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MMT-004

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. SC. (MACS)]

Term-End Examination

December, 2025

MMT-004 : REAL ANALYSIS

Time : 2 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions out of
Question Nos. 2 to 6.*

(iii) *Use of calculator is not allowed.*

(iv) *Notations as in the study material.*

A-409/MMT-004

P. T. O.

1. State whether the following statements are True or False. Justify your answers :

$$5 \times 2 = 10$$

- (a) If $T = [a, b]$ and $B(T)$ denote the set of all bounded real valued functions on T , for $f, g, \in B(T)$ define :

$$d(f, g) = | f(t_0) - g(t_0) |$$

for some fixed $t_0 \in T$. Then d is a metric on $B(T)$.

- (b) The sequence $\left\{ \frac{1}{n} \right\}$ is a convergent

Cauchy sequence in $(0, 1)$ under the discrete metric on $(0, 1)$.

- (c) If a subset of a metric space is bounded, then it is totally bounded.

- (d) The function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ given by $f(x, y) = (e^x, e^y, x + 4)$ is continuously differentiable.
- (e) The sequence $f_n : \chi_{[n, n+1]}$, $n = 1, 2, 3, \dots$ satisfy all the conditions of the monotone convergence theorem.
2. (a) Suppose A and B are non-empty disjoint closed subsets of a metric space (X, d) . Prove that there exist open sets U and V such that $A \subset U$, $B \subset V$ and $U \cap V = \phi$. 3
- (b) Consider $C[0, 1]$ with the metric defined by the Riemann integral. Show that the sequence given by :

$$f_n(x) = \begin{cases} 1 & , \text{ if } 0 \leq x \leq \frac{1}{2} \\ -2^n \left(x - \frac{1}{2} \right) + 1, & \text{ if } \frac{1}{2} \leq x \leq \frac{1}{2} + \frac{1}{2^n} \\ 0 & , \text{ if } \frac{1}{2} + \frac{1}{2^n} \leq x \leq 1 \end{cases}$$

is a Cauchy sequence which is not convergent. 4

(c) Define the following with an example of each : 3

(i) A compact set in a metric space

(ii) A connected set in a metric space

3. (a) Let $f : X \rightarrow \mathbb{R}$ be a continuous function when X is a metric space. Suppose X is connected. Prove that $f(X)$ is an interval. 2

(b) Define the differentiability of a map $f : E \rightarrow \mathbb{R}^m$, E is an open set in $\mathbb{R}^n$, at a point $x \in E$. Prove that if the derivative exists, then it is unique.

Check whether the function

$f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by :

$$f(x, y) = \left(\frac{1}{x}, y \right), (x, y) \neq (0, 0)$$

$$(0, 0), (x, y) = (0, 0)$$

is differentiable or not at $(0, 0)$. 4

- (c) Define an algebra and a σ -algebra on a non-empty set X . Prove through an example, that an algebra may fail to be a σ -algebra. 4

4. (a) Compute the derivative of the function

$f : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ given by :

$$f(x_1, x_2, x_3, x_4)$$

$$= (x_1^2 - x_2^2, 2x_1x_2, x_1x_3, x_1^2x_3^2x_4^2)$$

at $(1, 2, -1, -2)$. Find also the directional derivative $D_v f(1, 2, -1, -2)$ with $v = (2, 1, -2, -1)$. 4

- (b) Check whether the function

$f : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ defined by :

$$f(x, y, z, w) = (3x + 2y, x^2 - y^2, wz, y + w)$$

is locally invertible at the point

$$(1, 2, 1, 1). \quad 3$$

- (c) Define Lebesgue outer measure and show that the Lebesgue outer measure of a countable set is 0. 3

5. (a) State the Implicit Function Theorem for a function $f : E \subset \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n$, where E is open. 2

- (b) Show that : 2

$$\overline{A} = \{x \in X : B(x, r) \cap A \neq \emptyset$$

for every $n > 0\}$

- (c) Use the dominated convergence theorem to find :

$$\lim_{n \rightarrow \infty} \int_1^{\infty} f_n(x) dx$$

where :

$$f_n(x) = \frac{\sqrt{x}}{1 + nx^5} \quad 4$$

- (d) Is a continuous function measurable ?

What about the converse ? Justify.

(The function is real valued with a measurable domain). 2

6. (a) Prove that the continuous image of a path connected metric space is connected. 3

- (b) Find the Fourier series for $f(t) = t$ on $[-\pi, \pi]$. 3

- (c) Find the critical points of the function

$$f : \mathbb{R}^3 \rightarrow \mathbb{R} \text{ given by :} \quad 4$$

$$f(x, y, z) = x^2y^2 + z^2 + 2x - 4y + z$$

and classify them.

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