No. of Printed Pages : 7 **MPH–004**

M. SC. (PHYSICS)

(MSCPH)

Term-End Examination

June, 2025

MPH-004 : QUANTUM MECHANICS–I

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any* ***five*** *questions.*

(ii) Symbols have their usual meanings.

(iii) Values of physical constants are given at the end.

(iv) You may use a calculator.

(v) The marks for each question are indicated against it.

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1. (a) A particle of mass m is constrained to move in a one-dimensional space between two infinitely high potential

barriers located a distance $2a$ apart. Using the uncertainty principle, derive the zero-point energy of the particle. 4

(b) Using the relativistic expression for the energy of the electron : 6

$$E^2 = p^2c^2 + m^2c^4,$$

Calculate the phase velocity and group velocities of the electron wave.

2. (a) Define a Hermitian operator. Show that Hermitian operators have real expectation values . 5

(b) The wave function of a particle of mass m is given by : 5

$$\psi(x,t) = N \exp\left(-\frac{\alpha m}{\hbar}x^2 - i\alpha t\right)$$

where α and N are constants. Use the Schrödinger equation to determine the potential $V(x)$ in which the particle is moving.

3. Solve the Schrödinger equation for the following symmetric infinite potential well :

$$V(x) = \begin{cases} \infty, & \text{for } x < -L \\ 0, & \text{for } -L \le x \le L \\ \infty, & \text{for } x > L \end{cases}$$

Determine the eigen functions and eigen energies. What is the parity of ground state and first excited state ? 4+4+2

4. Calculate the mean potential and kinetic energy of the ground state of the simple harmonic oscillator defined by the wave function : 6+4

$$\psi_0(x) = \left(\frac{a}{\sqrt{\pi}}\right)^{1/2} \exp\left(-\frac{a^2x^2}{2}\right)$$

where $a^2 = \frac{m\omega}{\hbar}$. Given that

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a^3}} \ .$$

5. Write the stationary state Schrödinger equation in spherical polar coordinates for the symmetric rigid rotator which has the classical Hamiltonian $H = \frac{L^2}{2I_0}$, where $\vec{L}$ is the angular momentum and I_0 is the moment of inertia. Deduce the form of the wave function $\psi\ (r,\ \theta,\ \phi)$ and obtain the energy eigen functions and energy eigen values. What is the degeneracy of each energy eigen value ? 2+6+2

6. (a) The state of a system at time $t = 0$ is $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$, where $|1\rangle$ and $|2\rangle$ are the normalized energy eigen kets of the system corresponding to the energy eigen values E_1 and E_2 respectively ($E_2 > E_1$). Calculate the shortest time in which the state vector $|\psi(t)\rangle$ will be orthogonal to $|\psi(0)\rangle$. 5

(b) Define the projection operator for a quantum system. Write the identity operator in terms of the projection operator and hence show that in a complete orthonormal basis $(|\phi_1\rangle, |\phi_2\rangle ...... |\phi_n\rangle)$ an operator $\hat{O}$ can be written as : 5

$$\hat{O} = \sum_{i=1}^{n} \sum_{j=1}^{n} O_{ij} |\phi_i\rangle \langle \phi_j|$$

7. (a) Define the ladder operators $\hat{a}$ and $\hat{a}^+$ for the simple harmonic oscillator and show that $\left[\hat{a}, \hat{a}^+\right] = 1$. 5

(b) Using the expression : 5

$$\hat{H} = \hbar\omega\left(\hat{a}\hat{a}^+ + \frac{1}{2}\right)$$

show that :

$$\hat{H}\left(\hat{a}|E\rangle\right) = (E - \hbar\omega)\left(\hat{a}|E\rangle\right)$$

where $|E\rangle$ is an energy eigen ket of the simple harmonic oscillator with energy eigen value E.

8. (a) Use the expressions : 5

$$\left(\mathrm{J}^2\right)_{-j'jm_{j'}m_j} = \left\langle j', m_{j'}, \left|\hat{\mathrm{J}}^2\right| j, m_j \right\rangle$$

$$= j(j+1)\hbar^2 \delta_{jj'}\, \delta m_{j'}\, m_j$$

and

$$(\mathrm{J}_z)_{j'jm_{j'}m_j} = \left\langle j', m_{j'} \left| \hat{\mathrm{J}}_z \right| j, m_j \right\rangle$$

$$= m_j \hbar\, \delta_{j'j}\, \delta m_{j'}\, m_j$$

to write the angular momentum matrices J^2 and J_z for $j = 1$.

(b) Using the basis vectors $\left|\uparrow\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\left|\downarrow\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for the spin half system and the Pauli's spin matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, show that : 5

$$\hat{\mathrm{S}}_y = \frac{i\hbar}{2}\left[-\left|\uparrow\right\rangle\left\langle\downarrow\right| + \left|\downarrow\right\rangle\left\langle\uparrow\right|\right]$$

Physical constants :

$h = 6.63 \times 10^{-34}\,\mathrm{Js}$

$\hbar = 1.05 \times 10^{-34}\,\mathrm{Js}$

$m_e = 9.1 \times 10^{-31}\,\mathrm{kg}$

$e = 1.6 \times 10^{-19}\,\mathrm{C}$

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