

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

June, 2025

MST-018 : MULTIVARIATE ANALYSIS

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining question nos. 2 to 6.*

(iii) *Use of scientific (non-programmable) calculator is allowed.*

(iv) *Symbols have their usual meanings.*

1. State whether the following statements are True or False. Give reasons in support of your answers : 5×2=10

(a) The eigen values of a positive definite matrix are less than equal to zero.

(b) Multiple correlation is the minimum correlation between the linear combination of the components of x and the linear combination of the components of y .

(c) $A = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is an orthogonal and idempotent matrix.

(d) A factor model postulates that a random vector $\tilde{X}$ is linearly dependent upon a few observable common factors.

(e) If $\tilde{X} \sim N_3(\tilde{\mu}, \Sigma)$ with $\tilde{\mu} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and

$\Sigma = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 2 & -1 \\ 0 & -1 & 4 \end{pmatrix}$, then the distribution

of $\begin{pmatrix} \tilde{X} - \tilde{\mu} \\ \tilde{\mu} \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} \tilde{X} - \tilde{\mu} \\ \tilde{\mu} \end{pmatrix}$ is non-central Chi-square with 3 degrees of freedom.

2. Let $\tilde{X}_{P \times 1} \sim N_P(\tilde{\mu}, \Sigma)$. Also, let $\tilde{X}$, $\tilde{\mu}$ and Σ be partitioned as :

$$\tilde{X}_{P \times 1} = \begin{pmatrix} \tilde{X}_{K \times 1}^{(1)} \\ \tilde{X}_{(P-K) \times 1}^{(2)} \end{pmatrix}, \quad \tilde{\mu}_{P \times 1} = \begin{pmatrix} \tilde{\mu}_{K \times 1}^{(1)} \\ \tilde{\mu}_{(P-K) \times 1}^{(2)} \end{pmatrix}$$

and

$$\Sigma_{P \times P} = \begin{pmatrix} \Sigma_{11 K \times K} & \Sigma_{12 K(P-K)} \\ \Sigma_{12 (P-K) \times K} & \Sigma_{22 (P-K) \times (P-K)} \end{pmatrix}$$

Then prove that the conditional distribution :

$$\tilde{X}^{(1)} | \tilde{X}^{(2)} = \tilde{X}^{(2)} \sim N_K$$

$$\left(\tilde{\mu}^{(1)} + \Sigma_{12} \Sigma_{22}^{-1} \left(\tilde{X}^{(2)} - \tilde{\mu}^{(2)} \right), \Sigma_{11.2} \right),$$

where $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$.

Also, let $\tilde{X} = \begin{pmatrix} \tilde{X}^{(1)} \\ \tilde{X}^{(2)} \end{pmatrix} \sim N_4(\tilde{\mu}, \Sigma),$

where $\tilde{\mu} = \begin{pmatrix} -4 \\ 1 \\ 4 \\ 0 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 2 & 6 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Then find $E\left(\tilde{X}^{(1)} \mid \tilde{X}^{(2)} = \tilde{x}^{(2)}\right)$ and
 $\text{Cov}\left(\tilde{X}^{(1)} \mid \tilde{X}^{(2)} = \tilde{x}^{(2)}\right).$ 10

3. If :

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix},$$

then :

- (i) Obtain the square root matrix corresponding to the matrix A and verify that $A^{1/2} A^{1/2} = A$. 5
- (ii) Determine the first principal component of matrix A and the proportion of the total variability that is explained. 5

4. If : a

$$\tilde{X} \sim N_4 \left(\tilde{\mu}, \tilde{\Sigma} \right)$$

with

$$\tilde{\mu} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ -1 \end{pmatrix} \text{ and } \begin{pmatrix} 5 & 0 & 2 & 0 \\ 0 & 4 & 0 & -1 \\ 2 & 0 & 3 & 0 \\ 0 & -1 & 0 & 2 \end{pmatrix},$$

then :

- (i) Check whether the variance-covariance matrix Σ is positive definite or not. 2
 - (ii) Obtain the marginal distribution of $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$. 4
 - (iii) Comment on the independence of the subvectors $\begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ and $\begin{pmatrix} X_2 \\ X_4 \end{pmatrix}$. 4
5. Define principal component analysis. Write the stepwise procedure for obtaining the principal components. 10
 6. (a) Define Hotelling's T^2 and mention *two* applications of it. Also, give the proof of *one* of them. 5
 - (b) Define clustering. Differentiate between the single linkage and complete linkage method of clustering. 5

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