

**M. SC. (PHYSICS)**  
**(MSCPH)**

**Term-End Examination**

**December, 2024**

**MPH-001 : MATHEMATICAL METHODS**  
**IN PHYSICS**

*Time : 2 Hours*

*Maximum Marks : 50*

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**Note :** Attempt any **five** questions. Marks are indicated against each question. Symbols have their usual meanings. You may use calculator.

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1. Using the generating function for Hermite polynomials : 7+3

$$g(x, t) = e^{2xt - t^2} = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!}$$

obtain (i) the recurrence relation :

$$H_{n+2}(x) = 2x H_{n+1}(x) - 2(n+1)H_n(x)$$

- (ii) establish the relation between  $H_n(x)$  and  $H_n(-x)$  as :

$$H_n(-x) = (-1)^n H_n(x)$$

2. (a) The one-dimensional differential equation for the function  $E_y$  is given by : 5

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2} = 0$$

Solve this equation if  $E_y = 0$  at  $x = 0$  and  $x = L$ .

- (b) Write the Rodrigue's formula for Legendre polynomials. Using this formula, obtain the value of  $P_3(x)$  and  $P_4(x)$ . 1+4
3. (a) What are symmetric and anti-symmetric matrices ? Separate the real and imaginary parts of a Hermitian matrix  $H$  by writing it as the sum of a real matrix  $A$  and  $i$  (the imaginary unit) times another real matrix  $B$  : 2+3

$$H = A + iB$$

What is the nature of the matrices  $A$  and  $B$  ?

- (b) Obtain the eigen values for the real symmetric matrix : 2+3

$$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & -\cos \theta & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Determine the eigen vector for any one eigen value.

4. (a) Show that  $g^{ij}$  is a contravariant second rank tensor. 5
- (b) Obtain the Taylor's series expansion of  $\cos^2 z$  about  $z = 0$ . 5
5. Using the method of Residues (Jordan's lemma), show that : 10

$$\int_{-\infty}^{\infty} \frac{\cos x \, dx}{x^2 + 4} = \frac{\pi e^{-2}}{2}$$

6. (a) Consider a triangle P in the  $z$ -plane with vertices at  $i$ ,  $1 - i$ ,  $1 + i$ . Obtain the triangle  $P_0$  which mapped P under the transformation  $w = 3z + 4 - 2i$ . What is the relation between P and  $P_0$  ? Also calculate the area magnification. 5

- (b) Obtain the Fourier transform of the function : 5

$$f(t) = \begin{cases} 0 & , \quad t < 0 \\ e^{-at} & , \quad t \geq 0, a > 0 \end{cases}$$

7. Obtain the inverse Laplace transform of : 10

$$F(s) = \frac{s^2 + 2s + 3}{(s + 2)(s + 1)^2}$$

8. (a) Define the symmetric group  $S_3$  and give its geometrical illustration. 5
- (b) Show that all  $n \times n$  unitary matrices form a group. 5

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