposition of the form 'All S are P' or 'No S are P' reveals that S and P are merely class-names. If the concept of denotation is closely examined, then it becomes clear that all class-indicators include or exclude a certain number of elements known as members of a particular class, otherwise called sets. Therefore every set represented by a term in the proposition is very much similar to denumerable set which is a set of positive integers. A set is said to be denumerable when it is a set of positive integers because only then members are countable. If members are countable, then denotation makes sense, not otherwise. Similarly, the concept of intension reveals that to be a well-defined member the member must possess a definite set of properties without which it ceases to be a member of that particular set.

Against this background, we should try to know what the difference or differences between universal and singular on the one hand and particular and singular propositions on the other signify. First let us consider universal and singular propositions. The proposition 'All men are mortal' has both contrary and contradictory relations. However, the proposition 'Socrates is mortal' has only contradictory relation, but not contrary. It may be necessary to point out that, though it amounts to repetition, two propositions are contraries only when two conditions are satisfied; when p is true, q is false and when p is false q is doubtful. On the other hand, contradiction arises when p is true, q is false and when p is false q is true and *vice versa*. Suppose that the second proposition, 'Socrates is mortal' is negated. We get 'Socrates is not mortal'. When the first statement is true, the second statement is false. Though the first condition is satisfied, the second condition is not satisfied because when the first statement is false, the second statement is not doubtful, but turns out to be true. If logical relations matter, then the distinction between universal and singular proposition also ought to matter. This is a point which Aristotle failed to notice. Further, both particular and singular are existential propositions which make matters still worse. Like universal propositions, particular propositions also have two distinct relations which distinguish them from singular propositions. Instead of contrary, sub-contrary explains one type of relation between two particular propositions. If 'some men are

mortal' is true, then 'some men are not mortal' is doubtful and if 'some men are mortal' is false, then 'some men are not mortal' is true. Of course, contradiction explains the relation between universal and particular. Here is the difference. Though both particular and singular propositions are existential, sub-contrary relation is not common to both. This means that universal and particular propositions, on the one hand, and particular and singular, on the other, deserve to be classified separately. They are called general propositions distinct from singular propositions because the subject of such propositions is a general term. A term which refers to an indefinite number of things is a general term which is called common noun in grammar. What we call quantifiers are applicable to general propositions, but not to singular propositions.

This is one difference. Second difference is crucial. In this context, the emphasis is on the word *existence*. If a certain proposition is characterized as existential, how do we understand such characterization? When we discussed Venn's diagram in connection with the distribution of terms, we learnt that universal propositions do not carry existential import whereas particular propositions carry existential import. The statement 'All men are mortal' does not affirm the existence of men whereas 'Some men are....' affirm the existence of men irrespective of the quality of proposition. Same is the case with 'No men are...' No assertion is made about the existence of men. Existence presupposes the presence of members in a given class. If existence makes sense, then in negative sense nonexistence also must make some sense. Suppose that a set does not contain a single member. Then what is its status? Till nineteenth century this question did not occur to anyone. In other words, the concept of null set paved the way for further progress in Aristotelian logic. How did it happen?

The concept of null set plays crucial role in distinguishing Aristotelian system from modern logic. Let us recall the very first statement of introduction; 'Predicate logic, is a branch of logic, which is concerned with predicates or with predication of properties, and also with *things or objects to which the predicates may be ascribed*'. In the strict sense of the term, predicate may be ascribed to only things or individuals actually existing. Otherwise, the commonplace difference between fact and fiction will be completely obliterated. Therefore in a restricted sense existential propositions make matters of fact relevant. When matters of fact become relevant, purely formal character of formal logic makes room for the relevance of content to a certain extent. But generalization, which is a characteristic of deductive inference, does not lose its significance. The only requirement is that the content of the argument must be factual, but not fictitious.

Where does null set figure in this discussion? One fundamental relation between propositions with which we are concerned, presently, is contradiction. The law of contradiction holds good when terms include members as a matter of fact. However, the situation is different when the terms represent null sets. Consider this proposition.

1. All female philosophers of Karnataka are the residents of New York.

This sentence is obviously false. Therefore according to the law of its contradiction, the statement mentioned below must be true.

2. Some female philosophers of Karnataka are not the residents of New York.

Statements 1 and 2 are supposed to be contradictories. The second statement ought to be true according to the law of contradiction since the first statement is false. But, in reality, this statement is also false. But two contradictories cannot be false. This problem arises because we are dealing with nonexistent members. Therefore in the strict sense of the word second statement also, like universal statement, does not carry existential import. Within the framework of traditional logic this problem remains unnoticed because there was no concept of set at all—whether null set or non-null set. Modern logic corrected this mistake by making null set a distinct entity. The underlying principle is that all existential propositions should include only non-null sets. This stipulation marks one difference between traditional and modern systems.

Equivalence is second major factor. Consider these propositions.

3. All triangles are plane figures.

4. All equilateral triangles are equiangular triangles.

Statements 3 and 4 assume the forms as follows.

3a. If any figure is a triangle, then it is a plane figure.

4a. A figure is equilateral triangle if and only if it is equiangular.

Let us symbolize these statements.

$$3a \equiv F \Rightarrow P$$

$$3b \equiv F \Leftrightarrow P$$

Again, traditional logic did not distinguish these propositions. The difference between 3a and 3b becomes clear only within the framework of modern logic. This is another important progress made by modern logic over traditional logic. Such differences matter in quantification logic.

This aspect has something to do with the difference between subset and proper subset. Proposition 3 discloses that the set of triangles is a proper subset of the set of plane figures. However, the set of equilateral triangles is equivalent to the set of equiangular triangles. This explains why the sentential connectives differ from 3a to 3b.

Traditional logic made one type of distinction among propositions; conditional and unconditional and within the latter four kinds of propositions. Modern logic not only discovered a new aspect in conditional proposition but also it added new kinds of propositions which were not considered by traditional logic. Hence it could evolve new set of rules. But these rules had limitations. In the absence of further augmentation of new rules they could not be applied to arguments which consisted of singular propositions. So the search for relevant rules did not stop.

Let us turn to basic difference between propositional logic and predicate logic. This difference lies in dealing with the internal structure of simple as well as compound propositions. Therefore predicate logic includes rules hitherto used and also a new set of rules. Predicate logic is

concerned with the internal structure of propositions. It is not the case with propositional logic. In strict technical terms, this is also known as Quantification Theory or the Predicate Calculus. It has its own syntax, which helps us to devise statements, which are considered well-formed statements. We are now concerned with this new syntax.

1.2 SYMBOLIZATION OF PROPOSITIONS

How do we symbolize the statement 'Socrates is a philosopher'? A unique method is devised which is merely a convention. The subject term is represented by first letter of the same which is always a small letter and predicate is represented by the first letter of the same which is always a capital letter. The proposition considered above now becomes

Ps

The singular terms are represented in predicate logic by the individual constants. These are small letters from 'a' to 'w', with or without numerical subscripts. Their function is to denote only one, unique individual or object from the domain of discourse. Since their reference remains fixed or constant within a given context, they are called individual constants.

Predicates are linguistic expressions of properties. In other words, predicates are words or expressions that we use to refer to properties or attributes that things have. For example, we may use the predicate term 'red' to refer to the property of 'being red' that a flower has. Predicate logic we are discussing is called the First Order Predicate Logic. Within the limits of this order only simpler predications such as properties of individuals or objects are considered. There can be complicated predications where we need to consider properties of properties and quantity of properties, and that would be the Second Order Predicate Logic. In higher order logic, we have variables standing for properties such as 'F', 'G', etc., and property constants.

There are three ways in which change can be effected; i) change S or ii) change P or iii) change both S and P. accordingly we have the following possibilities.

- i) $Pa, Pb, Pc, \dots$ etc.
- ii) $Gs, Ga, Gb, \dots$ etc.
- iii) $Ab, Cd, Ef, \dots$ etc.

It is easy to notice that any kind of change is just indefinite in the sense that the list can be extended to include the whole of humanity though it is not intended. This process is just simplified by using the variable 'x' in place of constants. When we do so, we obtain Px . When variable is used in place of individual constants, we get what is called *propositional function*. Propositional function is neither true nor false. Truth-value can be assigned only when constants replace the variable. Consider the following replacements.

1. Pa where a stands for Aristotle
2. Pb where b stands for Berkeley
3. Ph where h stands for Hitler
4. Pc where c stands for Churchill

It is evident that 1 and 2 are true whereas 3 and 4 are false. That 1 and 2 are true is known only when we know what a and b stand for. Similar is the case with 3 and 4. Therefore in quantification logic we should ascertain the actual truth-status of propositions. If h stands for Himalayas, then the statement does not make any sense. Pa, Pb, etc. result from propositional function by an operation called instantiation. Accordingly, a, b, c, etc. are called substitution instances. Further, a and b are true substitution instances whereas h is not a true substitution instance.

We learnt the way to symbolize singular propositions. There is a different way of symbolizing general statements which are, doubtless, compound propositions. Quantifiers are used in this connection. A general proposition is of two types; universal and particular. So we have two quantifiers denoting these two. Since each of them may be affirmative or negative, we have four kinds of propositions, which are represented as follows:

1. All Indians are mortal.: $(x) Mx$
2. No Indians are mortal.: $(x) \neg Mx$
3. Some Indians are mortal.: $(\exists x) Mx$
4. Some Indian are not mortal.: $(\exists x) \neg Mx$

‘ $\forall$ ’ also can be used in place of (x) . The symbols used on the R. H. S. need some explanation.

The symbol (x) is expanded in several ways. It can read ‘for all values of x’ or ‘Given any x’ or simply ‘for every x’, etc. where ‘x’ stands for individual constant, ‘Indians’ and ‘M’ stands for mortal. Therefore $\neg Mx$ is read ‘x is not mortal’. The symbol $\exists x$ is read ‘there exists at least one x such that’ (x) is called universal quantifier and $\exists$ is called existential quantifier. If we substitute I (Indians) or P (Pakistanis) for x then we get a proposition, which may be true or false. It may be noted that universal quantifier is true only when every substitution instance of the same is true or it has only true substitutions whereas the existential quantifier is true when at least one substitution instance of the same is true.

Just as x is used as individual variable to denote the subject, two Greek letters ‘ Φ ’ (Phi) and ‘ Ψ ’ (Psi) are used to denote predicate. So they are called predicate variables. Using these variables A, E, I and O propositions can be represented as follows:

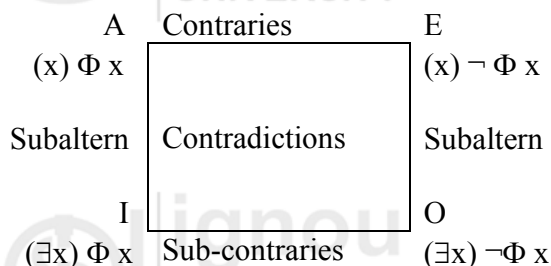
1. All Indians are mortal.: (A) $(x) \Phi x$
2. No Indians are mortal.: (E) $(x) \neg \Phi x$
3. Some Indians are mortal.: (I) $(\exists x) \Phi x$
4. Some Indians are not mortal.: (O) $(\exists x) \neg \Phi x$

Using class membership relation, general propositions are represented as follows:

1. $(x) \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \in \Psi\}$ Where $\in$ is read ‘element of’
2. $(x) \neg \Phi x \equiv (x)\{x \in \Phi \Rightarrow x \notin \Psi\}$ Where $\notin$ is read ‘not an element of’
3. $(\exists x) \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \in \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x)\{x \in \Phi \wedge x \notin \Psi\}$

1.3 LOGICAL RELATIONS INVOLVING QUANTIFIERS

Our study begins with traditional square of opposition, which does not need any explanation. We know how A, E, I and O are denoted with the help of quantification theory. Let us replace A, E, I and O by these quantifiers in the square:



Later we will learn that this square is altered when we make the assumption that there is only one individual. Therefore all relations are not discussed. Against this background, we shall restrict ourselves to only two important logical relations; viz., equivalence and contradiction. They are represented as follows:

1. Equivalence:

- 1) $(x) \Phi x \equiv \{\neg (\exists x) \neg \Phi x\}$
- 2) $(x) \neg \Phi x \equiv \{\neg (\exists x) \Phi x\}$
- 3) $(\exists x) \Phi x \equiv \{\neg (x) \neg \Phi x\}$
- 4) $(\exists x) \neg \Phi x \equiv \{\neg (x) \Phi x\}$

2. Contradiction:

- 1) $(x) \Phi x \quad (\exists x) \neg \Phi x$
- 2) $(x) \neg \Phi x \quad (\exists x) \Phi x$
- 3) $\exists x \Phi x \quad (x) \neg \Phi x$
- 4) $\exists x \neg \Phi x \quad (x) \Phi x$

When we use predicate variable, the propositional forms are expressed as follows:

- 1) $(x) \Phi x \equiv (x) \{ \Phi x \Rightarrow \Psi x \}$
- 2) $(x) \neg \Phi x \equiv (x) \{ \Phi x \Rightarrow \neg \Psi x \}$
- 3) $(\exists x) \Phi x \equiv (\exists x) \{ \Phi x \wedge \Psi x \}$
- 4) $(\exists x) \neg \Phi x \equiv (\exists x) \{ \Phi x \wedge \neg \Psi x \}$

When we represent A, E, I & O with this new set, their equivalent forms also undergo changes.

- 1) $(x) \{ \Phi x \Rightarrow \Psi x \} \equiv \neg \exists x \{ \Phi x \wedge \neg \Psi x \}$
- 2) $(x) \{ \Phi x \Rightarrow \neg \Psi x \} \equiv \neg \exists x \{ \Phi x \wedge \Psi x \}$
- 3) $(\exists x) \{ \Phi x \wedge \Psi x \} \equiv \neg (x) \{ \Phi x \Rightarrow \neg \Psi x \}$
- 4) $\exists x \{ \Phi x \wedge \neg \Psi x \} \equiv \neg (x) \{ \Phi x \Rightarrow \Psi x \}$

If negations placed behind the quantifiers on the R. H. S. are removed, then automatically they become contradictories of respective propositions.

A predicate like mortal is called simple predicate because the propositional function which, if used, has true and false substitutions. All substitutions to variable are called 'substitution instances'. When such predicates are negated, such formula or statement is called 'normal form formula'.

What is the function of quantifiers? Quantifiers are expression in Predicate logic which state that a certain number of the individuals or objects have the property in question. They do not state which one of the individuals have the property. A quantifier consists of:

- A left parenthesis '('
- A quantifier symbol
- One of the individual variable symbols
- A right parenthesis ')'
- In predicate logic, we have two quantifier symbols: '(x)' or ' $\forall$ ' and ' $\exists$ '

These quantifiers are in non-natural language the symbols of quantity indicators; 'all', 'some', and 'no', which may occur in statements about predications. Predicate Logic uses only two kinds of quantifier symbols: universal quantifier and existential quantifier.

