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MMT–005

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2024

MMT–005 : COMPLEX ANALYSIS

Time : 1½ Hours *Maximum Marks : 25*

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***three*** *questions from Question No.* ***2*** *to* ***5****. Use of calculator is not allowed.*

1. State, giving reasons, whether the following statements are True *or* False : 5×2=10

   (a) The intersection of two open connected sets of complex numbers is an open connected set.

(b) $\cos z$ is a bounded function.

(c) All the singular points of the function :

$$f(z) = \frac{1}{(2z-1)\,(3z-2)}$$

are isolated singular points.

(d) The radius of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{n^{50}(z-5)^n}{n!}$$

is 5.

(e) The function $f(z) = 9z - \frac{4}{z}$ is conformal at $z = \pm\frac{2}{3}i$.

2. (a) Find the harmonic conjugate of the function : 2

$$u(x, y) = e^{-x} \cos y$$

(b) Find the Laurent's series expansion in powers of z for the function : 3

$$f(z) = \frac{1}{z^2 - 7z + 12}$$

which is valid in the region $|z| > 4$.

3. (a) Let function f be analytic everywhere inside and on a positive oriented circle C_R centred at Z_0 with radius R. Show that : 2

$$\left|f^{(n)}(z_0)\right| \leq \frac{n!\,M}{R^n},\ n = 1, 2, 3, \ldots\ldots$$

where M is maximum value of $f(z)$ on the circle $|z - z_0| = R$.

(b) Find, for the following function : 3

$$f(z) = \frac{\cos(z^2) - 1}{z^6}$$

(i) The principal part of $f(z)$ at $z = 0$.

(ii) The nature of the singular point $z = 0$.

(iii) The residue at $z = 0$.

4. (a) Find the Mobius transformation which maps $-i$, 1, i to $-i$, 0 and i respectively. 3

   (b) Find all the solutions of : 2

$$e^z = (1 + \sqrt{3}\, i)$$

5. Evaluate the following improper integral : 5

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$$

using the method of contour integration.

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