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M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

December, 2025

MPH-006 : CLASSICAL MECHANICS–II

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any* ***five*** *questions.*

(ii) The marks for each question are indicated against it.

(iii) Symbols have their usual meanings.

(iv) You may use a calculator.

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1. (a) The Hamiltonian of a simple harmonic oscillator is given as : 5

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

Show that the velocity vector is tangential to the curve defined by the Hamiltonian.

(b) Consider two Lagrangians, L (q, p, t) and L′ (q, p, t) such that : 5

$$L'(q, p, t) = L(q, p, t) + \frac{dF}{dt}(q, p, t)$$

Show that the variation of action is the same for both the Lagrangians that is $\delta S = \delta S'$.

2. (a) Consider a particle of mass m moving under a uniform force $\vec{F} = F\hat{j}$. If the Lagrangian is given as : 5

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - F_y$$

Determine the conservation laws under rotation.

(b) A particle of mass m has the Lagrangian : 5

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + ax^5$$

Obtain the Hamiltonian of the system and the corresponding Hamilton's equations of motion. Take k and a as constants.

3. (a) Write the bilinear invariant condition for canonical transformation. Using the condition, show whether the transformation : 1+4

$$Q = \frac{q^2}{\sqrt{2}}, \; P = \frac{p^2}{\sqrt{2}}$$

is a canonical transformation.

(b) Consider a dynamical system described by a Hamiltonian : 5

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

Obtain the new Hamiltonian k (Q, P) using the generating function, $F_2 = \dfrac{q\mathrm{P}}{\sqrt{mw/\hbar}}$.

4. Write down the fundamental Poisson brackets for two functions $f\ (q,\ p,\ t)$ and $g\ (q,\ p,\ t)$. If the functions $f\ (q,\ p,\ t)$ and $g\ (q,\ p,\ t)$ are constants of motion, show that the Poisson bracket of the two constants of motion is also a constant of motion. 2+8

5. The Hamiltonian for a system of two degrees of freedom is given as : 10

$$H(q_1, q_2, p_1, p_2) = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \cos(q_1 + 4q_2)$$

Obtain the constants of motion for the system in Q, P variables. Hence obtain the new Hamiltonian for $Q_1 = q_1 + 4q_2$ and $Q_2 = q_2$.

6. The Lagrangian of a damped harmonic oscillator is given as :

$$L = e^{\lambda k}\left(\frac{m\dot{q}^2}{2} - \frac{m\omega^2}{2}q^2\right)$$

(a) Obtain the Hamiltonian of the system and write the Hamilton-Jacobi (H-J) equation. 4

(b) Using the generating function : 6

$$F_2(q, P, t) = e^{\lambda t/2}qP,$$

obtain the new Hamiltonian K (Q, P). Hence write the H-J equation associated with the new Hamiltonian.

7. The Hamiltonian of a particle of mass m in a two-dimensional simple harmonic potential is : 10

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m(w_x^2 x^2 + w_y^2 y^2)$$

Determine the action-angle variables J_x and J_y (in integral form) if the total energy (E) of the system is conserved.

8. Write the Euler's equation for a rigid body. Give the conditions for a torque free motion of a symmetric top. Hence obtain the equations of motion of the rigid body. 10

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