Check Your Progress I

Note: Use the space provided for your answers.

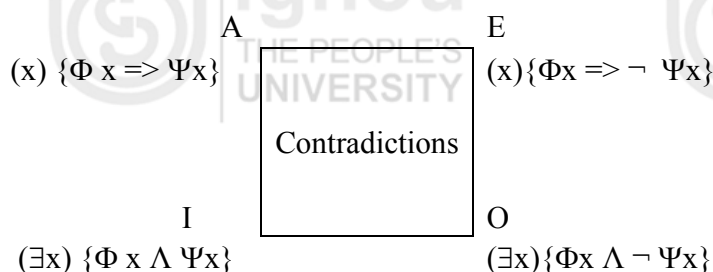
1. Explain the quantification theory.

2. How do we classify the quantifiers?

1.4 FALL-OUT OF UNIVERSAL AND EXISTENTIAL QUANTIFIERS

An important aspect needs to be clarified at this stage. Suppose that there is only one individual in the universe, which is something not logically impossible. There is a difference between saying that 'there is only one individual in the universe' and saying that 'there is at least one individual'. In the latter case we commit ourselves to the possibility of a second or third thing

which exists. Let us exclude this possibility. Then traditional square of opposition undergoes change automatically. Let us see how it happens. We shall use two predicates instead of one and consequently implication steps in. First let us draw the figure.



According to this figure of four relations mentioned earlier only contradiction holds good. We only know that there is x . Suppose that Φx does not have true substitution instance. Then both A and E are true because Φx , i. e., antecedent is false and a false antecedent makes all implications in which it occurs always true. Therefore they are not contraries. Similarly, I and O are not subcontraries. When Φx does not have true substitution, both I and O are false whether Ψ is the predicate or not because one of the conjuncts in I and O is false. Subaltern (or superaltern) disappears in a different way.

When we assume that there is only one individual in the universe, it results in a unique implication. $(x) \{ \Phi x \Rightarrow \Psi x \}$ implies in a unique way the statement $(\exists x) \{ \Phi x \Rightarrow \Psi x \}$. This statement actually means that there is definitely one x and this x has the property Ψ or does not have the property Φ . Here care is required. Suppose that instead of making this disjunctive assertion we assert that x has Φ or it does not have Ψ . Then the implication becomes false. A true implication cannot imply a false consequence with true antecedent. Such an assertion is, evidently, different from the assertion $(\exists x) \{ \Phi x \wedge \Psi x \}$.

Modern logic admits different types of general proposition with which we have already become familiar while learning the technique of testing arguments. A proposition like 'All A are B or C' is not the same as the proposition, 'All A are B or All A are C'. It only means that if x is A, then x is B or C whereas the second statement means that if x is A, then x is B or if x is A, then x is C. Again, we must construct truth-table to know how they differ. (This is left as an exercise for the student). On the other hand, the proposition, 'Some A are B or C' means the same as saying 'Some A are B or some A are C'. This means that when the kind of quantification of a proposition changes, correct transformation of proposition also changes.

Check Your Progress II

Note: Use the space provided for your answers.

1. Explain the importance of universal quantifier.

2. Expose the worth of existential quantifier.

1.5 EXAMPLES

1. Symbolize the following using universal quantifier.

a) All physical things are temporary.

$$(x) \{Px \Rightarrow Tx\}$$

b) No system is permanent.

$$(x) \{Sx \Rightarrow \neg Tx\}$$

c) All dogs are mammal.

$$(x) \{(Dx \Rightarrow Mx)\}$$

d) No dogs are fish.

$$(x) \{(Dx \Rightarrow \neg Mx)\}$$

e) No men are immortal.

$$(x) \{(Mx \Rightarrow \neg Ix)\}$$

2. Symbolize the following using Existential quantifier.

a) Some jobs are temporary.

$$(\exists x) \{Jx \wedge Tx\}$$

b) Some fish are not mammals.

$$(\exists x) \{Fx \wedge \neg Mx\}$$

c) Some dogs are Black.

$$(\exists x) \{Dx \wedge Bx\}$$

d) Some dogs are not fish.

$$(\exists x) \{Dx \wedge \neg Fx\}$$

e) Some men are not tall.

$$(\exists x) \{Mx \wedge \neg Tx\}$$

1.6 EXERCISES

Symbolize the following using quantifier.

1. All books are interesting.

2. No good books are useless.

3. Not every book is expensive.

4. No expensive things are good.

5. Ram is an ambitious person.

6. Ravi is a ruthless person.

7. Some horses are not black.
8. Rose is a colourful flower.
9. All horses are four legged.
10. Some good books are rare and expensive.

Check Your Progress III

Note: Use the space provided for your answer.

1. Symbolize the following using quantifier.
 - a) Some sheep are not white.
 - b) Every flower is beautiful and attractive.
 - c) All cattle are four legged.
 - d) Some good books are also not expensive.
 - e) All Humans are mortal.

1.7 LET US SUM UP

In this unit we have discussed the function of quantifiers. The universal quantification of a propositional function is true if and only if all of its substitution instances are true, and that the existential quantification of a propositional function is true if and only if it has at least one true substitution instance. If we grant that there is at least one individual, then every propositional function has at least one substitution instance (true or false). Under this assumption, we can say that if the universal quantification of a propositional function is true then its existential quantification must be true also. According to our discussion of quantifiers, contraries and sub-contraries do not stand; only contradiction remains acceptable.

1.8 KEY WORDS

Predicate logic: Logic of predicates or properties, and things or objects to which the predicates may be ascribed.

First order predicate logic: It is the elementary kind of predicate logic, in which only simpler predications such as properties of individuals or objects are considered.

Quantifiers: Symbols that state how many of the individuals have the property in question.

Universal quantifier: Universal quantifier is the symbol in non-natural language which represents universal proposition (both affirmative and negative)

Existential quantifier: It is the symbol in non-natural language which represents particular proposition (both affirmative and negative).

1.9 FURTHER READINGS AND REFERENCES

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UNIT 2 RULES OF UNIVERSAL INSTANTIATION AND GENERALIZATION, EXISTENTIAL INSTANTIATION AND GENERALIZATION, AND RULES OF QUANTIFIER EQUIVALENCE

Contents

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- 2.1 Introduction
- 2.2 Rules of Quantification
- 2.3 Rules of Quantifier Equivalence
- 2.4 Application of the Quantification Rules
- 2.5 Examples
- 2.6 Exercises
- 2.7 Quantification Rules and Arguments
- 2.8 Let Us Sum Up
- 2.9 Key Words
- 2.10 Further Readings and References

2.0 OBJECTIVES

The first objective of this unit is to emphasize the need to have a new set of rules. Before we apply these rules a brief explanation of these rules is needed. Therefore there is another important objective. This second objective is to provide a theoretical basis for the application of these rules. However, these rules only supplement the rules discussed in detail in the preceding unit. Hence the third objective is to demonstrate how the integration of the new set of rules with the rules with which we are already familiar is necessary. Thus by the end of this unit we should be able to have a basic understanding of the significance of quantifiers. Secondly, we must succeed in identifying the quantified statements. Finally, we must succeed in testing successfully the validity of arguments with the help of quantifiers

2.1 INTRODUCTION

A quantifier is a tool with the help of which we will be in a position to measure the *magnitude* of the subject of a proposition. What do we mean by 'magnitude'? The term magnitude applies to any physical quantity which is measurable. In this particular case the measurable physical quantity is *things* to which the proposition in question refers. If we translate this term to traditional mould, then it just signifies the concept of the distribution of terms. The difference, however, is that in traditional analysis this concept turns out to be quite clumsy and ambiguous. Various synonymous words make matters further worse. The use of quantifiers in non-natural language resolves all these difficulties at one stroke. It is also possible to do away with mathematical interpretation of distribution of terms if we wish so. Further, this technique renders the application of distribution to the predicate term superfluous. The significance of the rules of quantification must be understood against this background.

There are four rules of quantification. They are as follows; Universal Instantiation (UI), Universal generalization (UG), Existential Instantiation (EI) and Existential generalization (EG). The first two rules involve the quantifier which is called Universal quantifier which has definite application. Whenever an affirmative proposition contains words like 'all', 'every' 'each', etc. and propositions contain words like 'No', 'None', etc., universal quantifier replaces all such words. This sort of economy also achieves simplicity. This is the distinct advantage of the use of quantifiers. On the other hand, whenever propositions irrespective of quality contain words like 'someone', 'many things', 'a few, etc., existential quantifier is used. In symbolic logic these quantifiers are symbolized as follows. ' (x) ' or ' $(\forall x)$ ' is the symbol for universal quantifier and ' $(\exists x)$ ' is the symbol for existential quantifier. The symbolic representation of these quantifiers removes ambiguity in addition to achieving economy and simplicity. The difference between the instantiation and generalization rules with respect to both the quantifiers is that for universal quantifier UI allows the elimination of the universal quantifier whereas UG allows us to introduce a universal quantifier and similarly, for existential quantifier EI allows the elimination of an existential quantifier and EG allows us to introduce the same.

Every quantifier has a certain range. The range of ' (x) ' or ' $(\forall x)$ ' is indefinite whereas the range of ' $(\exists x)$ ' is definite in the sense that in the latter case we are definite that there is at least one member whereas in the former case we are not.

2.2 RULES OF QUANTIFICATION

In predicate calculus the letter 'x' signify individual variable. The aforementioned four rules permit the transformation of non-compound into equivalent compound propositions to which the Rules of Inference and Equivalence are applicable. They also permit the transformation of compound propositions into equivalent non-compound propositions. These additional rules thus make it possible to construct formal proofs of validity for arguments whose validity depends upon the inner structure of some non-compound statements contained in those arguments. These rules stand in need of brief explanation.

Universal Instantiation (UI)

This rule says that any substitution instance of a proposition function can be validly deduced from a universal proposition. A universal proposition is true only when it has only true substitution instances. This is the necessary and sufficient condition for any true universal proposition. Therefore any true substitution instance can be validly deduced from the respective universal proposition. A propositional function always consists of variable 'x'. At times z also is used as a variable and y has a definite role to play other than that of constant. Therefore any instance which is a substitution for x is regarded as a constant and letters from 'a' through 'w' are symbols for constants. These letters signify subject in traditional sense, and in modern sense, an 'instance of a form'. To obtain such an instance of a form 'x' is replaced by another Greek letter ' ν ' (*nu*) which is another symbol for an individual constant. It is also an example for universal instantiation because the universal quantifier is instantiated here.

This rule is symbolically represented as follows:

$$\frac{(x) \Phi x}{\therefore \Phi \nu}$$

This rule requires a little elaboration. Let us replace v by a more familiar constant, say, a , and Φ by F . When Fa is inferred from $(x) Fx$, the quantifier (x) is dropped. The reason is that the universal quantifier has indefinite extension whereas constant is restricted to one particular individual. Therefore in this context it is wrong to use universal quantifier. The rule of UI allows such of those instances where we replace all variables bound by a universal quantifier with individual constant. Thus $(x) (Sx \Rightarrow Px)$ will yield $(Sa \Rightarrow Pa)$ where a is the constant used in place of the variable. The application of UI goes with a few stipulations. The quantifier (x) in $(x) Fx$ should not be within the range of a negation $(\neg)$. It should not also be within the extent of another quantifier. The span of (x) in $(x) Fx$ must extend to the complete expression. A violation of any one of these limitations will lead to an incorrect utilization of UI. In other words, if we say that $\neg (x) Fx$ implies $\neg Fa$, then it must be viewed as a wrong understanding of UI.

We may use the UI rule in the following way:

- a) First, remove the universal quantifier.
- b) Next, replace the resulting free variable by a constant.

Universal Generalization (UG)

This rule helps us to proceed to generalization after an arbitrary selection is made to substitute for x . In UG, 'arbitrary selection' is very important because as the name itself suggests, generalization always proceeds from individual instances. Arbitrary selection always means 'any'. And there is no specific choice involved. In this sense, selection is 'random' or arbitrary. The letter y is the symbol of 'arbitrary' selection. This is the reason why ' y ' is not regarded as a constant. This process is called generalization because the conclusion is a universal proposition. The underlying principle is that what holds good in the case of any variable selected at random must hold good in all instances. In other words, the variable y is equivalent to saying 'any'. If we recall the traditional rules of syllogism, universal conclusion follows from universal premises only. It only means that we need prior universal proposition. Let us club UI with this step. Then we are allowed to say that if universal proposition is true then any variable selected at random must be true. Therefore it must be understood that in this case the process is from universal to universal through an individual. When ' x ' replaces ' y ' there is generalization. When universal quantifier describes the proposition it becomes UG. The procedure is as follows.

$$\begin{array}{c} \Phi y \\ \therefore (x) \Phi x \end{array}$$

In the above given rule the letter ' y ' in Φy (or Fy) stands for any arbitrarily selected individual. It is only a pseudo name and not the name of a particular individual. This letter ' y ' in UG is not a constant but an individual variable only. But it is different from x in the sense that it is an indefinite replacement for x . In UG we substitute first all pseudo names with variables and then bind them with universal quantifiers.

We apply UG to the statement in the following manner:

- a) First add the universal quantifier.

b) Then ensure that in the conclusion the variable is bound by this newly introduced universal quantifier.

Existential Instantiation (EI)

This rule is applicable when the proposition has existential quantifier and in this case any symbol ranging from a through w is used as a substitute for the individual variable x. We can infer the truth of any substitution instance from existential quantification because existential quantification is true only when there is at least one true substitution instance. However, this rule has a clause when it is applied to an argument. The constant, say 'a' which we use to substitute for x should not have occurred anywhere earlier in that argument. It only means that in the same argument EI cannot be used twice when it is assumed that there is only one true substitution instance. The rule is represented as follows.

$$\frac{(\exists x) \Phi x}{\therefore \Phi v}$$

This formula says that there is at least one or some unspecified number of members in the domain in question have a certain property, say, 'Φ'. The letter 'v' in Φv stands for that unspecified number of members and hence it is in a sense sort of pseudo name and not the name of any particular individual. Therefore this may be called an existential pseudo name. It is necessary to adhere to certain stipulations while implementing this rule. In the first place, the formula (∃x) in (∃x) Φx should not be within the range of negation (¬). Secondly, (∃x) in (∃x) Φx should not be within the extent of another quantifier. Thirdly, the quantifier (∃x) must cover the complete expression. We may use the rule EI in a statement in the following way:

- a) First, remove the existential quantifier.
- b) Next, replace the resulting free variable with a constant.

