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MPH–004

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2025

MPH–004 : QUANTUM MECHANICS–I

Time : 2 Hours *Maximum Marks : 50*

Note *: (i) Attempt any* ***five*** *questions.*

(ii) Symbols have their usual meanings.

(iii) The values of physical constants are given at the end.

(iv) You may use a calculator.

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1. (a) A thermal neutron with a speed v corresponding to the average thermal energy $\frac{3}{2}k_{\mathrm{B}}\mathrm{T}$ at T = 300 K is incident on a crystal. Will a diffraction pattern be observed ? Explain. 5

(b) Use the uncertainty principle to derive an expression for the average size of the Hydrogen atom. 5

2. (a) In a region of space a particle of mass m and zero energy has a wave function $\psi(x) = \mathrm{N}xe^{-x^2/9}$ where N is a constant. Use the time-independent Schrödinger equation to derive the potential energy $\mathrm{V}(x)$ of the particle. 5

(b) Show that : 5

$$\left[\hat{x}^2, \hat{p}_x^2\right] = 2i\hbar\left(\hat{x}\hat{p}_x + \hat{p}_x\hat{x}\right).$$

3. Consider a one-dimensional potential well with an infinite barrier at $x = 0$ and a finite potential V for $x \geq \mathrm{L}$. (a) State the boundary conditions. (b) For a particle of mass m inside the well with an energy $\mathrm{E} < \mathrm{V}$, derive the quantization condition for the energy.

3+7

4. Write down the eigen value equations for the operators $\hat{H}, \hat{L}^2$ and $\hat{L}_z$ for the hydrogen atom eigen function $\psi_{n_l m_l}$. List the eigen-functions for $n = 3$. What is the number of degenerate eigen funtions for $n = 3$? For the ground state of the hydrogen atom which has the wave function :

$$\psi_{100} = \frac{1}{\sqrt{\pi} a_0^{3/2}} \exp\left(\frac{-r}{a_0}\right)$$

calculate the expectation value of the potential energy $V(r) = \frac{-e^2}{r}$. You may use

$\int_0^{\infty} e^{-ax} x^m dx = \frac{1}{a^{m+1}} m!$. 10

5. (a) Explain the importance of the simple harmonic oscillator potential in Physics. 2

(b) Write down the Schrödinger equation for a one-dimensional simple harmonic oscillator of frequency ω and write the expression for its eigen energy E_n. 3

(c) If the initial wave function for a simple harmonic oscillator is : 5

$$\psi(x,0) = \frac{1}{\sqrt{3}}\psi_2(x) + i\sqrt{\frac{2}{3}}\psi_0(x)$$

determine $\psi(x,t)$.

6. (a) State any *two* properties of linear operators in Hilbert space. 2

(b) Show that a linear combination of the degenerate eigen vectors of an operator belonging to a particular eigen value of the operator is also an eigen vector belonging to the same eigen value. 4

(c) Define Hermitian and anti-Hermitian operator. For an operator $\hat{A}$, show that $\left(\hat{A}+\hat{A}^{+}\right)$ is Hermitian and $\left(\hat{A}-\hat{A}^{+}\right)$ is anti-Hermitian. 4

7. (a) $|\phi_1\rangle$ and $|\phi_2\rangle$ are the othonormal basis states of a two-dimensional Hilbert space. The Hamiltonian $\hat{H}$ in

this basis is given by the spectral representation :

$$\hat{\mathrm{H}} = \mathrm{E}_0 \left[|\phi_1\rangle \langle\phi_2| + |\phi_2\rangle \langle\phi_1| \right]$$

Determine the eigen values and normalized eigen vectors of $\hat{\mathrm{H}}$. 5

(b) Using the definition of the lowering operator $\hat{a}\frac{-}{\sqrt{2\hbar m\omega}}(m\omega\hat{x} + i\hat{p})$ derive the ground state wave function for the simple harmonic oscillator. 5

8. (a) Write the angular momentum eigen kets $\left(|lj, m_j\rangle\right)$ for $j = \frac{1}{2}$. Derive the matrix elements for J^2 and J_z for $j = \frac{1}{2}$. 5

(b) Describe briefly the Stern-Gerlach experiment and explain its principal result. 5

Physical constants :

$h = 6.626 \times 10^{-34}$ Js

$\hbar = 1.015 \times 10^{-34}$ Js

$m_p = 1.67 \times 10^{-27}$ kg

$m_n = 1.675 \times 10^{-27}$ kg

$k_B = 1.38 \times 10^{-23}$ JK^{-1}

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