No. of Printed Pages : 3 **MST–022**

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2024

MST-022 : LINEAR ALGEBRA AND MULTIVARIATE CALCULUS

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Attempt any **four** questions from the remaining question nos. **2** to **6**.*

(iii) Use of scientific calculator (non-programmable) is allowed.

(iv) Symbols have their usual meanings.

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1. (a) Explain the meaning of a vector from data science perspective. 2

(b) Which of the following matrices, $A \in \mathbf{R}^{2 \times 2}$ will not satisfy $A^4 = I_{2\times 2}$? 3

(i) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) Evaluate : 2

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x - y}$$

(d) Find the linearisation of the function $f(x, y) = x^2 + y^2 + 1$ at the point (1, 1). 3

2. Find singular value decomposition (SVD) of the matrix A, where $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. 10

3. (a) Find the equation of the level surface for the function $f(x, y, z) = \dfrac{x + y - z}{2x - y - z}$ passing through the point P(1, – 1, 1). 2

(b) Find the pseudo-inverse of the matrix : 8

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

4. (a) Give an example of a linear transformation which is self-inverse of itself. 4

(b) Give example of two linear transformations which commute. 4

(c) Write any *two* permutation matrices. 2

5. (a) Obtain gradiant of the function $2x^2 - \frac{y^2}{3}$ at the point (1, 2). 3

(b) Using Taylor's formula for $f(x, y) = xe^y$, find the quadratic approximation of $f(x, y)$ near the origin. 7

6. (a) Find trace of the Hessian matrix H of the function $f : \mathbf{R}^2 \to \mathbf{R}$: 5

$$f(x, y, z) = 3x^2 + 2y^2 + 5z^2 - xz - 3yz.$$

(b) Let $z = f(x, y)$, where $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$, α is a constant. Find the value of $f_u^2 + f_v^2$ in terms of f_x^2 and f_y^2. 5

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