Existential Generalization (EG)

This rule states that from any true substitution instance of a propositional function, the existential quantification of that function can be validly deduced. Only then the existential quantification can become true. When the existential quantification is so deduced, the individual constant which appeared in earlier steps is replaced by x in the conclusion. The unique feature of this rule is that though there is generalization, the conclusion continues to be existential. The rule is represented as follows.

$$\frac{\Phi v}{\therefore (\exists x) \Phi x}$$

In the rule EG the letter 'v' may be the name of a particular individual or again a pseudo name. (∃x) Φx states that there is at least one x such that x is Φ. When we apply EG, we have to follow some conditions. Each happening of 'v' in Φ x must be substituted by 'x' and the scope of (∃x) must extend to the entire expression.

The method of application of EG is simple:

- a) Insert an existential quantifier.
- b) Ensure that at least one occurrence of the individual variable which we have generalized is bound by the newly introduced existential quantifier.

Earlier, it was stated that the EI should not have occurred earlier in any argument. But no explanation was given for this stipulation. We should know why there is this particular restriction on the use of EI. Suppose that 'a' is the constant whose existence is definite. We are sure of the existence of a, but we are not sure whether there is any other constant. In an argument in an earlier step a constant, say, 'a' is regarded as 'b'. The fact that 'a' is 'b' is not adequate enough to conclude in some other step that 'a is c' when there is no reference of any kind to 'c' in the premise. Since the logical constant 'a' is used in existential mode, it is mandatory that EI should be used in the very first step of the proof. If it occupies any other position, then it is a mistake.

Check Your Progress I

Note: Use the space provided for your answers.

1. Explain UI and EG.

2. Explain the significance of random variable.

2.3 RULES OF QUANTIFIER EQUIVALENCE

In the previous unit we learnt the technique of framing the equivalence for quantifiers. In this unit we shall consider examples for equivalence forms. The standard forms of categorical statements are A, E, I and O. Let us start with A statement: "Every cat is a mammal." In this statement there are two individuals, viz., 'a cat' and 'a mammal'. We also know that these propositions are symbolized as follows.

Cx: x is a cat.

Mx: x is a mammal.

We should also note that the predicate of 'being a mammal' is predicated to every member which has the property of being a cat. We need universal quantifier in this case. On Boolean

interpretation, universal statements are actually conditional statements with no existential commitment. We may paraphrase it as: For every x , if x is a cat then x is a mammal.

The expression 'if x is a cat, then x is a mammal' is translated with a truth-functional symbol ' $\Rightarrow$ '. Thus translated, it becomes $(x) (Cx \Rightarrow Mx)$. Using the same symbolization key, we symbolize the 'E' statement 'No cat is a mammal' as, $(x) (Cx \Rightarrow \neg Mx)$. It must be remembered that "No cat is a mammal" can be paraphrased as: For every x , if x is a cat, then x is not a mammal. This proposition does not deny the predication of the property of 'being a cat', but it denies the property of 'being a mammal' to any cat. This is the reason why the negation sign is placed before ' Mx ' in the statement ' $(x) (Cx \Rightarrow \neg Mx)$ '.

Let us consider the 'I' and 'O' statements. 'Some cats are mammals' is an illustration for 'I'. When paraphrased, we have to admit the existential import and also the property predication. It has to be stronger in assertion than the conditional we state for the universal statements. Considering these aspects, we translate I and O propositions to symbolic form.

There is (exists) at least one x such that x is a cat and x is a mammal. This is symbolized as follows:

$$(\exists x) (Cx \wedge Mx)$$

Note that a truth-functional symbol ' $\wedge$ ' from propositional logic has been used to denote that both properties belong to ' x '.

We translate the 'O' statement, 'Some cats are not mammals' in the following way.

There is (exists) at least one x such that x is a cat and x is not a mammal; and this expression is symbolized as follows:

$$(\exists x) (Cx \wedge \neg Mx)$$

As a matter of convention these propositions are represented symbolically as follows:

1. (A) $(x) \Phi x$
2. (E) $(x) \neg \Phi x$
3. (I) $(\exists x) \Phi x$
4. (O) $(\exists x) \neg \Phi x$

Using class membership relation, these propositions are represented as follows:

1. $(x) \Phi x \equiv (x) \{x \in \Phi \Rightarrow x \in \Psi\}$ Where $\in$ is read 'element of'
2. $(x) \neg \Phi x \equiv (x) \{x \in \Phi \Rightarrow x \notin \Psi\}$ Where $\notin$ is read 'not an element of'
3. $(\exists x) \Phi x \equiv (\exists x) \{x \in \Phi \wedge x \in \Psi\}$
4. $(\exists x) \neg \Phi x \equiv (\exists x) \{x \in \Phi \wedge x \notin \Psi\}$

Where Φ and Ψ are the symbols for attributes. These are the four rules of quantifier equivalence. They are also known as Quantifier Negation Rules because if negation is placed behind quantifier, then it becomes the contradiction of the original statement.

2.4 APPLICATION OF THE QUANTIFICATION RULES

Before we proceed further let us recapitulate what we learnt in the previous unit. In order to relearn we shall apply these rules to statements. This will be a good introduction to the next stage of our learning.

I. We shall apply 'UI' for the statements mentioned below and remove the quantifier.

a) $(x) (Hx \Rightarrow Mx)$

$$\frac{}{(Ha \Rightarrow Ma)}$$

b) $(x) (Hx \Rightarrow \neg Mx)$

$$\frac{}{(Ha \Rightarrow \neg Ma)}$$

c) $(x) (Mx \Rightarrow \neg Ix)$

$$\frac{}{(Ma \Rightarrow \neg Ia)}$$

II. We shall now apply 'UG' for the statements mentioned below to add quantifier and then generalize. Very soon we ought to discover that this is really the reverse process.

d) $(Ha \Rightarrow Ma)$

$$\frac{}{(x) (Hx \Rightarrow Mx)}$$

e) $(Ha \Rightarrow \neg Ma)$

$$\frac{}{(x) (Hx \Rightarrow \neg Mx)}$$

f) $(Ma \Rightarrow \neg Ia)$

$$\frac{}{(x) (Mx \Rightarrow \neg Ix)}$$

III. We shall apply 'EI' for the statements mentioned below to remove the quantifier.

g) $(\exists x) (Hx \wedge Mx)$

$$\frac{}{(Ha \wedge Ma)}$$

$$h) (\exists x) (Hx \wedge \neg Mx)$$

$$\frac{}{(Ha \wedge \neg Ma)}$$

$$i) (\exists x) (Mx \wedge \neg Ix)$$

$$\frac{}{(Ma \wedge \neg Ia)}$$

IV. We shall apply EG for the statements mentioned below to add quantifier and to generalize:

$$j) (Ha \wedge Ma)$$

$$\frac{}{(\exists x) (Hx \wedge Mx)}$$

$$k) (Ha \wedge \neg Ma)$$

$$\frac{}{(\exists x) (Hx \wedge \neg Mx)}$$

$$l) (Ma \wedge \neg Ia)$$

$$\frac{}{(\exists x) (Mx \wedge \neg Ix)}$$

[Note: Whenever universal quantifier has to be symbolized instead of (x) we can also use ($\forall x$). However, the former is more extensively used.]

Check Your Progress II

Note: Use the space provided for your answers.

1. Explain the significance of rules quantifier equivalence.

2. Briefly explain the uses of instantiation and generalization.

2.5 EXAMPLES

1. Symbolize the following using universal quantifier.

a) Every Human is a mammal

$$(x) (Hx \Rightarrow Mx)$$

b) No Horse is a mammal.

$$(x) (Hx \Rightarrow \neg Mx)$$

c) All dogs are four legged.

$$(x) (Dx \Rightarrow Fx)$$

d) No donkeys are birds.

$$(x) (Dx \Rightarrow \neg Bx)$$

e) No men are immortal.

$$(x) (Mx \Rightarrow \neg Ix)$$

2. Symbolize the following using existential quantifier.

a) Some flowers are red.

$$(\exists x) (Fx \wedge Rx)$$

b) Some flowers are not red.

$$(\exists x) (Fx \wedge \neg Rx)$$

c) Some birds are White.

$$(\exists x) (Bx \wedge Wx)$$

d) Some fish are not snake.

$$(\exists x) (Fx \wedge \neg Sx)$$

e) Some men are tall.

$$(\exists x) (Mx \wedge Tx)$$

2.6 EXERCISES

Symbolize the following using quantifiers.

1. Some animals are dogs.

2. All cats are mammals.

3. No donkeys are blues.

4. All crows are black.
5. All parrots are not black.
6. Some philosophers are not Indians.
7. All Indians are not Tamilians.
8. All Tamilians are Indians.
9. All horses are not four legged.
10. Some good books are not expensive.

(Note: when a statement contains the words All.... not the corresponding proposition is 'O'.)

2.7 QUANTIFICATION RULES AND ARGUMENTS

It is quite interesting and also rewarding to learn how quantification rules can be applied to various arguments studied by traditional logic. This will also help us to discover the limits of traditional logic.

As mentioned earlier, out of five relations under square of opposition only contradiction survives and this has already been explained. Therefore let us concentrate on conversion and obversion among equivalence relation and categorical syllogism among mediate inference. While doing so, let us remind ourselves of the restriction imposed by modern logic which stipulates that from universal quantifier alone existential quantifier cannot be deduced and *vice versa*. Therefore A – I and I – A are excluded. However, a special form of A is considered where both S and P are equivalent sets and is examined with the help of quantification rules.

Consider the following proposition.

1. All spinsters are unmarried female persons.
 $\therefore$ All unmarried female persons are spinsters.

Since the rule of distribution is adhered to, the argument is valid. Let us see how the rule of quantification can be applied to this example.

There are two ways of symbolizing. They are as follows

- a) $(x)\{x \in S \Rightarrow x \in U\}$
- b) $(x)\{x \in S \Leftrightarrow x \in U\}$

a) does not completely convey the meaning of 1. Therefore we have to consider b). It can be reformulated as follows:

- c) $(x)\{x \in S \Rightarrow x \in U\} \wedge \{x \in U \Rightarrow x \in S\}$

For the sake of simplicity let us drop the quantifier. Applying commutative law, we get

- d) $\{x \in U \Rightarrow x \in S\} \wedge \{x \in S \Rightarrow x \in U\}$

This is an instance of simple conversion. Now apply simplification law.

$$e) \{ x \in U \Rightarrow x \in S \}$$

Translate 'e' to natural language. We get

All unmarried female persons are spinsters.

It may be noted that if this method is followed, we do not get the existential conclusion. Therefore conversion by limitation does not find place in this interpretation. (It is possible to get the identical result if commutative law is not used. But then it will not be clear to an untrained mind that the proposition is converted. It must be noted that commutative law is nothing but conversion). Examination of E proposition is left for the student as an exercise.

The case of existential proposition is simple. Examine this statement.

2. Some bananas are sweet.

Translate this statement to symbolic form.

$$a) (\exists x) \{ (x \in B) \wedge (x \in S) \}$$

Again, drop the quantifier for the sake of simplicity and apply commutative law. We get

$$b) \{ (x \in S) \wedge (x \in B) \}$$

When b) is translated to natural language, we get conversion in traditional sense.

Some sweet objects are bananas.

The case of O is a special case. It is quite illuminating to apply the quantification rules to know why it does not have conversion.

3. Some bananas are not sweet.

Translate this statement to symbolic form.

$$a) (\exists x) \{ Bx \wedge \neg Sx \}$$

(For the sake of simplicity class-membership is not considered in this particular case).

In this case simple conversion is possible in a different way altogether. We can only say

$$(\exists x) \{ \neg Sx \wedge Bx \}$$

If we reflect for a while we easily discover that this is, in reality, the symbolic form of partial contraposition. Suppose that we restrict ourselves to conversion. We are only entitled to convert (or commute) Bx and Sx. The negation sign ought to remain unaffected. For further clarity, let us compare the scene with algebra. $a + b = -c$ becomes $c = -(a + b)$. Just as negative sign is not disturbed in algebra while interchanging, so also in modern logic the negation sign remains undisturbed when we interchange the terms. But this is not conversion. In the case of algebraic equation, signs on both sides change. Therefore it is not equivalent to commutation. Commutation and conversion are technically identical. The upshot of the argument is that when there is negation on any one side or term, conversion or commutation is not possible. In other words, commutation holds good only when the relation is symmetric.

On similar lines, obversion can be explained. If x is not an element of S, then it means that x is an element of the complement of S.

The position of equivalence relation has now become clear. We have learnt that along with quantification rules we also require Rules of Inference and Rules of Replacement. We use the same technique to test the validity of syllogism. It is a matter of great interest to know that the rules of quantification project syllogism in a new perspective, which helps us to abandon the rule of distribution of terms, which is not only cumbersome in presentation but also time consuming. Further, quantification rules can be used to test non-syllogistic arguments also subject to the condition that general and singular propositions find place in such arguments. Let us use the following arguments to illustrate these rules.

1.

- 1) All Indians are Asians.
- 2) Tendulkar is an Indian.
- 3) $\therefore$ Tendulkar is an Asian.

This is symbolized as follows: $(x) \{Ix \Rightarrow Ax\}$

It

$\therefore At$

The formal proof is constructed as follows:

- 1) $(x) \{Ix \Rightarrow Ax\}$
- 2) It / $\therefore At$
- 3) $It \Rightarrow At$ 1, UI
- 4) $\therefore At$ 3, 2, M.P.

In this particular argument only one premise is general. However, the argument may consist of only general proposition in which case slightly different procedure has to be followed. Consider this argument.

2.

- 1) All politicians are voters.
- 2) All ministers are politicians.
- 3) $\therefore$ All ministers are voters.

When symbolized it becomes:

- 1) $(x) \{Px \Rightarrow Vx\}$
- 2) $(x) \{Mx \Rightarrow Px\} / \therefore (x) \{Mx \Rightarrow Vx\}$

The formal proof is as follows:

- 1) $(x) \{Px \Rightarrow Vx\}$
- 2) $(x) \{Mx \Rightarrow (Px)\} / \therefore (x) \{Mx \Rightarrow Vx\}$
- 3) $Pa \Rightarrow Va$ 1, UI
- 4) $Ma \Rightarrow Pa$ 2, UI
- 5) $Ma \Rightarrow Va$ 4, 3, H.S.
- 6) $\therefore (x) \{Mx \Rightarrow Vx\}$ 5, UG

When the individual variable x is instantiated by any constant, then quantifier goes and we do not quantify individual or individuals. Now coming to the 6th step, it may be mentioned that if one substitution instance is true for a given structure then all substitution instances must be true for that structure. Further the universal quantification of a propositional function is true if and only if all substitution instances are true. (The 6th line is not a part of the proof).

In the third and the fourth steps we have applied universal instantiation because both premises are universal and therefore we have substituted the constants for variables.

