

No. of Printed Pages : 6

**MPH-006**

**M. SC. (PHYSICS)**

**(MSCPH)**

**Term-End Examination**

**June, 2025**

**MPH-006 : CLASSICAL MECHANICS-II**

*Time : 2 Hours*

*Maximum Marks : 50*

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**Note :** (i) Attempt any **five** questions.

(ii) The marks for each question are indicated against it.

(iii) Symbols have their usual meanings.

(iv) You may use a calculator.

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**B-1503/MPH-006**

**P. T. O.**

1. Consider a particle of mass  $m$  with the Lagrangian :

$$L(x, y, z) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(|\vec{r} - \vec{\mu}t|)$$

Show that a transformation  $\vec{r}' = \vec{r} + \varepsilon \vec{\mu}$  leaves the Lagrangian invariant. Here  $\varepsilon$  is an infinitesimal parameter. 10

2. State Liouville's theorem. Assuming that a system in two-dimensional phase space obeys Hamiltonian dynamics, show that the volume in the phase space remains constant as the phase points evolve with time. 10

3. (a) Using the conditions  $\delta_{q_i} = \delta_{p_i} = 0$  at the end points, show that : 5

$$\delta S' = \delta \int_{t_1}^{t_2} \left[ p_i \dot{q}_i - H(q_i, p_i, t) + \frac{dF(q_i, p_i, t)}{dt} \right] dt = 0$$

is equivalent to the condition :

$$\delta S = \delta \int_{t_1}^{t_2} [p_i \dot{q}_i - H(q_i, p_i, t)] dt = 0$$

- (b) Consider an Atwood's machine consisting of two masses  $m_1$  and  $m_2$ , connected by a string of constant length  $l_0$ . The Lagrangian of the system is :

$$L = \frac{1}{2}(m_1 + m_2)\dot{y}_1^2 + (m_1 - m_2)gy_1 + m_2gl_0$$

Obtain the Hamiltonian and Hamilton's equation of motion. 3+2

4. (a) Write the condition for extended canonical transformation in symplectic form. 1
- (b) Using symplectic approach, show whether the transformation : 7

$$Q = \alpha^2 q \text{ and } P = \beta^2 p$$

is canonical. Here  $\alpha$  and  $\beta$  are non-zero constants.

- (c) Using Poisson brackets, obtain the relation for  $\alpha$  and  $\beta$ , and hence the value of the scale parameter. 2
5. (a) Write the bilinear invariant condition for canonical transformation. 2
- (b) Consider a dynamical system described by the Hamiltonian : 4+4

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$$

where  $\omega$  is some constant.

- (i) Obtain the new Hamiltonian  $K(Q, P)$ , using the generating

$$\text{function } F_1(q, Q) = -\frac{Q}{q}.$$

- (ii) Obtain Hamilton's equations of motion in the new variables.

6. A particle of mass  $m$ , charge  $e$  in an EM field is described by the Lagrangian :

$$L(\vec{r}, \dot{\vec{r}}) = \frac{1}{2} m \dot{\vec{r}}^2 + e(\dot{\vec{r}} \cdot \vec{A}) - e\phi$$

where  $\phi$  and  $\vec{A}$  are the scalar and the vector potentials.

(i) Determine the canonical momenta. 2

(ii) Obtain the Hamiltonian for the system. 4

(iii) Show that : 4

$$[\vec{r}, H] = \dot{\vec{r}}$$

7. The Hamiltonian of a particle with mass  $m$  moving in a central force field is : 8+2

$$H = \frac{1}{2m} \left( p_r^2 + \frac{p_\theta^2}{r^2} \right) - \frac{k}{r}$$

where  $k$  is a constant.

Obtain Hamilton's characteristic function  $W(r, \theta)$  and the action-angle variables  $J_\theta$ .

8. (a) Starting from  $\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{N}$ , derive the Euler's equations for a rigid body in body axis for  $\vec{L} = \omega_1 \hat{x} + \omega_2 \hat{y} + \omega_3 \hat{z}$ . Here  $\vec{L}$  is the angular momentum,  $\vec{N}$  is the torque, and  $\vec{\omega}$  is the angular velocity. 7
- (b) Write Euler's equations for torque free motion. 3

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