

No. of Printed Pages : 4

MMT-002

**M. SC. (MATHEMATICS WITH
APPLICATIONS TO COMPUTER
SCIENCE) [M. SC. (MACS)]**

Term-End Examination

June, 2025

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours

Maximum Marks :25

Note : (i) *Question No. 5 is compulsory.*

(ii) *Do any **three** questions from Q. Nos.*

1 Q. 4.

(iii) *Use of calculator is not allowed.*

A-321/MMT-002

P. T. O.

1. (a) Let $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ be a linear transformation defined by :

$$T(x, y, z) = (x + y - z, x + y + 3z)$$

Find the matrix of T relative to the ordered bases $\{(1, 1, 1), (1, 1, 0), (1, 0, 1)\}$ of $\mathbf{R}^3$ and $\{(1, 1), (0, 1)\}$ of $\mathbf{R}^2$. 2

- (b) Find the quadratic polynomial that best fits the data $(-2, -4), (-1, -4), \left(1, \frac{11}{5}\right)$ and $(2, 8)$. 3

2. Let :

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$

Find a matrix P such that $P^{-1}AP$ is in block diagonal form. 5

3. Find the singular value decomposition of A , where : 5

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$

4. (a) Prove that if N is a non-zero nilpotent operator, then N is not diagonalisable.

2

(b) Check whether or not $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$

is unitarily diagonalisable. If it is, find a unitary matrix U such that U^*AU is diagonal. If A is not unitarily diagonalisable, obtain the Schur decomposition of A .

3

5. Which of the following statements are true and which are false ? Justify your answers with a short proof or a counter-example, whichever is appropriate :

10

- (i) If two matrices have the same characteristic polynomial, they are similar.
- (ii) An invertible matrix must be positive definite.
- (iii) A and AA^t have the same rank for any matrix A .

- (iv) For every matrix $S \in M_n(\mathbf{R})$, there is an $n \times n$ orthogonal matrix O such that $O'SO \in M_n(\mathbf{R})$ is upper triangular.
- (v) Any non-zero square matrix over $\mathbf{R}$ has positive spectral radius.

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