UG can be applied in the following manner. Add the sixth line to the proof system after we replace x by y at all stages. Then we have the application of UG

- 1) $(x)\{Px \Rightarrow Vx\}$
- 2) $(x)\{Mx \Rightarrow Px\} / \therefore (x)\{Mx \Rightarrow Vx\}$
- 3) $P_y \Rightarrow V_y$ 1, UI
- 4) $M_y \Rightarrow P_y$ 2, UI
- 5) $M_y \Rightarrow V_y$ 3, 4, H.S.
- 6) $\therefore (x)\{Mx \Rightarrow Vx\}$ 5, UG

These two examples suggest that while testing the validity of arguments in general, UI has to be used necessarily though EI may not be necessary. The situation is similar to the traditional formation of rules of syllogism, which hint that without particular propositions it is possible to construct an argument, but not without universal propositions.

Now consider an argument, which has a particular proposition. Since one proposition is particular, it is imperative that the conclusion must be particular.

3.

- 1) All politicians are voters.
- 2) Some ministers are politicians.
- $\therefore$ Some ministers are voters.

By now the method of symbolization should be familiar.

- 1) $(x)\{Px \Rightarrow Vx\}$
- 2) $(\exists x)\{Mx \wedge Px\} / \therefore (\exists x)\{Mx \wedge Vx\}$
- 3) $Ma \wedge Pa$ 2, E.I.
- 4) $Pa \Rightarrow Va$ 1, UI
- 5) $Pa \wedge Ma$ 3, Com.
- 6) Pa 5, Simp.
- 7) Ma 5, Simp.
- 8) Va 4, 6, M.P.
- 9) $Ma \wedge Va$ 7, 8, Conj.
- 10) $\therefore (\exists x)(Mx \wedge Vx)$ 9, UG

Let us examine why the restriction of EI must be honoured. Consider a fallacious argument.

- 1) Some animals are herbivorous.
- 2) Some animals are men.
- $\therefore$ Some men are herbivorous.

When symbolized the argument becomes:

- 1) $(\exists x)\{Ax \wedge Hx\}$
- 2) $(\exists x)\{Ax \wedge Mx\} / \therefore (\exists x)(Mx \wedge Hx)$
- 3) $Aa \wedge Ha$ 1, E.I.
- 4) $Aa \wedge Ma$ 2, E.I. (Error)

4th Step is erroneous. The second premise tells us that there is at least one thing that is both an animal and herbivorous. It does not permit us to conclude it should also be regarded as man. Therefore a second use of EI leads to error.

Check Your Progress III

Note: Use the space provided for your answers.

Employ proper method to remove the quantifiers in the following statements.

1. $(x)(Hx \Rightarrow Mx)$

2. $(x)(Ax \Rightarrow \neg Lx)$

2.8 LET US SUM UP

In this unit we have presented a new set of rules called quantification rules which supplement the existing rules. UI, UG, EI and EG are these rules. We learnt the art of testing arguments which consist of singular and general propositions and also the art of translating general statements to truth-functionally compound statements. Rules of quantifier equivalence are also discussed. These rules will allow us to replace a quantified expression by its equivalent expression whenever there is need. Application of Rules of Instantiation and Generalization were discussed. Contradiction and conversion were presented in terms of the theory of quantification. Syllogism is another type of argument tested within the frame-work of this theory.

2.9 KEY WORDS

Quantified statement: A statement which does it refer to a particular person or a object. It refers to the quantity or magnitude of subject term.

Scope of a quantifier: The extent of the interpretive power of a quantifier.

Rules of Quantifier Equivalence: It is between A, E, I, O, which follow from the Square of opposition, but phrased as quantified expressions and their negations in predicate logic.

Bound variable: If the variable is either part of the quantifier or lies within the scope of a quantifier, then it is called bound variable.

Free variable: Any variable is free if and only if the variable is not bound.

2.10 FURTHER READINGS AND REFERENCES

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UNIT 3 PROOFS OF VALIDITY

Contents

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3.0 OBJECTIVES

The main objective of this unit is to give a perspective of the technique of proving validity. Evidently, this is in continuation of what we learnt in the previous unit. In order to reinforce what we have already learnt, arguments in verbal form are considered in detail in this unit in addition to symbolic expressions. Application of the technique of proving validity to arguments in verbal form is the second objective of this unit. At the end of this unit we should be able to apply the technique of proving validity to arguments in natural language. Secondly, we must learn to compare and evaluate different methods of proof of validity in terms of merits and demerits

3.1 INTRODUCTION

There are several ways of testing arguments. Every method is unique in the sense that it has its own advantages and disadvantages. Disadvantage means only limitation. Simplicity, economy, etc. are only matter of degree. Even when we consider complex methods, they have certain advantages. One such advantage is the development of insight into the inner structure of statements which are often hidden. Second most important advantage is that our ability to reason is sharpened. In philosophical discourse these two factors play decisive role. Therefore it is imperative that we explore all possible and plausible avenues to test the validity of arguments. Against this background, we must understand that the proliferation of plurality of methods of testing is imperative.

One such method is natural deduction method. Natural deduction method is also known as derivation by substitution or formal proof of validity. Formal proof of validity is relevant while proving the validity of a given argument. This is also an example for direct proof wherein we derive a given conclusion from a set of premises by using such accepted rules, such as Rules of Inference and Equivalence or Replacement. Arguments expressed in natural language can be subjected to the formal proof of validity when translated to symbolic language. In this context we should consider what is known as *decision procedure*. Decision procedure means the way of testing an expression in order to know whether it is a tautology or not. We say that an expression

is a tautology when it is true in all instances. If an expression is false in all instances, then it is said to be a contradiction. Lastly, if an expression is true in some instances and false in one or more than one instance, then it is said to be contingent. An expression is nothing but an argument or argument form expressed in the form of a single statement which is necessarily a compound statement. Consider this argument and corresponding expression to understand what we mean by an expression and also what tautology, contradiction and contingent mean in terms of truth-value.

$$p \Rightarrow q$$

$$p / \therefore q$$

The argument is in accepted form. This is translated to the form of an expression in this manner.

$$\{(p \Rightarrow q) \wedge p\} \Rightarrow q$$

Now we will construct truth-table for the aforementioned expression in the following manner.

p	$\neg p$	q	$\neg q$	$\{(p \Rightarrow q) \wedge p\}$	$\Rightarrow$	q
1	0	1	0	1	1	1
1	0	0	1	0	1	1
0	1	1	0	0	1	1
0	1	0	1	0	1	0

How do we know that this expression is tautological? This is true in all instances. The truth-value in the main column is the standard of determination. Consider this argument.

$$p \Rightarrow q$$

$$\therefore p \wedge \neg q$$

When we put it in the form of an expression, it becomes

$$\{(p \Rightarrow q) \wedge (p \wedge \neg q)\}$$

We can construct truth-table using the same method (which is left as an exercise for the student) to find out that this expression is a contradiction. Likewise, the argument

$$p \Rightarrow q$$

$$\neg p$$

$$\therefore \neg q$$

becomes $\{(p \Rightarrow q) \wedge (\neg p \wedge \neg q)\}$ in the form of an expression and the construction of truth-table shows that it is contingent because it is true in some instances only.

Why do we need an alternate method? We already know that if there are two variables, we will have four rows according to the formula 2^n where n is the number of variables. Suppose that there are six variables. Then the number of rows is $2^6 = 64$. Evidently, construction of truth-table in this case is cumbersome. Naturally, we do not attempt to prove that the given expression is tautological using truth-table method. Therefore we search for alternate methods which are shorter. Only trial-and-error method helps us to know which is shorter and which is not. Rules of quantification help in discovering one such short method.

One important aspect to be remembered is that all valid arguments are tautological. Therefore when such an argument is put in the form of an expression, it must be true in all instances.

3.2 QUANTIFICATION AND EQUIVALENCE RELATION

In the previous unit we examined conversion. The same discussion is now extended to cover another important relation. Obversion is the one to be examined now.

1. All players are well-paid.

$\therefore$ No players are nonwell-paid.

As usual, transform this argument to symbolic form.

1. $(x) \{Px \Rightarrow Wx\}$

2. $\therefore (x) \{Px \Rightarrow \neg(\neg Wx)\}$

The negation within parentheses symbolizes the complement of predicate whereas the negation which precedes the parentheses symbolizes the quality of the conclusion. Now apply UI to the symbolic form.

3. $Pa \Rightarrow Wa$ 1, UI

4. $Pa \Rightarrow \neg(\neg Wa)$ 3, DN

4 corresponds to the conclusion of 1. Applying UG we get

5. $\therefore (x) \{Px \Rightarrow \neg(\neg Wx)\}$

Let us recall the method of obtaining contraposition. In the case of 'A' we convert obversion. In the language of modern logic it means that we have to commute double negation because double negation is another word for obversion. Now Commute 4.

6. $(\neg Wa) \Rightarrow \neg(Pa)$ 4, Com.

6 is partial contrapositive of 1. In natural language 6 means 'No nonwell-paid persons are players'. Next steps is self-explanatory.

$$7. (\neg Wa) \Rightarrow (\neg Pa)$$

7 is full contrapositive of 1 in virtue of the law of contradiction, the only survivor among opposition relations. If an element does not belong to a set, it ought to belong to its complement. In natural language it becomes 'All nonwell-paid persons are nonplayers'. The process of obtaining the conclusion stops at this point since 'A' does not have conversion by limitation. Using the same technique, obversion and other relations for the remaining propositions can be determined by the student. It must be remembered that according to the stipulations laid down by modern logic, inversion, whether partial or full, is invalid since it is always particular whereas the premises (A or E) are universal.

Check Your Progress I

Note: Use the space provided for your answer.

1. Explain formal proof method.

2. Examine the following arguments with the help of quantification rules.

a) Some women are not sages.

Therefore some non sages are not non women.

b) Some scientists are mathematicians.

Therefore some mathematicians are not nonscientists.

3.3 RULES OF QUANTIFICATION AND NONSYLLOGISM

All arguments need not be syllogistic though they consist of two premises and a conclusion. Relational argument is one such example.

1. Bangalore is to the west of Chennai.
Mangalore is to the west of Bangalore.

∴ Mangalore is to the west of Chennai.

Aristotelian system does not regard this class of arguments as syllogistic though this can be shown to be valid if the structure of propositions are modified, but it results in the distortion of the meaning of statements. If we try to retain the meaning, then it becomes impossible to demonstrate the validity or invalidity, as the case may be, of such argument.

Apart from relational arguments, there is another class of arguments which consists of more than three terms and propositions. Consider this argument.

- | | | |
|------------------------------------|---|---|
| 1 | 2 | 3 |
| Men are both stupid and dishonest. | | |
| | 4 | |
| Some men are irritable. | | |

∴ Some dishonest persons are irritable.

Terms are numbered. So there is no confusion regarding the nonsyllogistic nature of arguments. However, the statements are misleading. If we regard a conjunctive proposition as one proposition, then in this argument there are three propositions. If we give priority to simple propositions then the first premise has two simple propositions. Then we will have four propositions. Therefore this type of argument is classified as nonsyllogistic. To test this kind of argument we do not require any additional rule. Proper symbolization of this class of argument is important. The symbolization is as follows:

- | | |
|---|-------------|
| 1. $(x) [Mx \Rightarrow (Sx \wedge Dx)]$ | |
| 2. $(\exists x) [Mx \wedge Ix] / \therefore (\exists x) (Ix \wedge Sx)$ | |
| 3. $[Ma \wedge Ia]$ | 2, E. I. |
| 4. $Ma \Rightarrow (Sa \wedge Da)$ | 1, U. I. |
| 5. Ma | 3, Simp. |
| 6. $(Sa \wedge Da)$ | 4, 5, M. P. |
| 7. Sa | 6, Simp. |
| 8. Ia | 3, Simp. |
| 9. $Ia \wedge Sa$ | 8, 7, Conj. |
| 10. $(\exists x) (Ix \wedge Sx)$ | 9, E.G. |

The status of (1) calls for our attention. Had the first premise been regarded as a conjunctive proposition, then (1) ought to have been symbolized as

11. $Sm \wedge Dm$

It is a well known fact that conjunction does not have any equivalent form. Therefore (1) is not equivalent to (11), which means that we have arrived at a proposition very different from the first premise.

Consider another statement, which has a very different structure.

Americans and Germans are pioneers in science.

This statement actually means that a pioneer in science may be an American or a German. Surely, it does not mean that a pioneer in science is both an American and a German. Hence

when this innocuous statement is translated into logical language, it becomes a disjunctive proposition with exclusive 'Or'. Nor is it a conjunctive proposition of the form

Americans are pioneers in science and Germans are pioneers in science.

This is so because a conjunctive proposition of this form means the same as saying that a pioneer in science is both an American and a German, which is absurd. Consider this argument:

Americans and Germans are scientists.

Some white men are Americans.

Therefore, some white men are scientists.

This argument is symbolized as follows:

- | | |
|--|---|
| 1. $(x) [(Ax \vee Gx) \Rightarrow Sx]$ | |
| 2. $(\exists x) [Wx \wedge Ax]$ | / $\therefore (\exists x) [Wx \wedge Sx]$ |
| 3. $Wa \wedge Aa$ | 2, E.I. |
| 4. Aa | 3, Simp. |
| 5. $(Aa \vee Ga)$ | 4, Add. |
| 6. $(Aa \vee Ga) \Rightarrow Sa$ | 1, U.I. |
| 7. Sa | 6, 5, M.P. |
| 8. Wa | 3, Simp. |
| 9. $Wa \wedge Sa$ | 8, 7, Conj. |
| 10 $(\exists x) [Wx \wedge Sx]$ | 9, E.G. |

In one particular sense, nonsyllogistic arguments are more significant than traditional syllogism for the simple reason that in any debate, whether based in science or politics syllogism is seldom used. Application of nonsyllogistic arguments is widespread and more useful. Therefore there is greater need to become familiar with nonsyllogistic arguments.

