

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

June, 2025

**MST-012 : PROBABILITY AND PROBABILITY
DISTRIBUTIONS**

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from question numbers 2 to 6.*

(iii) *Use of scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

1. State whether the following statements are TRUE or FALSE. Give reason in support of your answer :

5×2=10

- (a) “Let $(\Omega, \mathcal{F}, P)$ be a probability space and X be a discrete random variable which assumes countably infinite values, then we can define uniform probability law on X .”
- (b) If $\Omega = \{HH, HT, TH, TT\}$, then write largest σ -field on Ω .
- (c) Describe the Binomial distribution.
- (d) Define gamma distribution with two parameters.
- (e) Let X be a random variable with $E(X) = 3$, then which one of the following is true and why ?

$$E\left(\frac{1}{X+2}\right) \geq \frac{1}{5} \text{ or } E\left(\frac{1}{X+2}\right) < \frac{1}{5} \forall x > -2$$

2. State Monty Hall problem and solve it. Also, write its intuitive solution. 10
3. (a) A bag contains 4 red and 12 black balls. Three balls are drawn one-by-one with replacement. Plot PMF and CDF of number of red balls. 6

- (b) Let $(\Omega, \mathcal{F}, P)$ be a probability space and E be an event, then find the probability distribution of the indicator random variable $I_E : \Omega \rightarrow \{0, 1\}$ of event E . 4
4. (a) Suppose a biased coin having p as probability of success is given to you, where p is unknown. Initially, to give full freedom to the value of p , you are considering p as a random variable, where $p \sim \text{unif}(0, 1)$. You tossed the coin 100 times and observed 98 times head and 2 times tail. With this information, find the probability that p is greater than 0.8. 5
- (b) If each of the random variables X , Y and Z follow exponential distribution with parameter 2, then find $P(X + Y + Z \leq 1)$. 5
5. State and prove Borel-Cantelli lemmas 1 and 2. 10

6. The PDF of jointly continuous random variable (X and Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} k(4-x)(5-y), & 0 \leq x \leq 4, 0 \leq y \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

Obtain : 1+1+5+1+1+1

- (i) The value of k
- (ii) Marginal PDFs of X and Y
- (iii) Joint CDF of (X, Y)
- (iv) Marginal CDFs of X and Y
- (v) Conditional PDF of X given Y
- (vi) Are random variable X and Y independent ?

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