

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

December, 2025

MST-003 : PROBABILITY THEORY

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 7.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*

(v) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers : 5×2=10

- (a) According to subjective approach to probability theory, the probability of a student to get first division will always be the same.
- (b) If Ω is the sample space of a random experiment, the conditional probability $P(\Omega/\Omega) = 0$.
- (c) If X and Y are two random variables having joint probability mass function $f_{X,Y}(x, y)$ such that :

$$f_{X,Y}(x, y) = f_X(x)f_Y(y)$$

then the random variables X and Y will be dependent.

- (d) If X following exponential distribution is a discrete random variable then :

$$P[X > t + h \mid X > h] > P[X > t]$$

- (e) If a random variable X is such that :

$$f(x) = \begin{cases} \frac{1}{3}, & 2 < x < 5 \\ 0, & \text{otherwise} \end{cases}$$

then $E(X) = 3$.

2. (a) Three coins are tossed simultaneously. Find the probability of getting two heads given that at least *one* coin has turn up a head.
- (b) A sample of 3 items is selected at random from a bag containing 10 items of which 4 are defective. Find the expected number of defective items.
- (c) Explain the difference between independence and mutually exclusiveness of two events, by giving examples. 3+4+3
3. (a) What are restrictions in classical approach to probability and what are drawbacks of subjective probability.
- (b) Write *two* advantages of a random variable.

- (c) Define binomial and Poisson distributions and give interpretation of their means. 2+2+6
4. (a) A machine is known to produce 3% defective items. What is the probability that at least 5 items are to be examined in order to get 2 defective items ?
- (b) Arnav is a football player. His success rate of goal hitting is 70%. What is the probability that Arnav hits his third goal on his seven attempt ?
- (c) State and prove memoryless property of geometric distribution. 4+3+3
5. All manufactured devices and machines fail to work sooner or later. Suppose that the failure rate is constant and the time (in

hours) to failure 'X' follows exponential distribution with parameter λ :

- (a) Measurements show that the probability that the time to failure for device (computer memory chips) in a given class exceeds 10^4 hours is e^{-1} (≈ 0.368). Calculate the value of the parameter λ .
- (b) Using value of the parameter λ determined in part (a), calculate the time x_0 such that the probability that the time to failure is less than x_0 is 0.05.

5+5

6. A production line manufactures 1000-ohm (Ω) resistors that have 10 percent tolerance. Let X denote the resistance of a resistor. Assuming that X is a normal random variable with mean 1000-ohm and variance 2500-(ohm)^2 , find the probability that a resistor selected at random will be rejected.

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7. An information source generates symbols at random from a four-letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{4}$ and $P(c) = P(d) = \frac{1}{8}$. A coding scheme encodes these symbols into binary codes as follows :

a 0

b 1 0

c 1 1 0

d 1 1 1

Let X be the random variable denoting the length of the code, that is, the number of binary symbols (bits).

- (i) What is the range of X ?
- (ii) Assuming that the generations of symbols are independent, find the probabilities $P[X = 1]$, $P[X = 2]$, $P[X = 3]$ and $P[X > 3]$.
- (iii) Obtain cdf of the random variable X .
Also sketch its graph. 10

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