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MMTE-005

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. SC. (MACS)]

Term-End Examination

December, 2025

MMTE-005 : CODING THEORY

Time : 2 Hours

Maximum Marks : 50

Note : (i) *There are six questions in this paper.*

(ii) *The **sixth** question is compulsory.*

(iii) *Do any **four** questions from Question
Nos. 1 to 5.*

(iv) *Show all the relevant steps. Do the
rough work at the bottom or at the
side of the page only.*

(v) *Calculators are **not** allowed.*

C-2760/MMTE-005

P. T. O.

1. (a) When do we say that the parity check matrix of a $[n, k]$ linear code is in standard form. Check whether the parity matrix :

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

of a linear code C is in standard form or not. What are the length and dimension of C ? 3

- (b) Define the Hamming distance between two code words. What is the Hamming distance between the two code words 100110 and 110101 ? 2

- (c) What is a repetition code ? If, in a repetition code in which the message of

length three is sent thrice, the code word 110 110 100 is received, decode the message assuming there is at most one error. 2

- (d) Let $\mathbf{C}$ be a binary cyclic code of length nine with generator polynomial $x^2 + x + 1$. Find the generator matrix and the parity check matrix of the code. 3

2. (a) Let $n \in \mathbf{N}$, q be a power of a prime and $0 \leq s < n$. Define the q -cyclotomic coset of s modulo n . Find the 8-cyclotomic set of 1 modulo 17. 3

- (b) Find the gcd of $x^5 + 2x^3 + 2x^2 + 3x + 1$, $x^4 + 2x^3 + x^2 + 3x + 1 \in \mathbf{F}_5[x]$. 4

- (c) Let C_1 be the $[4, 3]$ binary linear code generated by :

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

and let C_2 be the $[4, 1]$ -binary linear code generated by $[1010]$. Let C be the code obtained through using $(u \mid u + v)$ construction on the codes C_1 and C_2 . Find the generator matrix of C . Also, give the length and the dimension of C . 3

3. (a) Prove that the integers modulo n do not form a field if n is not a prime. 2

- (b) The systematic generator matrix for a $[6, 3]$ linear block code is

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}. \text{ Compute the standard}$$

array for syndrome decoding. 5

(c) Check whether the polynomial

$x^3 + x + 1 \in \mathbf{F}_{32}$ is primitive. You are

given that $x^{30} \equiv x^2 + 1 \pmod{x^3 + x + 1}$.

You may assume that the polynomial is

irreducible. 3

4. (a) Let $\mathbf{C}$ be the narrow-sense binary BCH

code of designed distance $\delta = 5$, which

has a defining set $T = \{1, 2, 3, 4, 6, 8,$

$9, 12\}$. Let α be a primitive 15th root of

unity, where $\alpha^4 = 1 + \alpha$ and let the

generator polynomial of $\mathbf{C}$ be

$g(x) = x^8 + x^7 + x^6 + x^4 + 1$. If $y(x) = x$

$+ x^4 + x^7 + x^8 + x^{11} + x^{13}$ received, find

the transmitted code word. You can use

Table 1. 6

Table 1

0000	0
0001	1
0010	α
0100	α^2
1000	α^3
0011	α^4
0110	α^5
1100	α^6
1011	α^7
0101	α^8
1010	α^9
0111	α^{10}
1110	α^{11}
1111	α^{12}
1101	α^{13}
1001	α^{14}

- (b) Find the convolutional code (2, 1) with generator matrix $G = [1, 1 + D]$, for the message $m = 1 + D + D^3$. 4

5. (a) Let C be the $[7, 4, 2]$ binary code with the following parity check matrix : 5

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- (i) Give the Tanner graph for this code.
- (ii) List all the code words of C and hence find its minimum distance.
- (b) List all the code words of the code C over $\mathbf{Z}_4$ generated by :

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Also, find the Lee weight distribution of the code. 5

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter example.

$$5 \times 2 = 10$$

- (a) If F is a field and the polynomial $p(x) \in F[x]$ has no roots in F , then $p(x)$ is irreducible over F .
- (b) The code with generator matrix :

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

has a unique code word of weight three.

- (c) A quadratic code of length seven exists over $\mathbf{F}_3$.
- (d) The parity check matrix of a turbo code can be the identity matrix.
- (e) Every perfect code is self dual.

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