3.4 EXERCISES

I. Construct formal proofs of validity.

1. $(x) [Qx \Rightarrow Rx]$
 $(\exists x) (Qx)$
 $\therefore (\exists x) Rx$
2. $(x) [Sx \Rightarrow (Tx \Rightarrow Ux)]$

$$\begin{aligned} & (x)[Ux \Rightarrow (Vx \wedge Wx)] \\ \therefore (x) [Sx \Rightarrow (Tx \Rightarrow (Vx \wedge Wx))] \end{aligned}$$

$$\begin{aligned} 3. \quad & (x)[Dx \Rightarrow \neg Ex] \\ & (x)[Fx \Rightarrow Ex] \\ \therefore (x) [Fx \Rightarrow \neg Dx] \end{aligned}$$

$$\begin{aligned} 4. \quad & (\exists x) [Jx \wedge Kx] \\ & (x) [Jx \Rightarrow Lx] \\ \therefore (\exists x) [Lx \wedge Kx] \end{aligned}$$

$$\begin{aligned} 5. \quad & (x) [(Ix \Rightarrow Ax)] \\ & (\exists x) [(Px \wedge Ix)] \\ \therefore (\exists x) [(Px \wedge Ax)] \end{aligned}$$

$$\begin{aligned} 6. \quad & (x) [(Mx \Rightarrow Nx)] \\ & (\exists x) [(Mx \wedge Ox)] \\ \therefore (\exists x) [(Nx \wedge Ox)] \end{aligned}$$

$$\begin{aligned} 7. \quad & (x) [(Dx \Rightarrow Cx)] \\ & (\exists x)[(Ax \wedge Dx)] \\ \therefore (\exists x)[(Ax \wedge Cx)] \end{aligned}$$

$$\begin{aligned} 8. \quad & (\forall x) [(Px \Rightarrow Qx)] \\ & (\exists x) [(Rx \wedge Px)] \\ \therefore (\exists x) [(Rx \wedge Qx)] \end{aligned}$$

$$\begin{aligned} 9. \quad & (\forall x) [(Tx \Rightarrow Ux)] \\ & \neg Ut \\ \therefore \neg Tt \end{aligned}$$

$$\begin{aligned} 10. \quad & (\forall x) [(Dx \Rightarrow Cx)] \\ & (\forall x) [(Ex \Rightarrow \neg Cx)] \\ \therefore (\forall x) [(Dx \Rightarrow \neg Ex)] \end{aligned}$$

$$\begin{aligned} 11. \quad & (\exists x) [(Hx \Rightarrow Ix) \wedge (Jx \Rightarrow Kx)] \\ & (\forall x) [(Ix \wedge Kx) \Rightarrow Lx] \\ & \neg Lx \\ \therefore (\exists x) (\neg Hx \vee \neg Jx) \end{aligned}$$

$$\begin{aligned} 12. \quad & (\forall x) (\neg Qx \Rightarrow \neg Px) \\ & (\exists x) (Px \wedge Rx) \\ \therefore (\exists x) (Qx \wedge Rx) \end{aligned}$$

$$13. \quad (\forall x) (\neg Px \Rightarrow \neg Qx)$$

$$\neg Px$$

$$\therefore \neg Qx$$

$$\begin{aligned} 14. \quad & (\forall x) (\neg Ax \Rightarrow \neg Bx) \\ & (\forall x) (Ex \Rightarrow \neg Ax) \\ & \therefore (\forall x) (\neg Ex \Rightarrow \neg Bx) \end{aligned}$$

$$\begin{aligned} 15. \quad & (\exists x) [(\neg Ax \Rightarrow \neg Bx) \wedge (\neg Kx \Rightarrow \neg Jx)] \\ & (\forall x) [\neg Lx \Rightarrow \neg (Kx \wedge Ax)] \\ & \neg Lx \\ & \therefore (\exists x) (\neg Bx \vee \neg Jx) \end{aligned}$$

$$\begin{aligned} 16. \quad & (\forall x) [(Qx \Rightarrow Px)] \\ & (\exists x) [(\neg Px \wedge \neg Rx)] \\ & \therefore (\exists x) [(\neg Qx \wedge \neg Rx)] \end{aligned}$$

Check Your Progress II

Note: a) Use the space provided for your answer.

Apply quantification rules to test the validity of the following syllogistic arguments which were explained in the previous unit.

1. All fish swim.

No swimming creatures are mammals.

Therefore no mammals are fish.

2. Some historians are Marxists.

All historians are women.

Therefore some women are Marxists.

3. No birds are harmful.

All cheats are harmful.

Therefore no birds are cheats.

3.5 MULTIPLY GENERAL PROPOSITIONS

There are two types of general proposition; singly general and multiply general. If a general proposition has only one quantifier, then it is called singly general. Up till now, we considered only propositions of former kind. If a general proposition consists of two or more than two quantifiers, then such a proposition is called multiply general propositions. Consider, for example, this proposition:

“If all Indians play cricket, then there are at least some Asians who play cricket.”

Its symbolization is as follows:

- | | | |
|----|--|-------------------------------|
| 1) | All Indians play cricket: | $(x)\{Ix \Rightarrow Px\}$ |
| 2) | There are at least some Asians who play cricket: | $(\exists x)\{Ax \wedge Px\}$ |

Now the symbolization of the whole sentence is as follows:

$$(x)\{Ix \Rightarrow Px\} \Rightarrow (\exists x)\{Ax \wedge Px\}$$

Depending upon the complexity of the given statement quantifiers may occur any number of times.

3.6 THE STRENGTHENED RULE OF C.P. AND QUANTIFICATION

In an earlier unit, we learnt that assumption is different from C. P. and that assumption does not include the conclusion. It depends solely on the premise. A few examples will illustrate how an argument can be tested using these techniques.

- | | | |
|----|----|--|
| 1. | 1) | (x)[Cx $\Rightarrow$ Dx] |
| | 2) | (x)[Ex $\Rightarrow$ $\neg$ Dx] |
| | | $\therefore$ (x)[Ex $\Rightarrow$ $\neg$ Cx] |

The argument is written in standard form;

- | | | | | |
|--|----|---------------------------------|-------|--|
| | 1) | (x)[Cx $\Rightarrow$ Dx] | | |
| | 2) | (x)[Ex $\Rightarrow$ $\neg$ Dx] | / | $\therefore$ (x)[Ex $\Rightarrow$ $\neg$ Dx] |
| | 3) | Cy | | |
| | 4) | Cy $\Rightarrow$ Dy | 1, | U.I. |
| | 5) | Ey $\Rightarrow$ $\neg$ Dy | 2, | U.I. |
| | 6) | Dy | 4, 3, | M.P. |
| | 7) | $\neg$ Ey | 5, 6, | M.T. |
| | 8) | Dy $\Rightarrow$ $\neg$ Ey | 6, 7, | C.P. |

$$9) \quad (x)[Ex \Rightarrow \neg x] \quad 9, \quad \text{U.G.}$$

From (1) two aspects become clear. The limit of assumption ends, when CP is used. So it does not depend upon the assumption. Second, since we are making an assumption, in place of 'x' only 'y'; an arbitrary chosen symbol can be used. This explanation holds good whenever the strengthened rule of CP is used.

$$\begin{array}{llll}
 2. & 1) & (x)[Nx \Rightarrow Ox] & \\
 & 2) & (x)[Px \Rightarrow \neg Ox] & / \therefore (x)(Nx \wedge \neg Px) \Rightarrow Ox \\
 \rightarrow & 3) & Ny & \\
 & 4) & Ny \Rightarrow Oy & 1, \quad \text{U.I.} \\
 & 5) & Py \Rightarrow \neg Oy & 2, \quad \text{U.I.} \\
 & 6) & Oy & 4, 3, \quad \text{M.P.} \\
 & 7) & \neg Py & 5, 6, \quad \text{M.T.} \\
 & 8) & Ny \wedge \neg Py & 3, 8, \quad \text{Conj.} \\
 & 9) & (Ny \wedge \neg Px) \Rightarrow Oy & 8, 6, \quad \text{C.P.} \\
 & 10) & \therefore (x)(Nx \wedge \neg Px) \Rightarrow Ox & \text{U.G.}
 \end{array}$$

Check Your Progress III

Note: Use the space provided for your answer.

Using the method of derivation by substitution prove the following arguments.

$$\begin{array}{l}
 1 \quad (\exists x) [(\neg Hx \Rightarrow Ix) \wedge (Mx \Rightarrow Rx)] \\
 \quad (\forall x) [(Ix \wedge Rx) \Rightarrow Lx] \\
 \quad \neg Lx \quad / \therefore (\exists x) \neg (\neg Hx \wedge \neg Mx)
 \end{array}$$

$$\begin{array}{l}
 2 \quad (\exists x) [(\neg Ax \Rightarrow Bx) \wedge (Cx \Rightarrow Dx)] \\
 \quad (\forall x) [(Bx \wedge Dx) \Rightarrow \neg Lx] \\
 \quad Lx \quad / \therefore (\exists x) \neg (\neg Ax \wedge Cx)
 \end{array}$$

3.7 LET US SUM UP

The most important vehicle for a formal system is its component for ‘derivation’ or ‘proof’. The derivation procedure that looked into this unit is called formal derivation because it relies on the valid argument forms, and not on the invalid arguments. On the whole, function of formal derivation is to infer the logical consequences from the premises. It uses the valid argument forms as the logical rules to determine which consequences can be correctly or validly drawn from the premises. The logical rules, being valid argument forms themselves, have a special quality. They are all truth-preserving.

3.8 KEY WORDS

Formal derivation: Deductive proof procedures that establish the validity of an argument by inferring the logical consequences from given premises by valid argument forms.

Nonsyllogism: It is a kind of mediate inference which consists of propositions with complex structure.

3.9 FURTHER READINGS AND REFERENCES

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- 4.0 Objectives
- 4.1 Introduction
- 4.2 Methods of Proving Invalidity - 1
- 4.3 Methods of proving Invalidity - 2
- 4.4 Exercises
- 4.5 Let Us Sum Up
- 4.6 Key Words
- 4.7 Further Readings and References

In our study of logic this is the second unit dealing with the role played by the technique of proving the invalidity of arguments. Therefore the objective of this unit is a repetition of the earlier unit on proving invalidity. However, there is no harm in reiterating our earlier goal. Proving invalidity is significant not in negative sense, but in positive sense. The singular objective of this unit is very clear. If we know what is wrong, we will know what is right in the *right* sense of the word and we will avoid consciously the pitfalls of illogical ways of arguing. Otherwise, we may walk into the trap of fallacies. Thus by the end of this unit one should be in a position to establish the invalidity of seemingly valid arguments and also to identify the difference between proving tautology and proving invalidity.

Building up of a consistent proof system is an efficient mode to show that an argument is valid. Suppose that an argument happens to be invalid. Then it is not possible to construct proof of its validity using any rule applied so far. The reason is simple. There are rules to govern the *right* path, but there are no rules to govern the wrong paths. This maxim holds good in all walks of life. Surely, logic is no exception. Therefore when compared with the technique of proving validity, the technique of determining or proving invalidity turns out to be somewhat indirect. The difference is more pronounced when we compare this method with the methods of proving validity or tautology. We became familiar with one aspect of proving invalidity when in the previous unit we established the invalidity of arguments consisting of truth-functionally compound statements. We accomplished this task by assigning the truth-values to those simple statements, which constitute the structure of respective truth-functionally compound propositions, in such a way as to make their premises true and their conclusions false. We use exactly the same method to prove the invalidity of arguments involving quantifiers with an additional assumption which becomes clear shortly.

4.2 METHOD OF PROVING INVALIDITY -1

Before we embark upon our task we have to be sure of the worthiness of our starting point and also very clear about what that starting point amounts to. Otherwise, we may go astray. The foundation of quantification logic is characterized by the assumption that we are dealing with nonempty sets. This turns out to be the model for which the arguments which we are testing must conform. In other words, the presupposition means that there is *at least* one individual. Again, it must be pointed out, and there is a good deal of value in repeating what was pointed out earlier, that there is a difference between saying *at least* one and saying there is *only* one.... 'At least one' surely does not exclude (it does not mean that it includes) the possibility of two or more than two members whereas the latter prohibits. Our study of the technique of proving invalidity is based upon the assumption that there are two or more than two members in the universe of discourse though we begin with the assumption that there is exactly one member.

So far, we were concerned with the assumption which we have made. Now we should turn to the basic logical principle which has to be followed. In the previous unit on proving invalidity we dealt with truth-functionally compound propositions which constituted the arguments. Quantification logic deals with arguments consisting of general and singular statements to which familiar Rules cannot be applied directly. Hence there is need to try a different method. The logical principle involved here is that there is equivalence between general propositions which are non-compound and truth-functional singular propositions. These singular propositions can be said to be the units or constituents of non-compound general propositions. The equivalence between these kinds can be established very easily when we use the principles of truth-functional logic. There is a certain method of establishing the same. Suppose that there is only one member, viz., 'a' or two members, a and b in the nonempty universe of discourse. Then we do not have any reason to apply the rule UI. Therefore this technique is independent of the rules of quantification. This is an important aspect to remember. In such a case we get the following result.

$$\begin{aligned} 1 \quad & (x) (\Phi x) \equiv \Phi a \\ 2 \quad & (\exists x) (\Phi x) \equiv \Phi a \end{aligned}$$

It is obvious that in a nonempty universe of discourse with exactly one member, the proposition with universal quantifier is logically equivalent to proposition with existential quantifier. Therefore the following equation holds good.

$$3 \quad (x) (\Phi x) \equiv \Phi a \equiv (\exists x) (\Phi x)$$

However, the result alters when we consider two members, say a and b. Then proposition with universal quantifier takes this form.

$$4. \quad (x) (\Phi x) \equiv [\Phi a \wedge \Phi b]$$

$$5. \quad (\exists x) (\Phi x) \equiv [\Phi a \vee \Phi b]$$

It is not possible to say that $1 \equiv 2$ without further evidence. 4 is true only when Φa is true and also Φb is true. However, for 5 to be true it is sufficient if at least one of the two components on

R.H. S. is true. We shall generalize our conclusion. If there are 'n' members, then the respective equations become

$$6. (x) (\Phi x) \equiv [\Phi a \wedge \Phi b \wedge \Phi c \wedge \dots \Phi n]$$

$$7. (\exists x) (\Phi x) \equiv [\Phi a \vee \Phi b \vee \Phi c \dots \Phi n]$$

It is obvious that every equation is a biconditional proposition. Therefore both sides of the equation must have the same truth-value. Since we are dealing with non-empty universe of discourse, 0 cannot be the value assigned to any side. From this stage onwards we restrict ourselves to the method of assigning truth-values.

Now our task is very much simplified. An argument involving quantifiers must possess such a structure that after assigning 0 to the conclusion it must be possible to assign 1 to every premise. We must remember that the conjunction of an indefinite number of true premises with just one false premise produces a false conjunction. Therefore in an invalid argument it is imperative that all premises are true while the conclusion is false. This is the first step in testing the invalidity of arguments. In the next step we prove the invalidity of a given argument by displaying or describing a model in which the given argument is logically equivalent to an invalid truth-functional argument. This is achieved by translating the given argument involving quantifiers to a logically equivalent argument involving only singular propositions and truth-functional compounds of them, and then using the method of assigning truth-values to prove that the argument is invalid. This is the actual method of proving the invalidity of arguments. We shall examine a few arguments to become familiar with this technique. Consider this argument:

1.

All sharks are dangerous.

All lions are dangerous.

Therefore all sharks are lions.

