No. of Printed Pages : 5 **MMT–004**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2024

MMT–004 : REAL ANALYSIS

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***four*** *questions out of question nos.* ***2*** *to* ***6****. Use of calculator is not allowed.*

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1. State whether the following statements are True *or* False. Give reasons for your answers :

5×2=10

(a) The product of two discrete metrics is a discrete metric.

(b) The set of integers is neither dense nor nowhere dense in **R** with respect to the usual metric on **R**.

(c) $\mathbf{R}^2/\{(0, 1)\}$ is compact but not connected.

(d) For the function :

$$f(x, y, z) = x^2 y + y^2 z + z^2,$$

$$f''(1, 1, -1/2) \text{ is } \begin{bmatrix} 2 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

(e) All continuous real valued functions defined on [0, 1] are measurable but not integrable.

2. (a) Find the interior, closure and boundary of the set : 2

$$B = \{(x, y) \in \mathbf{R}^2 : x = 0\}$$

(b) Define uniform continuity of functions between metric spaces. Prove that a uniformly continuous image of a Cauchy sequence is a Cauchy sequence. 4

(c) For the function $F : \mathbf{R}^4 \to \mathbf{R}^3$ given by :

$$F(x, y, z, w) = (x^2y, xyz, x^2 + y^2 + zw^2)$$

Write down the component functions. Find the derivative of F at (0, 0, 0, 0). 4

3. (a) Prove that a compact subset of a metric space is closed and bounded. Is the converse true ? Justify. 4

(b) Prove that the Lebesgue measure is countably additive. 3

(c) Check whether the function : 3

$$f(x, y) = (x^2 - y^2, 2xy)$$

is locally invertible on $E = \{(x, y) \mid x > 0\}$.

4. (a) Prove that a non-empty subset of **R** is connected if and only if it is an interval (**R** with the usual metric). 4

(b) Find and classify the stationary points of the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by : 3

$$f(x, y) = (y - x^2)(y - 2x^2)$$

(c) Suppose f is a non-negative measurable function and $E \in M$. Define $\int_E f\, dm$. For $A, B \in M$ with $A \cap B = \phi$, prove that : 3

$$\int_{A \cup B} f\, dm = \int_A f\, dm + \int_B f\, dm$$

5. (a) Prove that a continuous function from a compact metric space to any other metric space is uniformly continuous. 4

(b) (i) State and prove the Monotonic convergence theorem. 4

(ii) Does the sequence :

$$f_n = \chi_{[n,\, n+1]},\ n = 1, 2.....$$

satisfy all the conditions of the above theorem ? Check. 2

6. (a) For a real function $f \in L,[-\pi, \pi]$, define the nth Fourier coefficient of f, the exponential and the trigonometric form of the Fourier series of f.

Find the Fourier series of the function : 4

$$f(t) = \begin{cases} 0, & -\pi \le t < 0 \\ 1, & 0 \le t \le \pi \end{cases}$$

(b) State the Riemann-Lebesgue lemma. 2

(c) Find the extreme values of the function : 4

$$Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$$

Subject to the constraint :

$$x_1 + x_2 + x_3 = 20,$$

$$x_1,\ x_2,\ x_3 \geq 0\,.$$

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