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MSTE-012

M. SC. (APPLIED STATISTICS)

(MSCAST)

Term-End Examination

December, 2025

MSTE-012 : STOCHASTIC PROCESSES

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining question nos. 2 to 6.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Symbols have their usual meanings.*

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P. T. O.

1. State whether the following statements are True or False. Give reasons in support of your answers :

$$5 \times 2 = 10$$

- (a) If parametric space 'T' is continuous and state space 'S' is discrete, then the stochastic process is a Discrete valued stochastic process.
- (b) The sum of two independent Poisson processes $X_1(t)$ and $X_2(t)$ with parameters $\lambda_1(t)$ and $\lambda_2(t)$, respectively is also a Poisson process with parameter $(\lambda_1 + \lambda_2)t$.
- (c) The average waiting time of a customer in the queue $E(W_q)$ is $\left(\frac{\lambda}{4}\right)$ times of the average length of the queue L_q .
- (d) The complex random process $Z(t) = X(t) + iY(t)$ is said to be Wide

Sense Stationary if $\{X(t)\}$ and $\{Y(t)\}$ are jointly stationary in strict sense.

- (e) If the one-step transition probability does not depend on the step, i.e., $p_{ij}(n-1, n) = p_{ij}(m-1, m)$, the Markov chain is called a heterogeneous Markov chain.

2. (a) What is meant by Transition Probability in stochastic processes ? Considering a discrete-valued stochastic process, define (i) one-step, (ii) three-step and (iii) r-step transition probabilities. 5

- (b) Find the probability that a random walk of 12 steps with probability $p = 0.8$ ends at (i) position $x = 6$, (ii) position $x = -4$. 5

3. (a) Explain what is meant by a pure birth process. Obtain the probability distribution of pure birth process. 5

(b) In the standard gambler's ruin problem, with total stake ' a ' and gambler's stake ' k ' and gambler's probability of winning at each play is p , calculate the probability of ruin in the following case :

$$a = 80, k = 70 \text{ and } p = 0.45.$$

Also find the expected duration in each case. 5

4. (a) Why there are two types of renewal processes ? What are the basic differences between them ? 5

(b) Find the mean and variance of the stationary process $\{X(t)\}$, whose Auto Correlation Function is given by : 5

$$R_{XX}(\tau) = \frac{25\tau^2 + 36}{6.25\tau^2 + 4}$$

5. (a) What do the letters in the symbolic representation $(a/b/c) : (d/e)$ of a queueing model represent ? Also write down the formulas for P_0 and P_n in a Poisson queue system in the steady state. 4
- (b) Customer arrive at a one-man barber shop according to a Poisson process with a mean inter-arrival time 12 minutes. Customers spend an average of 10 minutes in the barber's chair. Then : 6
- (i) What is the expected number of customers in the barber shop and in the queue ?
- (ii) Calculate the percentage of time, an arrival can walk straight into the barber's chair without waiting.
- (iii) How much time can a customer expect to spend in the barber's shop ?

- (iv) What is the average time customers spend in queue ?
- (v) What is the probability that the waiting time in the system is greater than 30 minutes ?
6. (a) Derive the mean and variance of the Pure-Birth Process. 5
- (b) The Transition Probability Matrix of a Markov chain $\{X_n\}$, $n = 1, 2, 3, \dots$, having 3 states 1, 2 and 3 is : 5

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

and the initial distribution is

$$P^0 = (0.7, 0.2, 0.1).$$

Find (i) $P\{X_2 = 3\}$ and

(ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

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