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MMT-007

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. SC. (MACS)]

Term-End Examination

June, 2025

**MMT-007 : DIFFERENTIAL EQUATIONS AND
NUMERICAL SOLUTIONS**

Time : 2 Hours

Maximum Marks : 50

Weightage : 50%

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions out of
Q. Nos. 2 to 7.*

(iii) *Use of scientific/non-programmable
calculator is allowed.*

B-1498/MMT-007

P. T. O.

1. State whether the following statements are True or False. Justify your answer with the help of a short proof or a counter-example :

$$5 \times 2 = 10$$

- (a) For initial value problem :

$$y' = \begin{cases} \frac{2y}{x}, & x > 0 \\ 0, & x = 0 \end{cases}, \quad y(0) = 0$$

the Lipschitz condition is not satisfied in any closed rectangle containing $(0, 0)$.

(b) $L\{e^{2at} \cos 3\omega t\} = \frac{s}{s^2 + 9\omega^2}$, where L is

the Laplace transformation.

(c) The initial value problem $y' = \frac{y-x}{y+x}$,

$y(1) = 2$ has a solution 2.0596 at $x = 1.2$, using Runge-Kutta second order method with $h = 0.1$.

$$(d) \quad f(x) = \begin{cases} e^{-ax}; & x \geq 0, a > 0 \\ 0; & x < 0 \end{cases}$$

The Fourier transform of the function $f(x)$ is a complex function.

- (e) Finite element Galerkin method is a weighted residual method and does not require the variational form of the given differential equation.

2. (a) Prove that : 5

$$\int_{-1}^1 \frac{P_n(x)}{\sqrt{1-2xt+t^2}} dx = \frac{2t^n}{2n+1}$$

- (b) Prove the following relation for Hermite polynomials $H_n(x)$: 5

$$xH'_n(x) = nH'_{n-1}(x) + nH_n(x)$$

3. (a) Obtain a series solution about $x = 0$ of the equation : 5

$$x^2 y'' + 6xy' + (6 + x^2)y = 0$$

- (b) Using Green's function, solve the boundary-value problem : 5

$$\left(\frac{d^2 y}{dx^2} + y \right) = -x, \quad y(0) = y\left(\frac{\pi}{2}\right) = 0.$$

4. (a) Using Laplace transform, solve the PDE : 6

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0$$

subject to the conditions :

$$u(0, t) = 10 \sin 2t$$

$$u(x, 0) = 0$$

$$u_x(x, 0) = 0,$$

$$\lim_{x \rightarrow \infty} u(x, t) = 0.$$

- (b) Solve the initial value problem
 $y' = x^2 + y^2$, $y(0) = 1$ upto $x = 0.4$ using
fourth order Taylor's series method
with $h = 0.2$. 4

5. (a) Show that the Milne-Simpson
method : 6

$$y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n+1} + 4y'_n + y'_{n-1})$$

is not absolutely stable for any h and for
any initial value problem.

- (b) Using the generating function for
 $J_n(x)$, prove that : 4

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$$

for integer values of n . Here J_n is the
Bessel's function of the first kind of
order n .

6. (a) Find the solution of the following initial boundary value problem, subject to the given initial and boundary conditions : 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1,$$

$$u(x, 0) = \sin(2\pi x)$$

$$u(0, t) = 0 = u(1, t)$$

Assume step length in x -direction $h = 0.25$, solve using Laasonen method with mesh ratio $\lambda = 0.6$. Integrate for two time levels.

- (b) Find the inverse Laplace transform of : 4

$$\frac{21s - 33}{(s+1)(s-2)^3}$$

7. (a) The heat conduction equation $u_t = u_{xx}$ is approximated by : 5

$$\frac{1}{2k} \left(u_m^{n+1} - u_m^{n-1} \right) = \frac{1}{h^2} \left(u_{m-1}^n - 2u_m^n + u_{m+1}^n \right)$$

Investigate the stability using the Von Neumann method.

(b) Solve the boundary value problem :

$$y'' - 4y' + 3y = 0; \quad y(0) = 1, y(1) = 0$$

using second order finite difference

method with $h = \frac{1}{3}$. 5

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