No. of Printed Pages : 5 **MMT–003**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question No. **1** is compulsory.*

*(ii) Answer any **four** questions from Q. Nos. **2** to **6**.*

(iii) Calculators are not allowed.

(iv) Show all the steps involved in every solution.

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1. Which of the following statements are True and which are False ? Justify your answers with a short proof or counter-example, whichever is appropriate : 10

(i) If a group of order 7 acts on a set with 13 elements, then there must be at least 6 fixed points.

(ii) There is a unique (upto isomorphism) group of order 119.

(iii) In the ring $\mathbf{Z}+\mathbf{Z}_w \subseteq \mathbf{C}$, $w = e\frac{2\pi i}{3}$, 7 is an irreducible element.

(iv) $\mathrm{SO}(n)(\mathbf{R}) = \mathrm{SL}_n(\mathbf{R})$, where $n \in \mathbf{N}$.

(v) If a field extension F/K is separable, then it is normal.

2. (a) Show that all the roots of $x^{31} + \bar{i} \in \mathbf{Z}_5[x]$ lie in $\mathbf{F}_{5^3}$. Hence determine the splitting field of this polynomial. 5

(b) How many isomorphism classes of abelian groups are there of order $5^4 \times 7^3$? 2

(c) Show that if $AT = A'T'$, where $A, A' \in O(n)$ and T, T' are upper triangular matrices with positive diagonal entries, then $A = A'$. 3

3. (a) Find an integer x, $720 < x < 1440$, such that $x \equiv 1 \pmod 5, x \equiv 3 \pmod{16}$ and $x \equiv 5 \pmod 9$. 5

(b) Check whether or not a group of order 12 is simple. 5

4. (a) Give an example, with justification, of a domain which has an irreducible element that is not prime. Is it possible to find a prime element in a domain which is not irreducible ? Justify your answer. 5

(b) (i) State Eisenstein's criterion. 1

(ii) How many field homomorphisms are there from $\mathbf{Q}\left(\sqrt[8]{2}\right)$ to $\mathbf{C}$? How many of these will have their images in $\mathbf{R}$? Justify your answers.

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5. (a) Describe the group with presentation $\left\langle a, b \mid a^5, b^2, baba \right\rangle$. What is its order ? Justify your answer. 5

(b) Show that the number of distinct monic irreducible polynomials of degree 3 in $\mathbf{Z}_7[x]$ is 112. 5

6. (a) Let $G = GL_n(\mathbf{R})$ operate on the set $X = \mathbf{R}^n$ by left multiplication. Describe the decomposition of X into orbits under this operation. Further, find stab(e_1). 5

(b) Let * be a binary operation on **N**, defined by $a * b$ = minimum of a and b, for a, $b \in \mathbf{N}$. Check whether or not (**N**, *) is a semigroup. 2

(c) Find all the ring homomorphisms from $\mathbf{Z}_{36}$ to $\mathbf{Z}_6$. 3

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