

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M. SC. (MACS)]
Term-End Examination
December, 2025**

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours Maximum Marks : 25

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **three** questions from
Question Nos. 2 to 5.*

(iii) *Use of calculator is not allowed.*

1. State, giving reasons whether the following
statements are True or False : $2 \times 5 = 10$

(a) The set $\left\{z \in \mathbf{C} : 0 < \arg z < \frac{\pi}{4}\right\}$ is a
domain.

- (b) The function $f(z) = |z|^2$ is nowhere differentiable.
- (c) The identity :

$$\log \left(\frac{z_1}{z_2} \right) = \log z_1 - \log z_2$$

holds for every pair of non-zero complex numbers z_1 and z_2 .

- (d) $\int_C \frac{1}{z^2 + 5z + 6} dz = 0$, where C denotes the positive oriented circle $|z| = 1$.
- (e) The function $f(z) = \cos z$ is conformal everywhere.

2. (a) Prove that the function : 3

$$u(x, y) = y^3 - 3x^2y$$

is harmonic. Find its harmonic conjugate.

- (b) Consider the region $R = \{z : |z| \leq 3\}$. If $f(z) = 3 - z$ in R , then find a point in R , where $|f(z)|$ attains its maximum value.

2

3. (a) Find the value of $\log(-1 - i)$. 2

- (b) (i) Prove that the Mobius transformation $w = \frac{2z-1}{2-z}$ maps unit disc to itself. $1\frac{1}{2}$

- (ii) Prove that if the sequence (z_n) converges to z , then the sequence $(|z_n|)$ converges to $|z|$. $1\frac{1}{2}$

4. (a) Find the Laurent's series expansion in powers of z for the function : 3

$$f(z) = \frac{1}{z^4 + 3z^2 + 2}$$

in the region $1 < |z| < \sqrt{2}$. Also write its principal part at $z = 0$.

- (b) Without evaluating the integral, show that : 2

$$\left| \int_C \frac{1}{z^4 - 5z^2 + 4} dz \right| \leq \frac{3\pi}{40}$$

where C is upper half of the circle $|z| = 3$.

5. (a) Use Cauchy Residue theorem to evaluate : 3

$$\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta}, \quad a > 0$$

- (b) Find the radius of convergence of the power series : 2

$$\sum_{n=1}^{\infty} [5 + 2(-1)^n]^n z^n$$

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