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MPH-001

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2025

**MPH-001 : MATHEMATICAL METHODS IN
PHYSICS**

Time : 2 Hours

Maximum Marks : 50

Note : (i) *Answer any **five** questions.*

(ii) *The marks are indicated against
each question.*

(iii) *Symbols have their usual meanings.*

(iv) *You may use calculator.*

A-203/MPH-001

P. T. O.

1. Using the generating function for Legendre polynomials and their orthogonality property, show that : 10

$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$$

2. (a) Write the Laplace's equation in spherical coordinates (r, θ, ϕ) . Solve the following ODE : 2+8

$$r^2 \frac{d^2 R}{dr^2} + 2r \frac{dR}{dr} - n^2 R = 0$$

3. (a) When is a set of vectors called linearly independent ? Show that the vectors :

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } \vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

in the Cartesian space are linearly independent. 2+3

- (b) For the Pauli matrix : 5

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

calculate :

$$u_1(\theta) = \exp(i\theta\sigma_2) = 1 + i\theta\sigma_2 - \frac{\theta^2}{2!}\sigma_2^2 - \dots$$

For real θ , show that :

$$u_1(\theta) = \cos\theta + i\sin\theta\sigma_2$$

4. (a) (i) Show that $\alpha_i = g_{ij} v^j$ transforms covariantly, where g_{ij} are the components of the metric tensor of rank 2 and v^i the components of a contravariant vector. 3
- (ii) Show that the determinant of an orthogonal matrix is either +1 or -1. 2

(b) Show that :

$$u = 2x(1 - y)$$

is harmonic in some domain. Calculate its harmonic conjugate V . 2+3

5. (a) Using the method of residues, evaluate the integral : 5

$$\int_0^\pi \frac{d\theta}{1 + \sin^2 \theta}$$

(b) Obtain the Taylor's series expansion of : 5

$$f(z) = \frac{1}{z-1} \text{ about } z = -1 \text{ and } z = i.$$

6. (a) Prove that the series $\sum_{n=1}^{\infty} \frac{z^{n-1}}{3^n}$ converges for $|z| < 3$. 3

(b) Using the Laplace transforms, solve the initial value problem : 7

$$y'' - 4y' + 3y = 0;$$

$$y(0) = 3,$$

$$y'(0) = 7.$$

7. Determine the Fourier transform of the normalised Gaussian distribution :

$$f(t) = \frac{1}{\sqrt{2\pi}} \frac{1}{\tau} \exp\left(-\frac{t^2}{2\tau^2}\right) - \infty < t < \infty$$

Also draw the diagrams of the Fourier transform of Gaussian distribution for small τ and large τ . 10

8. (a) Define a continuous group giving an example. 5
- (b) Show that $n \times n$ unitary matrices form a group. 5

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