

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

December, 2025

**MST-022 : LINEAR ALGEBRA AND
MULTIVARIATE CALCULUS**

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 6.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers : 5×2=10

- (a) The equation of the level surface of the function $f(x, y, z) = \sqrt{y^2 - xz} + 3y$ through the point $(1, 4, 7)$ is $\sqrt{y^2 - xz} + 3y = -3$.

- (b) The function :

$$W = \{(x, y, 2x + 3y) : x, y \in \mathbf{R}\}$$

is a subspace of $\mathbf{R}^3(\mathbf{R})$.

- (c) The value of $h \in \mathbf{R}$ for which the set of vectors $\{(2, h, 4), (5, h, 2), (3, h, 0)\}$ is linearly dependent is 0.

- (d) The directional derivative of the function $f : \mathbf{R}^3 \rightarrow \mathbf{R}$ is defined by $f(x, y, z) = 2xy^2 + yz + z^2$ at $Q(1, 1, 0)$ in the direction of $\vec{a} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$ equals to -1 .

- (e) The matrix :

$$A = \begin{Bmatrix} 1 & -1 \\ 0 & 1 \end{Bmatrix}$$

satisfies $A^4 = I_{2 \times 2}$.

2. Find SVD of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. 10

3. Find an orthonormal basis for the subspace of $\mathbf{R}^4$ having basis : 10

$$w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, w_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, w_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 4 \end{bmatrix}.$$

4. (a) If $\ln z = z^2 + 2y - x$, then find the value of $\frac{\partial z}{\partial y}$ at $(1, 0, 2)$. 5

(b) Find $\frac{\partial w}{\partial r}$ when $r = 1$, $s = -1$, if

$$w = (x + y + z)^2, x = r - s, y = \cos(r + s), \\ z = \sin(r + s). \quad 5$$

5. (a) If :

$$U(x, y) = x^2 y \tan^{-1}\left(\frac{y}{x}\right) + xy^2 \sin^{-1}\left(\frac{x}{y}\right); x, y > 0,$$

then find the value of : 5

$$x^2 U_{xx} - x U_x + 2xy U_{xy} - y U_y + y^2 U_{yy}.$$

- (b) Using steepest descent algorithm, find the value of the function :

$$f(x) = 2x^2 + 2xy + y^2 + x - y$$

at the second iteration, starting with initial guess as $X_0 = (0, 0)^T$. 5

6. Explain how principle of gradient descent works. 10

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