

No. of Printed Pages : 6

MMT-003

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-003 : ALGEBRA

Time : 2 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Answer any **four** questions from*

Q. Nos. 2 to 6.

(iii) *Calculator is not allowed.*

B-1494/MMT-003

P. T. O.

1. State whether the following statements are true or false, giving reasons for your answer :

(a) If G is a group of order p^5q , p, q distinct primes and if $q \not\equiv 1 \pmod{p}$, then G has a unique sylow p -subgroup. 2

(b) The field $\mathbb{Q}(\sqrt[3]{2}, \sqrt{7}, \sqrt[4]{3})$ is a radical extension of $\mathbb{Q}$. 2

(c) There are 3 units in $\mathbb{Z} + \mathbb{Z}_w \subseteq \mathbb{C}$, where

$$w = e^{\frac{2\pi i}{3}}. \quad 2$$

(d) If a group of order 21 acts on a set with 8 elements, then it must fix at least one element of the set. 2

(e) If $a, b \in \mathbb{Z}$ and ab is a square mod p , then a and b are squares mod p . 2

2. (a) Give an example of an element of order 35 in the permutation group S_{13} . 2
- (b) Let K be a Galois extension of a field F with Galois group the quaternion group of order 8. How many fields L are there such that $F \subsetneq L \subsetneq K$ and how many of these L will be normal over F ? Further, give the degrees of each of the subfields. Justify your answers. 5
- (c) What is the units digit (in the decimal system) of 13^{1025} ? 3
3. (a) Solve the following simultaneous system of congruences : 4
- $$x \equiv 2 \pmod{5}$$
- $$x \equiv 1 \pmod{7}$$
- $$x \equiv 3 \pmod{11}.$$

(b) Is $\sqrt[4]{5} + \sqrt[4]{7}$ a constructible number ?

Justify your answer. 3

(c) Write the class equation of a finite group G . Is there a group of order 10 with class equation $10 = 1 + 2 + 2 + 5$. If yes, exhibit the group and its conjugacy classes. If no, give the class equation of S_3 . 3

4. (a) Define the symplectic group $SP_{2n}(\mathbf{R})$ and show that $SP_2(\mathbf{R}) = SL_2(\mathbf{R})$. Is it true that in general $SP_{2n}(\mathbf{R})^2 = SL_{2n}(\mathbf{R})$? 3

(b) Find an element α such that $Q(\alpha) = Q(\sqrt{3}, \sqrt{7})$. Justify your choice of α . 3

- (c) If G is a finite group and P is a Sylow p -subgroup of G , prove that : 4

$$N_G(N_G(P)) = N_G(P)$$

5. (a) Define the characteristic of a field.
Show that the characteristic of a field is zero or a prime number. 3
- (b) Determine all possible abelian groups of order 500 (upto isomorphism) giving the invariant factors of each group. 4
- (c) Define nilpotent element in a ring.
Check whether $\bar{3}$ and $\bar{15}$ are nilpotent in $\mathbb{Z}_{45}$. 3
6. (a) What is the degree of the field $\mathbb{Q}[\alpha]$ over $\mathbb{Q}$, where α is a root of $x^8 - 2$?
Justify your answer. 2

- (b) Let G be a group. Show that $\text{Inn}(G)$, the group of inner automorphisms of G , is a normal subgroup of $\text{Aut}(G)$, the group of all automorphisms of G . 3
- (c) Show that $\langle 5, x \rangle$ is not a principal ideal in $\mathbb{Z}[x]$. 5

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