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MMT-004

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. SC. (MACS)]

Term-End Examination

June, 2025

MMT-004 : REAL ANALYSIS

Time : 2 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions out of
question nos. 2 to 6.*

(iii) *Use of calculator is not allowed.*

B-1495/MMT-004

P. T. O.

1. State whether the following statements are True or False. Give reason for your answers :

$$5 \times 2 = 10$$

- (a) If A is a subset of a metric space (X, d) and $x \notin A$, then $d(x, A) \neq 0$.

- (b) The sequence $\left\{ \left(\frac{1}{n}, \frac{1}{n} \right) \mid n = 1, 2, \dots \right\}$ is not convergent in $\mathbf{R}^2$ under the discrete metric on $\mathbf{R}^2$.

- (c) $\mathbf{R}^2 \setminus \{(0, 1)\}$ is path connected.

- (d) The partial derivative of the function $f(x, y) = (x^2 - y, x + 2y)$ at $(1, 1)$ is $(2, 1)$.

- (e) Every Borel set is measurable.

2. (a) Let A be a subset of a metric space (X, d) . Prove that :

2

$$\overline{A} = \{x \in A : d(x, A) = 0\}$$

- (b) Prove that a Cauchy sequence in a metric space is convergent if and only if it has a convergent subsequence.

Is the continuous image of a Cauchy sequence a Cauchy sequence. Justify your answer.

5

- (c) For the function $F: \mathbf{R}^2 \rightarrow \mathbf{R}^3$ given by

$$F(x_1, x_2) = (e^{x_1}, e^{x_2}, x_1 + x_2), \quad \text{find the}$$

directional derivative in the direction of $(1, 1)$ at $(0, 0)$.

3

3. (a) Prove that a finite union of compact sets in a metric space is compact. What about arbitrary union ? Justify your answer.

5

- (b) Define the outer measure in $*$ and measure m of a subset A of $\mathbf{R}$. Prove that for any given $\varepsilon > 0$, we can find an open set O such that $A \subset O$ and $m(O) \leq m^*(A) + \varepsilon$. 5

4. (a) Apply the implicit function theorem to the function $f(x, y, z) = x^2 + y^3 - xy \sin z$ at the point $(1, -1, 0)$. 3

- (b) Prove that a metric X is connected if every continuous function f on X to the discrete metric space $\{1, -1\}$ reduces to a constant function. 3

- (c) Find the critical points of the function :

$$f(x, y, z) = x^2y + y^2z + z^2 - zx$$

and check whether they are extreme points. 4

5. (a) Prove that a path connected metric space is connected. What about the converse ? Justify your answer. 4

(b) State and prove the dominated convergence theorem. 6

6. (a) For a real function $f \in L_1[-\pi, \pi]$, define the n th Fourier coefficient of f , the exponential and the trigonometric forms of the Fourier series of f .

Find the Fourier series of the function : 4

$$g(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \leq t < \pi \end{cases}$$

(b) Suppose that f and g are two integrable functions such that $\int_A f dm \leq \int_A g dm$ for all $A \in \mathcal{m}$. Show that $f \leq g$ a.e. 3

- (c) Find and classify the extreme values of
the function : 3

$$f(x_1, x_2) = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$$

subject to the constraint :

$$x_1 + 2x_2 = 2, \quad x_1, x_2 \geq 0.$$

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