No. of Printed Pages : 5 **MMTE–001**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination

December, 2025

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No.* ***1*** *is compulsory. Answer any* ***four*** *questions from Q. Nos.* ***2*** *to* ***7****. Symbols have their usual meanings.*

1. State whether the following statements are true *or* false. Justify your answers with a short proof or a counter-example : 2×5=10

   (i) If S is a vertex-cover of a graph G, then $\overline{S}$ is also a vertex-cover of G.

   (ii) Every acyclic graph is planar.

   (iii) $L(K_4)$ is 2-colourable.

   (iv) Every 2-connected graph is Hamiltonian.

   (v) For every subgraph H of a graph G, $\alpha(H) \leq \alpha(G)$.

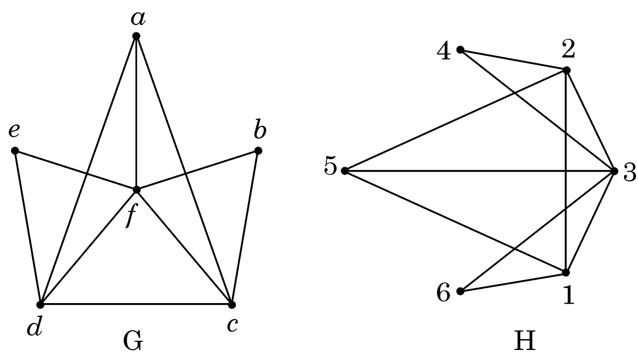
2. (a) Check whether the sequence :

   $$(6, 4, 4, 4, 3, 3, 2, 2, 1, 1, 1, 1)$$

   is a graphic sequence. 3

   (b) Find the girth of the Peterson graph. 4

(c) Check whether the following graphs are isomorphic or not : 3



3. (a) Show that the Grötzsch graph is 4-chromatic. 5

(b) Let G be a planar graph with at least 11 vertices. Show that $\overline{G}$ is non-planar. 5

4. (a) Prove that : 6

$$\tau(k_n) = n^{n-2}$$

(b) Define a semi-Eulerian graph. Show that a graph having exactly two odd vertices is semi-Eulerian. 4

5. (a) Define a k-critical graph. Find a 3-critical subgraph of the Peterson graph. 3

   (b) Show that if G is a regular graph, then so is L(G). 2

   (c) Let G be a planar graph with n vertices, m edges, r faces and k components. Show that : 5

$$n - m + r = k + 1$$

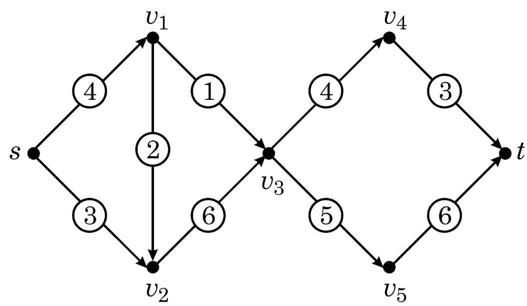
6. (a) State and prove Hall's theorem. 7

   (b) Draw the Cartesian product of K_3 and C_4. Is the product a regular graph ? Justify. 3

7. (a) Let G be a connected graph with blocks $B_1, B_2, \ldots B_k$. Show that : 5

$$n(G) = \sum_{i=1}^{k} n(B_i) - k + 1.$$

(b) Define a flow on the following network having value at least 5 : 5



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