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# M. SC. (PHYSICS)

# (MSCPH)

## Term-End Examination

## December, 2024

### MPH-006 : CLASSICAL MECHANICS–II

*Time : 2 Hours* *Maximum Marks : 50*

***Note :*** *Attempt any **five** questions. The marks for each question are indicated against it. Symbols have their usual meanings. You may use a calculator.*

1. Consider a particle of mass $m$ moving in a central potential V($r$), described by the Lagrangian : 10

$$L = \frac{1}{2} m \vec{r}^{\,2} - V(r)$$

Show that the $z$-component of the angular momentum is conserved if the rotation is along $z$-axis.

2. Consider a simple harmonic oscillator which has the Lagrangian : 5+5

$$L(q, \dot{q}) = \frac{1}{2} m\dot{q}^2 - \frac{1}{2} m\omega^2 q^2$$

(a) Obtain the Hamiltonian and show that the path evolution of the system is phase space is an ellipse.

(b) Obtain Hamilton's equations of motion and hence determine the amplitude of oscillation for the initial conditions :

$$q(t = 0) = q_0,\ p(t = 0) = p_0$$

3. (a) Write down the Lagrangian for a relativistic free particle. Show that the Hamilton's principal function is Lorentz invariant. 5

(b) The Lagrangian of a system is : 5

$$L = \frac{1}{2} m\dot{x}^2 + m\dot{x}\,\dot{y}$$

Obtain the Hamiltonian and the Hamilton's equation of motion.

4. Write down the condition for canonical transformation when functional dependence are given. Show whether the transformation : 1+9

$$Q = q\cos 2\theta - \frac{p}{m\omega}\sin\theta$$

$$P = m\omega q\,\sin 2\theta + p\cos 2\theta$$

is canonical.

5. Consider a particle of mass $m$ in a uniform gravitational field, $g$. The Hamiltonian of the system is given by :

$$H = \frac{p^2}{2m} + mgq$$

using the generating function :

$$F_2(q, P) = \frac{2}{3}(2m^2 g)^{\frac{1}{2}}(P - q)^{\frac{3}{2}}$$

(i) Obtain the new Hamiltonian K(Q, P). 6

(ii) Identify the cyclic coordinate and the constant of motion. 2

(iii) Obtain the equation of motion in the new coordinate. 2

6. (a) Show that if $f(q, p, t)$ and $g(q, p, t)$ are constants of motion, then : 5

$$\frac{d}{dt}[f, g] = 0$$

(b) Consider a transformation $Q = pq^{2\alpha}$ $P = \beta q^{-\alpha-1}$ , where $\alpha$ and $\beta$ are constants. Using Poisson brackets, obtain the value of $\alpha$ and $\beta$ such that the transformation is canonical. 5

7. Consider a particle of mass $m$ falling vertically in a uniform gravitational field, $g$. The Hamiltonian of the system is : 10

$$H = \frac{p^2}{2m} + mgz$$

Using Hamilton-Jacobi equation, obtain $z$ as a function of time.

8. Consider a uniform square plate of length $x = y = a$ and mass M. Using the secular equation, obtain the three principal moments of inertia ($I_1$, $I_2$, $I_3$). 10

Take :

$$I_{xx} = I_{yy} = \frac{1}{3} M a^2,$$

$$I_{zz} = \frac{2}{3} M a^2,$$

$$I_{xy} = I_{yx} = -\frac{1}{4} M a^2$$

and $$I_{zx} = I_{xz} = I_{yz} = I_{zy} = 0 .$$

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