No. of Printed Pages : 4 **MMT–002**

MASTERS IN MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE [MSC(MACS)]

Term-End Examination

June, 2026

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks :25*

Weightage : 70%

Note : *(i) There are five question in this paper.*

*(ii) The **fifth** question is compulsory.*

*(iii) Do any **three** questions from Q. 1 to Q. 4.*

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1. (a) Let $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ be a linear operator defined by $T(x, y, z) = (2x + z, y - z)$. Find the matrix of the operator with respect to the bases $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right\}$ and $B' = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ of $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively. 2

(b) Find the spectral decomposition of the matrix $\begin{bmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{bmatrix}$. 3

2. Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 0 \\ 1 & -1 \end{bmatrix}$. Find the SVD of A. Use the SVD to find the Moore-Penrose inverse of A. 5

3. Let A and B be a city and its suburb, respectively. Assume that each year $\frac{2}{5}$ of the population of A moves to B and $\frac{1}{5}$ of the population of B moves to A. What is the long term effect of this on the populations of A and B ? Are they likely to stabilise ? 5

4. (a) Find the Jordan canonical form of the matrix $A = \begin{bmatrix} 4 & 1 & 0 \\ -4 & 0 & 0 \\ 5 & 2 & 1 \end{bmatrix}$. Also find an invertible P such that $P^{-1}AP$ is in Jordan canonical form. 3

(b) Find the QR decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 2 & 1 \end{bmatrix}$. 2

5. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter example as appropriate : 2×5=10

(i) A 3 × 3 matrix with minimal polynomial $(x-3)^2(x-1)$ has $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ as its Jordan canonical form.

(ii) If A is any $n \times n$ symmetric matrix, A^2 is positive semi definite.

(iii) If A is a $n \times n$ diagonalisable matrix, than the characteristic polynomial has distinct roots.

(iv) If A is any matrix, e^A is an invertible matrix.

(v) The square root of any positive definite matrix is unique.

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