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MST-022

M. SC. (APPLIED STATISTICS)
(MSCAST)

Term-End Examination

June, 2025

MST-022 : LINEAR ALGEBRA AND
MULTIVARIATE CALCULUS

Time : 3 Hours

Maximum Marks : 50

Note : (i) *Question No. 1 is compulsory.*

(ii) *Attempt any **four** questions from the remaining Question Nos. 2 to 6.*

(iii) *Use of Scientific calculator (non-programmable) is allowed.*

(iv) *Symbols have their usual meanings.*

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P. T. O.

1. State whether the following statements are true *or* false. Give reasons in support of your answers :

$$5 \times 2 = 10$$

- (a) The number of critical points of the function :

$$f(x, y) = x^3 + y^3 - 3xy + 625$$

is 2.

- (b) The directional derivative of the function :

$$f(x, y, z) = x^3 + y^2 + z^2$$

at point (1, 0, 1) in the direction of the vector (4, 3, 0) is $18/5$.

- (c) Consider a matrix A of order 5×7 such that its rank is 3. The dimension of the null space of A is 2.

- (d) Let W_1 and W_2 be two subspaces of a vector space V of dimension 10 such that $\dim(W_1) = 4$, $\dim(W_2) = 5$ and $5 \leq \dim(W_1 + W_2) \leq 9$. Then $\dim(W_1 \cap W_2)$ lies between 0 and 4.

- (e) The equation of the level surface of the function $f(x, y, z) = \sqrt{y^2 - xz} + 3y$ through the point (1, 4, 7) is $\sqrt{y^2 - xz} + 3y = 15$.

2. (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$, then find algebraic

and geometric multiplicities of the smallest eigen value of matrix A. 4

- (b) Consider a basis $B = \{(1, 1, 0), (1, 2, 0), (0, 1, 2)\}$ of the vector $\mathbf{R}^3$. Then using the Gram-Schmidt process, obtain an orthonormal basis of $\mathbf{R}^3$ corresponding to B. 6

3. (a) Find the least square solution of the over determined linear system : 5

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$$

(b) If :

$$u = x + y^2 + z^3,$$

$$v = xyz$$

$$w = x^2 - yz$$

then at the point $(x, y, z) = (1, 1, 1)$, find

$$\text{the value of } \frac{\partial(u, v, w)}{\partial(x, y, z)}.$$

5

4. Let M denote the set of all 2×2 real symmetric matrices. Show that M is a vector space over $\mathbf{R}$, under the vector addition and scalar multiplication, defined as follows :

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} + \begin{bmatrix} d & e \\ e & f \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix}$$

$$\alpha \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha b & \alpha c \end{bmatrix}$$

$$\text{where } a, b, c, d, e, f, \alpha \in \mathbf{R}.$$

10

5. (a) Consider the function

$$f(x, y) = \sin(\pi x - x^2 y).$$

Find the Hessian matrix H_f that is associated with the second order term in the Taylor's expansion of f around $(1, \pi)$. 5

- (b) Find the first four non-zero terms of the Taylor's expansion for the function : 5
- $$f(x) = e^x + x + \sin x; \text{ about } x = 0.$$

6. (a) If $w = xy + z^2$, $x = t^2 e^s$, $y = t \cos s$ and $z = s \sin t$, then find the value of $\frac{\partial w}{\partial t}$ at the point $s = 0, t = \pi$. 4

- (b) Show that the function :

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is not continuous at origin but its partial derivatives f_x and f_y exist at $(0, 0)$. 6

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