No. of Printed Pages : 5 **MST–018**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2024

MST–018 : MULTIVARIATE ANALYSIS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. 1 is compulsory.*

*(ii) Attempt any **four** questions from the remaining Question Nos. **2** to **6**.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. State whether the following statements are True or False. Give reasons in support of your answers : $2\times5=10$

   (a) Multivariate normal distribution is an analogue of univariate normal distribution.

(b) The first and second principal components in principal component analysis depend on each other.

(c) If :

$$A = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 8 & -1 \\ 2 & -1 & 4 \end{pmatrix}$$

and $$B = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & 0 \\ -2 & 0 & 6 \end{pmatrix},$$

then the trace of matrix A is greater than trace of matrix B.

(d) If the variance-covariance matrix Σ of a random vector $\underset{\sim}{X}$ is partitioned as :

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

then $\Sigma_{21} = \Sigma_{12}$.

(e) Canonical correlation is the maximum correlation between the variates and linear combination of the variates.

2. (a) Let $\underset{\sim}{X}_{p\times 1} \sim N_p(\underset{\sim}{\mu}, \Sigma)$. Also let $\underset{\sim}{X}$, $\underset{\sim}{\mu}$ and Σ be partitioned as :

$$\underset{\sim}{X}_{p\times 1} = \begin{pmatrix} \underset{\sim}{X}^{(1)}_{k\times 1} \\ \underset{\sim}{X}^{(2)}_{(p-k)\times 1} \end{pmatrix}, \quad \underset{\sim}{\mu}_{p\times 1} = \begin{pmatrix} \underset{\sim}{\mu}^{(1)}_{k\times 1} \\ \underset{\sim}{X}^{(2)}_{(p-k)\times 1} \end{pmatrix} \text{ and}$$

$$\Sigma_{p\times p} = \begin{pmatrix} \Sigma_{11\ k\times k} & \Sigma_{12\ k\times(p-k)} \\ \Sigma_{21\ (p-k)\times k} & \Sigma_{22\ (p-k)\times(p-k)} \end{pmatrix}.$$

Then prove that the sub-vectors $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ are independently normally distributed if and only if $\Sigma_{12} = 0$, i.e., $\text{Cov}\left(\underset{\sim}{X}_{k\times 1}\right), \left(\underset{\sim}{X}^{2}_{(p-k)\times 1}\right) = 0$, where 0 is the null matrix. 5

(b) Let $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ has the following joint density function :

$$f(x_1, x_2) = \begin{cases} 2\left(x_1 + x_2 - 4x_1^3 x_2\right), & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & , \text{ otherwise} \end{cases}$$

Then find the mean vector of $\underset{\sim}{X}$ and the diagonal elements of the variance-covariance matrix of $\underset{\sim}{X}$. 5

3. (a) Let $\underset{\sim}{X}$ be a 3-dimensional vector with dispersion matrix $\Sigma = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. Then determine the first principal component and the proportion of the total variability that it explains. 5

(b) If $\underset{\sim}{X}_{p \times 1} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$, where Σ is a diagonal matrix, then the components of $\underset{\sim}{X}_{p \times 1} = \begin{pmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ \cdot \\ X_p \end{pmatrix}$ are independently normally distributed as $X_i \overset{\text{Ind.}}{\sim} N_1\left(\mu_i, \sigma_{ii}\right)$ for $i = 1, 2, \ldots\ldots, p$. 5

4. (a) If $\Sigma = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the variance-covariance matrix of a random vector $\underset{\sim}{X}$, then obtain the corresponding correlation coefficient's matrix. 5

(b) If $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$, then prove that :

$$\underset{\sim}{Y}_{q\times 1} = C_{q\times p}\underset{\sim}{X}_{p\times 1} + \underset{\sim}{V}_{p\times 1} \sim N_q\left(C\underset{\sim}{\mu} + \underset{\sim}{V}, C\Sigma C'\right),$$

where C is any $q \times p$ matrix of rank $q(\leq p)$ of constant elements and $\underset{\sim}{V}_{p\times 1}$ is a p component vector of constants. 5

5. (a) Define Wishart distribution. If $A_1 \sim \omega(n_1, p, \Sigma)$ and $A_2 \sim \omega(n_2, p, \Sigma)$, where A_2 is independent of A_1, then prove that : 5

$$A_1 + A_2 \sim \omega(n_1 + n_2, p, \Sigma).$$

(b) Define sample discriminant function. Write the stepwise classification procedure using sample discriminant function. 5

6. Obtain and draw the dendrogram for the following distance matrix using single linkage method : 10

$$D = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & & & & \\ 7 & 0 & & & \\ 2 & 5 & 0 & & \\ 4 & 3 & 7 & 0 & \\ 9 & 8 & 1 & 6 & 0 \end{pmatrix} \end{array}$$

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