We symbolize this argument as follows.

$$(x) (Sx \Rightarrow Dx)$$

$$(x) (Lx \Rightarrow Dx)$$

$$\therefore (x) (Sx \Rightarrow Lx)$$

In the case of a model containing exactly one individual, say, (a) the given argument is logically equivalent to:

$$(Sa \Rightarrow Da)$$

$$(La \Rightarrow Da)$$

$$\therefore (Sa \Rightarrow La)$$

Assign truth-values to the propositions beginning with the conclusion according to the norm. While assigning the 0 value to the conclusion we must remember that it should be assigned to the sentential connective.

$$(Sa \Rightarrow Da)$$

$$1 \quad 1 \quad 1$$

$$(La \Rightarrow Da)$$

$$0 \quad 1$$

$$\therefore (Sa \Rightarrow La)$$

$$1 \quad 0 \quad 0$$

Since the premises are true when the conclusion is false, the argument is invalid. In the second premise there is no reason to assign the value to the consequent since the premise is known to be true beforehand.

Now suppose that there are two members, say a and b, in the universe of discourse. Then the argument takes this form.

$$(Sa \Rightarrow Da) \wedge (Sb \Rightarrow Db)$$

$$(La \Rightarrow Da) \wedge (Lb \Rightarrow Db)$$

$$\therefore (Sa \Rightarrow La) \wedge (Sb \Rightarrow Lb)$$

Now assign the truth-values as we did in the previous case.

$$(Sa \Rightarrow Da) \wedge (Sb \Rightarrow Db)$$

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$(La \Rightarrow Da) \wedge (Lb \Rightarrow Db)$$

$$0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$\therefore (Sa \Rightarrow La) \wedge (Sb \Rightarrow Lb)$$

$$1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1$$

The argument turns out to be invalid according to this fixation of values. The student is advised to obtain the result when the second component of the conclusion takes 0.

One more example is considered.

2.

All snakes are animals.

All elephants are animals.

Therefore all snakes are elephants.

We symbolize the argument as follows.

$$\begin{aligned} & (Sa \Rightarrow Aa) \\ & (Ea \Rightarrow Aa) \\ \therefore & (Sa \Rightarrow Ea) \end{aligned}$$

When the truth-value is assigned to the logically equivalent form of this argument with exactly one member, the result is as follows.

$$\begin{aligned} & (Sa \Rightarrow Ua) \\ & \begin{array}{ccc} 1 & 1 & 1 \end{array} \\ & (Ea \Rightarrow Ua) \\ & \begin{array}{ccc} 0 & 1 & 1 \end{array} \\ \therefore & (Sa \Rightarrow Ea) \\ & \begin{array}{ccc} 1 & 0 & 0 \end{array} \end{aligned}$$

There is no need to consider a non-empty universe of discourse with two members. The method of testing is the same as the one for the first example.

Consider an example where two premises are universal and conclusion is particular.

3.

All flowers are attractive.

All attractive things are temporary.

Therefore some flowers are temporary.

Assuming that in non-empty universe of discourse there is only one member, we shall assign the truth-values. Let us assign 0 to Fa.

$$\begin{aligned} & (Fa \Rightarrow Aa) \\ & \begin{array}{ccc} 0 & 1 & 0 \end{array} \\ & (Aa \Rightarrow Ta) \\ & \begin{array}{ccc} 0 & & 1 \end{array} \\ \therefore & (Fa \wedge Ta) \\ & \begin{array}{ccc} 0 & & 0 \end{array} \end{aligned}$$

With this set of values the argument turns out to be invalid no matter what value is assigned to Ta in the conclusion and in the premise. Of course, if Aa takes 1, then Ta must take 1. We must remember that nowhere did we use any rule of quantification.

Let us consider a non-empty universe of discourse consisting of three individuals. This is being discussed only with the intention of reinforcing what we have learnt so far. Once we arrive at desired result, it is possible to safely assume that the result holds good for n number of members.

Suppose that there are only three men in the model of men, viz. a, b and c. In such a case the proposition 'A' can be represented in the following manner.

$$1. (x) (\Phi x) \equiv (\Phi a \wedge \Phi b \wedge \Phi c)$$

The LHS is true if and only if Φa is true, Φb is true and Φc is true. If any one of them is false, then the LHS is false. Similarly, the proposition 'E' becomes

$$2. (x) (\neg \Phi x) \equiv (\neg \Phi a \wedge \neg \Phi b \wedge \neg \Phi c)$$

If a, b and c are the only men in the model of men, then as in the previous case, in the present case also the LHS is true. This is because a conjunction is true (RHS) if and only if everyone of the three components is true. If any one of them is false then LHS also is false.

While the propositions with universal quantifiers are translated to the conjunction mode, those with existential quantifiers are translated to the disjunction mode. If we persist with the same model, then

$$3. (\exists x) (\Phi x) \equiv (\Phi a \vee \Phi b \vee \Phi c)$$

$$4. (\exists x) (\neg \Phi x) \equiv (\neg \Phi a \vee \neg \Phi b \vee \neg \Phi c)$$

These four equations are in the form of biconditional proposition and a biconditional proposition is true only when both sides of the equation have the same truth-value. From these four equations, it is clear that the truth-status of propositions with quantifiers is determined by the truth-conditions of corresponding compound proposition. This relation is in perfect consonance with the definition of universal and existential quantifiers. We always assume that the propositions are true as long as we are dealing with the set of premises. Therefore both sides of the equations representing the premises must necessarily be true.

On logical grounds, there is a qualitative difference between a model containing only one individual and another model containing two or more than two individuals. For the sake of convenience let us call the first model monadic and the second one polyadic or n-adic model for n number of members. For example, if there are two individuals, then the model is dyadic and if there are three members, then triadic and so on. There is a qualitative difference because in a monadic model an invalid argument may correspond to a valid truth-functional argument whereas the very same argument in any other model may correspond to an invalid truth-functional argument. Let us consider an argument which is invalid from traditional angle.

4.

All politicians are lawyers.

All judges are lawyers.

$\therefore$ All judges are politicians.

1. $p1: (x) \{Px \Rightarrow Lx\}$

2. $p2: (x) \{Jx \Rightarrow Lx\}$

$\therefore (x) \{Jx \Rightarrow Px\}$

Since there is only member, this argument is logically equivalent to

3. $p1: \{Pa \Rightarrow La\}$

4. $p2: \{Ja \Rightarrow La\}$

$\therefore Ja \Rightarrow Pa$

In a monadic model $(x) (\Phi x) \equiv \Phi a \equiv (\exists x) (\Phi x)$

$\therefore$ The argument is logically equivalent to

5. $Pa \wedge La$

6. $Ja \wedge La$

$\therefore Ja \wedge Pa$

If we assign the value 0 to any one of the components of the conclusion, then not only the conclusion is false but also one of the premises becomes false. However, according to definition, the premises must be true. It is logically impossible to derive a false conclusion from true premises. Therefore the argument is valid. However, the same argument is invalid in a dyadic model.

We shall symbolize the previous argument.

1. $p1: (x) \{Px \Rightarrow Lx\}$

2. $p2: (x) \{Jx \Rightarrow Lx\}$

$\therefore (x) \{Jx \Rightarrow Px\}$

Since we are considering a dyadic model the symbolic presentation is logically equivalent to

3. $(Pa \Rightarrow La) \wedge (Pb \Rightarrow Lb)$

4. $(Ja \Rightarrow La) \wedge (Jb \Rightarrow Lb)$

$\therefore (Ja \Rightarrow Pa) \wedge (Jb \Rightarrow Pb)$

Assign 0 to Pa and 1 to the rest. The result can be computed as follows:

5. $(Pa \Rightarrow La) \wedge (Pb \Rightarrow Lb)$

0 1 1 1 1 1 1

6. $(Ja \Rightarrow La) \wedge (Jb \Rightarrow Lb)$

1 1 1 1 1 1 1

$\therefore (Ja \Rightarrow Pa) \wedge (Jb \Rightarrow Pb)$

1 0 0 0 1 1 1

The conjunction of the truth-values in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalized to include other polyadic

models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model. In order to become familiar with this method let us work with some more problems.

5. $(x) (Dx \Rightarrow \neg Ex)$
 $(x) (Ex \Rightarrow Fx)$
 $\therefore (x) (Fx \Rightarrow \neg Dx)$

Let us restrict this argument to a dyadic model. If this argument is invalid in this model, then it is invalid in all other polyadic models. The logically equivalent form of 3 is as follows.

1. $(Da \Rightarrow \neg Ea) \wedge (Db \Rightarrow \neg Eb)$
 2. $(Ea \Rightarrow Fa) \wedge (Eb \Rightarrow Fb)$
 $\therefore (Fa \Rightarrow \neg Da) \wedge (Fb \Rightarrow \neg Db)$

Assign 0 to $\neg Da$. Accordance to the law of contradiction $Da = 1$. Similarly, $\neg Db$ is assigned 0. Therefore $Db = 1$. Assign 1 to $\neg Ea$. Ea takes 0. Assign 1 to $\neg Eb$. Eb takes 0. Assign 1 to Fa and Fb . The result can be computed as follows.

3. $(Da \Rightarrow \neg Ea) \wedge (Db \Rightarrow \neg Eb)$
 1 1 1 1 1 1 1
 4. $(Ea \Rightarrow Fa) \wedge (Eb \Rightarrow Fb)$
 0 1 1 1 0 1 1
 $\therefore (Fa \Rightarrow \neg Da) \wedge (Fb \Rightarrow \neg Db)$
 1 0 0 0 1 0 0

In this argument also the conjunction of the truth-values in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalized to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

6. 1 $(\exists x) (Jx \wedge Kx)$
 2 $(\exists x) (Kx \wedge Lx)$
 $\therefore (\exists x) (Lx \wedge Jx)$

We shall consider this argument also in a dyadic model. This is logically equivalent to

- 3 $(Ja \wedge Ka) \vee (Jb \wedge Kb)$
 4 $(Ka \wedge La) \vee (Kb \wedge Lb)$
 $\therefore (La \wedge Ja) \vee (Lb \wedge Jb)$

There is a difference between this argument and the previous arguments. In this argument the premises and conclusion are disjunctive unlike the previous arguments which have conjunctive statements. The difference is due to quantifiers. In case of universal quantifiers conjunction is the connective whereas in case of existential quantifiers disjunction is the connective.

Assign the truth-values as follows; 0 to La and Jb and 1 to the rest. The result is computed as follows.

- 5 $(Ja \wedge Ka) \vee (Jb \wedge Kb)$

$$\begin{array}{cccccc}
 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
 6 & (Ka \wedge La) \vee (Kb \wedge Lb) \\
 1 & 0 & 0 & 1 & 1 & 1 & 1 \\
 \therefore & (La \wedge Ja) \vee (Lb \wedge Jb) \\
 0 & 0 & 1 & 0 & 1 & 0 & 0
 \end{array}$$

In this argument also the conjunction of the truth-values in 5 and 6 yields true premises whereas the conclusion is false. Hence the argument is invalid. This result can be generalized to include other polyadic models with 3 or more than 3 members. Whatever holds good to a dyadic model in this case also holds good to any other polyadic model.

4.3 METHOD OF PROVING INVALIDITY -2

We shall examine those immediate inferences which are regarded as valid by traditional logic, but invalid according to modern logic. Let us begin with conversion per limitation for a dyadic model.

7.

All women are pious.

Therefore some pious persons are women.

Symbolize the argument and assign truth-values:

$$(Wa \Rightarrow Pa) \wedge (Wb \Rightarrow Pb)$$

$$\begin{array}{cccccc}
 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
 \therefore & (Pa \wedge Wa) \wedge (Pb \Rightarrow Wb) \\
 1 & 1 & 1 & 0 & 1 & 0 & 0
 \end{array}$$

Consider O proposition.

8.

Some soldiers are not graduates.

Therefore some graduates are not soldiers.

Again, we consider this argument for a dyadic model. Then the rest follows.

$$\begin{array}{cccccc}
 (Sa \wedge \neg Ga) \wedge (Sb \wedge \neg Gb) \\
 1 & 1 & 1 & 1 & 1 & 1 & 1
 \end{array}$$

$$\therefore (Ga \wedge \neg Sa) \wedge (Gb \wedge \neg Sb)$$

$$0 \ 0 \ 0 \quad 0 \ 0 \ 0 \ 0$$

Consider opposition of relations. Let us begin with superaltern.

9.

All graduates are university educated.

Therefore some graduates are university educated.

we shall examine this argument for a dyadic model following the familiar norms.

$$(Ga \Rightarrow Ua) \wedge (Gb \Rightarrow Ub)$$

$$0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1$$

$$\therefore (Ga \wedge Ua) \wedge (Gb \wedge Ub)$$

$$0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1$$

The argument is invalid for a dyadic model. Therefore it is invalid for polyadic model.

There is no need to work out the status of subaltern relation. It is sufficient if the statements of 9 are reversed. This is left as an exercise for the student.

Contrary is the relation to be considered now.

10.

All swimmers are medalists.

$\therefore$ No swimmers are medalists.

Symbolize the statements and assign the truth-values as usual.

$$Sa \Rightarrow Ma$$

$$1 \ 1 \ 1$$

$$\therefore Sa \Rightarrow \neg Ma$$

$$1 \ 0 \ 0$$

Subcontrary relation is left out for the student as an exercise.

This method of proving invalidity in this manner is, perhaps, decisive since the invalidity of those arguments, which are held by traditional logic as valid, becomes clear from the last two examples.

Check Your Progress I

Note: Use the space provided for you answers.

1 Give examples to prove that subcontrary and subaltern relations are invalid.

2 Distinguish between monadic and dyadic models.

4.4 EXERCISES

Using the method of assigning truth-values prove that the following arguments are invalid.

1.

$$\begin{aligned} &(x) \{Ax \Rightarrow (Bx \vee Cx)\} \\ &(x) \{Dx \Rightarrow (Ex \vee Fx)\} \\ &(x) \{\neg Bx \Rightarrow (Fx \vee Gx)\} \\ &(\exists x) \{(Fx \Rightarrow Dx) \wedge (\neg Ex \Rightarrow \neg Dx)\} \\ &\quad \neg Gx \end{aligned}$$

$$\therefore (x) \{Ax \Rightarrow (Dx \vee Fx)\}$$

2.

$$\begin{aligned} &\{Aa \Rightarrow (Ba \vee Ca)\} \\ &\{Da \Rightarrow (Ea \vee Fa)\} \\ &\{Ba \Rightarrow (Fa \vee Ga)\} \\ &\{(Fa \Rightarrow Da) \wedge (\neg Ea \Rightarrow \neg Da)\} \\ &\quad \neg Ga \end{aligned}$$

$$\therefore \{Aa \Rightarrow (Da \vee Fa)\}$$

3.

$$\begin{aligned} &Aa \Rightarrow (Ba \vee Ca) \\ &Da \Rightarrow (Ea \vee Fa) \\ &\neg Ba \Rightarrow (Fa \vee Ga) \\ &(Fa \Rightarrow Da) \wedge (\neg Ea \Rightarrow \neg Da) \\ &\quad \neg Ga \end{aligned}$$

$$\therefore Aa \Rightarrow (Da \vee Fa)$$

4.

$$\begin{aligned}
 &(x)\{Px \Rightarrow (Qx \vee Rx)\} \\
 &(x)\{Sx \Rightarrow (Tx \vee Ux)\} \\
 &(x)\{\neg Qx \Rightarrow (Ux \vee Vx)\} \\
 &(\exists x)\{[(Ux \Rightarrow Sx) \wedge (\neg Tx \Rightarrow \neg Sx)] \\
 &\quad \neg Vx\}
 \end{aligned}$$

$$\therefore (x)\{Px \Rightarrow (Sx \vee Ux)\}$$

Check Your Progress II

Note: Use the space provided for your answer.

