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MPH–007

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2024

MPH-007 : CLASSICAL ELECTRODYNAMICS

Time : 2 Hours | *Maximum Marks : 50*

Note : *Answer any **five** questions. Marks for each question are indicated against it. Symbols have their usual meaning. You can use calculator.*

1. Using Maxwell's equations in vacuum with no source charge or currents, show that electric field and magnetic field satisfy wave equation and hence calculate the speed of light. 4+4+2

2. In a region, where $\rho = 0$ and $\vec{j} = \vec{0}$, the electric and magnetic fields are given as :

$$\vec{E}(x, y, z, t) = E_0 \sin(k_z z) e^{i(\omega t - k_x x)} \hat{y}$$

$$\vec{B}(x, y, z, t) = \vec{B}_0(y, z) e^{i(\omega t - k_x x)}$$

(a) What condition must k_x and k_y satisfy so that :

$$\vec{E}(x, y, z, t) = E_0 \sin(k_z z) e^{i(\omega t - k_x x)} \hat{y}$$

satisfy the wave equation for an electromagnetic wave ? 2

(b) Determine $\vec{B}_0(y, z)$ so that $\vec{E}$ and $\vec{B}$ satisfy Maxwell's equations. 8

3. (a) Determine the electric and magnetic fields corresponding to scalar potential $\phi(x, y, z, t) = -ax$ and vector potential :

$$\vec{A}(x, y, z, t) = at\,\hat{x} + f(x - bt)\hat{y},$$

where a and b are constants. 5

(b) The electric field of an electromagnetic wave propagating in a rectangular waveguide has the form :

$$\vec{E} = E_0 \sin(3z) \cos(1.5 \times 10^9 t - k_x x)\, \hat{y},$$

where x and z are in metres and t is in seconds. Determine k_x and phase velocity for the wave. 3+2

4. Consider a plane electromagnetic wave incident obliquely at the interface of two dielectric media. Using the boundary conditions, derive the laws of reflection and refraction in geometrical optics. 10

5. (a) Write Maxwell's equations in a conductor and obtain the wave equation for the electric fields. 2+3

(b) Starting from the wave equation for the electric field in a conductor, show that the electric field associated with the electromagnetic waves in a conductor is given by : 5

$$\vec{E} = \vec{E}_c\, e^{-\kappa x} e^{i(kx-\omega t)}$$

6. Show that the retarded scalar potential given by : 10

$$\phi(\vec{r}, t) = \frac{1}{4\pi \epsilon_0} \int \frac{1}{|\vec{r} - \vec{r}'|} \rho\left(\vec{r}', t - \frac{|\vec{r} - \vec{r}'|}{c}\right) dV'$$

satisfies the inhomogeneous wave equation for scalar potential :

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

7. (a) Show that the retarded vector potential is given by : 5

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{1}{|\vec{r} - \vec{r}'|} \vec{j}(\vec{r}', t_r)\, dV'$$

(b) Consider the equation :

$$\vec{A}.\vec{B} = 0,$$

where $\vec{A}$ and $\vec{B}$ are 3-dimensional vectors. Does the equation retain its form under a Lorentz transformation ? Justify your answer. 5

8. Consider two inertial frames of reference S and S′ such that frame S′ moves with a velocity v in the positive x-direction with respect to S. Obtain Lorentz transformation equations and the inverse Lorentz transformation equations. 7+3

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