No. of Printed Pages : 4 **MMT–003**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. SC. (MACS)]

Term-End Examination

June, 2026

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Answer any ***four*** *questions from Q. Nos.* ***2*** *to* ***6****.*

(iii) Calculators are not allowed.

1. State whether the following statements are True *or* False giving reasons for your answers :

(a) If G is a group of order $7^{100} \times 11$, then the Sylow 7-subgroup is normal. 2

(b) The polynomial $x^n - 1$ is not solvable by radicals over Q. 2

(c) $\dfrac{Q[X]}{(X^2+1)}$ is a unique factorization domain. 2

(d) S_{11} (permutation group) has no elements of order 30. 2

(e) The group $G = \left\{ \begin{pmatrix} a & a \\ 0 & 0 \end{pmatrix} \middle| a \in \mathbf{R}^* \right\}$ is a subgroup of $GL_2(\mathbf{R})$. 2

2. (a) Show that a group of order 30 must have an element of order 15. 5

(b) Let K be a Galois extension of a field F whose Galois group G(K/F) is cyclic of order 16. How many fields L will there be such that $F \subset L \subset K$ and the degree of L over F is 4 ? Will such L be normal extensions of F ? 3

(c) What is the units digit of 2015 in the decimal system ? 2

3. (a) How many elements are there in $\mathbf{Z}_{550}$ whose fifth power is 1 ? 3

(b) Is $\sqrt[4]{5}+\sqrt[4]{11}$ a constructible number ? Justify your answer. 3

(c) Let G be a p-group and let X be a set. Show that $|X| \equiv |X^G| \pmod{p}$. Deduce that, if $(|X|, p) = 1$, then there is a fixed point for the action of G. 4

4. (a) Is the group $\dfrac{F(a,b)}{< a^b, b^2, bab\,a >}$ finite, where $F(a,b)$ is the free group on the letters a and b ? Justify your answer. 4

(b) Define the Euclidean algorithm on an integral domain. 2

(c) Prove that $Q\left(\sqrt{3}, \sqrt{5}\right)$ is an extension of degree four over Q. 4

5. (a) Give an example of a prime ideal in Q[X]. Is the ideal also maximal ? Justify your answer. 3

(b) Determine all possible abelian groups of order 600 (upto isomorphism) and write down the invariant factors of each group. 7

6. (a) Determine the Legendre symbol $\left(\frac{66}{157}\right)$. 2

(b) Let G = S_4, the symmetric group on 4 symbols. Let G act on G by conjugation, i.e., if $g, a \in \mathrm{G}$, then $g * a = g\, a\, g^{-1}$. What is the orbit of the cycle (3, 4) and what is its stabilizer ? 4

(c) Factor $\mathrm{X}^9 - \mathrm{X}$ into monic, irreducible polynomials in $\mathbf{Z}_3[\mathrm{X}]$. 4

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