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MST-012

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

December, 2025

MST-012 : PROBABILITY AND PROBABILITY
DISTRIBUTIONS

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining question numbers 2 to 6.*

(iii) *Use of scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

B-1765/MST-012

P. T. O.

1. (a) Let Ω be the sample space of the random experiment of tossing a coin twice. Let X denotes the number of heads in two tosses and Y denotes the number of tails before the first head. Prepare the joint probability distribution of X and Y . 2
- (b) Suppose in a party there are n (< 365) persons. Assume that each of them is born in a non-leap year. Also, assume that birth rate is uniform throughout the 365 days of a year. Find the probability of at least one sharing birthday. 2
- (c) Obtain MGF of continuous uniform distribution. 3

- (d) If $X \sim \text{Expo}(2)$, then find upper bound of $P(X \geq 2)$. 3

2. Define induced probability space by a random variable on the real line. 10

3. (a) Joint PDF of the jointly continuous random variable (X, Y) is given by :

$$f_{X,Y}(x,y) = \begin{cases} \lambda_1, \lambda_2 e^{-(\lambda_1 x + \lambda_2 y)}; & x, y > 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

Find the PDF of the random variable

$$\frac{X + Y}{2}. \quad 8$$

(b) If PMF of a random variable X is given by :

X	$p(x)$
0	0.1
1	0.4
2	0.3
3	0.2

find the PMF of the random variable

$$2X + 7. \quad 2$$

4. (a) If $X \sim \text{Pois}(2)$ and $Y \sim \text{Pois}(3)$ are two independent random variables, then find $P(X + Y < 4)$. 4

- (b) If $X \sim \text{Beta}(4, 5)$, then write the PDF and CDF of X . 3

- (c) If $X \sim \text{Gamma}(100, 5)$, then write the PDF and CDF of X . 3

5. (a) If $X \sim \text{Lap}(30, 5)$, then find probability that the random variable X is less than 28. 5

- (b) If $X \sim \text{LN}(\mu, \sigma^2)$, then prove that : 5

$$F_X(\mu) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)$$

6. Before the result of the election between two candidates A and B, we do not know what percent of votes candidates A and B will get. Suppose candidate A gets p percent of votes and candidate B gets q percent of votes, where $p + q = 100$ percent. Values of p and q are unknown before the results of the election. Suppose you being a statistician are given the job to estimate the value of p prior to election. Suppose your organisation wants that your estimate of p can be at the most 3 percent away from the true value of p . To meet the requirement of your organisation you need to conduct a random survey of the voters where you have to record the responses as 'yes' or 'no' in favour

of candidate A (say). Assume that every person of your sample gives his/her responses as 'yes' or 'no' only. How large size of the sample you will need to make your estimate within 3 percent of the true value of p with a probability of at least 0.95 ? 10

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