1. Using the technique of proving invalidity show that the following arguments are invalid:

- a) $Ix \vee (Kx \wedge Jx)$
 $\neg (Ix \vee Jx) \vee (Lx \equiv \neg Mx)$
 $\neg (Lx \Rightarrow \neg Mx) \vee (\neg Nx \wedge Mx)$
 $(Nx \Rightarrow Ox) \wedge (Ox \Rightarrow Mx)$
 $\neg (\neg Jx \vee Kx) \vee Ox \therefore Oa$
- b)
- $$\begin{aligned}
 &(Bx \wedge Cx) \vee Ax \\
 &(Ax \vee Cx) \Rightarrow (Lx \equiv \neg Mx) \\
 &(Lx \supset \neg Mx) \Rightarrow (\neg Nx \wedge Mx) \\
 &(Nx \Rightarrow Zx) \wedge (Zx \supset Mx) \\
 &(\neg Cx \vee Bx) \Rightarrow Ox \therefore Za \vee Kx
 \end{aligned}$$

4.5 LET US SUM UP

In this unit we have learnt about the significance of proving invalidity. The method followed is assigning of truth-values to the statements. We have also learnt that the conclusion must be always assigned the value 0 while all premises must invariably take the value 1. Some inferences held as valid in traditional logic are shown conclusively to be invalid with the help of this method. The importance of nonempty universe of discourse is demonstrated.

4.6 KEY WORDS

Nonempty universe of discourse or model: A universe of discourse with at least one member.

Monadic model: A model with exactly one member.

Polyadic or n-adic model: A model with an indefinite number of (n) members.

4.7 FURTHER READINGS AND REFERENCES

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UNIT 5 APPLICATIONS OF SYMBOLIC LOGIC

Contents

- 5.0.Objectives
 - 5.1.Introduction
 - 5.2.Application of Symbolic Logic with Digital World
 - 5.3.Booleen Algebra
 - 5.4.Logic Gates
 - 5.5.Role of symbolic logic in Multi – Value logic
 - 5.6.Application of Fuzzy Logic.
 - 5.7.Let Us Sum Up
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5.0. OBJECTIVES

This unit is to limelight the application of Symbolic Logic in the Modern Era. The instrumental value of logic is well known in many disciplines, such as Philosophy, Mathematics, and Computer Science. Mathematics teaches logic almost as an extension of algebra or calculus with lemmas and proofs. And also, Computer science teaches it with more emphasis on its applicability for enriching programming power or for building ‘thinking machines’. But Philosophy, of which logic has always been a fundamental part, approaches logic somewhat differently. For then its key concerns are different kinds. We present a range of binary systems suitable for representing information in digital components. The binary number system is explained and binary codes are illustrated to show the representation of decimal and alphanumeric information. We introduce the concepts of Boolean algebra from a fundamental point of view, co-relating with Symbolic Logic. The correlation between a Boolean expressions and its equivalent interconnection of gates is emphasized. At present, in the field of communication, entertainment, medical electronics, and digital electronics has taken giant strides. Here fundamental ideas about implementing a logic circuit for a logic expression or writing a logic expression for a given logic circuit are discussed.

All possible logic operations for two variables are investigated and from that, the most useful logic gates are derived. The characteristics of digital gates available in integrated circuit form are presented. This unit supplies the diagram and tabulation methods for simplifying Boolean functions. The diagram is used to simplify digital circuits constructed with AND, OR, NAND, NOR, and wired-logic gates. The chief endeavor of this chapter is to utter that the Digital logic is not based on numbers but they are based on the sentences. More specifically, it is based on the connectivity of the propositions. Primary purpose of this unit is to facilitate education in the increasingly important areas

of multi-value logic suitable for representing information in Fuzzy logic. The role of symbolic logic is decorated in the multi-value logic. Truth status of propositions is challenging and is not restricting the future events. The fundamental of fuzzy propositions is also discussed in this chapter.

5.1. INTRODUCTION

In the history of western logic, Symbolic logic is relatively recent development. It is the study of human thoughts through symbols. It is learning towards mathematics and symbolization. It would be a well high hopeless task to discuss modern considerations of logic by the use of only ordinary language. A symbolic language has become necessary in order to achieve the required exact scientific treatment of the subject. Because of the presence of such symbolism, the resulting treatment is known as symbolic logic.

5.2. APPLICATION OF SYMBOLIC LOGIC WITH DIGITAL LOGIC

The word ‘digit’ is resultant from the Latin term “*digitus*” which means finger or toe. The fact that this system has ten digits is commonly attributed to the ten fingers of a human being. The most commonly used number system is the decimal number system; it is composed of digits ‘0’ to ‘9’. Digital logic is concerned with the interconnection among digital components and modulus. Digital computer has made possible many scientific industrial and commercial advances that would not be attainable otherwise. Our space program would have been unfeasible without real-time, continuous computer monitoring and many business enterprises functions efficiently only with the aid of automatic data processing. Computers are used in scientific calculations, commercial and business data processing, air traffic control, space guidance, the educational field, and many other areas. A digital computer is a programmable machine that processes binary data which is represented by binary number system. The simplest number system employing positional notation is the binary system. A binary number system uses only two symbols or digits namely 0 and 1. They are the binary number system which has a base or radix of 2. A binary digit 0 and 1 are called “*bits*”. 4-bit binary word is called “*nibble*”, and 8-bit binary word is called “*byte*”.

Check Your Progress I

Note: Use the space provided for your answer

1. What do you mean by “*digitus*”? Explain.

2. Explain the application of Symbolic Logic in the digital world.

5.3. BOOLEAN ALGEBRA

George Boole urbanized Boolean algebra in 1847 and used it to crack in Mathematical logic or Symbolic logic. In 1938 Claude Shannon introduced a two-valued Boolean algebra called “Switching Algebra”, in which he demonstrated that the properties of bi-table electrical circuits can be represented by this algebra. Claude Shannon previously applied Boolean algebra to design of switching network in 1939. The values used in Boolean algebra are representing the symbols ‘0’ and ‘1’. They have no numeric values. A binary ‘1’ will represent a High level, and a binary ‘0’ will represent a Low level in Boolean Equations.

Basic Operation: Boolean algebra has only three operators AND ($\bullet$), OR ($+$) and NOT ($'$) or Complement or Inverse.

AND Operator: The logical operation of AND can be articulated with symbols as follows. Let one input variable is A, the other input variable is B and the output variable is C. Subsequently the Boolean expression of this basic operator function is $C = A \wedge B$ (or) $C = AB$. The table for the Boolean expression $C = A \wedge B$, is as follows.

A	B	$A \wedge B$	C
0	0	$0 \wedge 0$	0
0	1	$0 \wedge 1$	0
1	0	$1 \wedge 0$	0
1	1	$1 \wedge 1$	1

OR Operator: The logical operation of OR can be articulated with symbols as follows. Let one input variable is A, the other input variable is B and the output variable is C. In that case the Boolean expression of this basic operator function is $C = A + B$. The table for the Boolean expression $C = A + B$, is as follows.

A	B	$A + B$	C
0	0	$0 + 0$	0
0	1	$0 + 1$	1
1	0	$1 + 0$	1

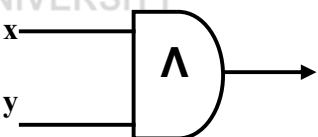
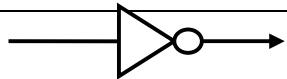
1	1	1+1	1
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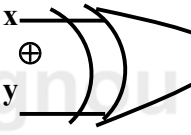
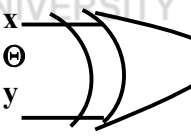
NOT Operator: The Complement of '0' is '1' and the Complement of '1' is '0'. Symbolically, we write $0' = 1$ and $1' = 0$. The logical operation of an inverter (NOT) can be expressed with symbols as follows: If the input variable is 'A' and the output variable is called X, then $X = A'$. This expression states that the output is the complement of input, so if $A=0$ then $X=1$ and $A=1$ then $X=0$. The table for the Boolean expression $A = X'$, is as follows.

A	X
0	1
1	0

5.4 LOGIC GATES

A logic gate is an electronic circuit, which takes numerous inputs and produces a single output. Logic gates form the fundamental building blocks for all the digital circuits. AND gate, OR gate and NOT gate are called "basic gates". NAND gate and NOR gate are called "Universal Gates". All the gates are obtainable in Integrated Circuit (IC) form. The different IC families differ in their speed, power dissipation, propagation delay, etc. There are eight functions to be considered as candidates for logic gates: AND, OR, NAND, NOR, XOR, XNOR, INVERTER, BUFFER. The graphic symbols and truth tables of the eight gates are show in below.

Name	Graphic Symbol	Algebraic Function	Truth Table	
AND		$f = x \cdot y$	x y	f
			0 0	0
			0 1	0
			1 0	0
			1 1	1
OR		$f = x + y$	x y	F
			0 0	0
			0 1	1
			1 0	1
			1 1	1
			x	F

INVERTER	x	$f = x'$	0 1	1 0
BUFFER	x 	$f = x$	x 0 1	F 0 1
NAND	x y 	$f = (xy)'$	$x \ y$ 0 0 0 1 1 0 1 1	F 1 1 1 0
NOR	x $+$ y 	$f = (x+y)'$	$x \ y$ 0 0 0 1 1 0 1 1	F 1 0 0 0
XOR	x $\oplus$ y 	$f = x \oplus y$	$x \ y$ 0 0 0 1 1 0 1 1	F 0 1 1 0
XNOR	x $\ominus$ y 	$f = x \ominus y$	$x \ y$ 0 0 0 1 1 0 1 1	F 1 0 0 1

Each gate has one or two inputs nominated by x , y , etc and the out put is designated by 'f'.

Modern Classification of Propositions and Digital Logic Gates: Modern logicians categorize propositions into three types. They are Simple, Compound and General. Compound proposition is classify further into:

1. Conjunctive
2. Implicative (Conditional)
3. Disjunctive
 - (a) Inclusive, (b) Exclusive
4. Equivalence (Bi- Conditional)
5. Negation

Conjunctive: A conjunctive proposition is a compound proposition containing two or more simple propositions, conjoined by the word 'and'. Two or more propositions so

conjoined are called conjuncts. 'Sankara is philosopher and Ramanuja is a philosopher' is one such conjunctive proposition. The symbol ' * ' or ' $\wedge$ ' (dot) is used to represent the function of conjunctive proposition. If we substitute the variable 'p' for 'Sankara is philosopher' and 'q' for 'Ramanuja is a philosopher' then the two conjuncts are symbolized as (p $\wedge$ q). The truth value and truth functions of these conjuncts will be as such:

p	$\wedge$	q	
T	1	T	
T	0	F	
F	0	T	
F	0	F	Equi - 1

Among the Logic gates, AND gate is one of the basic gates; its truth table is stated below and let the equitation be "2". The evaluation of this equation says that output 'f' of an AND gate is High (1) only both inputs are High (1)

p	$\wedge$	q	
0	0	0	
0	0	1	
1	0	0	
1	1	1	Equi - 2

This gate is compared with the equitation "1", Conjunctive proposition of the compound proposition in Symbolic Logic which states that when the antecedent and consequent of the proposition is True the validity of the whole proposition will be true, if not it is invalid. This equitation (1) is rewritten without affecting the truth values and truth-functions for our better understanding and is stated as equitation "3".

p	$\wedge$	q	
F	0	F	
F	0	T	
T	0	F	
T	1	T	Equi - 3

Let F=0, and T=1, substitute in Equitation "3"

p	$\wedge$	q	
0	0	0	
0	0	1	
1	0	0	
1	1	1	Equi - 4
$\Rightarrow$ Equi - 4 = Equi - 2			
$\Rightarrow$ [Equi - 4 = Equi - 3] = Equi - 1			
$\Rightarrow$ Equi - 4 = Equi - 1			
$\Rightarrow \therefore$ Equi - 1 = Equi - 2			

Equitation “1” interprets the basic truth table of a conjunctive proposition of a compound proposition containing two or more simple proposition, conjoined by the word ‘and’. The Equitation “2” just reveals us that output of an AND gate is High (1) only when both inputs are High (1). The above truth table specifies the out put values of every possible combination of values of the variables in the expression. In this way other gates can be proved.

Check Your Progress II

Note: Use the space provided for your answer

1. Define Logic Gate.

2. Write a short note on Boolean operators.

5.5 ROLE OF SYMBOLIC LOGIC IN MULTI – VALUE LOGIC

In 1936 K.Michalski discovered that three-valued logics had actually been anticipated as early as the 14th century by the medieval schoolman, William of Occam. The possibility of a three-valued logic has also been considered by the Philosophers Hegel and, in 1896, Hugh MacColl. It will be recalled that new geometries primary came about through the refutation of Euclid’s parallel postulate, and that new algebras first came about through the rejection of the commutative law of multiplication. Similarly, the new so-called “many-valued logics” first came about by denying Aristotle’s law of excluded middle.

In crisp logics, such as binary logic, variables are either true or false, black or white, 1 or 0. An extension to binary logic is multi-value logic, where variables may have many crisp values. In 1921, in a short two-page paper, J.Lukasiewicz considered a three-valued logic, or a logic in which proposition ‘p’ may possess any one of the three possible truth values. Very shortly after and independent of Lukasiewicz’s work, E.L.Post considered m-Valued logics, in which proposition ‘p’ may possess any one of ‘m’ possible truth values, where ‘m’ is greater than 1. If ‘m’ exceeds 2, the logic is said to be many-valued.

