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**MMT-009**

**M. Sc. (MATHEMATICS WITH  
APPLICATIONS IN COMPUTER  
SCIENCE)**

**[M. Sc. (MACS)]**

**Term-End Examination**

**December, 2024**

**MMT-009 : MATHEMATICAL MODELLING**

*Time : 1½ Hours*

*Maximum Marks : 25*

*Weightage : 70%*

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***Note :*** Attempt any ***five*** questions. Use of scientific  
(non-programmable) calculator is allowed.

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1. (a) Explain each of the following with examples : 2
  - (i) Reaction-diffusion model Vs. Advection-reaction-diffusion model.
  - (ii) Variational matrix of a system of  $n$ -dimensional equations.

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- (b) Find a linear demand equation that best fits the following data and use it to predict annual sales of homes priced at ₹ 14,00,000 : 3

|                                                    |     |     |    |    |    |    |    |
|----------------------------------------------------|-----|-----|----|----|----|----|----|
| $x = \text{price}$<br>(lakhs<br>of ₹)              | 16  | 18  | 20 | 22 | 24 | 26 | 28 |
| $y = \text{sales}$<br>of new<br>homes<br>this year | 126 | 103 | 82 | 75 | 82 | 40 | 20 |

2. Do the stability analysis of the following model formulated to study the effect of toxicant on prey-predator population : 5

$$\frac{dN_1}{dt} = r_0N_1 - r_1C_0N_1 - bN_1N_2$$

$$\frac{dN_2}{dt} = -d_0N_2 - d_1V_0N_2 + \beta_0bN_1N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dV_0}{dt} = k_2P - g_2V_0 - m_2V_0$$

Here  $r_0$ ,  $d_0$ ,  $b$ ,  $\beta_0$ ,  $m_1$ ,  $m_2$ ,  $P$ ,  $k_1$ ,  $k_2$ ,  $g_1$ ,  $g_2$ ,  $r_1$  and  $d_1$  are all positive constants.

$N_1(t)$  = Density of prey population

$N_2(t)$  = Density of predator population

$C_0(t)$  = Concentration of the toxicant in the individuals of the prey population

$V_0(t)$  = Concentration of the toxicant in the individuals of the predator population

$r_0$  = Growth rate or birth rate

$d_0$  = Death rate

$b$  = Predation rate

$\beta_0$  = Conversion coefficient

$m_1$ ,  $m_2$  = Depuration rates

$k_1$ ,  $k_2$  = Uptake rates

$g_1$ ,  $g_2$  = Loss rates

$r_1$  = Death rate due to  $C_0$

$d_1$  = Death rate due to  $V_0$

$P$  = Constant environmental toxicant concentration

3. (a) A telephone exchange has two long distance operators. The telephone company observes that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. 3
- (i) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day ?
- (ii) If the subscriber will wait and serviced in turn, what is the expected waiting time ?
- (b) Find the number of quantities required for estimating the expected return and standard deviation for 300 securities in Markowitz model. How many estimates are required for these securities while using single index Sharpe model ? 2

4. Solve the integer programming problem : 5

$$\text{Max. : } Z = 7x_1 + 9x_2$$

$$\text{subject to : } -x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$x_1 \geq 0, x_2 \geq 0$  and integers.

5. (a) A discrete model for a population  $N_t$  consists of :

$$N_{t+1} = \frac{rN_t}{1 + bN_t^2}$$

where  $t$  is the discrete time and  $r$  and  $b$  are positive parameters. Determine the non-negative steady state and discuss the linear stability of the model. 3

- (b) What are the factors to describe the mathematical models in population dynamics ? Write any *four* of them with explanation. 2

6. Consider the budworm population dynamics governed by the equation  $\frac{dx}{dt} = rx \left( 1 - \frac{x}{k} \right) - x$ , where  $k$ , the carrying capacity and  $r$ , the birth rate of the budworm population  $x(t)$ , are positive parameters. Find the steady states and use the perturbation to do the stability analysis of the equation for  $0 < r < 1$ . 5

7. (a) The deviation  $g(t)$  of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation :

$$4 \frac{d^2 g}{dt^2} + 8\alpha \frac{dg}{dt} + (2\alpha)^2 g = 0$$

for  $\alpha$ , a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time  $t$  is measured in minutes. Identify the type (overdamped) of this differential equation. Find the condition on  $\alpha$  for which the patient is normal.  $2\frac{1}{2}$

- (b) In a tumour region, the control parameters of growth and decay are in  $2 : 1$ . Emigration occurs at a constant rate of  $3 \times 10^3$  cells per month. There is decay of 35 cells every month. Use these assumptions to formulate the logistic model of tumour size. Solve the formulated equation if the initial size of the tumour is  $5 \times 10^6$  cells. When does the proliferation of tumour cells stabilize for this model ?  $2\frac{1}{2}$

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