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MMTE-001

**M. SC. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE) [M.SC. (MACS)]**

Term-End Examination

June, 2025

MMTE-001 : GRAPH THEORY

Time : 2 Hours

Maximum Marks : 50

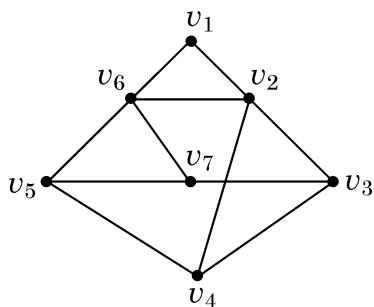
Note : *Question No. 1 is compulsory. Answer
any **four** questions from Q. Nos. 2 to 7.
Symbols have their usual meanings.*

1. State whether the following statements are true or false. Justify your answers with a short proof or a counter-example : $5 \times 2 = 10$
 - (i) Every tree is a bipartite graph.

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- (ii) Q_n is Eulerian for all $n \geq 2$.
 - (iii) If G is a triangle-free graph, then $\chi(G) \leq 2$.
 - (iv) $K_{3,4}$ is non-planar.
 - (v) If G has a cut-vertex, then so does $L(G)$.
2. (a) Use Whitney's Theorem or Expansion Lemma to show that the following graph is 2-connected : 3

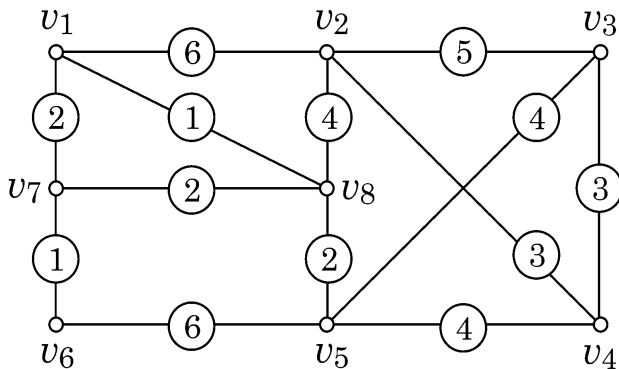


- (b) Check whether the graph given in part (a) is Hamiltonian. 2
- (c) Find the number of (v_3, v_5) -walks of length 3 in the graph given in part (a) by computing the powers of the adjacency matrix. 5

3. (a) Let $G = (V, E)$ be a connected graph with at least two vertices and degree sequence $(d_1, d_2, \dots, d_n)$, $d_i \geq 1$ for each i . Then show that G is a tree if and only

if $\sum_{i=1}^n d_i = 2(n-1)$, where, $n = |V|$. 4

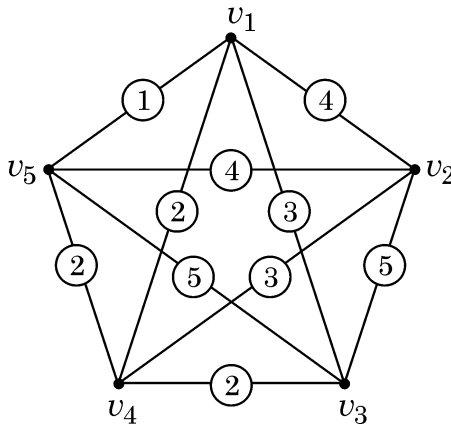
- (b) Use Prim's algorithm on the following graph to find a minimum-weight spanning tree starting at v_1 : 4



- (c) What is the thickness of the Peterson graph ? Justify your answer. 2
4. (a) If G is a bipartite graph, then prove that $\chi'(G) = \Delta(G)$. 7
- (b) Show that every proper subgraph of K_5 is planar. 3
5. (a) Show that every k -regular bipartite graph has a perfect matching, where $k \geq 1$. 5
- (b) Let G be a connected graph and $v \in V(G)$. Show that v is a cut-vertex of G iff there exist vertices $u, w \in V(G) \setminus \{v\}$ such that every (u, w) -path in G passes through v . 5
6. (a) Start with the cycle $C = (v_1, v_2, v_3, v_5, v_4, v_1)$ in the following weighted complete

graph K_5 , and perform one reduction step to get a Hamiltonian cycle with smaller weight :

4



- (b) Show that $K_{3,3}$ is a contraction of the Grötzsch graph. Hence conclude that Grötzsch graph is non-planar.

6

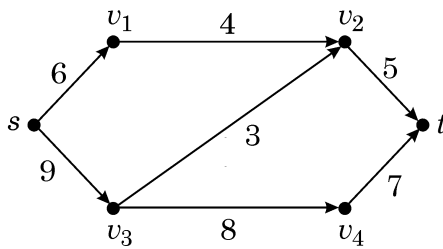
7. (a) Determine the minimum size of a maximal matching in C_n . Also determine the size of a maximum matching in C_n . Are the two quantities equal ?

3

- (b) The blocks of a tree are precisely its edges. (True or false)

Justify. 2

- (c) Define a flow on the following network with maximum possible value : 5



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