

**M. SC. (APPLIED STATISTICS)
(MSCAST)**

Term-End Examination

December, 2024

MST-016 : STATISTICAL INFERENCE

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining question nos. 2 to 6.*

(iii) *Use of scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers :

$$5 \times 2 = 10$$

- (a) If the sample size of a survey has decreased 4 times, then the standard error will be increased 2 times.
- (b) The moment estimators are generally more efficient than the maximum likelihood estimators.

- (c) If an estimator (T) is unbiased estimator of parameter θ , then the mean squared error (MSE) will be :

$$\text{MSE} = \text{Var} (T) + (\text{Bias})^2$$

- (d) If mean of a population is 30, the 90% confidence interval of variance based on sample of size 20 will be :

$$P \left[\frac{\sum_{i=1}^{20} (X - 30)^2}{\chi^2_{(19), 0.05}} \leq \sigma^2 \leq \frac{\sum_{i=1}^{20} (X - 30)^2}{\chi^2_{(19), 0.95}} \right] = 0.90$$

- (e) A patient suffering from fever visits to a doctor and suppose the doctor formulates the following hypotheses :

H_0 : The patient is suffering from COVID-19

H_1 : The patient is not suffering from COVID-19

If the doctor rejects H_0 when the patient is actually suffering from COVID-19, then the doctor commits Type-II error.

2. The age (in years) of five workers of a small manufacturing unit are given as follows : 10

28, 24, 30, 26, 32

- (i) How many samples of size 2 are possible without replacement ? Write them. Also prepare the sampling distribution of sample mean and plot it.
- (ii) Prepare the sampling distribution of the mean when samples of same size are drawn with replacement. Construct the graph.
- (iii) Compare the graphs under both conditions (i) and (ii).
3. The magnitude of earthquake (on the Richter scale) recorded in a region are given as follows :

10

6.5, 7.7, 5.6, 7.3, 6.7, 7.8, 6.7, 6.2, 5.2, 6.6, 6.0, 7.0, 7.2, 6.8, 7.2

It is observed that the earthquake follows an exponential distribution with parameter θ whose p.d.f. is given by :

$$f(x) = \frac{1}{\theta} e^{-x/\theta}; \quad x > 0, \theta > 0$$

Find :

- (i) Maximum likelihood estimator of the parameter θ .

- (ii) The maximum likelihood estimate of θ on the basis of the given data.
4. (a) A company made a new model of a car and is interested to estimate the average mileage of the model. For that purpose a statistician of the company, randomly selected 6 cars of new model and measured their mileages. He suggested two estimators (T_1 and T_2) for the same as :

$$T_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}{6}$$

$$T_2 = \frac{X_1 + 2X_2 + 3X_3 + 4X_4 + 5X_5 + 6X_6}{21}$$

where X_1, X_2, X_3, X_4, X_5 and X_6 represent the mileages of the selected 6 cars. It is known that the mileage of car follows normal distribution with mean μ and variance σ^2 . 7

- (i) Are both estimators unbiased ?
- (ii) Which one is more efficient ?
- (b) Define consistency of a estimator with example. 3
5. An experiment was conducted to compare the defective items produced by two different

machines A and B. The data on number of defective items produced by both machines were observed and are given in the following table :

Machine A	Machine B
26	19
37	22
40	24
35	27
30	24
30	18
40	20
26	19
30	25
35	
45	

Assuming that the parent populations are normal, test that the variance of the defective items produced by machine A is greater than machine B, at 1% level of significance. 10

6. Write short notes on the following : 3+3+4

- (i) Central limit theorem
- (ii) Properties of maximum likelihood estimator
- (iii) Types of errors in testing of hypothesis

Table values

(Some values for use, if required)

Z-value	<i>t</i> -value	F-value
$Z_{0.005} = 2.58$	$t_{10, 0.01} = 3.17$	$F_{(10, 8) 0.01} = 5.81$
$Z_{0.025} = 1.96$	$t_{8, 0.01} = 3.36$	$F_{(11, 9) 0.01} = 5.18$
$Z_{0.05} = 1.645$		

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