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MPH-001

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2025

MPH-001 : MATHEMATICAL METHODS

IN PHYSICS

Time : 2 Hours

Maximum Marks : 50

Note : (i) Answer any **five** questions.

(ii) The marks are indicated against each question.

(iii) Symbols have their usual meanings.

(iv) You may use a non-programmable calculator.

A-410/MPH-001

P. T. O.

1. Consider heat flow in a uniform bar of length L insulated along its length. The temperature of the bar is modelled by the one-dimensional heat diffusion equation :

4+6

$$\frac{\partial T(x, t)}{\partial t} = K \frac{\partial^2 T(x, t)}{\partial x^2} \quad (0 < x < L, t > 0)$$

- (i) Separate the equation into two ODEs.
- (ii) If the initial temperature distribution is given by :

$$T(x, 0) = (2L - x) \quad (0 < x < L)$$

solve the heat equation for the boundary conditions on $T(x, y)$ given as :

$$T(0,t)=0 \text{ and } \frac{\partial T(L,t)}{\partial x} = 0, \quad t \geq 0$$

2. (a) The generating function for Bessel's functions of the first kind and integral order is given by : 5

$$g(x,t) = \exp\left[\frac{x}{2}\left(t - \frac{1}{t}\right)\right] = \sum_{n=-\infty}^{+\infty} J_n(x)t^n$$

Show that :

$$J_0(x) + 2J_2(x) + 2J_4(x) + \dots$$

$$+ 2J_{2k}(x) + \dots = 1$$

- (b) The expression for Bessel's function of the first kind of order m is given by : 5

$$J_m(x) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k! \Gamma(m+k+1)} \left(\frac{x}{2}\right)^{2k+m}$$

Using this relation, derive the recurrence relation :

$$\frac{m}{x} J_m(x) + \frac{dJ_m}{dx} = J_{m-1}(x)$$

3. (a) Starting with generating function of Legendre polynomials of first kind : 4

$$g(x, t) = (1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) t^n$$

show that :

$$P_n(-x) = (-1)^n P_n(x)$$

- (b) Using the generating function of Hermite polynomials : 6

$$g(x, t) = e^{2xt - t^2} = \sum_{n=0}^{\infty} \frac{H_n(x) t^n}{n!}$$

evaluate the integral :

$$\int_{-\infty}^{+\infty} x e^{-x^2} H_n(x) H_m(x) dx$$

4. (a) Show that the vectors $(\bar{V}_1, \bar{V}_2, \bar{V}_3)$,

$$\bar{V}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \bar{V}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \bar{V}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

form the basis in $\mathbb{R}^3$, the vector space of three-dimensional vectors. 4

(b) Let $A = \begin{bmatrix} 1 & -a & -b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -d & -e \\ 0 & 1 & -f \\ 0 & 0 & 1 \end{bmatrix}$

Calculate AB and show that it is also in form of upper triangular matrix. Calculate the value of d, e, f , so that $AB = 1$. Find the condition that $AB = BA$. 6

5. (a) Obtain the normalized eigen vectors for a real symmetric matrix : 6

$$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & -\cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and find its diagonalizing matrix.

(b) Show that :

$$u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$$

is harmonic in some domain.

Calculate its conjugate harmonic

function $v(x, y)$ and construct the

complex function $f(z)$. 4

6. (a) Determine the value of Gamma function

$$\Gamma\left(\frac{1}{2}\right). \quad 6$$

(b) Determine and classify all the

singularities of the following complex

functions : 4

$$(i) \quad \frac{z}{e^{1/z} - 1}$$

$$(ii) \quad \frac{1}{(2\sin z - 1)^2}$$

7. Evaluate the integral $\int_{-\infty}^{+\infty} \frac{\cos x \, dx}{x^2 + a^2}$ by the method of Residues. 10

8. (a) Solve the initial value problem : 7

$$y'' + 5y' + 4y = f(t)$$

$$y(0) = 0, y'(0) = 0$$

$$\text{with } f(t) = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

using the methods of Laplace transforms.

- (b) Define equivalence classes of a group. 3

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