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MASTER'S DEGREE PROGRAMME IN PHYSICS (MSCPH)

Term-End Examination

December, 2024

MPH-004 : QUANTUM MECHANICS–I

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any **five** questions.*

(ii) Symbols have their usual meanings.

(iii) You may use a calculator.

(iv) The values of physical constants are given at the end.

(v) The marks for each question are indicated against it.

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1. (a) What should be the minimum kinetic energy of electrons in an electron microscope in order to observe an atom with a diameter of 0.1 nm ? 3

(b) The energy of a quantum particle of mass m is $E = \frac{P^2}{2m} + \alpha x^4$, where α is a positive constant. Use the uncertainty principle to calculate the minimum possible energy of the particle. 7

2. (a) Calculate the probability current density for a wave function : 6

$$\psi(x) = \phi(x) e^{iu(x)}$$

where $u(x), \phi(x)$ are real.

(b) Show that : 4

$$[\hat{L}_x, \hat{L}_y, \hat{L}_z] = i\hbar(\hat{L}_x^2 - \hat{L}_y^2) .$$

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3. Write down the potential function for a one-dimensional potential barrier of width a. State the boundary conditions for the wave function and obtain the general solution to the time-independent Schrödinger equation in all regions for a particle of mass m in this potential. Define the reflection and transmission coefficients when the energy of the particle is less than the height of the potential barrier. 2+2+4+2

4. Explain the importance of the simple harmonic potential problem in Physics. Write down the time-independent Schrödinger equation for the one-dimensional simple harmonic oscillator with the potential function $V(x) = \frac{1}{2} m \omega^2 x^2$.

Show that the Schrödinger equation can be

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written in terms of the dimensionless variables

$\lambda = \dfrac{2E}{\hbar\omega}$ and $a^2 = \dfrac{m\omega}{\hbar}$ as :

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\psi(\xi) = 0$$

Do the eigen functions of the simple harmonic oscillator have a definite parity ? Explain.

3+1+4+2

5. Define a spherically symmetric potential. Derive the time-independent Schrödinger equation in spherical polar coordinates for a spherically symmetric potential function and a wave function $\psi(r, \theta, \phi)$. Show that the wave function can be written as $\psi(r, \theta, \phi) = R(r)Y(\theta, \phi)$. Obtain the differential equations for $R(r)$ and $Y(\theta, \phi)$. 1+4+2+3

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6. A two-dimensional state space has the following basis vectors :

$$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \ |\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(i) Show that the states $|\psi_1\rangle, |\psi_2\rangle$ are orthonormal. 3

(ii) If the action of an operator $\hat{A}$ in this space is defined by :

$$\hat{A}|\psi_1\rangle = -\frac{i\hbar}{2}|\psi_2\rangle; \ \hat{A}|\psi_2\rangle = \frac{i\hbar}{2}|\psi_1\rangle$$

determine $\hat{A}|\phi_1\rangle$ and $\hat{A}|\phi_2\rangle$

where $|\phi_1\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle - i|\psi_2\rangle\right)$

and $|\phi_2\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle + i|\psi_2\rangle\right)$. 7

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7. Write the Heisenberg equation of motion for a Heisenberg operator $\hat{A}_H$ which does not depend explicitly on time. Determine the equations of motion for the operator $\langle \hat{X} \rangle$ and $\langle \hat{P} \rangle$ for a particle of mass *m* in a gravitational field with the Hamiltonian $\hat{H} = \frac{\hat{P}^2}{2m} + mg\,\hat{x}$. Solve the equations of motion to get $\hat{X}(t)$ and $\hat{P}(t)$, using the conditions $\hat{X}(t=0) = \hat{X}(0)$ and $\hat{P}(t=0) = \hat{P}(0)$. 2+4+4

8. Describe the Stern-Gerlach experiment with the help of a schematic diagram. Explain how the results of the experiment confirm the

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quantization of the z-component of the angular momentum and deduce the existence of spin. 10

Physical constants :

$h = 6.63 \times 10^{-34}\,\text{Js}$

$\hbar = 1.05 \times 10^{-34}\,\text{Js}$

$m_e = 9.11 \times 10^{-31}\,\text{kg}$

$e = 1.6 \times 10^{-19}\,\text{C}$

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