

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

December, 2024

**MST-011 : REAL ANALYSIS, CALCULUS AND
GEOMETRY**

Time : 2 Hours

Maximum Marks : 25

Note : *Question No. 1 is compulsory. Attempt any two questions from Question Nos. 2 to 4. Non-programmable scientific calculator is allowed. Symbols have their usual meanings.*

1. (a) Define appropriate function and then find the value of $\text{sgn}(-8.24)$. 2
- (b) Find the direction cosines of the normal to the plane $2x - 3y + 6z = 8$. 1
- (c) Without evaluating the integral $\int_0^\infty \frac{x^3}{(1+x)^9} dx$, find its value. 2

2. (a) Define inverse of a function. Show that exponential and logarithm functions are inverse of each other. Plot the graph of exponential and logarithm functions simultaneously on the same plot. 4
- (b) Define each term of the triplet (Ω, A, μ) . What is the name of this triplet ? Also, give *one* example to explain the meaning of each term of this triplet. 6
3. (a) Find the length/area/volume of the region formed by $2 \leq x \leq 7$ and $3 \leq y \leq 10$ using double integral. 2
- (b) Evaluate $\int_0^{\infty} x^9 e^{-8x} dx$ using properties of beta and gamma functions. 1
- (c) Prove that intersection of convex sets is also a convex set. 4
- (d) Find the equation of the hyperplane in $\mathbf{R}^{10}$ in Cartesian form which passes through the point A (1, 2, 3, 4, 5, 6, 7, 8, 9, 0) perpendicular to the direction having direction ratio $\langle 5, 4, 1, 7, -3, 2, 1, 6, -2, 3 \rangle$. 3

[3]

4. (a) If $f : [5, 15] \rightarrow \mathbf{R}$ be a function defined by $f(x) = 5x + 2, x \in [5, 15]$, show that f is Riemann integrable using definition 1 and definition 2. 5
- (b) State and prove connection between convex set and convex function. 5

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