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BCS–012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (REVISED)

Term-End Examination

December, 2024

BCS-012 : BASIC MATHEMATICS

Time : 3 Hours *Maximum Marks : 100*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **three** questions from the remaining questions.*

1. (a) Show that : 5

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$$

(b) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that : 5

$$A^2 - 4A - 5I_3 = O_3$$

(c) Use the principle of mathematical induction to show that $3^{2n}-1$ is divisible by 8 for each natural number n. 5

(d) If in an A. P. $a = 2$ and sum of the first five terms is one fourth of the sum of next five terms, show that $a_{20} = -112$. 5

(e) If $z \in \mathbf{C}$, where $\mathbf{C}$ is a set of complex numbers and $|z| = 1, z \neq -1$, show that $\frac{z-1}{z+1}$ is purely imaginary. 5

(f) If two roots r_1 and r_2 of the quadratic equation $x^2 + kx + 12 = 0$ are such that $(r_1 - r_2) = 1$, find k. 5

(g) Two tailors A and B, earn ₹ 600 per day and ₹ 800 per day respectively. Tailor A can stitch 6 shirts and 4 pants per day,

while Tailor B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum labour cost ? Also calculate the least cost.

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2. (a) Using elementary row operations, find inverse of the matrix : 5

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

(b) How many terms of the G. P. :

$$\sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots$$

add upto $39 + 13\sqrt{3}$? 5

(c) If $1, \omega, \omega^2$ are cube roots of unity, show that : 5

$$(2 + 3\omega + 2\omega^2)^9 \times (4 + 3\omega + 3\omega^2)^9 = 1$$

(d) If α and β are roots of the quadratic equation $3x^2 - 4x + 1 = 0$, form an equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. 5

3. (a) Find : 5

$$\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$$

(b) If $y = ae^{mx} + be^{-mx}$, show that : 5

$$\frac{d^2y}{dx^2} = m^2 y$$

(c) Determine the interval in which :

$$f(x) = -2x^3 + 24x + 7$$

is increasing. 5

(d) If a mothball evaporates at a rate proportional to its surface area $4\pi r^2$, show that its radius decreases at a constant rate. 5

4. (a) Evaluate : 5

$$\int \frac{x}{\sqrt{x+3}}\,dx$$

(b) Evaluate : 5

$$\int_a^b \frac{\log x}{x}\,dx\,, \quad a > b > 0$$

(c) Find the area bounded by $y = \sqrt{x}$ and $y = x$. 5

(d) If $\vec{a}$ and $\vec{b}$ are two vectors, show that : 5

$$|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

5. (a) Show that the lines : 5

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$$

and $$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

intersect.

(b) Solve the inequality : 5

$$\left|\frac{x-3}{2}\right| \leq 1$$

(c) Using determinants, find the area of a triangle whose vertices are (1, 2), (–2, 3) and (–3, –4). 5

(d) If $y = \log_e \left[e^x \left(\frac{x-2}{x+2} \right)^{3/4} \right]$, find $\frac{dy}{dx}$. 5

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