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MMT–007

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. MACS)]

Term-End Examination

December, 2024

MMT–007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Weightage : 50%

Note : *Question No.* ***1*** *is compulsory. Attempt any* ***four*** *questions out of Q. No.* ***2*** *to* ***7****. Use of Scientific/non-programmable calculator is allowed.*

1. State whether the following statements are True *or* False. Justify your answer with the help of short proof or a counter-example : 5×2=10

(a) Initial value problem :

$$\frac{dy}{dx} = x(y-1),\ y(0) = 1$$

has a unique solution.

(b) $L^{-1}\left\{\frac{s}{(s^2+a^2)}\right\} = \frac{t \sin at}{2a}$

where L^{-1} denotes Laplace inverse transformation.

(c) The IVP $y' = x + y^2$, $y(0) = 1$ on the interval [0, 0.4] using Runge-Kutta second order method with step length $h = 0.2$ has solution $y(0.4) \in (0, 1)$.

(d) $f^{-1}\left[\frac{e^{4i\alpha}}{3+i\alpha}\right] = \begin{cases} 0 , & x < -4 \\ e^{-3(x+4)}, & x \geq -4 \end{cases}$

where f^{-1} denotes Fourier inverse transformation.

(e) The partial differential equation :

$$\frac{\partial^2 u}{\partial x^2} + 4x\frac{\partial^2 u}{\partial y \partial x} + (1-y^2)\frac{\partial^2 u}{\partial y^2} = 0$$

is parabolic inside the ellipse $4x^2 + y^2 = 1$.

2. (a) Express :

$$f(x) = x^4 + 3x^3 + 4x^2 - x + 2$$

in terms of Legendre polynomials. 5

(b) Using the generating function, derive the relation : 5

$$x^n = \sum_{k=0}^{[n/2]} \frac{n!\, H_{n-2k}(x)}{2^n k!\, (n-2k)!}, \qquad n = 0, 1, 2.......$$

3. (a) Find the power series solution about the origin of the equation : 5

$$(1 - x^2)y'' - 4xy' + 2y = 0$$

(b) Construct Green's function for the following boundary value problem : 5

$$\frac{d^2y}{dx^2} - y = x, \text{ with } y(0) = y(1) = 0.$$

4. (a) Find the solution of $\nabla^2 u = 0$ in R subject to R : square $0 < x < 1, 0 < y < 1$ and $u(x, y) = x^2 - y^2$ on the boundary of square. Assume $h = \frac{1}{3}$ and use five-point formula. 6

(b) Find the interval of absolute stability for the second order Runge-Kutta method. 4

5. (a) Solve the initial value problem $y' = x^2 + y^2$, $y(0) = 1$ on the value $x = 0.3$ and obtain the approximate value of $y(0.3)$ using Milne's P-C method with the step length $h = 0.1$. Perform two corrector iterations per step, where : 6

P = Predictor : $y_{i+1}^{(P)} = y_i + hf_i$

C = Corrector :

$$y_{i+1}^{(C)} = y_i + \frac{h}{2}[f_i + f_i(x_{i+1}, y_{i+1})]$$

(b) For the Bessel's function, show that :

$$\int x\, J_0^2(x)\, dx = \frac{x^2}{2}[J_0^2(x) + J_1^2(x)] + C$$

where J_0 and J_1 are Bessel's function of the first kind of order 0 and 1 respectively. 4

6. (a) Find the solution of the initial boundary value problem :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1$$

$$u(x,0) = \sin(\pi x), \quad 0 < x < 1;$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, \quad 0 \leq x \leq 1, u(0, t) = 0,$$

$$u(1, t) = 0, t > 0,$$

by using second order implicit scheme method with $h = \frac{1}{4}$, $r = \frac{1}{3}$, where h is the step length in x-direction and r is the mesh ratio. Integrate for one time step. 6

(b) Find $L\{t \operatorname{erf}(2\sqrt{t})\}$, erf being error function. 4

7. (a) The wave equation $u_{tt} = u_{xx}$ is approximated by : 5

$$\delta_t^2 u_i^n = \frac{r^2}{2} \delta_x^2 \left[u_i^{n+1} + u_i^{n-1} \right]$$

where mesh ratio $r = \frac{k}{h}$, where k is the step length in the direction t and h is the step length in the direction x. Investigate the stability using the Von Neumann method.

(b) Solve the boundary value problem $y'' = xy + 1$, $y(0) + y'(0) = 1$, $y(1) = 1$ using the second order finite difference method with step length $h = \frac{1}{2}$. 5

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