

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

December, 2024

MST-003 : PROBABILITY THEORY

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 7.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Use of “Formulae and Statistical Tables Booklet” for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers :

$$5 \times 2 = 10$$

- (a) If a discrete random variable X has the probability distribution given as :

X	$p(x)$
0	c
1	$2c$
2	$3c$

then its cumulative distribution will be :

X	$F(x)$
0	$6c$
1	$3c$
2	c

- (b) If X is uniformly distributed over $(-5, 5)$, then $P[X > 3]$ will be $\frac{1}{5}$.
- (c) If $P[X = 3] = 16P[X = 7]$ for a binomial distribution with $n = 10$, then its mean will be $\frac{10}{3}$.
- (d) If $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$, then A and B are independent.

- (e) If a card is drawn from a well-shuffled pack of 52 cards, then the probability that it is either a heart or a queen will be $\frac{4}{13}$.
2. (a) A bag contains 10 gold and 8 silver coins. Two successive drawings of 3 coins are made. Find the probability that the first drawing will give 3 gold and the second 4 silver coins when :
- (i) the coins are replaced before the second draw.
- (ii) the coins are not replaced before the second draw.
- (b) In a certain competitive examination, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he/she makes a guess is 0.35 and the probability that he/she copies the answer is 0.20. The probability that his/her answer is correct given that he/she copied is 0.15. Find the probability that he/she guessed the answer of the question. Given that he/she correctly answered it.

3. (a) A random variable X has the following probability distribution : 6

X	$p(x)$
- 2	0.1
- 1	k
0	0.2
1	$2k$
2	0.3
3	$3k$

- (i) Find k .
 - (ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$.
 - (iii) Find cumulative probability distribution of X .
 - (iv) Find mean of X .
- (b) The number of mistakes counted in one hundred typed pages by a typist revealed that he made 2.8 mistakes on an average per page. Find the probability that in a page typed by him : 4
- (i) there is no mistake
 - (ii) there are two or less mistakes.

4. (a) In an examination of 5,000 students, the marks of the examinees in statistics were found to be distributed normally with mean 45 and standard deviation 14. Find the number of examinees whose marks out of 100 were : 5
- (i) less than 30
- (ii) between 30 and 70.
- (b) The joint p. d. f. of a two-dimensional random variable (X, Y) is given as : 5

$$f(x, y) = xy^2 + \frac{x^2}{8}; \quad 0 \leq x < 2, \quad 0 \leq y \leq 1$$

Compute :

- (i) $P(X > 1)$
- (ii) $P\left(Y < \frac{1}{2}\right)$
- (iii) $P\left(X > 1 \mid Y < \frac{1}{2}\right)$
5. (a) Buses arrive at a specified stop at 15 minutes interval starting at 7 A.M. If a passenger arrives at the stop at a random

time that is uniformly distributed between 7 A.M. and 7 : 30 A.M., then find the probability that he waits : 5

- (i) less than 5 minutes for a bus
- (ii) at least 12 minutes for a bus.

(b) An unbiased die is cast until 6 appears. What is the probability that it must be cast more than five times ? Also find its mean and variance. 5

6. (a) A multiple choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling an unbiased die and choose the first answer if he gets 1 or 2, the second answer if he gets 3 or 4 and the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction ? 5

(b) Suppose that on an average 1 customer per minute arrives at a shop. What is the probability that the shopkeeper will wait more than 3 minutes before the first two customers arrive ? Assume that waiting time follows gamma distribution. 5

7. (a) The joint probability distribution of a pair of random variable (X, Y) is given in the following table : 5

$\begin{matrix} \text{Y} \\ \text{X} \end{matrix}$	- 1	0	1
- 1	0	0	$\frac{1}{3}$
0	0	0	0
1	0	$\frac{1}{3}$	$\frac{1}{3}$

Find :

- (i) Marginal distribution of X and Y.
 - (ii) Conditional distribution of X given $Y = 1$.
- (b) A machine produces ball pens in a large number. The machine is known to produce 10 per cent defective ball pens. A quality control engineer is testing the produced ball pens randomly. What is the probability that at least 3 ball pens are examined in order to get 2 defectives ? 5

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