Bi-Value and Multi-Value Logic: We shall utilize the method of truth table, preliminary with a Basic truth table for conjunction. We, first of all, replicate the truth table for conjunction in the two-valued logic.

p	q		
	$\wedge$	T	F
	T	T	F
	F	F	F

Figure-1

Down the left-hand column emerge the possible truth values for proposition 'p' and across the top row show the possible truth values for the proposition 'q'. Now, knowing the truth value of 'p' and of 'q', one can find the truth value of ' $p \wedge q$ '. ' $p \wedge q$ ' is to be true when and only when both 'p' and 'q' are true, a T appear in the top left box of the table and F's come out in all the other boxes. We now ensue to the three-valued logic and again agree to take the conjunction ' $p \wedge q$ ' to be true when and only when both 'p' and 'q' are true. Denoting the three possible truth values of a proposition by T, ?, and F. We start to build a truth table.

p	q			
	$\wedge$	T	?	F
	T	T		
	?			
	F			

Figure-2

By our array ' $p \wedge q$ ', the top left box in the above table must contain as T, and no other box in the table is allowed to contain a T. Since there are eight remaining boxes and each may be filled in either of two possible ways, namely, with either F or ?, in sum $2^8 = 256$ possible ways of filling the eight boxes. It follows that there are 256 different ways of developing a truth table for conjunction in a three-valued logic. To illustrate two of the possible 256 truth tables for conjunction in a three-valued logic.

p	q			
	$\wedge$	T	?	F
	T	T	?	F

	?	?	?	F
	F	F	F	F

Figure-3a

In paradox: Let a proposition be 's' and the Negation of it is 'not-s', the proposition and its negation are same.

∴ "s=not-s" ----- Equi.1

The truth function of this Equitation is:

⇒ $t(s) = t(\text{not-s})$ ----- Equi.2

In a Bivalent Logic

⇒ $[t(s) = t(\text{not-s})] = 1$ ----- Equi.3

⇒ $t(\text{not-s}) = 1 - t(s)$ ----- Equi.4

Substitute Equi.2 in Equi.4

⇒ $t(s) = 1 - t(s)$ ----- Equi.5

⇒ If s= true, $t(s) = 1$ then $1=0$

⇒ If s= false, $t(s) = 0$ then $0=1$

Contradiction

When we test the same Equitation in Multi-value logic from Equi.5

⇒ $t(s) = 1 - t(s)$ ----- Equi.5

⇒ $t(s) + t(s) = 1$ ----- Equi.6

⇒ $2t(s) = 1$ ----- Equi.7

⇒ $t(s) = 1/2$ ----- Equi.8

The degree of truth lies in $\frac{1}{2}$ (Half-Truth)

∴ An extension to binary logic is multi-value logic.

Multi-Value Logic: The fundamental assumption, upon which classical logic (or two-valued logic) is based, that each proposition is either true or false – had been questioned since Aristotle. It is understood that propositions truth status is problematic are not restricted to future events. As an end result of the Heisenberg principle of uncertainty, for example, it is acknowledged that truth values of certain propositions in quantum mechanics are innately indetermined due to fundamental boundaries of measurements. In order to treat with such propositions, we must relax the true/false dichotomy of classical two-valued logic by allowing a third truth value, which may be called indetermined. The classical two-valued logic can be extended into three-valued logic in different ways. Several three-valued logics, each with its own rationale, are now well established. It is frequent in these logics to designate the truth, falsity, and indeterminacy by 1, 0, and $1/2$, respectively.

Once the range of three-valued logics was acknowledged as meaningful and useful, it became pleasing to explore generalization into n-valued logics for an arbitrary number of

truth values ($n \geq 2$). Several n -valued logics were, in fact, urbanized in the 1930s. For any given ' n ', the truth values in these generalized logics are usually labeled by rational numbers in the unit interval $[0,1]$. These values are obtained by uniformly dividing the interval between 0 and 1 exclusive. The set T_n of truth values of an n -valued logic is thus defined as:

$$T_n = \{0 = 0/n-1, 1/n-1, 2/n-1, \dots, n-2/n-1, n-1/n-1 = 1\}$$

These values can be interpreted as degrees of truth.

Fuzzy Logic: Fuzzy Sets as well as Fuzzy Logic is a factual magnum work. Fuzzy Logic addresses practically every significant topic in the broad expanse of fuzzy set theory. To us Fuzzy Sets and Fuzzy Logic is an astonishing achievement; it covers its immeasurable territory with impeccable authority, deep insight and a meticulous concentration to detail. To view Fuzzy Sets along with Fuzzy Logic in an appropriate perspective, it is compulsory to clarify a point of semantics which relates to the meanings of fuzzy sets and fuzzy logic. More exclusively, in a broad sense, fuzzy logic is a logical system which is an extension and generalization of classical multi-valued logics. However in a wider sense, fuzzy logic is almost identical with the theory of fuzzy sets.

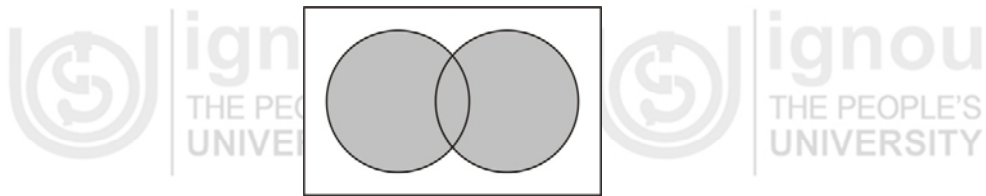
Fuzzy Propositions: The primary disparity between classical proposition and fuzzy propositions (Rule Base) is in the range of their truth values. At the same time as every classical proposition is mandatory to be either true or false, the truth or falsity of fuzzy proposition is a matter of degree. In this slice, we focus on simple fuzzy propositions, which we categorize into the subsequent four types:

1. Unconditional and unqualified proposition.
2. Unconditional and qualified proposition.
3. Conditional and unqualified proposition.
4. Conditional and qualified proposition.

Classical Set: The most vital and most fundamental term to be found in modern mathematics and logic is that of set, or class. Even though some modern studies make a technical dissimilarity between set and class, we do not do so in this treatment; it is recognized, however, that mathematicians are inclined to use the word set, whereas logicians universally refer to a class. The theory of sets, we shall see, forms one of the connecting links between mathematics on the one hand and logic on the other hand. We shall think of a set as simply a collection of well defined objects. The objects which make up a set will be called elements of the set or the items that are entered or considered as in the set are called the elements or members of the set. The important set operations can be defined as below:

- Complement of A is $A' = \{x \mid x \notin A\}$
- Intersection of A and B, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Union of A and B, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

The basic set operations, 'intersection', 'union' and 'complement', are illustrated using



therefore, it can be treated as one of the most fundamental activities. Decision-making is

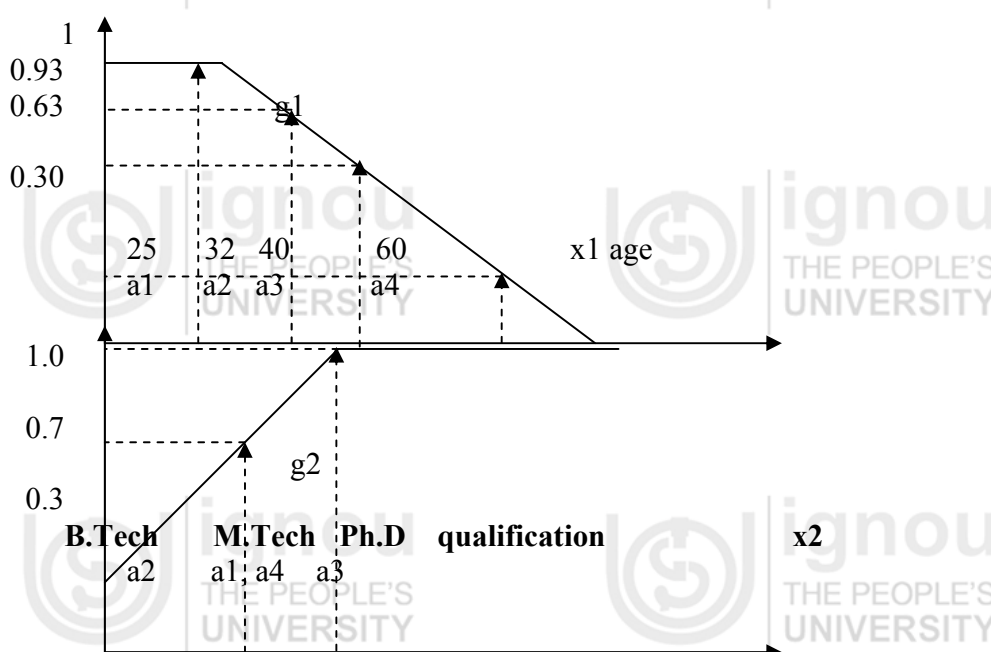
an area which studies about how decisions are actually made and how better they can be made successfully.

Problem: A research institute wants to recruit a young, dynamic and talented scientific officer to assist a team comprising three experts to work in the field of high voltage engineering. There are four applications and the details are listed below:

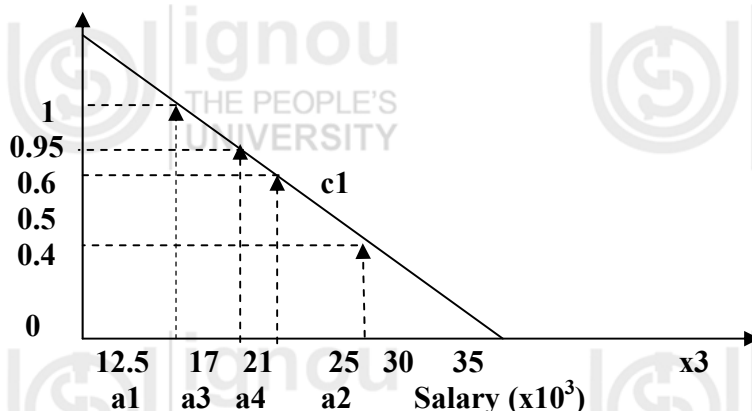
NAME	AGE	QUALIFICATION	SALARY DEMAND
a1	25	M.Tech	Rs.12,500/-
a2	55	B.Tech	Rs.25,000/-
a3	32	Ph.D	Rs.17,000/-
a4	40	M.Tech	Rs.21,000/-

The institute is in financial constraints. If the institute wishes to call only one person for interview, develop a fuzzy decision-making algorithm and give the result for the present problem.

Solution: In this problem, the institute is looking for young, dynamic (basically age decides) and talented (better qualification) candidates. Therefore, it is appropriate to consider age and qualification as goals, namely g_1 and g_2 . The Institute has financial impediments. Therefore, salary is the constraint c_1 . Thus the problem has two goals and a single constraint. Let us build simple fuzzy sets for g_1 , g_2 and c_1 suitably. It may be noted that an increase in age is less preferred. Similar is the case with salary too. As qualification is higher, it is better. Hence the three fuzzy sets are sketched as shown in the figures.



It is important to note that qualification cannot be expressed numerically. However, since a choice has to be made, this variable is assigned a suitable membership grade.



From the above fuzzy sets, we can find

$$\begin{aligned} &\mu(g1/a1), \mu(g1/a2), \mu(g1/a3), \mu(g1/a4), \\ &\mu(g2/a1), \mu(g2/a2), \mu(g2/a3), \mu(g2/a4), \\ &\mu(c1/a1), \mu(c1/a2), \mu(c1/a3), \mu(c1/a4), \\ &\mu(g1/ai) = [1/a1, 0.3/a2, 0.93/a3, 0.63/a4] \\ &\mu(g2/ai) = [0.7/a1, 0.3/a2, 1.0/a3, 0.7/a4] \\ &\mu(c1/ai) = [0.95/a1, 0.4/a2, 0.6/a3, 0.5/a4] \end{aligned}$$

Using the equation $D(ai) = \text{Min} \{a1, a2, a3, a4\}$ ie (i=total)

$$\Rightarrow D(ai) = \text{Min} \{0.7/a1, 0.3/a2, 0.6/a3, 0.5/a4\}$$

Now, applying the equation $D = \text{Max} \{D(ai)\}$

We get Fuzzy decision, $D = \text{Max} \{D(ai)\} = 0.7/a1$

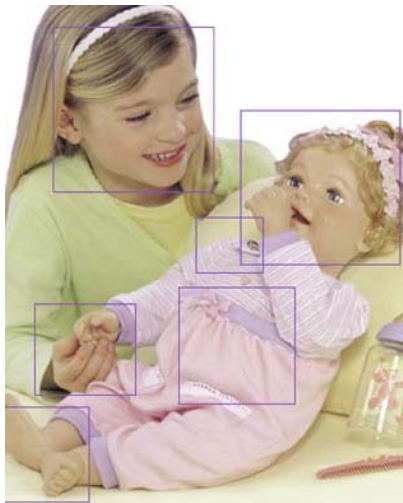
Thus, a1 is the suitable candidate to be called for interview.

Fuzzy-Machines: Fuzzy logic principles are extensively employed in various consumer products and the sale of these products is increasingly going up in recent years. A typical application is the use of fuzzy logic principle to automatic washing machines. While many manufacturers provide with a variety of features, the underlying principle of fuzzy logic-based washing is explained in this section. As long as the washing of clothes is concerned, removal of dirt particles is the objective. Thus the input of the fuzzy control system is the quantum of dirt and its rate of change. The weight of clothes is another input. With these three inputs, the following are the outputs:

1. Quantity of washing powder
2. Water quantity
3. Water flow rate
4. Washing time
5. Rinsing time and
6. Spinning time

Fuzzy-Genetic Algorithms: There are a number of ways in which Genetic Algorithms





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1) How many kinds of fuzzy proposition? Explain.

2) Explain the Basic Operation on Fuzzy Sets.

5.7 LET US SUM UP

In sum the intellectual capacity is to include an explicit cram on Multi-value logic. The role of symbolic logic is decorated in the multi-value logic. In order to deal with such propositions, we must relax the true/false dichotomy of classical two-valued logic by allowing a third truth value, which may be called indeterminacy. The fundamental difference between classical propositions and fuzzy propositions and the range of their truth values are also discussed in this chapter. Fuzzy Sets and Fuzzy Logic is a true magnum work. The fundamental distinction between classical proposition and fuzzy propositions is in the range of their truth values. While each classical proposition is required to be either true or false, the truth or falsity of fuzzy proposition is a matter of degree. Assuming that truth and falsity are expressed by values 1 and 0, respectively, the degree of truth of each fuzzy proposition is expressed. In metro trains, fuzzy logic is used to determine the proper start, stop, and cruising speed of the train; in washing machines it determines the amount of water and the number of rinses; in cameras and camcorders it adjusts the colour, contrast, brightness, focus, and son on; in vacuums it determines the suction power based on the amount and size of particles; in automobiles with automatic transmission it determine the proper gear; in intelligent vehicular systems it finds the best route and automatically guides an automobile; and in communication systems it processes signals, schedules and routes channels, and controls the system. In financial engineering the performance of stocks is being predicted. As a result we strongly conclude that the role of symbolic logic in the modern era is immensely significant.

5.8 KEY WORDS

Logic Gate: A logic gate is an electronic circuit, which takes numerous inputs and produces a single output.

Set: Set is a collection of well defined objects.

5.9 FURTHER READINGS AND REFERENCES

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